Top-bottom interference effects in Higgs plus jet production at the LHC

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Introduction

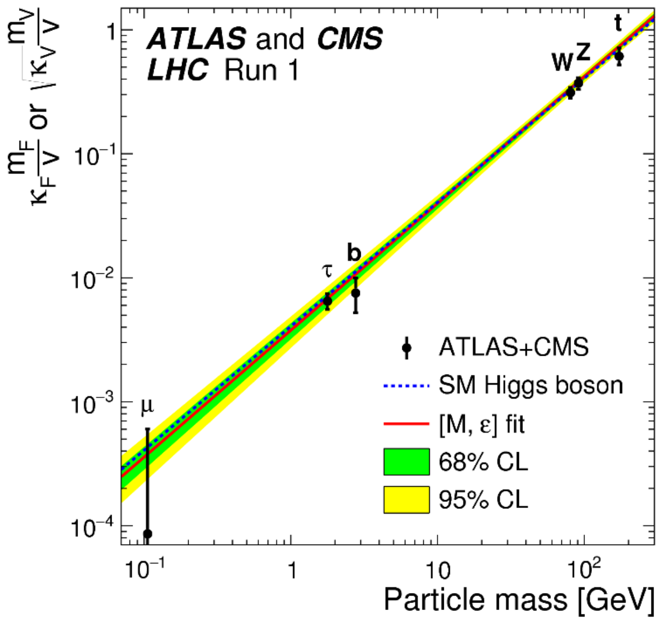
- Studies of Higgs boson properties are a crucial part of LHC physics program

- One important focus is the study of *Higgs couplings* to other particles

- After high-luminosity run it is expected that major Higgs couplings can be constrained to few percent level

- Higgs couplings to light generation quarks practically unconstrained

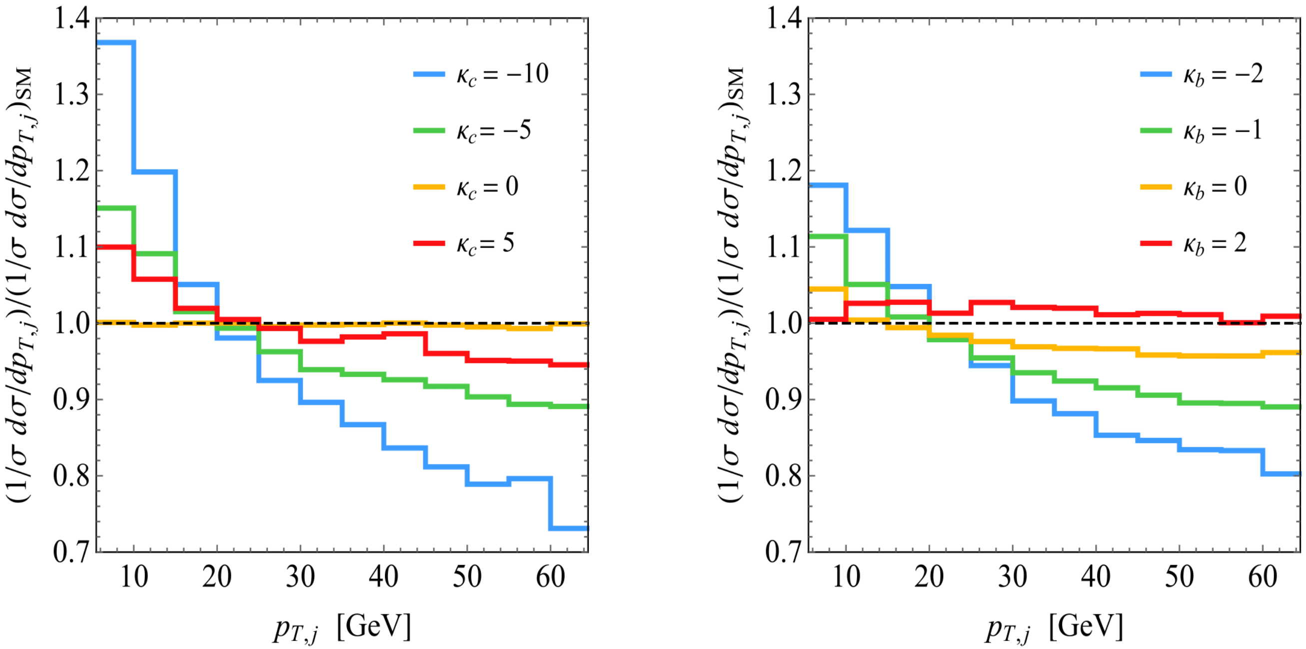
- Current bounds from global fits to inclusive Higgs production cross section and exclusive Higgs decays



[arXiv:1606.02266]

Introduction: $H + j$ production

- Non-trivial Higgs transverse momentum ($p_{T,H}$) distribution generated when extra jet is radiated: $H + j$
- Shape of $p_{T,H}$ distribution may put stronger constraints on light-quark Yukawa couplings [Bishara, Monni et al '16; Soreq et al '16]



$$\kappa_j = y_j / y_{j,SM}$$

- Bounds expected from HL-LHC $\kappa_c \in [-0.6, 3.0]$ $\kappa_b \in [0.7, 1.6]$ [Bishara, Monni et al '16]

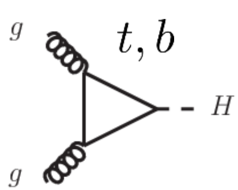
→ Reliable theoretical predictions for $H + j$ differential cross section required

Bottom corrections

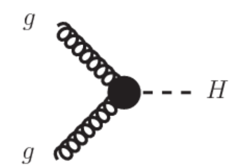
- QCD corrections to Higgs production known to be large, about hundred percent at NLO

- Inclusive production cross section at N3LO to few percent accuracy, using a point-like, top-loop induced ggH coupling (HEFT)

[Anastasiou, Duhr, Furlan, Mistlberger et al'16]



Top loop dominant: $\xrightarrow{m_t \to \infty}$



- At $\boldsymbol{p_{T,H}}$ larger than twice the bottom mass, the ggH coupling is not point-like

- Bottom corrections naively suppressed compared to top by factor

$$y_t\, y_b\, m_b/m_h \sim y_t\, m_b^2/m_h^2 \sim 10^{-3}$$

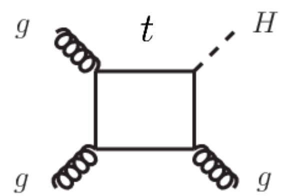
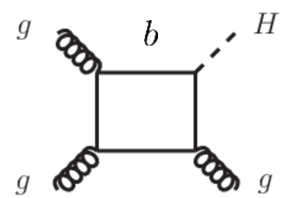
- Bottom amplitude *contains large Sudakov-like logarithms,* suppressed actually by

$$m_b^2/m_h^2\,\left(\log^2(m_h^2/m_b^2), \log^2(p_\perp^2/m_b^2)\right) \sim 10^{-1}$$

- In fact, LO bottom contribution ~ 5-10% of LO top contribution at $p_\perp \in [10, 40]\,\mathrm{GeV}$

Top-bottom interference

- Higgs plus jet production at LHC proceeds largely through quark loops

$\mathcal{A}^{LO}_{gg\to gH} \sim$  $+$ 

$\wr$ y_t $\qquad$ $\wr$ y_b

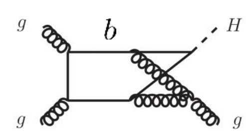
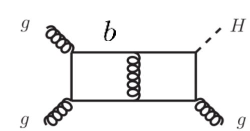
dominant bottom correction

- Differential cross section $\quad d\sigma \sim |\mathcal{A}|^2 \to d\sigma = d\sigma_{tt} + d\sigma_{tb} + d\sigma_{bb}, \quad d\sigma_{ij} \sim \mathcal{O}(y_i y_j)$

$$y_j \sim m_j \qquad m_b = 4.5\,\mathrm{GeV} \qquad m_t = 173\,\mathrm{GeV}$$

- NLO correction to $d\sigma_{tb}$ may be large, <u>as observed also for top contribution ~ 40%</u>, and relevant for reaching percent accuracy in differential cross section

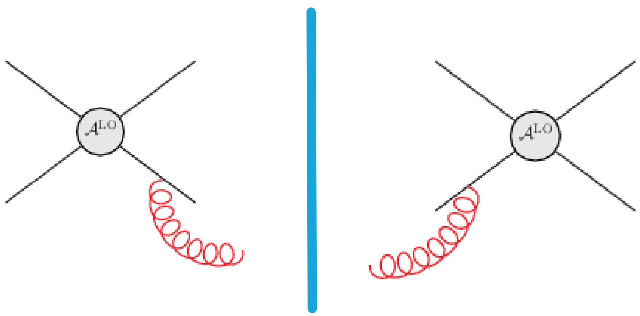
- Two-loop adds extra factor of $\quad \log^2(p_\perp^2/m_b^2),\ \log^2(m_h^2/m_b^2)$

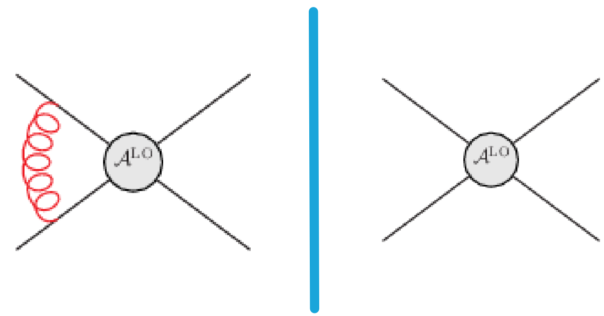
Quantitatively, how large are the bottom corrections at NLO?

Calculation at NLO

- Real (2 to 3) and virtual (2 to 2) contributions need to be combined, very well understood at NLO



Real corrections



Virtual corrections

- Peculiarity in this case: *LO is already 1-loop*

- Real corrections receive contributions from kinematical regions where one parton become soft or collinear to another parton, so a numerically stable approach required

- Real corrections computed in Openloops with exact top, bottom mass dependence

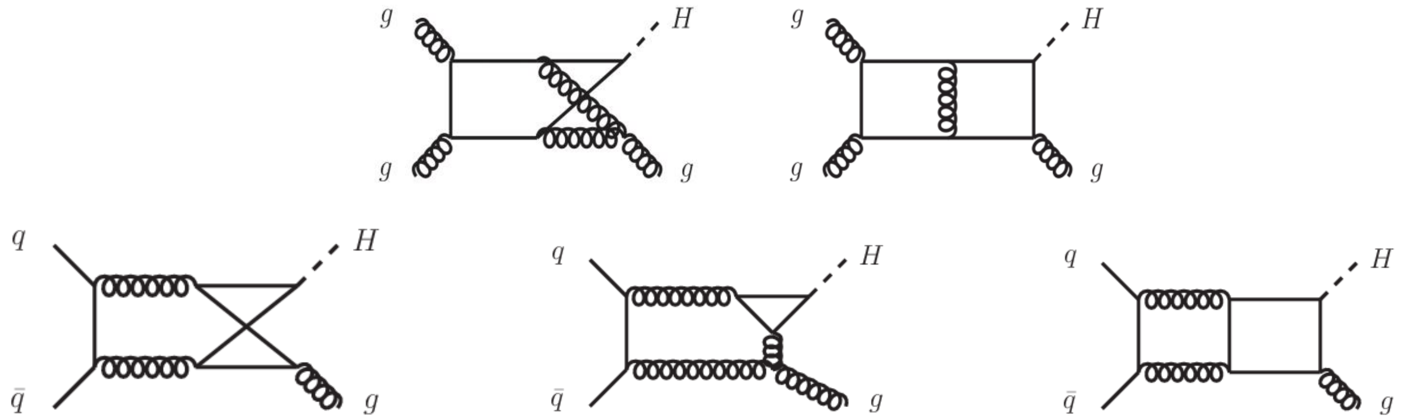
[Cascioli, Lindert, Pozzorini et al '12-17; Denner et al '03-'17]

- One new ingredient are two-loop virtual corrections

Virtual corrections

$$d\sigma_{tb}^{\text{virt}} \sim \text{Re}\left[\frac{\alpha_s}{2\pi}\left(A_t^{\text{NLO}} A_b^{\text{LO}*} + A_t^{\text{LO}} A_b^{\text{NLO}*}\right)\right]$$

- Typical two-loop Feynman diagrams are:



- Exact mass dependence in two-loop Feynman Integrals currently out of reach [planar diagrams: Bonciani et al '16]

Scale hierarchy: $m_b \ll p_\perp, m_h \ll m_t$ $\longrightarrow$

Top: *Infinite top mass limit*, well known how to be treated, expanded systematically via effective Lagrangian (HEFT)

Bottom: Small bottom mass expansion is different because loop is resolved $\longrightarrow$ new methods required

Two-loop *bottom amplitudes* expanded in *bottom mass* with differential equation method [Mueller & Ozturk '15; Melnikov, Tancredi, CW '16-'17]

Computing virtual bottom amplitudes

- Virtual amplitude made up of *complicated two-loop tensor Feynman integrals*

- Powerful tool for scalar integrals: *IBP reduction* to minimal set of *Master Integrals* (**MI**)

→ project amplitude onto form factors

$$\mathcal{A}_{H\to ggg}\,(p_1^{a_1}, p_2^{a_2}, p_3^{a_3}) = f^{a_1 a_2 a_3}\; \epsilon_1^\mu\, \epsilon_2^\nu\, \epsilon_3^\rho\; (F_1\, g^{\mu\nu}\, p_2^\rho + F_2\, g^{\mu\rho}\, p_1^\nu + F_3\, g^{\nu\rho}\, p_3^\mu + F_4\, p_3^\mu p_1^\nu p_2^\rho\,)$$

- Form factors F_i expressed in terms of *scalar integrals*

Three families flashing by

$$\mathcal{I}_{\text{top}}(a_1, a_2, ..., a_8, a_9) = \int \frac{\mathfrak{D}^d k \mathfrak{D}^d l}{[1]^{a_1}[2]^{a_2}[3]^{a_3}[4]^{a_4}[5]^{a_5}[6]^{a_6}[7]^{a_7}[8]^{a_8}[9]^{a_9}}$$

Prop.	Topology PL1	Topology PL2	Topology NPL
[1]	k^2	$k^2 - m_b^2$	$k^2 - m_b^2$
[2]	$(k-p_1)^2$	$(k-p_1)^2 - m_b^2$	$(k+p_1)^2 - m_b^2$
[3]	$(k-p_1-p_2)^2$	$(k-p_1-p_2)^2 - m_b^2$	$(k-p_2-p_3)^2 - m_b^2$
[4]	$(k-p_1-p_2-p_3)^2$	$(k-p_1-p_2-p_3)^2 - m_b^2$	$l^2 - m_b^2$
[5]	$l^2 - m_b^2$	$l^2 - m_b^2$	$(l+p_1)^2 - m_b^2$
[6]	$(l-p_1)^2 - m_b^2$	$(l-p_1)^2 - m_b^2$	$(l-p_3)^2 - m_b^2$
[7]	$(l-p_1-p_2)^2 - m_b^2$	$(l-p_1-p_2)^2 - m_b^2$	$(k-l)^2$
[8]	$(l-p_1-p_2-p_3)^2 - m_b^2$	$(l-p_1-p_2-p_3)^2 - m_b^2$	$(k-l-p_2)^2$
[9]	$(k-l)^2 - m_b^2$	$(k-l)^2$	$(k-l-p_2-p_3)^2$

Computing virtual bottom amplitudes

- Virtual amplitude made up of *complicated two-loop tensor Feynman integrals*

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$$\mathcal{A}_{H\to ggg}(p_1^{a_1}, p_2^{a_2}, p_3^{a_3}) = f^{a_1 a_2 a_3}\, \epsilon_1^\mu\, \epsilon_2^\nu\, \epsilon_3^\rho\, (F_1\, g^{\mu\nu}\, p_2^\rho + F_2\, g^{\mu\rho}\, p_1^\nu + F_3\, g^{\nu\rho}\, p_3^\mu + F_4\, p_3^\mu p_1^\nu p_2^\rho\,)$$

- Form factors F_i expressed in terms of *scalar integrals*

- *Integration by parts* (IBP) identities $\int \Big(\prod_i d^d k_i\Big) \frac{\partial}{\partial k_j^\mu}\Big(v^\mu I\Big) = \text{Boundary term} \overset{DR}{=} 0$

- Reduce to set of *MI* is *very* difficult, naïve reduction with public codes failed

- Performed in steps: top topology to subtopology reduction with Form+Reduze, then FIRE

$$\mathcal{I}_{a_1\cdots a_n}(s) = \sum_{(b_1\cdots b_n)\in\text{Master Integrals}} \text{Rational}_{a_1\cdots a_n}^{b_1\cdots b_n}(s,d)\text{MI}_{b_1\cdots b_n}(s)$$

MI with DE method for small m_b (1/2)

- System of partial differential equations (**DE**) in $\boldsymbol{m_b, s, t, m_h^2}$ with IBP relations

$$\frac{\partial}{\partial \tilde{s}_k} \vec{\mathcal{I}}^{MI}(\tilde{s},\epsilon) \overset{\text{IBP}}{=} \overline{\overline{M}}_k(\tilde{s},\epsilon).\vec{\mathcal{I}}^{MI}(\tilde{s},\epsilon)$$

- Interested in m_b expansion of Master integrals I^{MI}

→ expand homogeneous matrix M_k in small m_b

Step 1: solve DE in m_b

- Solve m_b DE with following ansatz

$$\mathcal{I}_i^{MI}(m_b^2, s, t, m_h^2, \epsilon) = \sum_{ijkn} c_{ijkn}(s, t, m_h^2, \epsilon) \left(\frac{m_b^2}{m_h^2}\right)^{j-k\epsilon} \log^n\left(\frac{m_b^2}{m_h^2}\right)$$

- Plug into m_b DE and get constraints on coefficients c_{ijkn}
- c_{i000} is $m_b = 0$ solution (hard region) and has been computed before [Gehrmann & Remiddi '00]

MI with DE method for small m_b (2/2)

- Ansatz $$\mathcal{I}_i^{MI}(m_b^2, s, t, m_h^2, \epsilon) = \sum_{ijkn} c_{ijkn}(s, t, m_h^2, \epsilon) \left(\frac{m_b^2}{m_h^2}\right)^{j-k\epsilon} \log^n\left(\frac{m_b^2}{m_h^2}\right)$$

Step 2: solve s, t, m_h^2 DE for $c_{ijkn}(s, t, m_h^2)$

- Solution expressed in extensions of usual polylogarithms: *Goncharov Polylogarithms*
- After solving DE for unknown c_{ijkn}, we are left with *unknown boundary constants* that only depend on ε

Step 3: fix ε dependence

- Determination of most boundary constants in ε by *imposing that unphysical singularities in solution* vanish
- Other constants in ε fixed by matching solution of DE to Master integrals computed via various methods (Mellin-Barnes, expansion by regions, numerical fits) in a specific point of s, t, m_h^2

Step 4: numerical checks with FIESTA

[A. Smirnov '14]

Numerical setup

[Lindert, Melnikov, Tancredi, CW '17]

- LHC 13 TeV
- PDF set and associated strong coupling constant: NNPDF3.0_lo for LO and NNPDF3.0_nlo for NLO
- Central scale is dynamical:

$$\mu_r = \mu_f = \mu_0 = H_T/2, \quad H_T = \sqrt{m_H^2 + p_\perp^2} + \sum_j p_{\perp,j}$$

$$m_H = 125\,\mathrm{GeV}, \quad m_t = 173.2\,\mathrm{GeV}$$

Theory uncertainties considered

- Scale variation: $\mu = \{1/2, 2\} * \mu_0$

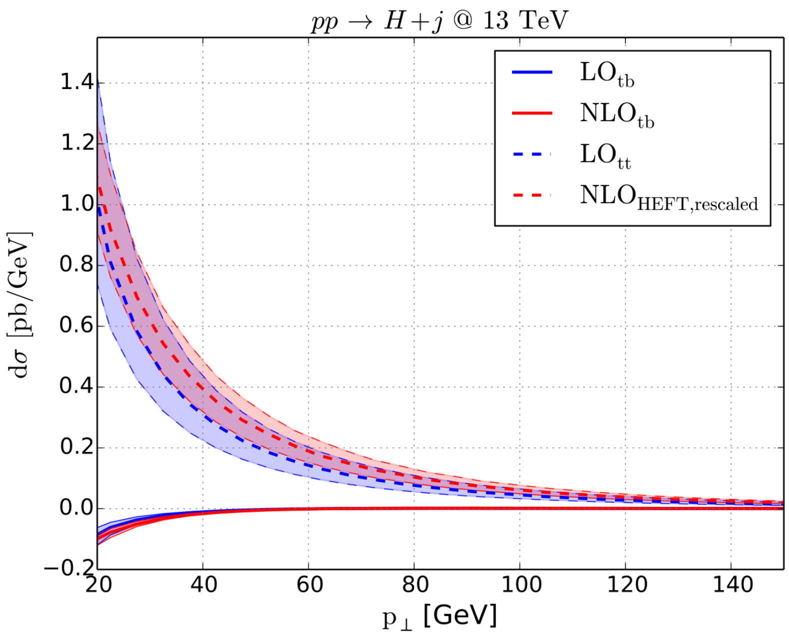
- Large ambiguity in bottom mass scheme: appropriate renormalization scheme for m_b from Yukawa coupling is MSbar scheme at $\mu{\sim}m_h$, while scheme for m_b from helicity flip might require on-shell bottom mass scheme instead. Two bottom mass schemes considered:

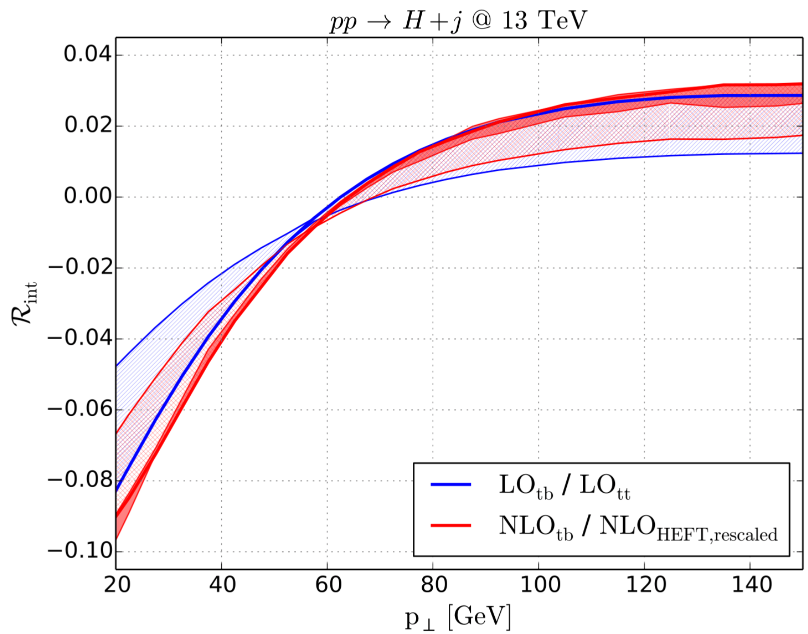
$$m_b^{\mathrm{OS}} = 4.75\,\mathrm{GeV}$$

$$m_b^{\overline{\mathrm{MS}}}(\mu = 100) = 3.07\,\mathrm{GeV}$$

Higgs transverse momentum distribution II

[Lindert, Melnikov, Tancredi, CW '17]

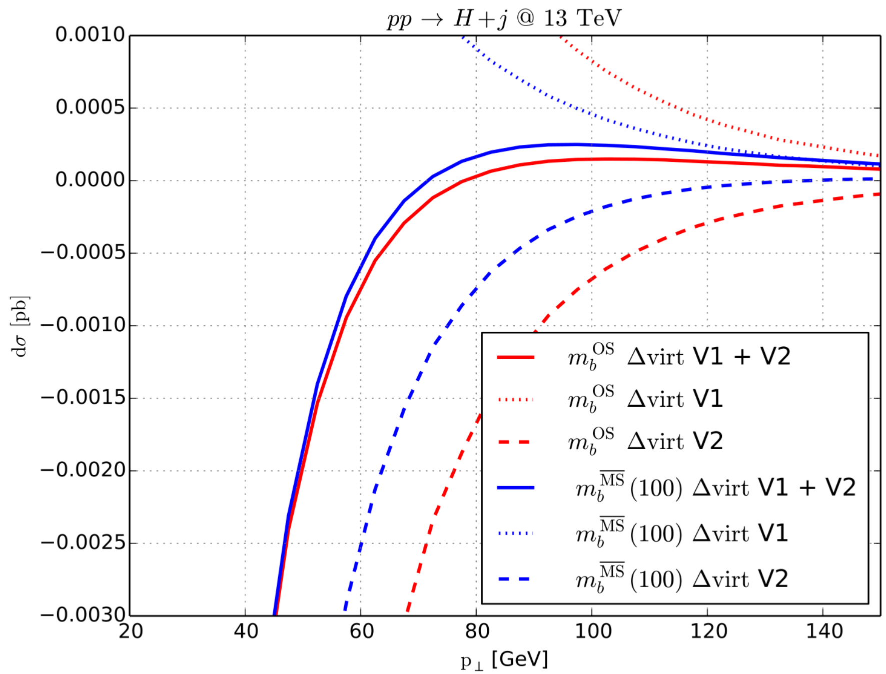




- Top-bottom interference at $p_{T,H}$=30 GeV: -6% at LO and -7% at NLO
- Large relative corrections to top-bottom interference ~ relative corrections to top-top ~ 40%
- Large mass renormalization-scheme ambiguity
- At small $p_{T,H}$ the ambiguity is reduced by a factor of two at NLO; less pronounced at larger $p_{T,H}$

V1:NLO(t)xLO(b) vs V2:LO(t)xNLO(b)

[Lindert, Melnikov, Tancredi, CW '17]



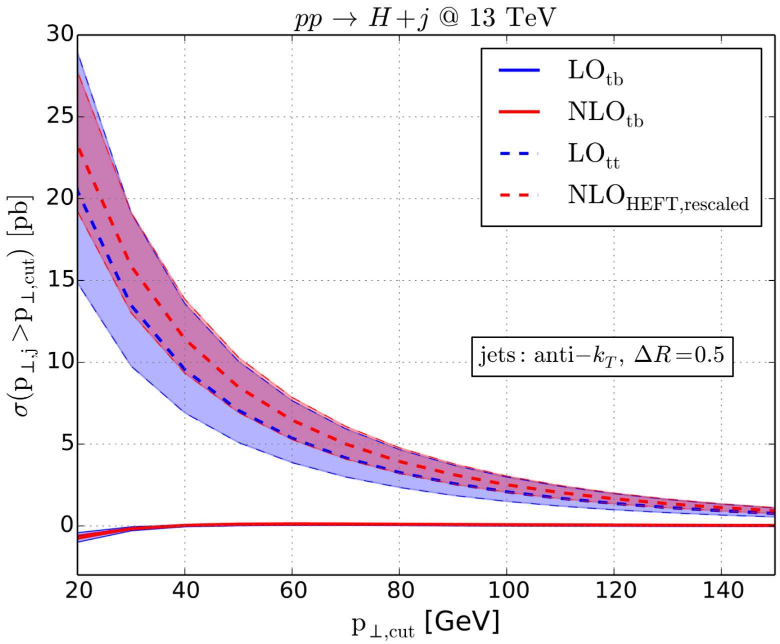
$$\mathrm{d}\sigma_{tb}^{\mathrm{virt}} \sim \mathrm{Re}\left[A_t^{\mathrm{LO}} A_b^{\mathrm{LO}*} + \frac{\alpha_s}{2\pi}\left(\underbrace{A_t^{\mathrm{NLO}} A_b^{\mathrm{LO}*}}_{\mathrm{V1}} + \underbrace{A_t^{\mathrm{LO}} A_b^{\mathrm{NLO}*}}_{\mathrm{V2}}\right)\right]$$

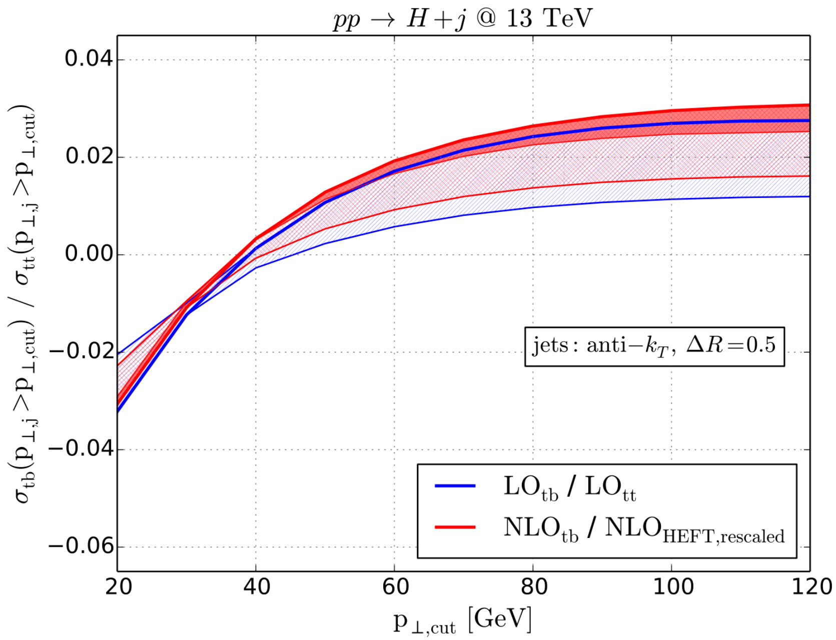
- Two contributions enter with opposite signs
- V2 is dominant at low $p_{T,H}$ ~ 20-50 GeV which reduces mass scheme ambiguity
- At large $p_{T,H}$ V1~V2 and V1 represents LO bottom mass scheme ambiguity

Total Higgs plus jet cross section

[Lindert, Melnikov, Tancredi, CW '17]

- Total integrated cross section as function of threshold on jet $p_{T,j}$





$$\sigma_{tb}/\sigma_{tt}(p_{T,j} > 20, 30, 40, 50)_{\mathrm{LO}} = -3.2, -1.2, +0.1, 1.1\%$$

$$\sigma_{tb}/\sigma_{tt}(p_{T,j} > 20, 30, 40, 50)_{\mathrm{NLO}} = -3.1, -1.1, +0.3, 1.3\%$$

- Total integrated NLO top-bottom interference contributes [-3% , 3%] of NLO top-top contribution
- Strong dependence on jet $p_{T,j}$ cut

Summary

- Fully differential NLO QCD corrections to top-bottom interference first time computed
- Two-loop integrals computed at first order in bottom mass expansion with DE method
- NLO bottom contribution ~ [-10, -4] % of NLO top contribution at lower range of Higgs $\boldsymbol{p_{T,H}}$
- Large relative NLO corrections to top-bottom interference similar to pure top NLO corrections ~ 40% for Higgs $\boldsymbol{p_{T,H}}$ and rapidity distributions
- On-shell vs MSbar bottom mass: large renormalization scheme ambiguity. Reduced at small $\boldsymbol{p_{T,H}}$ ~ 20-40 GeV, but unchanged at larger $\boldsymbol{p_{T,H}}$ ~ 60-100 GeV

Outlook

Combine various contributions to get best $H + j$ prediction:

- Low $\boldsymbol{p_{T,H}}$-resummation [Grazzini et al '13; Banfi et al '14,16; Bagnaschi et al '15]
- NNLO HEFT corrections [Boughezal et al '14,15; Gehrmann et al '15,16]
- NLO top-bottom interference [Lindert et al '17]

Backup slides

IBP reduction

[Melnikov, Tancredi, CW '16-'17]

- IBP reduction to Master Integrals

$$\mathcal{I}_{a_1\cdots a_n}(s) = \sum_{(b_1\cdots b_n)\in \text{Master Integrals}} \text{Rational}^{b_1\cdots b_n}_{a_1\cdots a_n}(s,d)\text{MI}_{b_1\cdots b_n}(s)$$

- _Reduction very non-trivial_: we were not able to reduce top non-planar integrals with $t=7$ denominators with FIRE5/Reduze
- Reduction fails because coefficients multiplying MI become too large to simplify ~ hundreds of Mb of text

- _Reduction for complicated t=7 non-planar integrals performed in two steps_:

1) FORM code reduction: $$\mathcal{I}^{\text{NPL}}_{t=7} = \sum c_i \text{MI}^i_{t=7} + \sum d_i \mathcal{I}^i_{t=6}$$

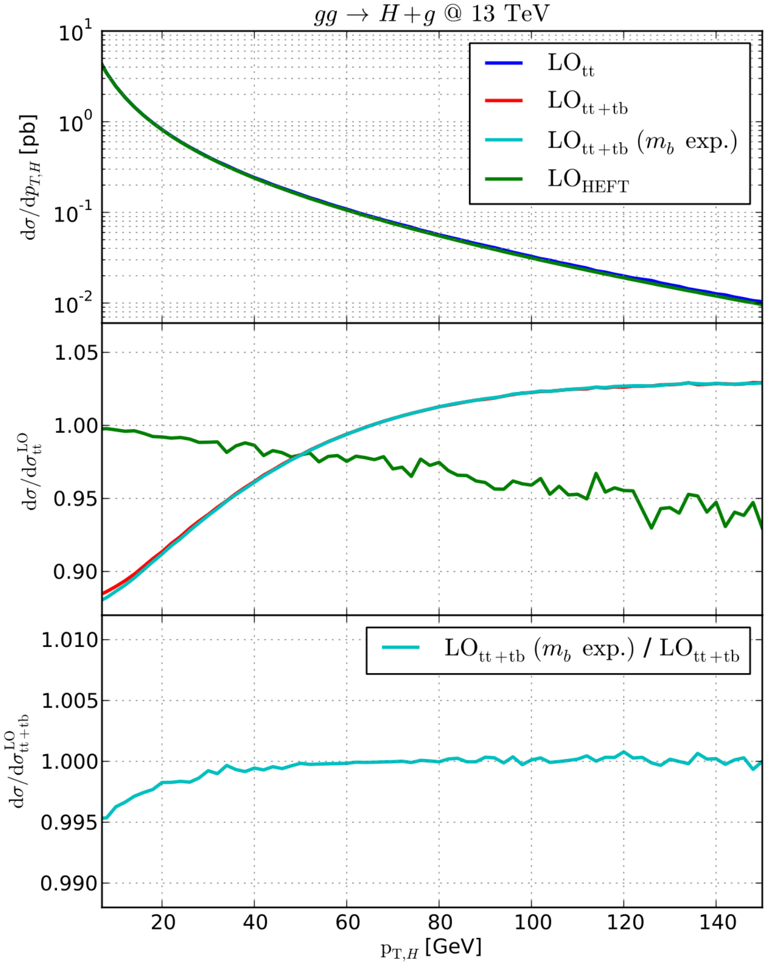
2) Plug reduced integrals into amplitude, expand coefficients c_i, d_i in m_b

3) Reduce with FIRE/Reduze: $t=6$ denominator integrals $\mathcal{I}_{t=6}$

- Exact m_b dependence kept at intermediate stages. Algorithm for _solving IBP identities directly expanded in small parameter is still an open problem_

- _Expansion in m_b occurs at last step_: solving with Master integrals with *differential equation method*

LO contributions



How useful is m_b expansion?

- NLO amplitudes require computing 2-loop Feynman integrals with massive quark loop
- If these integral are computed exactly in quark mass, results in *very complicated functions*

[planar diagrams: Bonciani et al '16]

$$
\begin{aligned}
&\log\left(x_3 x_1^2 - x_1^2 + x_2 x_1 - 4x_3 x_1 + R_1(x_1)R_2(x_1)R_7(x)\right)\,,\\
&\log\left(-x_2^2 + x_1 x_2 - x_1 x_3 x_2 + 2x_3 x_2 + 2x_1 x_3 + R_1(x_2)R_2(x_2)R_7(x)\right)\,,\\
&\log\left(-x_3^2 x_1^2 + 3x_3 x_1^2 + 4x_3^2 x_1 - 4x_2 x_3 x_1 + R_1(x_3)R_5(x)R_6(x)x_1\right)\,,\\
&\log\left(x_3 R_1(x_2)R_2(x_2) + x_2 R_1(x_3)R_2(x_3)\right)\,,\\
&\log\left(x_1 R_1(x_2)R_2(x_2) + x_2 R_1(x_1)R_2(x_1)\right)\,,\\
&\log\left(x_1 R_1(x_3)R_2(x_3) - R_1(x_1)R_1(x_3)R_5(x)\right)\,,\\
&\log\left(x_3 R_1(x_1)R_2(x_1) - R_1(x_1)R_1(x_3)R_5(x)\right)\,,\\
&\log\left(-x_2 R_1(x_1)R_2(x_1) + x_3 R_1(x_1)R_2(x_1) + x_1 R_3(x_3)R_4(x_3)\right)\,,\\
&\log\left(-x_2 R_1(x_2)R_2(x_2) + x_3 R_1(x_2)R_2(x_2) + x_2 R_3(x_3)R_4(x_3)\right)\,,\\
&\log\left(-x_2 R_1(x_3)R_2(x_3) + x_1 R_1(x_3)R_2(x_3) + x_3 R_3(x_1)R_4(x_1)\right)\,,\\
&\log\left(-x_2 R_1(x_2)R_2(x_2) + x_1 R_1(x_2)R_2(x_2) + x_2 R_3(x_1)R_4(x_1)\right)\,,\\
&\log\left(-x_3^2 x_1^2 + 3x_3 x_1^2 + 4x_3^2 x_1 - 3x_2 x_3 x_1 + R_1(x_1)R_1(x_3)R_5(x)R_7(x)\right)\,,\\
&\log\left(x_2 R_1(x_1)R_1(x_3)R_5(x) - x_1 x_3 R_1(x_2)R_2(x_2)\right)\,,\\
&\log\left(-x_2 x_3 + x_1 x_3 + R_1(x_2)R_2(x_2)x_3 - R_1(x_1)R_1(x_3)R_5(x)\right).
\end{aligned}
$$

$$
\begin{aligned}
&R_1(x_1) = \sqrt{-x_1}\,,\ R_1(x_3) = \sqrt{-x_3}\,,\ R_1(x_2) = \sqrt{-x_2}\,,\\
&R_2(x_1) = \sqrt{4-x_1}\,,\ R_2(x_3) = \sqrt{4-x_3}\,,\ R_2(x_2) = \sqrt{4-x_2}\,,\\
&R_3(x_1) = \sqrt{x_2-x_1}\,,\ R_3(x_3) = \sqrt{x_2-x_3}\,,\\
&R_4(x_1) = \sqrt{x_2-x_1-4}\,,\ R_4(x_3) = \sqrt{x_2-x_3-4}\,,\\
&R_5(x) = \sqrt{4x_2 + x_1 x_3 - 4(x_1+x_3)}\,,\\
&R_6(x) = \sqrt{2x_3(-2x_2 + x_1 + 2x_3) - x_1 x_3^2 - x_1}\,,\\
&R_7(x) = \sqrt{2x_1 x_3(x_2 - x_1) + (x_2 - x_1)^2 + (x_1 - 4)x_1 x_3^2}\,.
\end{aligned}
$$

- Starting from weight three not possible to express in terms of usual GPL's anymore
- *Expanding* in small bottom quark mass results in simple *2-dimensional harmonic polylogs*

[Vermaseren, Remiddi, Gehrmann]

Real corrections with Openloops

- Channels for real contribution to Higgs plus jet at NLO

$$gg \to Hgg,\ gg \to Hq\bar{q},\ qg \to Hqg,\ q\bar{q} \to Hgg,\ \cdots$$

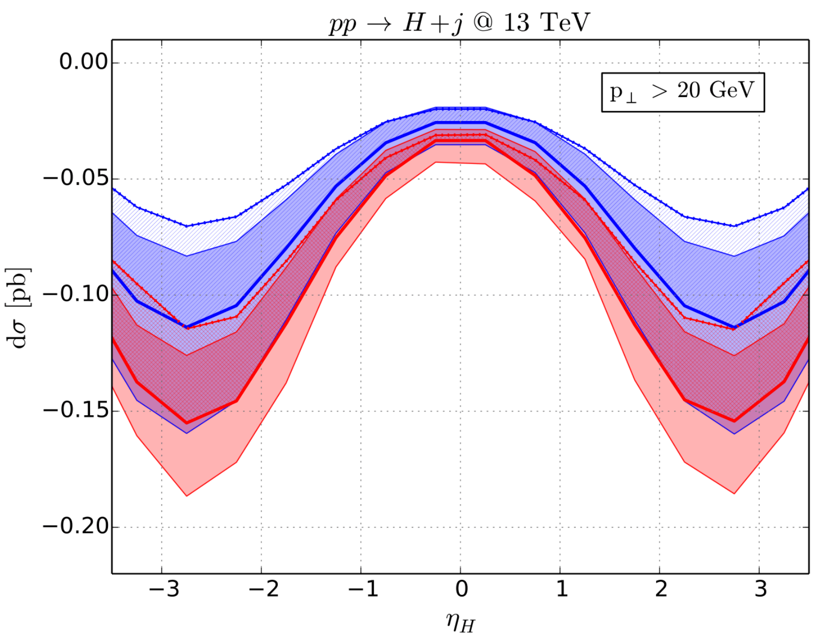
- Receives contributions from kinematical regions where one parton become soft or collinear to another parton
- This requires a delicate approach of these regions in phase space integral
- Openloops algorithm is publicly available program which is capable of dealing with these singular regions in a numerically stable way
- Crucial ingredient is tensor integral reduction performed via expansions in small Gram determinants: Collier

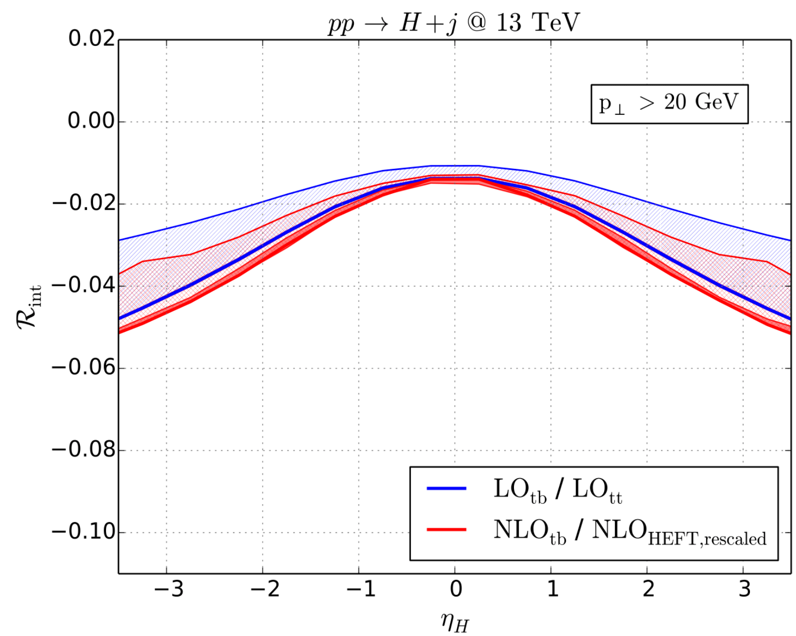
[Cascioli et al ’12, Denner et al ’03-’17]

- Exact top and bottom mass dependence kept throughout for both top-top and top-bottom contribution to differential cross section

Higgs pseudo-rapidity distribution

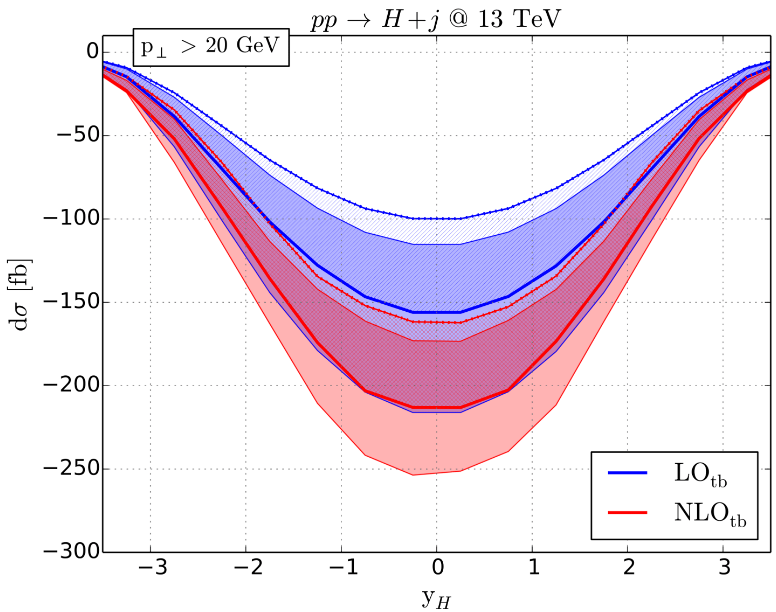
[Lindert, Melnikov, Tancredi, CW '17]

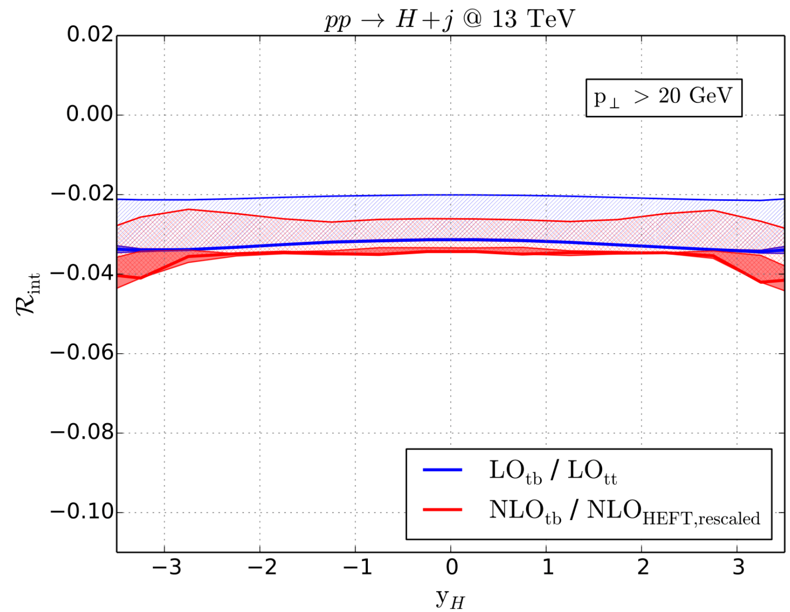




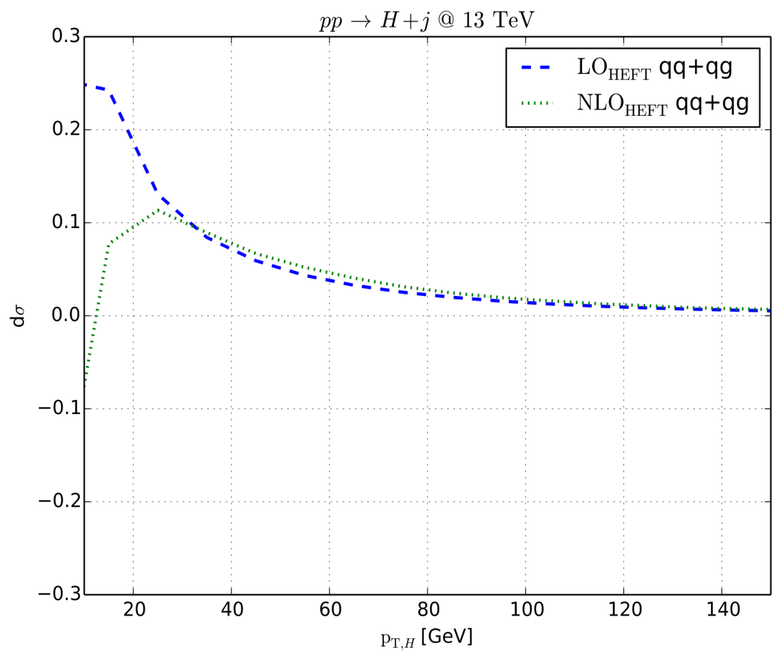
- Relative corrections to top-bottom interference ~ relative corrections to top-top
- At central rapidity (dominated by large $p_{T,H}$) mass scheme ambiguity similar between LO and NLO
- At larger absolute rapidity (dominated by small $p_{T,H}$) the mass scheme variation band is smaller for NLO

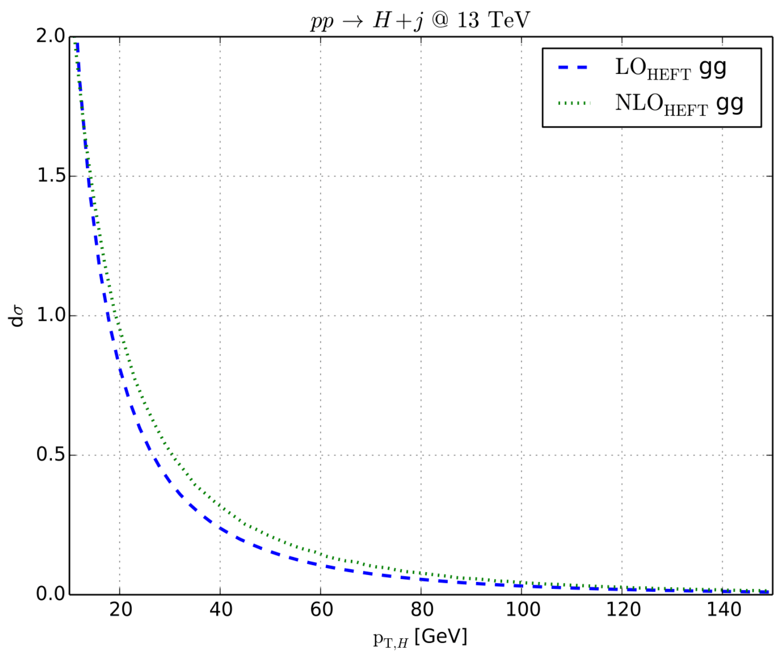
Higgs rapidity distribution





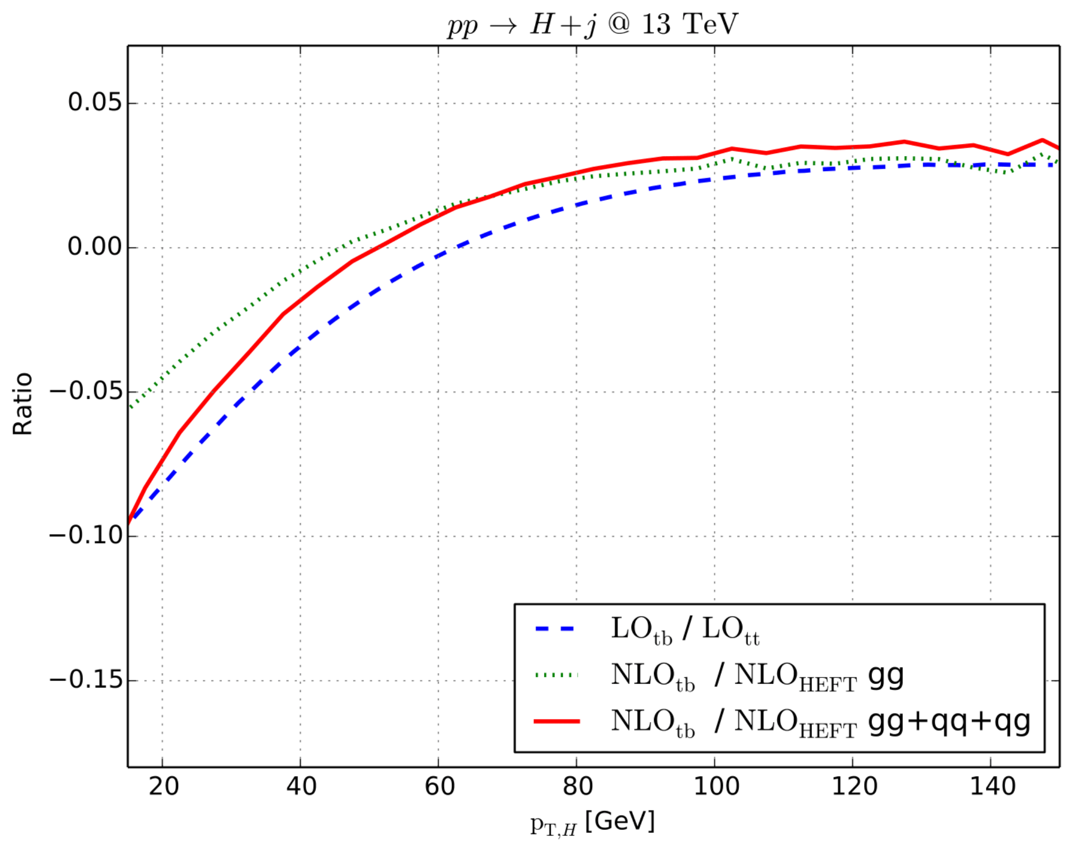
Channel contribution: tt





- gg fusion channel dominates

Channel contribution: tb



- gg fusion channel dominates