Soft and Coulomb effects in top-quark pair production beyond NNLO

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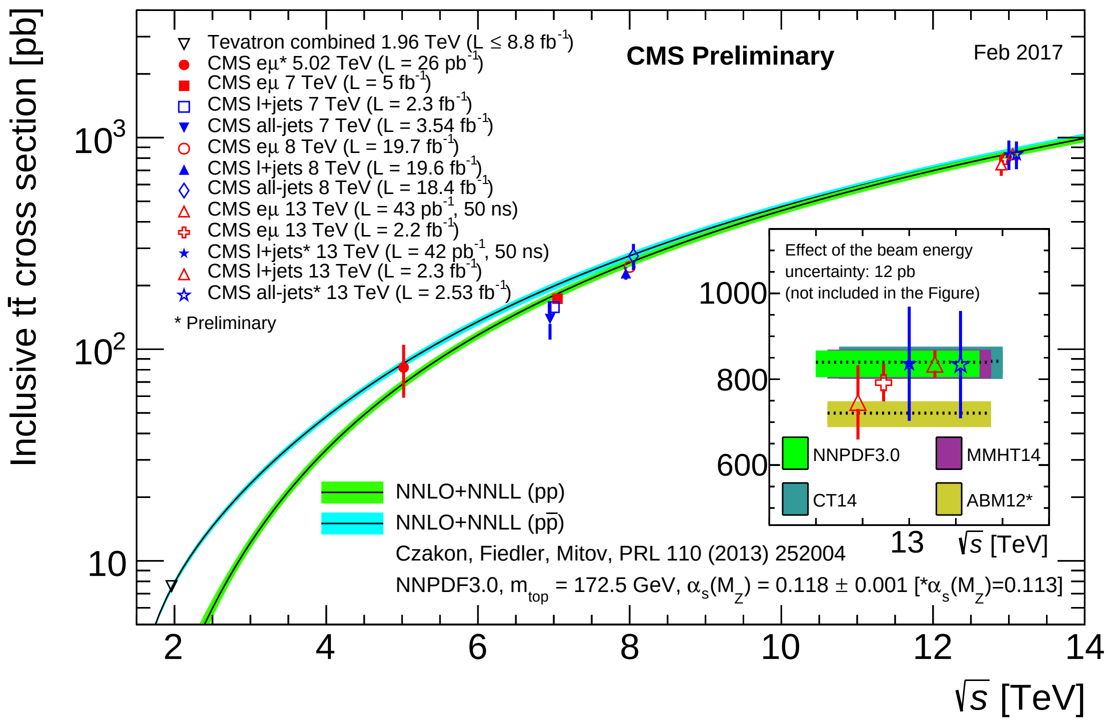
work in progress with Jan Piclum

Total $t\bar{t}$ cross section test of QCD and nature of top-quark:

- Experimental precision comparable to uncertainty of NNLO+NNLL prediction (Bärnreuther/Czakon/Fiedler/Mitov 12–13)



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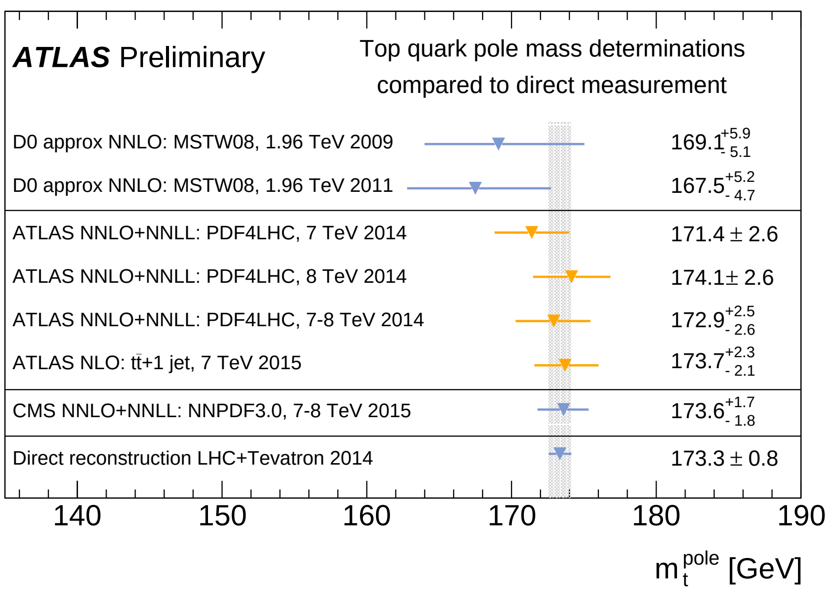
  - pole mass $m_t = 173.8^{+1.7}_{-1.8}\mathrm{GeV}$ from σ_{tt} measurement (CMS 16)
  - determination of $\alpha_s(M_Z) = 0.1177^{+0.0034}_{-0.0036}$ (Klijnsma/Bethke/Dissertori/Salam 17)



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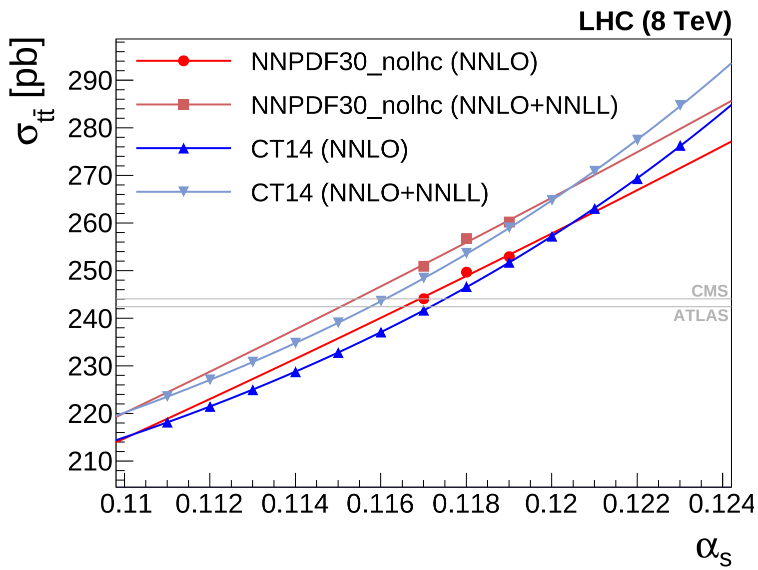
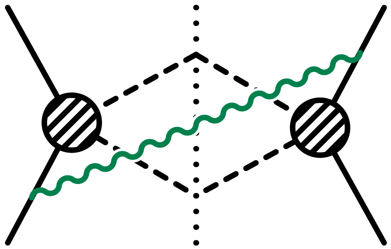
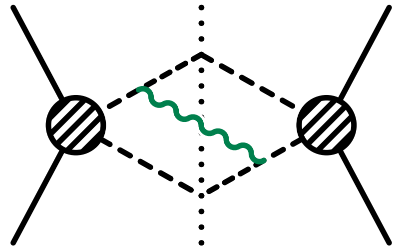


Threshold logarithms enhanced for $\beta = \sqrt{1 - \frac{4m_t^2}{\hat{s}}} \to 0$



$\Rightarrow \alpha_s \; \log^2(8\beta^2)$



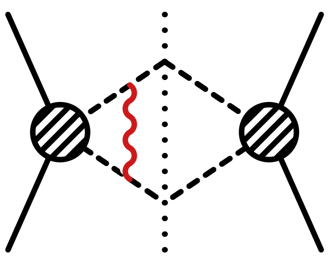
$\Rightarrow \alpha_s \; \log(8\beta^2)$

- remnants of cancellation of soft/collinear divergences between real and virtual corrections
- structure of soft-gluon emission universal

$\Rightarrow$ can predict threshold logs at higher orders

(Sterman 87; Catani, Trentadue 89, Kidonakis,Sterman 97, Bonciani et.al. 98, . . .)

Coulomb gluon corrections (Fadin, Khoze 87; Peskin, Strassler 90, NRQCD,. . .)



$\Rightarrow \alpha_s \, \frac{1}{\beta}$

Resummation of threshold-enhanced corrections, $\beta = \sqrt{1 - \frac{4m_t^2}{\hat{s}}} \to 0$

$$\hat{\sigma}_{pp'} \propto \sigma^{(0)} \exp\Big[\underbrace{\ln\beta\, g_0(\alpha_s \ln\beta)}_{\text{(LL)}} + \underbrace{g_1(\alpha_s \ln\beta)}_{\text{(NLL)}} + \underbrace{\alpha_s g_2(\alpha_s \ln\beta)}_{\text{(NNLL)}} + \underbrace{\alpha_s^2 g_3(\alpha_s \ln\beta)}_{\text{(N}^3\text{LL)}} + \dots\Big]$$

$$\times \sum_{k=0} \left(\frac{\alpha_s}{\beta}\right)^k \times \Big\{ \underbrace{1}_{\text{(LL,NLL)}} \; ; \; \underbrace{\alpha_s, \beta}_{\text{(NNLL)}} \; ; \; \underbrace{\alpha_s^2, \alpha_s\beta, \beta^2}_{\text{(NNLL', N}^3\text{LL)}} \; ; \dots \Big\} :$$

NNLL resummation

- Mellin-space resummation of threshold logarithms $\alpha_s \log\beta$

  (Czakon/Mitov/Sterman 09/Cacciari et al. 11)

  implemented in TOP++ (Czakon/Mitov)

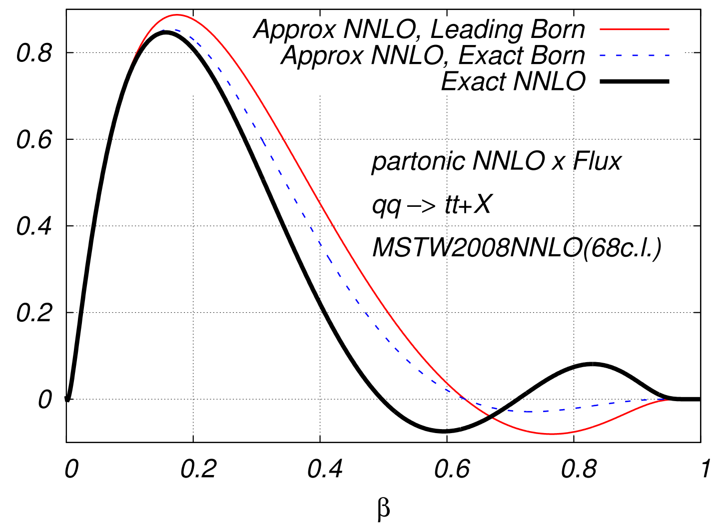
- Threshold logs and Coulomb corrections α_s/β

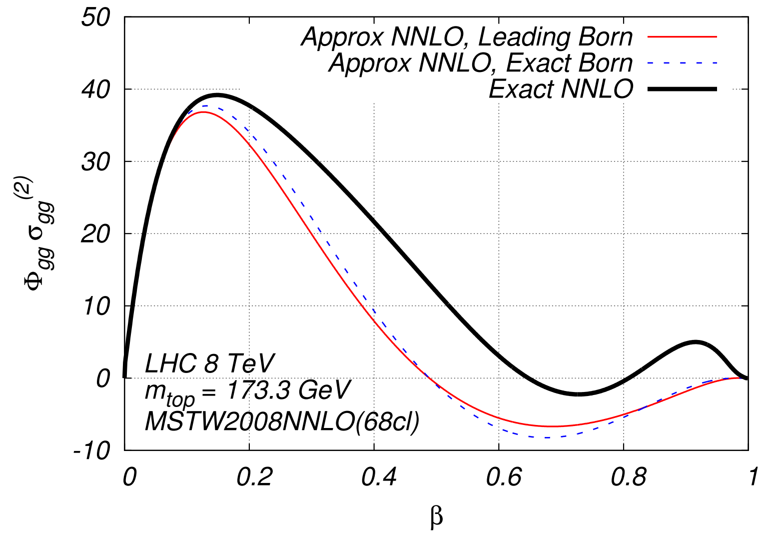
  (Beneke/Falgari/(Klein)/CS 09/11)

  implemented in TOPIXS (Beneke et al.)

- Resummation for p_T, $M_{t\bar{t}}$ distributions (Kidonakis; Ahrens et al.; low p_T: Zhu et al; Catani et al; boosted tops: Ferroglia et al.)

- Top-pair production dominated by $\beta \sim 0.6$
  $\Rightarrow$ justification of threshold approximation?





$\frac{d\sigma}{d\beta} = \frac{8\beta m_t^2}{s(1-\beta^2)^2} L(\beta,\mu_f)\hat{\sigma}$, (Bärnreuther/Czakon/Mitov 12; Czakon/Fiedler/Mitov 13)

$\Rightarrow$ threshold corrections give estimate of higher-order corrections

$\Rightarrow$ careful estimate of uncertainties necessary

- resummation not mandatory for $t\bar{t}$ production at LHC

$\Rightarrow$ compare resummed results to fixed-order expansions

Reduction of scale uncertainty from threshold resummation

$$\sigma_{t\bar{t}}^{\text{NNLO}}(13\text{TeV}) = 802.83^{+28.10(3.5\%)}_{-44.85(5.6\%)}\text{pb} \Rightarrow \begin{cases} \text{NNLL(top}++\text{)}: & 821.37^{+20.28(2.5\%)}_{-29.60(3.6\%)}\text{pb} \\ \text{NNLL(topixs)}: & 806.96^{+25.59(3.2\%)}_{-40.36(5.0\%)}\text{pb} \end{cases}$$

`top++`: Mellin space resummation (Sterman 87; Catani/Trentadue 89)

- Includes 2-loop constant term H_2 in threshold expansion

$$\sigma_{t\bar{t}}^{\text{NLLL}}|_{H_2=0} = 812.20 \text{ pb}$$

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`topixs:` combined soft/Coulomb resummation

- RGE for momentum-space resummation (Becher/Neubert 06)
- dependence on scales μ_f, $\mu_h \sim 2M$: $\Delta_{\rm scale}\sigma_{t\bar{t}}^{\rm NNLL} = {}^{+15.64}_{-37.71}$ pb
- resummation uncertainty: choice of $\mu_s \sim M\beta^2$, kinematic ambiguities, higher-order terms: $\Delta_{\rm res}\sigma_{t\bar{t}}^{\rm NNLL} = {}^{+20.26}_{-14.37}$ pb
- Includes bound-state effects $\sigma_{t\bar{t}}^{\rm NNLL}|_{\rm BS} = 2.8$ pb

NNLL vs NNLO

Reduction of scale uncertainty from threshold resummation

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`topixs:` combined soft/Coulomb resummation

- main source of numerical difference:
  treatment of $H_2 \Rightarrow \alpha_s^3 \log\beta^2$ terms (NNLL')

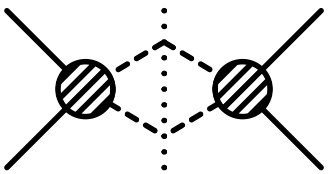
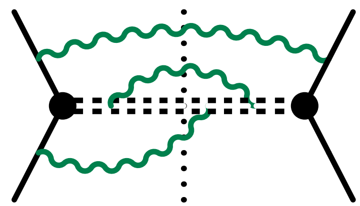
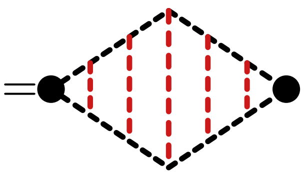
⇒ **Upgrade** `Topixs` to NNLL'/partial N³LL

- First step: expansion to N³LO

Factorization of cross section (Beneke, Falgari, CS 09/10)

$$\hat{\sigma}_{pp' \to HH'}|_{\hat{s} \to 4M^2} = \sum_R H_R W^R \otimes J^R$$

Hard, soft and Coulomb functions:

$H_R =$  , $W^R =$  , $J^R =$ 

Soft radiation "sees" only total colour state $R = 1, 8, \ldots$ of $t\bar{t}$

Factorization scale dependence of H, W cancels against PDFs:

$$\frac{d\sigma}{d\mu} = \frac{d}{d\mu}\left(f_1 \otimes f_2 \otimes H \otimes W \otimes J\right) = 0$$

- $\frac{df_i}{d\mu} \Rightarrow$ Altarelli-Parisi equation (3-loop: Moch/Vermaseren/Vogt 04/05)
- $\frac{dH}{d\mu} \Rightarrow$ IR singularities (2-loop: Becher, Neubert; Ferroglia et.al. 09)

$\Rightarrow$ RGE for soft function (NNLL: Beneke/Falgari/CS; Czakon/Mitov/Sterman 09)

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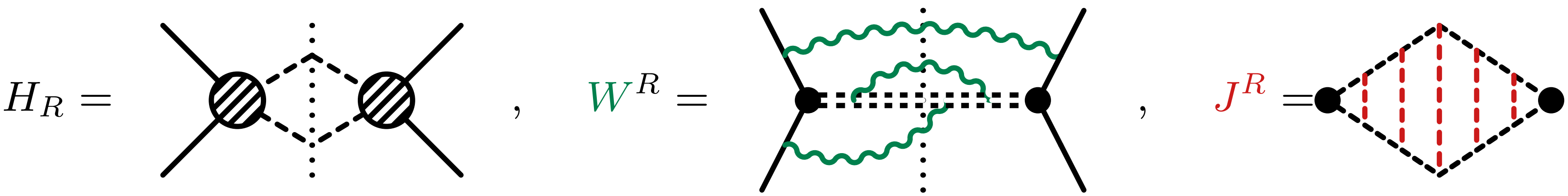


Soft radiation "sees" only total colour state $R = 1, 8, \ldots$ of $t\bar{t}$

Momentum-space solution to RGE (Becher/Neubert/Pecjak 07)

- evolve hard function from

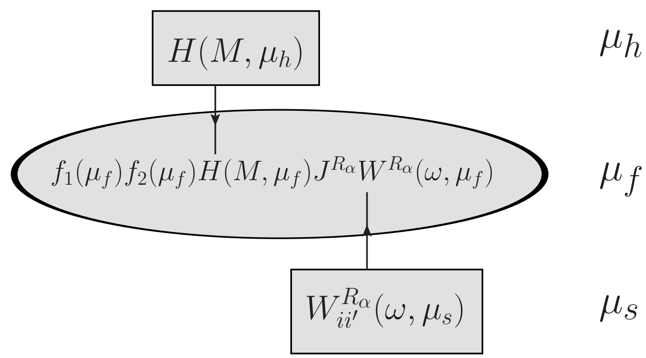
  $\mu_h \sim 2M$ to μ_f

- evolve soft function from

  $\mu_s \sim M\beta^2$ to μ_f

  choice of μ_s from relation to Mellin resummation

  (Sterman/Zeng 13; Bonvini et al. 14)

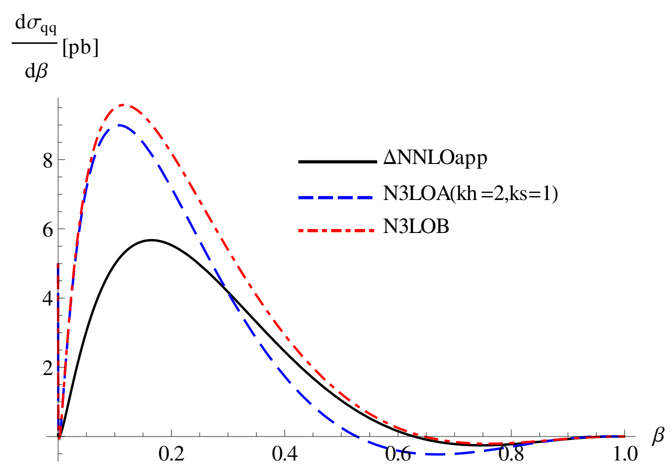


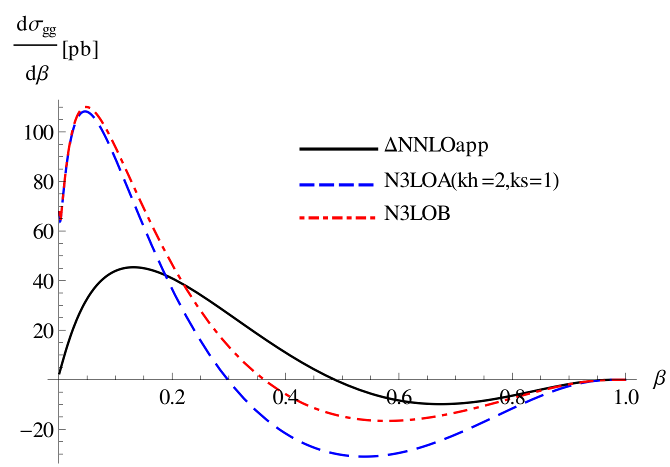
Expand NNLL to $\mathcal{O}(\alpha_s^3)$, e.g. (Beneke/Falgari/Klein/CS 11)

$$\Delta\sigma^{(3)}_{gg_8,\text{NNLL}} = \sigma^{(0)}_{gg_8}\frac{\alpha_s^3}{(4\pi)^3}\Big\{147456.\ln^6\beta - 169658.\ln^5\beta - 140834.\ln^4\beta + 524210.\ln^3\beta$$
$$+\frac{1}{\beta}\left[-15159.7\ln^4\beta - 5364.82\ln^3\beta + 19598.9\ln^2\beta - 17054.7\ln\beta\right]$$
$$+\frac{1}{\beta^2}\left[346.343\ln^2\beta + 522.978\ln\beta - 71.7884\right]\Big\} + \underbrace{\left\{\log\beta^{1,2}, 1/\beta, C^{(3)}\right\}}_{\text{not known exactly}} + \text{scale dep.}$$

N^3LO_A: keep all terms, including μ_s, μ_h-dependence and constants

N^3LO_B: only keep exactly known terms





LHC7

N3LO$_{\text{approx}}$?

Numerical results: (MSTW08)

$$\sigma_{t\bar{t}}^{\text{NNLO}}(8\text{TeV}) = 239.18^{+\ 9.29(3.9\%)}_{-14.85(6.2\%)}\text{pb} \Rightarrow \begin{cases} \text{N3LO}_A: & 244.87^{+3.5(1.5\%)}_{-6.7(2.8\%)}(\mu_f) + {}^{+3.8}_{-12.1}(\mu_s)\text{pb} \\ \text{N3LO}_B: & 245.90^{+6.7(2.7\%)}_{-5.0(2.0\%)}\text{pb} \end{cases}$$

- small μ_f-dependence
- strong dependence of incompletely known terms on soft scale

$\Rightarrow$ need input beyond NNLL

Other N^{3}LO approximations

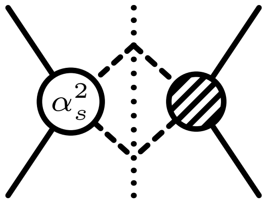
- Using NNLL in one-particle inclusive kinematics (Kidonakis 14)

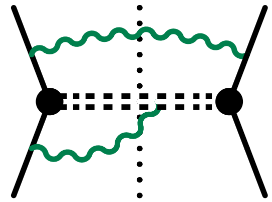
$$\sigma_{t\bar{t},1\text{PI}}^{\text{N3LOapp}}(8\text{TeV}) = 248^{+7(2.8\%)}_{-8(3.2\%)}\text{pb} \qquad \text{(MSTW08)}$$

- Combination of threshold resummation with subleading terms and large-x behaviour (Muselli et al. 15)

$$\sigma_{t\bar{t}}^{\text{N3LOapp}}(8\text{TeV}) = 253.98\text{pb} \pm 3.5\% \qquad \text{(NNPDF3.0)}$$

Input to resummation formula at N^{3}LL

$H_i^{(2)} =$ 

, $W_i^{(2)R} =$ 
, $J^R_{\rm NNLO} =$ 

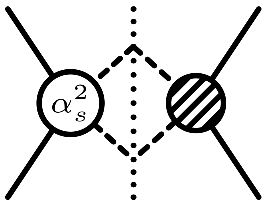
Hard function

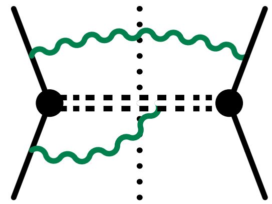
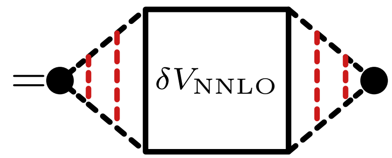
- NNLL: one-loop H_i (Czakon/Mitov 08; also Hagiwara et.al. 08)
- NNLL'/N^{3}LL: two-loop H_i
  from constant in NNLO threshold expansion
  (Bärnreuther/Czakon/Fiedler 13)

Soft function

- NNLL: 1-loop soft function for arbitrary R (Beneke/Falgari/CS 09)
- NNLL'/N^{3}LL: 2-loop soft function for singlet/octet
  (Belitzky 98;Becher/Neubert/Xu 07; Czakon/Fiedler 13)

Input to resummation formula at N³LL

$H_i^{(2)} =$ 

, $W_i^{(2)R} =$ 
, $J^R_{\rm NNLO} =$ 

Potential function

- NNLL:
  - NLO Coulomb-Green function sums terms $\alpha_s^n/\beta^n \times (\alpha_s v)$
  - spin-dependent $\alpha_s^2 \ln\beta$ term from NNLO Green function

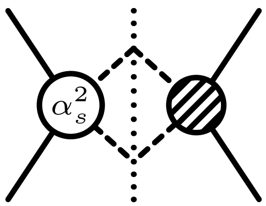
    (Beneke, Czakon,Falgari, Mitov, CS 09)

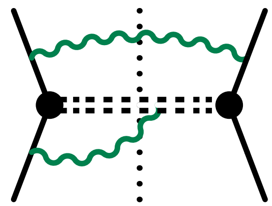
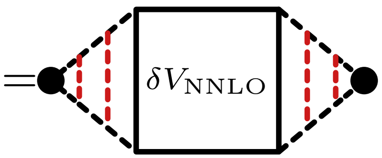
- N³LL:
  - NNLO Green function sums terms $\alpha_s^n/\beta^n \times (\alpha_s^2, \alpha_s v, v^2)$

    (Beneke/Signer/Smirnov; Hoang/Teubner 99,. . .)
  - spin-dependent $\alpha_s^3 \ln^{2,1}\beta$ terms from N³LO Green function, only known fully for $e^-e^+ \to t\bar{t}$

    (using implementation of Beneke/Kiyo/Maier/Piclum 16)

Input to resummation formula at N^3LL

$H_i^{(2)} =$ 

, $W_i^{(2)R} =$ 
, $J^R_{\text{NNLO}} =$ 

RGEs

- known for N^3LL:
  - 4-loop γ_{cusp} (Moch/Ruijl/Ueda/Vermaseren/Vogt 17; not needed for N^3LO_{app})
  - anomalous dimensions extracted from 3-loop splitting functions and quark and gluon form factors (Moch/Vermaseren/Vogt 04/05)
- missing: 3-loop massive soft anomalous dimension (Massless result: Almelid/Duhr/Gardi 15)

Conceptual issues

of soft/Coulomb resummation for $\alpha_s \log\beta \sim 1$, $\frac{\alpha_s}{\beta} \sim 1$

- interplay of Coulomb $(\alpha_s/\beta)^n$and power corrections $\sim \beta^l$
- no $\alpha_s/\beta \times \alpha_s \log^{2,1}\beta \times \beta$ corrections
  to soft NNLL resummation for σ_{tot}, $d\sigma/dM_{t\bar{t}'}$ (Beneke/Falgari/CS 10)
- Known corrections relevant for N³LO threshold expansion

$$\frac{\alpha_s^2}{\beta^2} \times \alpha_s\beta^2 \log\beta \sim \alpha_s^3 \log\beta$$

  – LL "next-to-eikonal" ISR effects similar to DY/Higgs

  (Krämer/Laenen/Spira 98; Laenen et al. 10)

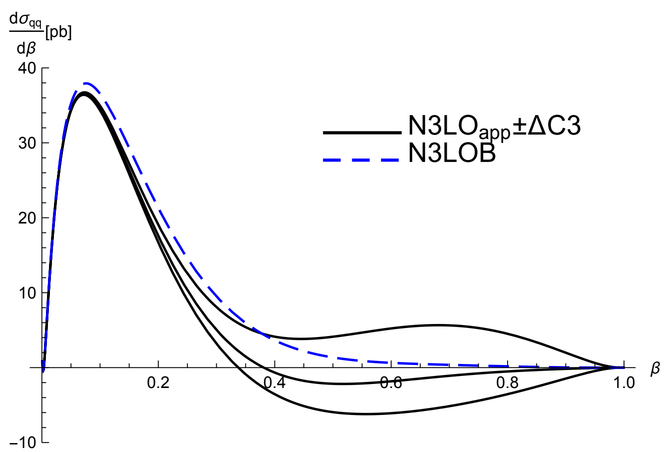
  Numerical effect $< 1\text{pb}$ at 8TeV

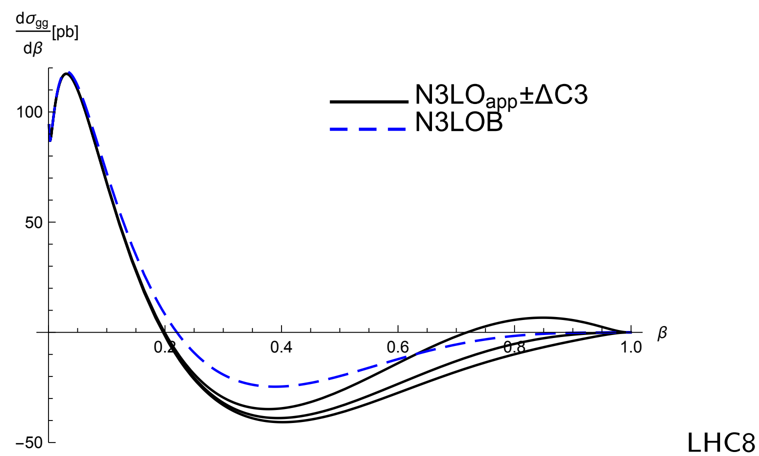
  – (ultra)-soft chromoelectric corrections

  (known only for $e^-e^+ \to t\bar{t}$ Beneke/Kiyo 08)

N³LO expansion of N³LL result: (preliminary)

$$\Delta\sigma^{(3)}_{gg_8,\mathrm{N^3LL}} = \Delta\sigma^{(3)}_{gg_8,\mathrm{NNLL}} + \sigma^{(0)}_{gg_8}\frac{\alpha_s^3}{(4\pi)^3}\Big\{ -2743.36\frac{1}{\beta} + (157.914 J^{S=0,(3)}_{L2,8} - 299282.)\ln^2\beta + (12\gamma^{(2)}_{H,s} + 157.914 J^{S=0,(3)}_{L,8} + 47748.4)\ln\beta \Big\}$$





large cancellations ⇒ (accidentally) small leftover correction

$$\Delta\sigma^{\mathrm{N3LOapp}}_{t\bar{t}}(8\mathrm{TeV}) = -1.29\mathrm{pb}$$ (unknown constants zero)

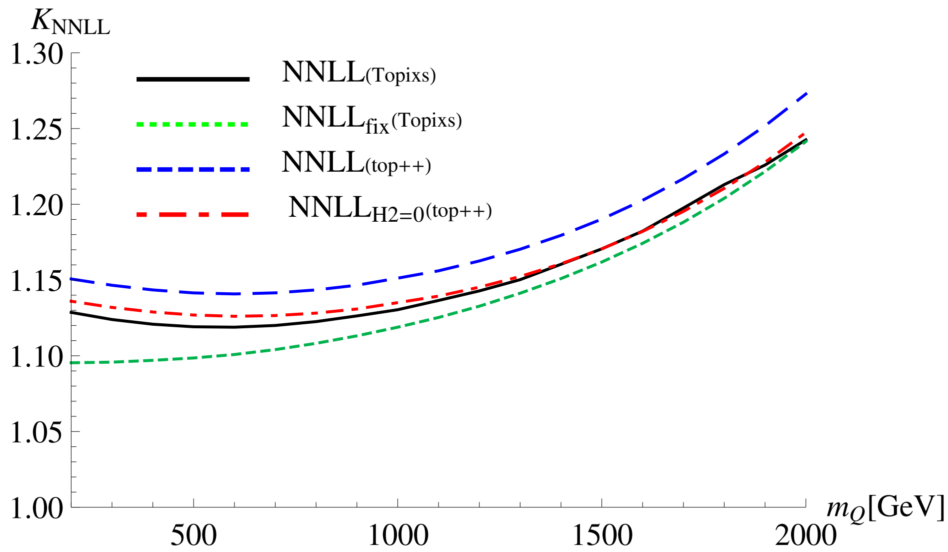
Estimate of $C^{(3)}$ by varying hard/soft scales: $^{+6.1}_{-2.8}\mathrm{pb}$

Estimate $J^{S,(3)}_{L2,R}$ from e^-e^+ results: effect $< 1\mathrm{pb}$

Summary and outlook

- **Experimental accuracy** of $\sigma_{t\bar{t}}$ comparable to NNLO+NNLL prediction
- construct partial N^{3}LL; approximate N^{3}LO
  - unknown: 3-loop anomalous dimensions, logarithmic terms in N^{3}LO Coulomb Green function
  - complete determination $\alpha_s^3 \ln^{2,1} \beta$ terms requires control over kinematically suppressed contributions
- **N^{3}LO$_{\rm app}$ results**
  - strong cancellations in integral over β
  - $\Rightarrow$ corrections sensitive to power suppressed effects
  - estimate size of possible N^{3}LO corrections $\sim 5\text{pb}$ at 8TeV.
- **Outlook**
  - estimate of uncertainties, μ_f dependence
  - implement NNLL'/N^{3}LL$_{\rm part}$ resummation.

Heavy Quarks as test case for resummation methods



$(K_{\mathrm{NNLL}} = \sigma^{\mathrm{NNLL}}/\sigma^{\mathrm{NLO}}$,

LHC $\sqrt{s} = 8$ TeV)

NNLL: momentum-space, running $\mu_s = 2m_Q\beta^2$ (`Topixs` default)

NNLL$_{\text{fix}}$: momentum-space, fixed μ_s (`Topixs`)

NNLL (top++): Mellin-space (Cacciari et al. 11; Czakon/Mitov 11-13)

NNLL$_{H_2=0}$ (top++): Mellin-space, two-loop constant term set to zero

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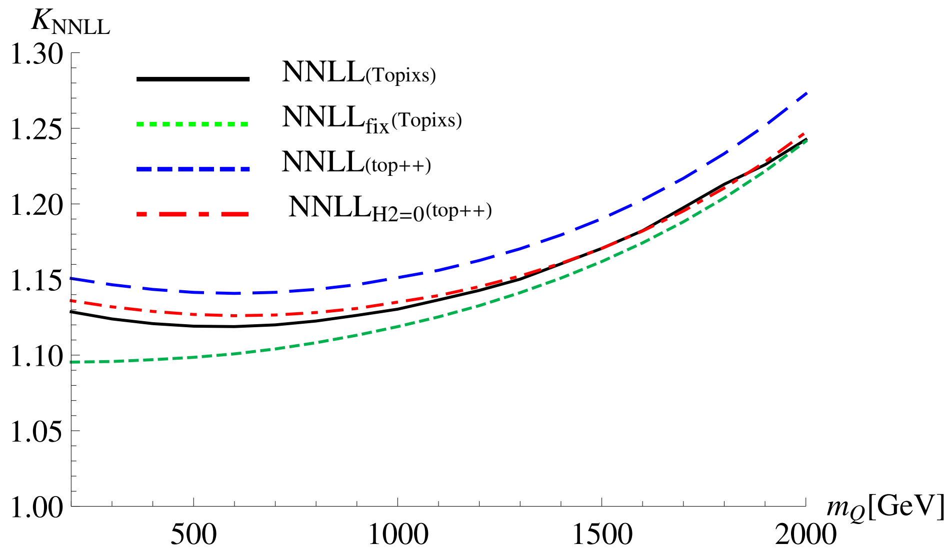


$(K_{\rm NNLL} = \sigma^{\rm NNLL}/\sigma^{\rm NLO}$,

LHC $\sqrt{s} = 8$ TeV)

⇒ resummation methods agree well for larger masses

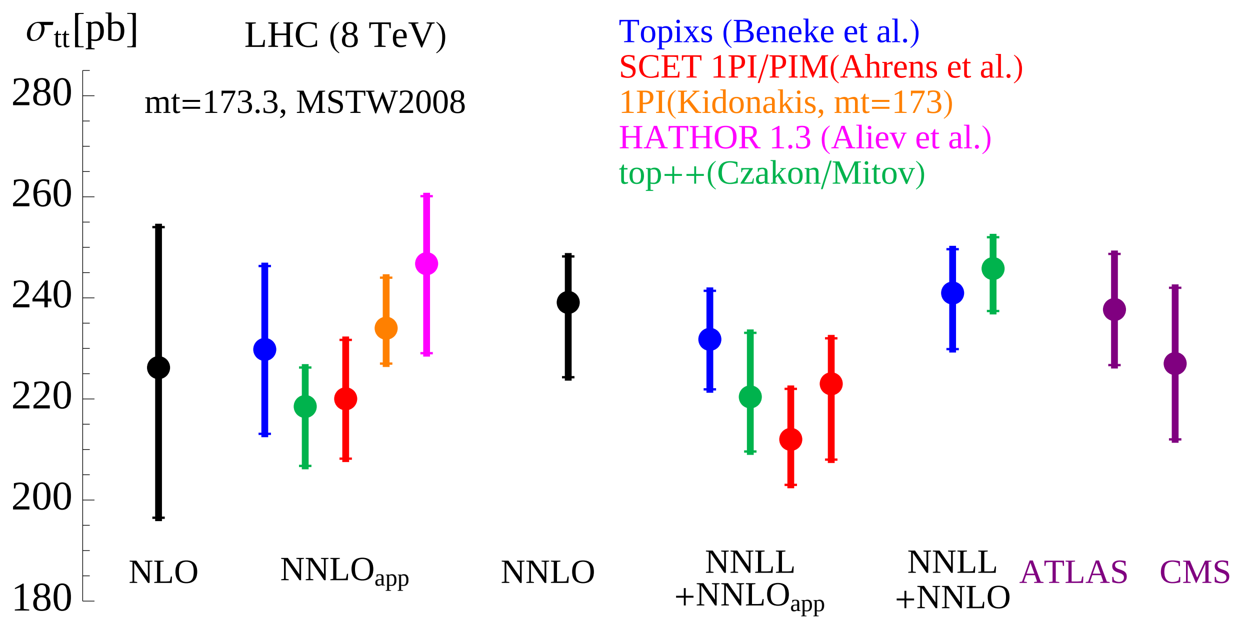
- differences at m_t: estimate of resummation ambiguities
- main difference: treatment of $H_2 \Rightarrow \alpha_s^3 \log\beta^2$ terms (NNLL')

⇒ **Upgrade** `Topixs` to NNLL'/partial N³LL

- First step: expansion to N³LO

Comparison of different approximations (excluding PDF+α_s uncertainties)

- $\pm 5\%$ scale uncertainty at NNLO; ± 3–4% at NNLL



Scale-dependence of approximate N^{3}LO cross section

$$\hat{\sigma}^{(3),\text{app}}_{pp',R}(\beta,\mu_f) = \hat{\sigma}^{(0)}_{pp',R}\left(\frac{\alpha_s(\mu_f)}{4\pi}\right)^3 \sum_{m=0}^{3} f^{(3,m)}_{pp'(R)} \ln^m\left(\frac{\mu_f}{m_t}\right)$$

Obtained in two ways:

- Expansion of resummation formula
- Direct computation in $x \to 1$ limit of splitting functions

$$f^{(3,3)}_{pp} = \frac{1}{3}\left[8\beta^{(0)} f^{(2,2)}_{pp} - 2\overline{f}^{(2,2)}_{pp} \otimes P^{(0)}_{p/p}\right]$$

$$f^{(3,2)}_{pp} = 4\beta^{(0)} f^{(2,1)}_{pp} + 3\beta^{(1)} f^{(1,1)}_{pp} - \overline{f}^{(2,1)}_{pp} \otimes P^{(0)}_{p/p} - \overline{f}^{(1,1)}_{pp} \otimes P^{(1)}_{p/p}$$

$$f^{(3,1)}_{pp} = 8\beta^{(0)} f^{(2,0)}_{pp} + 6\beta^{(1)} f^{(1,0)}_{pp} + 4\beta^{(2)} f^{(0,0)}_{pp} - \overline{f}^{(2,0)}_{pp} \otimes P^{(0)}_{p/p} - \overline{f}^{(1,0)}_{pp} \otimes P^{(1)}_{p/p} - \overline{f}^{(0,0)}_{pp} \otimes P^{(2)}_{p/p}$$

with $P_{p/\tilde{p}}(x) \approx \left(2\Gamma^r_{\text{cusp}}(\alpha_s)\frac{1}{[1-x]_+} + 2\gamma^{\phi,r}(\alpha_s)\delta(1-x)\right)\delta_{p\tilde{p}}$

$$(\overline{f}(z) \otimes P) = \frac{1}{\sqrt{1-z}}\int_z^1 \frac{dx}{x}\sqrt{1-\frac{z}{x}}\, f\left(\frac{z}{x}\right)P(x)$$

Factorization scale dependence

IR singularities of amplitude determine RGE of hard function

$$\frac{d}{d\ln\mu} H^{R,S}_{pp'}(\mu) = \left(\gamma_{\text{cusp}}(C_r + C_{r'}) \ln\left(\frac{4m_t^2}{\mu^2}\right) + 2(\gamma^p + \gamma^{p'} + \gamma^R_{H,s}) + \gamma^{R,S}_J\right) H^R_{pp'}(\mu).$$

RGE for soft function

$$\frac{d}{d\log\mu} W_i^{R_\alpha}(z^0,\mu) = \left(2\gamma_{\text{cusp}}(C_r + C_{r'}) \log\left(\frac{iz_0\bar{\mu}}{2}\right) - 2(\gamma^{R_\alpha}_{H.s} + \gamma^r_s + \gamma^{r'}_s\right) W_i^{R_\alpha}(z^0,\mu)$$

Known input

- 4-loop γ_{cusp} (Moch/Ruijl/Ueda/Vermaseren/Vogt 17; not needed for N^{3}LO$_{\text{app}}$)
- 3-loop γ^r_s γ^p(Moch, Vermaseren, Vogt 04/05); 2-loop $\gamma^{R_\alpha}_{H.s}$ (Beneke/Falgari/CS; Czakon/Mitov/Sterman 09)
- 2-loop $\gamma^{R,S}_J$ from IR divergences from potential factorization

Missing for N^{3}LL

- 3-loop $\gamma^{R_\alpha}_{H.s}$ (Massless result: Almelid/Duhr/Gardi 15)
- 3-loop $\gamma^{R,S}_J$ (colour singlet: Kniehl et al. 02/Hoang 03)

- No 3-loop Coulomb correction $\sim \alpha_s^3/\beta^3$ for $\Gamma_t \to 0$

  Careful treatment in distributional sense: (Beneke/Ruiz-Femenia 16)

$$\Delta J^{S(3)}_{R,\mathrm{LO}}(E) = \alpha_3 \frac{m_t^3}{8} \zeta_3 \, \delta(E)$$

  Small correction to cross section: $\Delta\sigma = 0.18$ pb at 8 TeV.

- Sub-leading soft corrections to DY/Higgs production:

  (Krämer/Laenen/Spira 96; Laenen et al. 10)

$$\left[\frac{\ln(1-x)}{1-x}\right]_+ \to \left[\frac{\ln(1-x)}{1-x}\right]_+ - \ln(1-x)$$

  enhancement by second Coulomb correction $\Rightarrow \sim \alpha_s^3 \ln\beta$ effect

  Numerical effect $< 1\mathrm{pb}$ at $8\mathrm{TeV}$