The top-quark mass: uncertainties due to b-quark fragmentation

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1. **Introduction**
2. **Resummed calculations and Monte Carlo codes for b-fragmentation in top events**
3. **Sensitivity of top mass to shower and hadronization parameters**
4. **Conclusions**

Frascati workshop on 'Top mass: challenges in definition and determination',
https://agenda.infn.it/conferenceDisplay.py?confId=9202

TOP 2015 and 2016 workshops: http://top2015.infn.it/, https://indico.cern.ch/event/486433/

G.C., PoS TOP2015 (2016) 037, arXiv:1511.08429

Work in progress with R.Franceschini and D.Kim

Reliable description of multiple radiation in top production and decay and b-fragmentation is fundamental in the measurement of the top properties, e.g. m_t

b-fragmentation enters in JES and MC uncertainties, typically evaluated by comparing two codes (HERWIG vs PYTHIA) or two different models in a given generator

World average: $\Delta m_t(\mathrm{tot}) \simeq 760$ MeV; $\Delta m_t(\mathrm{JES}) \simeq 250$ eV, $\Delta m_t(\mathrm{MC}) \simeq 380$ MeV

Several calculations and tools are available for bottom fragmentation in top decays

Perturbative-fragmentation approach: factorization of a massless hard scattering (being $m_b \ll m_t$) and universal perturbative fragmentation function (Mele-Nason,'91)

DGLAP evolution to resum (collinear) $\alpha_S^n \ln^k(m_t^2/m_b^2)$

NLO+NLL calculation for b and B-hadron energy spectra in top decays (NWA), including threshold resummation. Hadron corrections from e^+e^- data (G.C., M.Cacciari and A.Mitov '02)

Could be extended to NNLO+NNLL thanks to NNLO coefficient functions (Bruchenseifer, Caola, Melnikov '13), perturbative fragmentation function (Melnikov, Mitov '14) and time-like splitting functions (Moch, Mitov, Vogt '06)

NLO distributions with collinear resummation, NWA, hadronization as above (Biswas, Melnikov and Schulze, '10)

NNLO+NNLL fragmentation in SCET for $e^+e^- \to b\bar{b}$ Fickinger, Fleming, Kim and Mereghetti '16

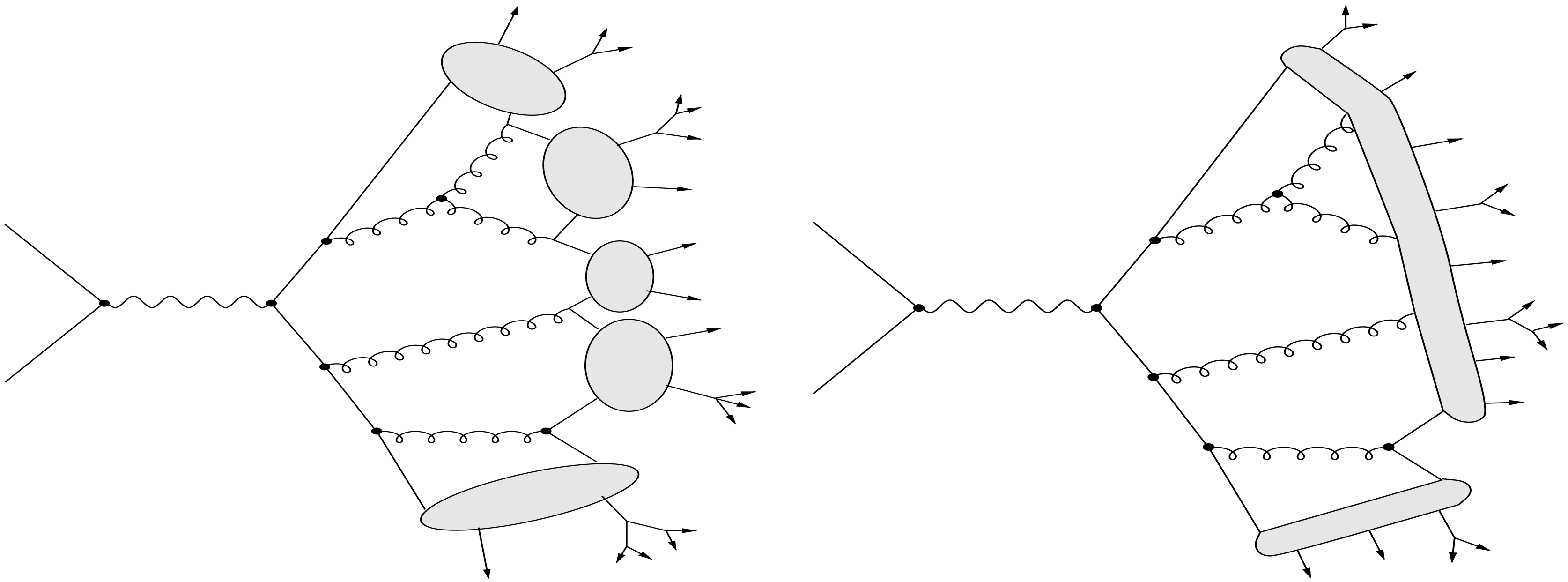
Standard generators (HERWIG, PYTHIA): LO matrix element, parton showers LL+(N)LL, tree-level matrix-element matching, string/cluster models for hadronization

Late progress in NLO+shower generators:

aMC@NLO: NLO single top, not yet $t\bar{t}$, though MadSpin includes some off-shell effects

POWHEG: NLO+PS for $t\bar{t}$ production and decay using OpenLoops, including non-resonant diagrams and interference top production/decay (Talk by T.Jezo)

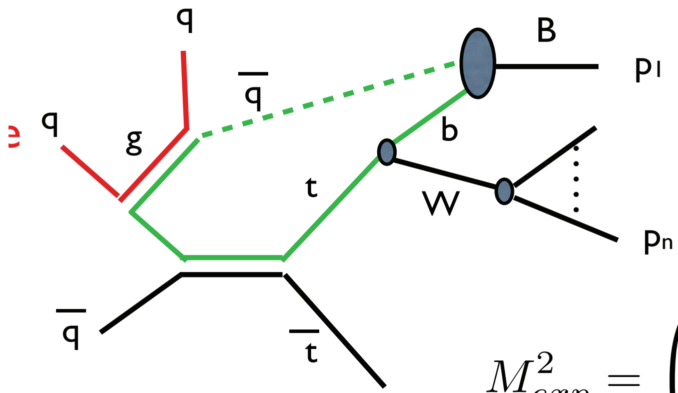
aMC@NLO and POWHEG rely on HERWIG and PYTHIA for shower and hadronization



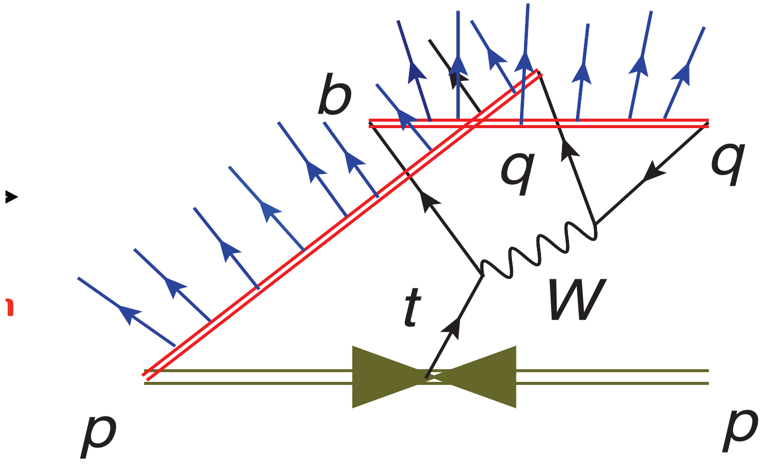
Left: HERWIG cluster model; Right: PYTHIA string model

Figure from Ellis, Stirling, Webber, 'QCD and Collider Physics'

b-fragmentation and colour reconnection (uncertainty on $m_t^{\rm pole}$) $\Delta m(\rm CR) \simeq 0.3$ GeV

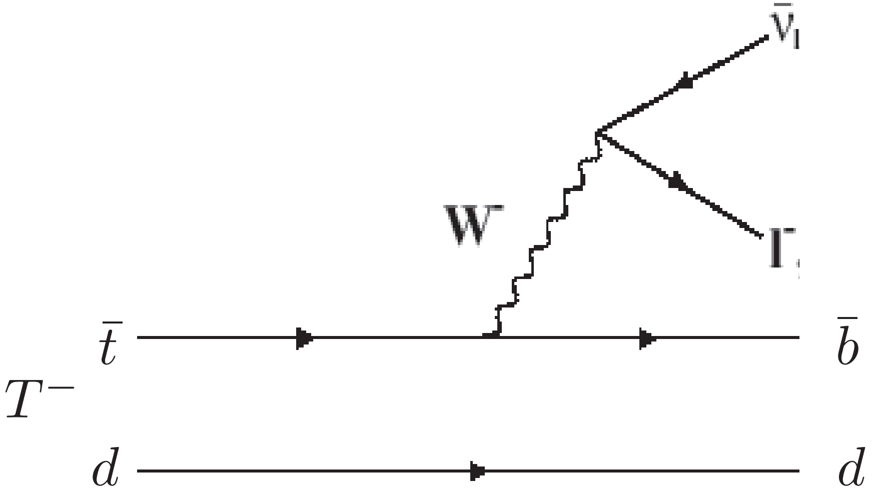


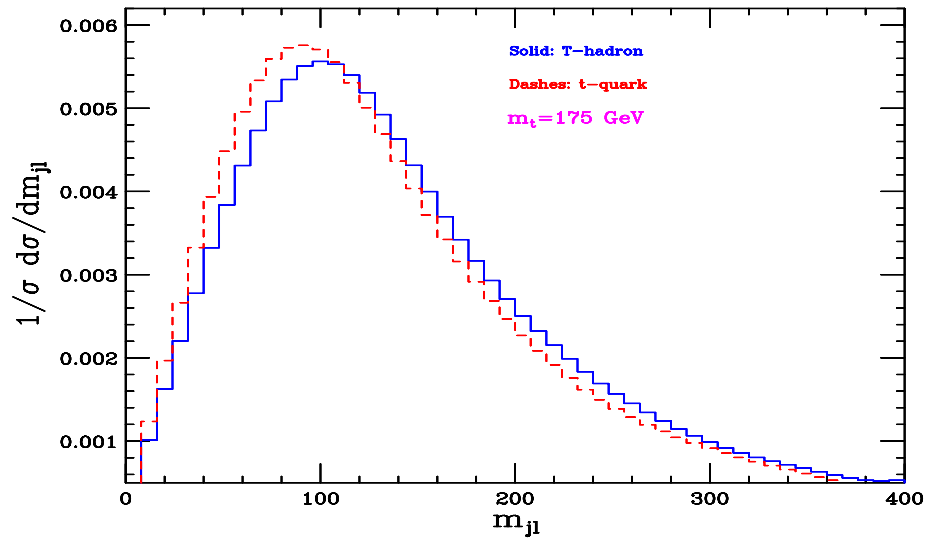
$$M^2_{exp} = \left(\sum_{i=1,\dots,n} p_i\right)^2$$



Left: M.Mangano, TOP'13 (HERWIG cluster model), Right: S.Argyropoulos, LNF'15 workshop (PYTHIA string model)

Simulation of fictitious T-hadrons (HERWIG 6, preliminary) (G.C. and M.Mangano, in progress)





Useful to study colour reconnection and uncertainty on measured m_t vs pole mass

$m_T^{\rm reco} = m_t^{\rm reco} + \Delta m^{\rm reco}$ with $m_T = m_t^{\rm pole} + m_q + \Delta m_{t,T}$ (lattice, NRQCD, etc.)

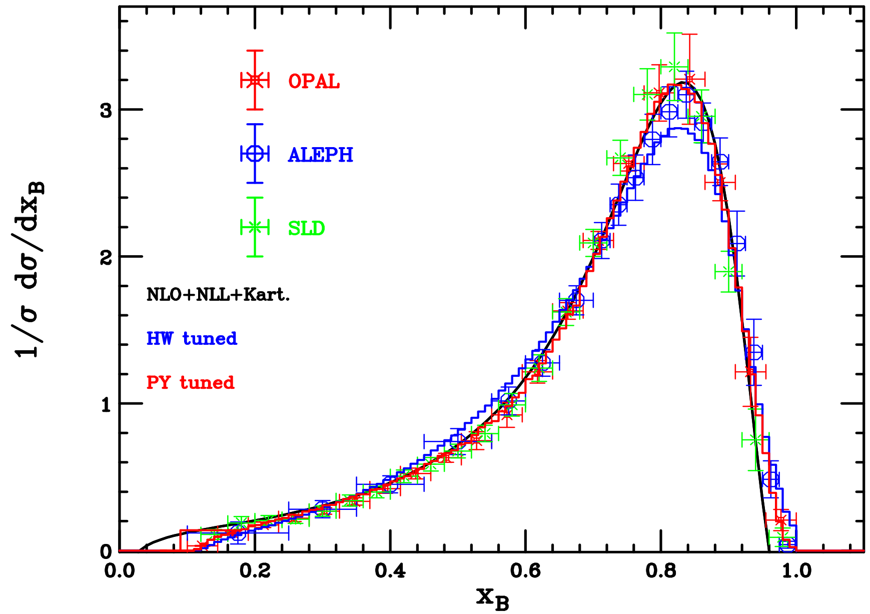
Tuning HERWIG and PYTHIA to e^+e^- data (G.C. and Drollinger '05, G.C. and F.Mescia '10)

HERWIG	PYTHIA
CLSMR(2) $=$ 0.3 (0.0)	PARJ(41) $=$ 0.85 (0.30)
DECWT $=$ 0.7 (1.0)	PARJ(42) $=$ 1.03 (0.58)
CLPOW $=$ 2.1 (2.0)	PARJ(46) $=$ 0.85 (1.00)
PSPLT(2) $=$ 0.33 (1.00)	
$\chi^2/\mathrm{dof} = 222.4/61$ (739.4/61)	$\chi^2/\mathrm{dof} = 45.7/61$ (467.9/61)

Lund/Bowler fragmentation function (PYTHIA): $f_B(z) \sim (1-z)^a \exp(-bm_T^2/z)/z^{1+brm_b^2}$

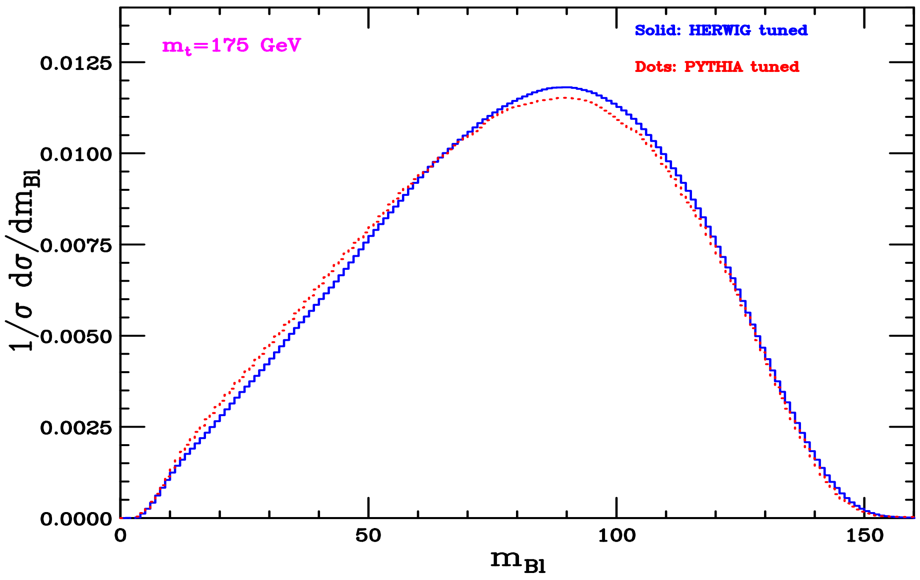
HERWIG parameters describe gaussian smearing (CLSMR), baryon/meson (CLPOW) and decuplet/octet (DECWT) ratios, mass spectrum of b-like clusters (PSPLT)

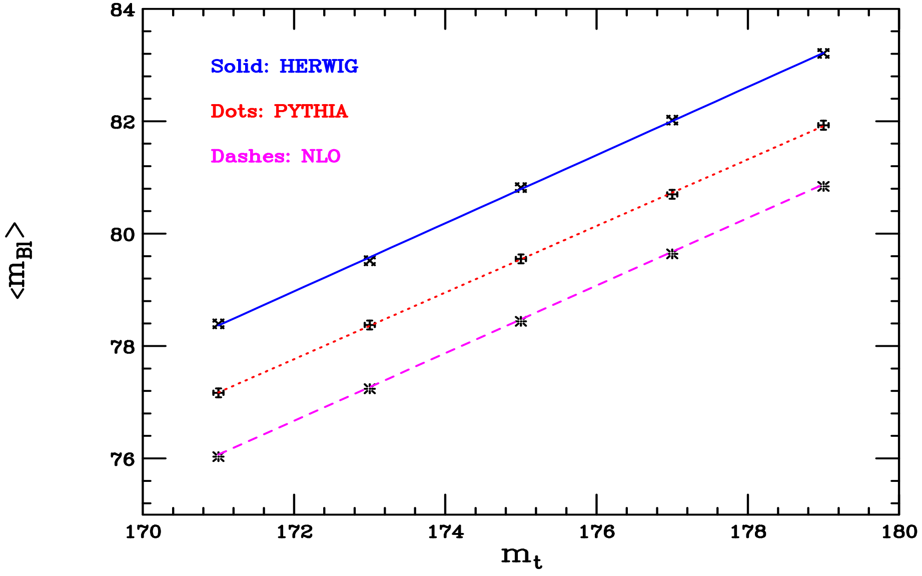
Our PYTHIA tuning in ATLAS jet-energy measurement and m_t world average



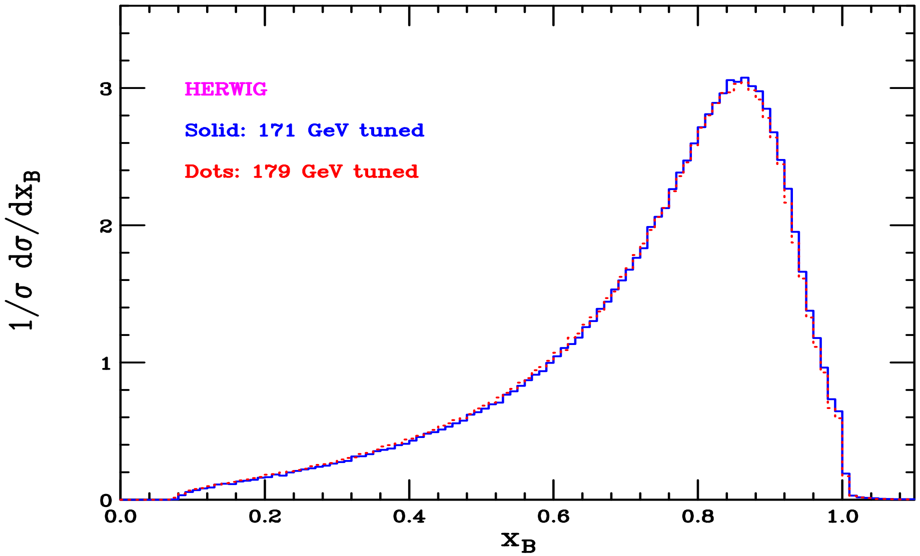
NLO+NLL calculation uses: $D(x) = Nx^\alpha(1-x)$ and fits α to data

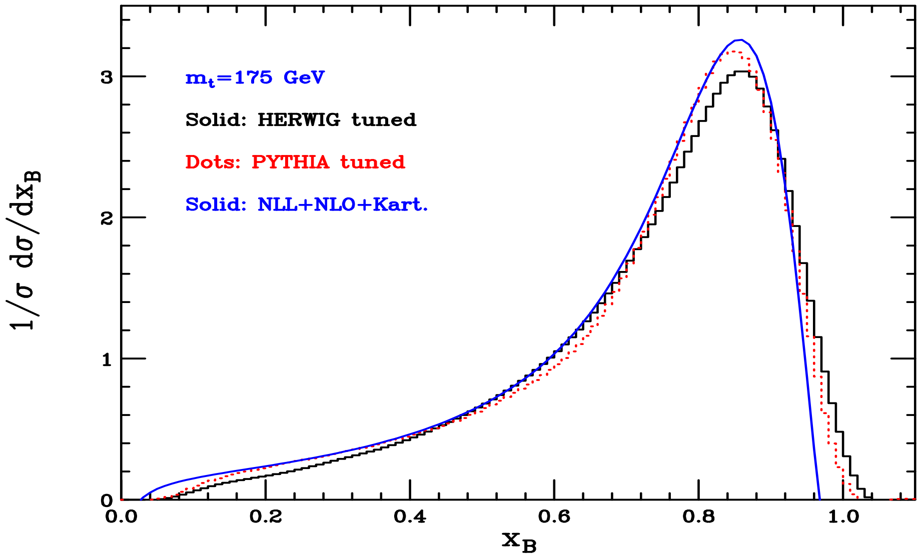
$m_{B\ell}$ according to tuned HERWIG and PYTHIA and NLO





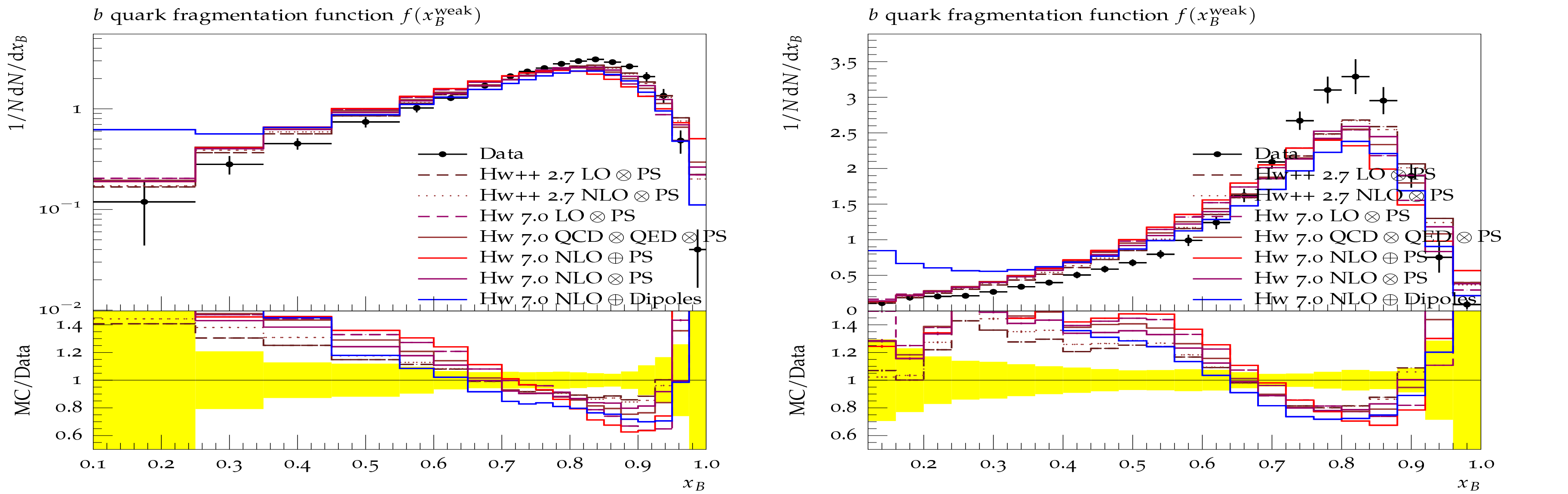
B-energy fraction $x_B = (2p_B \cdot p_t)/m_t^2$: mild dependence on m_t, but hard to measure





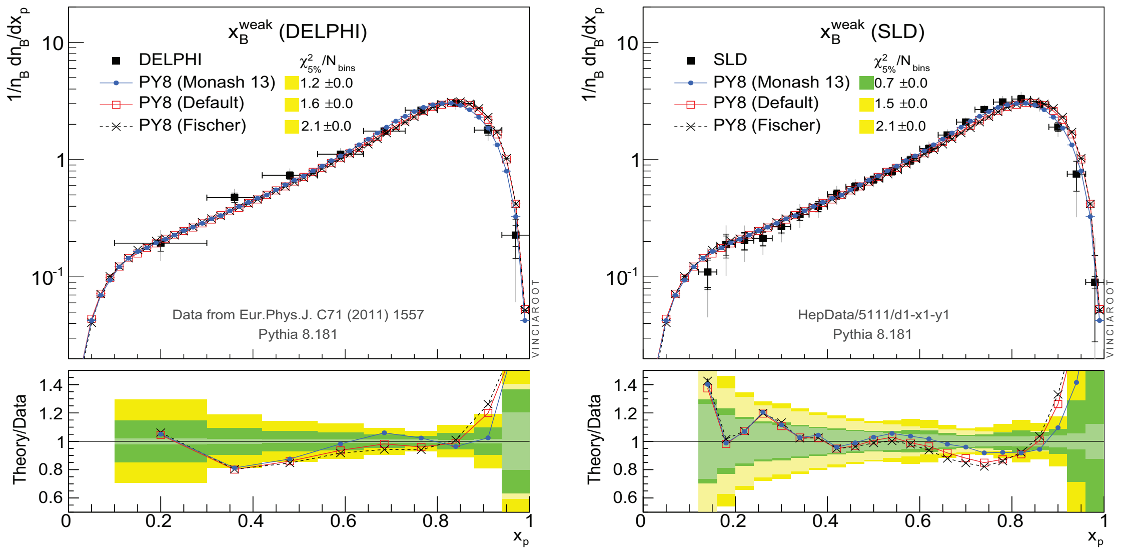
NLO+showers for top decays or C++ codes may shed light on $m_{B\ell}$ and x_B discrepancies

Results with HERWIG 7 (from https://herwig.hepforge.org/plots/herwig7.0/)



Left: ALEPH data; Right: SLD data

PYTHIA 8.1 with Monash tuning for b-fragmentation (Skands, Carrazza, Rojo, '14)

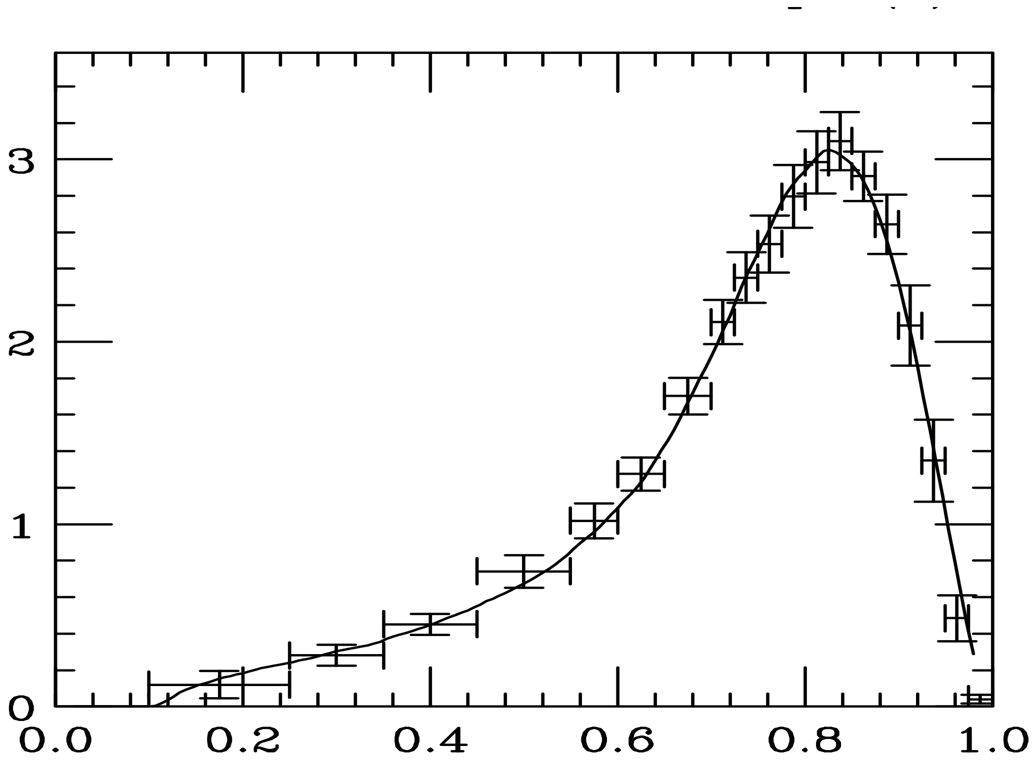


Preliminary results with POWHEG+PYTHIA 8 (E.Bagnaschi)

POWHEG has a private version with $e^+e^- \to q\bar{q}$ at NLO

Though it should be consistent with HERWIG and PYTHIA with matrix-element corrections, it may be worth to retune

Tuning to ALEPH data: $a = 0.8 \pm 0.19$; $b = 0.85 \pm 0.17$; $r = 0.85 \pm 0.02$; $\chi^2 \sim 10^{-3}$



In progress: extension to top decays and comparison with aMC@NLO and differences between of HERWIG and PYTHIA showers

Novel investigation on fragmentation uncertainties and *in situ* calibration of hadronization models (G.C., R.Franceschini and D.Kim)

Most m_t methods (template, $m_{b\ell}$, E_b-peak, endpoint, etc.) rely on b-jets: JES error

Confronting b-jets with B-hadrons: JES $\Leftrightarrow$ hadronization uncertainty:
$(m_{j\ell}, E_j, p_{T,j}) \Leftrightarrow (m_{B\ell}, E_B, p_{T,B})$

Sensitivities of observables O and top mass to Monte Carlo parameters θ:

$$\frac{dm_t}{m_t} = \Delta_O^m \frac{d\langle O\rangle}{\langle O\rangle} \quad ; \quad \frac{d\langle O\rangle}{\langle O\rangle} = \Delta_\theta^O \frac{d\theta}{\theta}$$

Precision of 0.3% on m_t ($\Delta m_t \leq 500$ MeV) from measurement of $\langle O\rangle$:

$$\frac{dm_t}{m_t} < 0.003 \ \Rightarrow \ \Delta_O^m \frac{d\langle O\rangle}{\langle O\rangle} < 0.003 \ \Rightarrow \ \Delta_O^m \, \Delta_\theta^O \, \frac{d\theta}{\theta} < 0.003$$

HERWIG 6: vary CLPOW, PSPLT, CLMSR, CLMAX, $\Lambda_{\rm MC} \simeq \exp(K/4\pi\beta_0)\Lambda_{\overline{\rm MS}}$ [CMW scheme for NLLs in the shower at large x]; shower cutoffs [VGCUT and VQCUT]; bottom and gluon effective masses [RMASS(5) and RMASS(13)]

PYTHIA 8: string model a, b, r_B parameters, $p_{T,\min}$, $\alpha_{S,\rm FSR}$

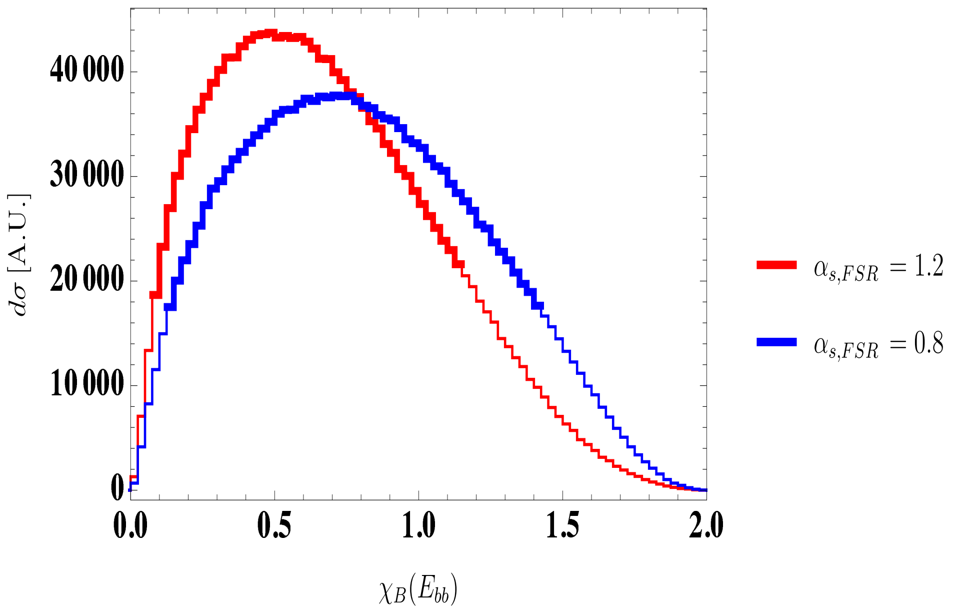
All parameters varied by $\sim 10\%$ around default values

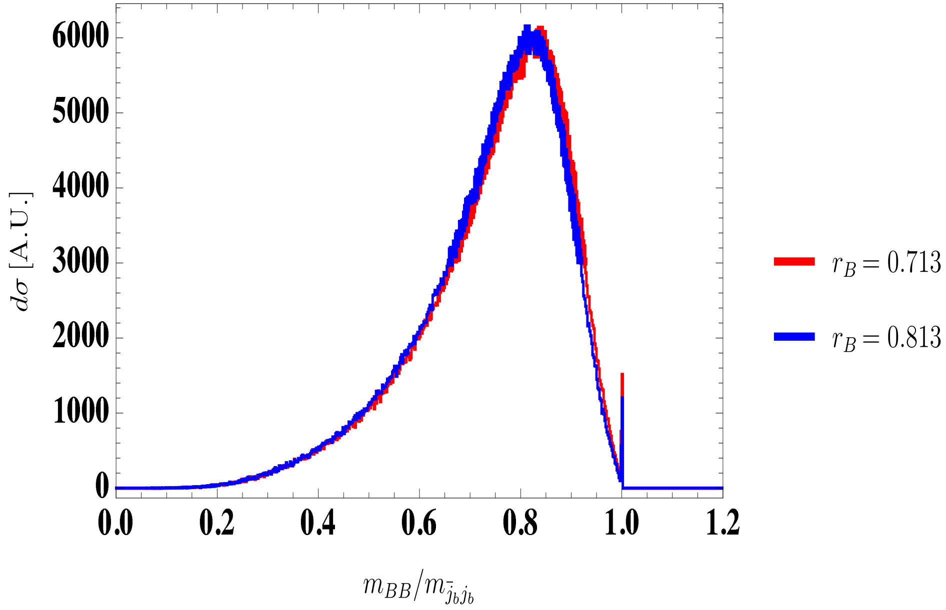
Calibration observables: quantities which may potentially exhibit mild dependence on m_t can be used to tune fragmentation parameters

$p_{T,B}/p_{T,j}$; $\rho(r) = \frac{1}{E_j \times \Delta R} \sum_{\rm tracks} E({\rm tracks})\Theta(|r - \Delta R|)$ (radial energy density);

$\Delta\phi(j\bar{j})$; $\Delta R(j\bar{j})$; $\Delta\phi(B\bar{B})$; $\Delta\phi(j\bar{j})$; $m_{B\bar{B}}/m_{j\bar{j}}$

$\chi_B(X_B) = 2E_B/X_B$, with $X_B = m_{j\bar{j}}$, $|p_{T,j}| + |p_{T,\bar{j}}|$ $E_j + E_{\bar{j}}$, $\sqrt{s^{j\bar{j}}_{\rm min}}$





Left: $\chi_B = E_B/(E_j + E_{\bar{j}})$ shows remarkable dependence on $\alpha_{S,\rm FSR}$

Right: $m_{B\bar{B}}/m_{j\bar{j}}$ exhibits mild dependence on r_B

Mellin moments: $\mathcal{M}_N = \frac{1}{x_{\rm max} - x_{\rm min}} \int_{x_{\rm min}}^{x_{\rm max}} dx \; x^{N-1} f(x)$; $\mathcal{M}_1 = \langle f \rangle$

$x_{\rm min}$ and $x_{\rm max}$ such that full width at half maximum (FWHM)

Sensitivity of calibration observables to top mass and PYTHIA parameters

$\mathcal{M}_1$	$\Delta_m^{(\mathcal{M}_1)}$	$\Delta_\theta^{(\mathcal{M}_1)}$					
		$\alpha_{s,FSR}$	m_b	$p_{T,\min}$	a	b	r_B
$\rho(r)$	-0.069(5)	0.52(4)	0.18(1)	-0.100(2)	0.033(3)	-0.055(8)	0.07(1)
$p_{T,B}/p_{T,j}$	-0.020(1)	-0.043(2)	-0.04(1)	0.013(7)	-0.011(1)	0.018(1)	-0.022(1)
m_j	0.28(1)	0.18(1)	0.011(3)	-0.018(2)	-0.0008(20)	0.0017(1)	-0.0004(40)
$\chi_B(\sqrt{s_{\min,b\bar{b}}})$	-0.045(2)	-0.24(2)	-0.072(1)	0.021(7)	-0.009(1)	0.022(1)	-0.025(6)
$\chi_B\left(E_j + E_{\bar{j}}\right)$	-0.10(1)	-0.30(2)	-0.10(1)	0.028(1)	-0.011(1)	0.029(1)	-0.033(5)
$\chi_B(m_{j\bar{j}})$	-0.19(3)	-0.50(5)	-0.18(1)	0.06(1)	-0.032(6)	0.05(2)	-0.06(5)
$\chi_B\left(\lvert p_{T,j}\rvert + \lvert p_{T,\bar{j}}\rvert\right)$	-0.24(3)	-0.49(6)	-0.19(1)	0.051(1)	-0.030(1)	0.05(1)	-0.024(2)
$\Delta\phi(j\bar{j})$	-0.011(3)	0.0036(1)	-0.0001(20)	-0.0006(6)	0.0003(4)	0.002(1)	0.003(7)
$\Delta R(j\bar{j})$	-0.007(2)	0.001(1)	-0.0007(4)	-0.0001(3)	0.0004(20)	0.0012(3)	0.0030(3)
$\Delta\phi(B\bar{B})$	-0.012(3)	0.0015(7)	-0.0005(30)	-0.0003(7)	-0.0001(30)	0.002(1)	0.0036(5)
$\Delta R(B\bar{B})$	-0.007(2)	0.0003(3)	-0.0006(5)	0.00002(40)	0.0002(20)	0.0008(2)	0.003(2)
$\lvert\Delta\phi(B\bar{B}) - \Delta\phi(j\bar{j})\rvert$	0.03(1)	0.66(1)	0.14(1)	-0.091(6)	0.013(5)	-0.020(8)	0.05(4)
$\lvert\Delta R(B\bar{B}) - \Delta R(j\bar{j})\rvert$	0.08(1)	0.88(1)	0.14(1)	-0.086(5)	0.015(5)	-0.020(5)	0.03(1)
$m_{B\bar{B}}/m_{j\bar{j}}$	-0.023(1)	-0.053(1)	-0.043(1)	0.013(1)	-0.008(1)	0.017(1)	-0.020(5)

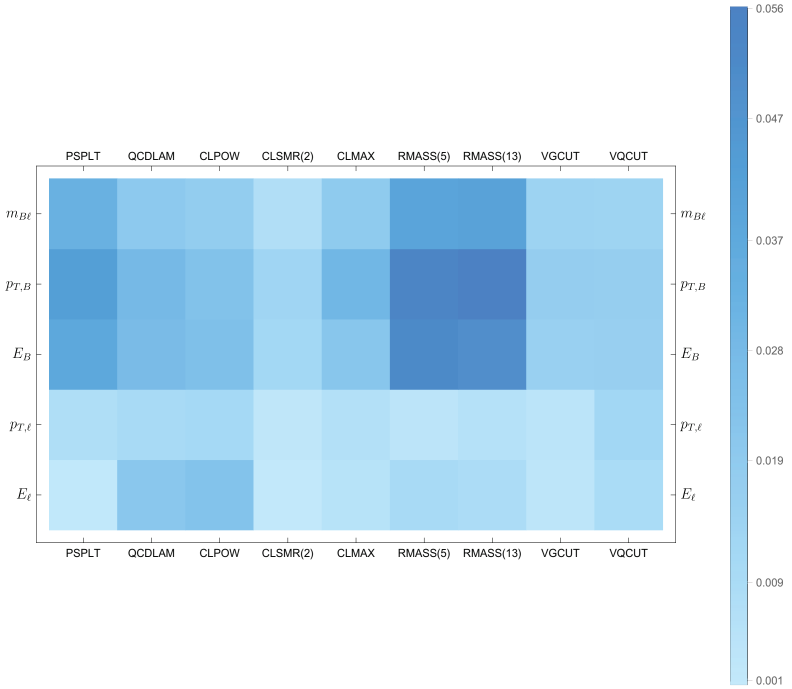
With the exception of m_j and χ_B, all observables have $\Delta_m^{\langle O\rangle} \leq \mathcal{O}(10^{-2})$

$\rho(r)$ and χ_B variables exhibit the largest dependence on PYTHIA parameters; mild dependence of ΔR, $\Delta\phi$ and B/j ratios

Results on m_t-determination sensitivity to Monte Carlo parameters

HERWIG: FHWM Mellin moments of $m_{B\ell}$, $p_{T,B}$, E_B, $p_{T,\ell}$, E_ℓ

Mellin-1	$\Delta_{m_t}^{(\mathcal{M}_1)}$	$\frac{dm_t}{d\mathcal{M}_1}$	$\Delta_\theta^{(\mathcal{M}_1)}$								
			PSPLT	QCDLAM	CLPOW	CLSMR(2)	CLMAX	RMASS(5)	RMASS(13)	VGCUT	VQCUT
$m_{B\ell,\mathrm{true}}$	1.2	1.63	0.019(1)	-0.0065(1)	-0.003(1)	0.0011(8)	-0.004(1)	0.03(4)	0.032(2)	0.0017(3)	-0.002(1)
$p_{T,B}$	1.2	3.3	0.033(3)	-0.013(4)	-0.007(3)	0.0017(2)	-0.008(4)	0.053(2)	0.0563(2)	0.0031(5)	-0.003(2)
E_B	1.1	2.4	0.028(2)	-0.012(1)	-0.0075(2)	0.0016(3)	-0.005(5)	0.049(3)	0.049(3)	0.003(8)	-0.003(2)
$p_{T\ell}$			0.002(2)	0.001(2)	-0.002(4)	0.0001(2)	-0.0007(4)	-0.0005(3)	-0.0005(2)	-0.0003(4)	0.002(1)
E_ℓ			-0.0005(4)	-0.006(3)	-0.003(3)	-0.00002(2)	-0.0002(5)	0.001(4)	0.001(2)	-0.0002(5)	0.001(2)



PSPLT, Λ_{MC}, m_b and m_g to be determined at $\mathcal{O}(10\%)$ for a 300 MeV accuracy on m_t

Other parameters: order of magnitude is enough

Sensitivity of m_t to PYTHIA parameters

Mellin moments of E_B, $E_B + E_{\bar{B}}$, $p_{T,B}$, $p_{T,B} + p_{T,\bar{B}}$, $m_{B\ell}$, m_{T2}

$$m_{T2} = \min_{\not{p}_1 + \not{p}_2 = \not{p}_T} \left\{ \max \left[m_T^2(p_{T,\ell^-}, \not{p}_1), m_T^2(p_{T,\ell^+}, \not{p}_2) \right] \right\}$$

Mellin-1	$\Delta_{m_t}^{(\mathcal{M}_1)}$	$\frac{dm_t}{d\mathcal{M}_1}$	$\Delta_\theta^{(m_t)}$					
			$\alpha_{s,FSR}$	$p_{T,\min}$	recoil	r_B	a	b
E_B	1.1	2.4	0.43	0.019	0.028	0.039	0.020	0.039
$E_B + E_B$	1.2	0.99	0.42	0.019	0.032	0.046	0.023	0.034
$p_{T,B}$	1.2	3.3	0.43	0.021	0.027	0.043	0.022	0.042
$p_{T,B} + p_{T,B}$	1.2	1.47	0.36	0.017	0.024	0.042	0.016	0.044
$m_{B\ell,\min}$	1.0	2.6	0.26	0.011	0.016	0.041	0.011	0.031
$m_{B\ell,\mathrm{true}}$	1.2	1.63	0.24	0.008	0.013	0.031	0.009	0.022
$m_{T2,B\ell,\mathrm{true}}^{(\mathrm{mET})}$	0.97	1.51	0.24	0.012	0.012	0.034	0.013	0.032
$m_{T2,B\ell,\min}^{(\mathrm{mET})}$	0.98	1.42	0.23	0.011	0.012	0.031	0.012	0.03
$m_{T2,B\ell,\min,\perp}^{(\mathrm{mET})}$		1.55	0.24	0.01	0.011	0.038	0.011	0.03
$m_{T2,B\ell,\mathrm{true}}^{(\mathrm{ISR})}$	0.99	1.45	0.21	0.007	0.01	0.022	0.008	0.022
$m_{T2,B\ell,\min}^{(\mathrm{ISR})}$	0.97	1.35	0.2	0.008	0.013	0.024	0.01	0.027
$m_{T2,B\ell,\min,\perp}^{(\mathrm{ISR})}$		1.52	0.22	0.01	0.01	0.03	0.013	0.03

Final-state strong coupling constant needed at 1%, other parameters at 10%

Stronger parameter dependence in PYTHIA than in HERWIG on m_t observables

In progress: checks with POWHEG and aMC@NLO to possibly decrease Δ's

Shape analysis: beyond Mellin moments (average values) peaks and endpoints ($\grave{O}$) may be useful to determine m_t

$\mathcal{M}_1$	$\Delta_{m_t}^{(\mathcal{M}_1)}$	$\frac{dm_t}{d\mathcal{M}_1}$	$\Delta_\theta^{(\mathcal{M}_1)}$					
			$\alpha_{s,FSR}$	$p_{T,\text{min}}$	recoil	r_B	a	b
$E_{B,\text{peak}}$	1.44	2.8	0.46	0.018	0.032	0.057	0.018	0.042
$\grave{m}_{B\ell,\text{min}}$	1.28	0.89	0.0021	$< 10^{-3}$	$< 10^{-3}$	0.0038	0.0021	0.0017
$\grave{m}_{B\ell,\text{true}}$	1.28	0.89	0.005	$< 10^{-3}$	$< 10^{-3}$	0.0035	0.0014	$< 10^{-3}$
$\grave{m}^{(\text{mET})}_{T2,B\ell,\text{true}}$	1.01	1	$< 10^{-3}$	$< 10^{-3}$	$< 10^{-3}$	0.0029	0.0015	0.0014
$\grave{m}^{(\text{mET})}_{T2,B\ell,\text{min}}$	1	1	0.0016	$< 10^{-3}$	$< 10^{-3}$	0.0035	0.0012	$< 10^{-3}$
$\grave{m}^{(\text{mET})}_{T2,B\ell,\text{min},\perp}$		0.96	0.016	$< 10^{-3}$	$< 10^{-3}$	0.0071	0.0019	0.0017
$\grave{m}^{(\text{ISR})}_{T2,B\ell,\text{true}}$	0.99	1	0.0067	0.0015	$< 10^{-3}$	0.0042	0.0021	0.0024
$\grave{m}^{(\text{ISR})}_{T2,B\ell,\text{min}}$	0.98	1	0.0063	0.0015	$< 10^{-3}$	0.0031	0.0021	0.0023
$\grave{m}^{(\text{ISR})}_{T2,B\ell,\text{min},\perp}$		0.98	0.0056	$< 10^{-3}$	$< 10^{-3}$	0.0042	0.0016	0.0014

Energy peaks exhibit substantial dependence on shower/hadronization parameters, especially on $\alpha_{S,\mathrm{FSR}}$

Very little sensitivity of endpoints on all parameters: $\Delta_\theta^{(\mathcal{M}_1)} \leq 10^{-3} - 10^{-2}$

Conclusions and outlook

Bottom fragmentation in top decays contributes to error on top mass: JES and MC uncertainties

Predictions for top decays yielded by the different codes exhibit some discrepancies, mostly driven by unsatisfactory tunings

Novel investigation on dependence of top-mass observables on MC parameters: determination may be needed to 1% accuracy to meet 0.3% precision on m_t

Calibration observables (angular variables, hadron/jet ratios, etc.) useful to constrain parameters thanks to mild dependence on m_t

Endpoints exhibit the littlest sensitivity on shower and hadronization parameters

Extending perturbative-fragmentation formalism to NNLO+NNLL for top decays

Re-tuning PYTHIA 8 and HERWIG++ can be a valuable strategy to pursue

Tuning fragmentation parameters directly to LHC data ($t\bar{t}$, $b\bar{b}$, $Z/\gamma+b$)

Calibration and sensitivity analysis with POWHEG/aMC@NLO

Monte Carlo parameters with errors (like pdfs) would be ultimately desirable