Energy-energy correlation in electron-positron annihilation at NNLL+NNLO accuracy

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Motivation

α_S is a fundamental parameter of the SM and must be determined precisely

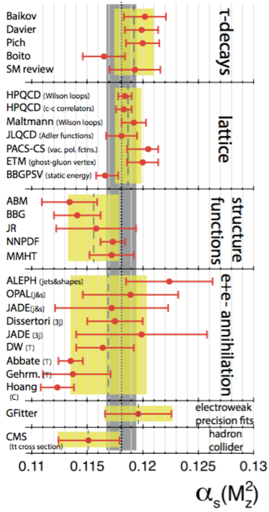
At colliders: obtained from fits to data

High precision measurements demand highly accurate theoretical predictions

One option: from 3-jet event shapes in e^+e^- collisions:

- extensively measured by multiple collaborations
- the Born contribution is proportional to α_S
- state-of-the-art theory: NNLO fixed-order and NNLL resummation (N^3LL for thrust and C-parameter)

α_S world average



Determination from e^+e^- annihilation based on

- jet rates (see also the talk by Z. Szőr)
- event shapes describing global topology (thrust, C-parameter, etc.)

Can also consider observables based on particle correlations

[*S. Bethke, Nucl. Part. Phys. Proc.* **282-284** *(2017) 149*]

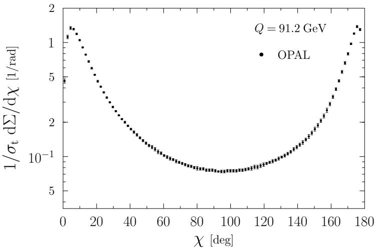
Energy-energy correlation

Energy-energy correlation is the normalized energy-weighted cross section:

$$\frac{1}{\sigma_t}\frac{d\Sigma}{d\cos\chi} = \frac{1}{\sigma_t}\int\sum_{i,j}\frac{E_i\,E_j}{Q^2}\,d\sigma_{e^+e^-\to ij+X}\,\delta(\cos\chi+\cos\theta_{ij})$$

E_i and E_j are particle energies, Q is the center-of-mass energy and $\theta_{ij} = \pi - \chi$ is the angle between the two particles

Was measured at LEP, SLC and PETRA



[*OPAL Collaboration, P.D. Acton et al. Z. Phys.* **C59** *(1993)*]

Energy-energy correlation

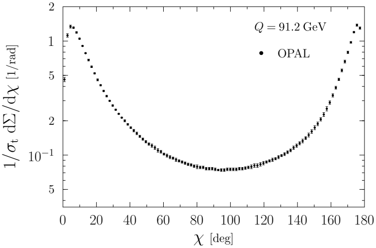
Goal: produce precise theoretical predictions for EEC

Fixed-order calculation:

- valid for medium angles
- available at NNLO accuracy

Resummation for EEC:

- for back-to-back region ($\chi \to 0$)
- computed at NNLL precision



To obtain a prediction valid on a wide kinematic range these computations must be combined through a matching procedure

Fixed-order calculation

The fixed-order expansion of EEC is

$$\left[\frac{1}{\sigma_t}\frac{d\Sigma}{d\cos\chi}\right]_{\text{(f.o.)}} = \frac{\alpha_{\rm S}}{2\pi}\frac{d\bar{A}}{d\cos\chi} + \left(\frac{\alpha_{\rm S}}{2\pi}\right)^2\frac{d\bar{B}}{d\cos\chi} + \left(\frac{\alpha_{\rm S}}{2\pi}\right)^3\frac{d\bar{C}}{d\cos\chi} + \mathcal{O}(\alpha_{\rm S}^4)$$

We performed perturbative calculations up to NNLO using the CoLoRFulNNLO scheme [*V. Del Duca, G. Somogyi, Z. Trócsányi*]

The scheme was implemented in the MCCSM package [*A. Kardos*]

Has already been tested on $H \to b\bar{b}$ and $e^+e^- \to 3$ jets

CoLoRFulNNLO scheme

Completely local subtraction for fully differential predictions at NNLO

The NNLO correction contains three separately divergent terms:

$$\sigma^{NNLO}[J] = \int_{m+2} d\sigma^{RR}_{m+2} J_{m+2} + \int_{m+1} d\sigma^{RV}_{m+1} J_{m+1} + \int_{m} d\sigma^{VV}_{m} J_{m}$$

In the $m+2$ parton line subtractions are needed to regularize 1- and 2-parton unresolved emissions:

$$\sigma^{NNLO}_{m+2} = \int_{m+2} \left\{ d\sigma^{RR}_{m+2} J_{m+2} - d\sigma^{RR,A_2}_{m+2} J_m - \left[d\sigma^{RR,A_1}_{m+2} J_{m+1} - d\sigma^{RR,A_{12}}_{m+2} J_m \right] \right\}_{d=4}$$

The $m+1$ parton line collects 1-parton emissions from the real-virtual term:

$$\sigma^{NNLO}_{m+1} = \int_{m+1} \left\{ \left(d\sigma^{RV}_{m+1} + \int_1 d\sigma^{RR,A_1}_{m+2} \right) J_{m+1} - \left[d\sigma^{RV,A_1}_{m+1} + \left(\int_1 d\sigma^{RR,A_1}_{m+2} \right)^{A_1} \right] J_m \right\}_{d=4}$$

The m parton line contains the double virtual term and integrated subtractions:

$$\sigma^{NNLO}_{m} = \int_{m} \left\{ d\sigma^{VV}_{m} + \int_2 \left[d\sigma^{RR,A_2}_{m+2} - d\sigma^{RR,A_{12}}_{m+2} \right] + \int_1 \left[d\sigma^{RV,A_1}_{m+1} + \left(d\sigma^{RR,A_1}_{m+2} \right)^{A_1} \right] \right\}_{d=4} J_m$$

CoLoRFulNNLO scheme

General features:

- fully local counterterms
  (mathematically well defined)

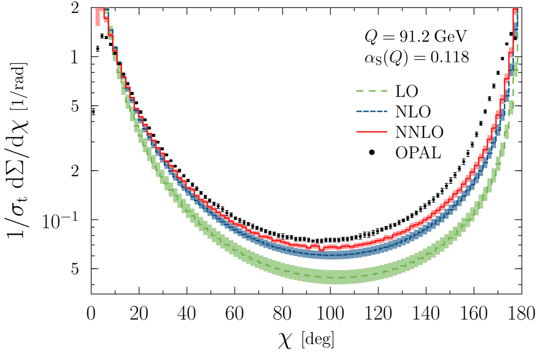
- fully differential predictions
  (with jet functions defined in four dimensions)

- explicit expressions including flavor and color
  (using color space notation)

- completely general construction
  (valid in any order of perturbation theory)

- option to constrain subtraction near singular regions (α_{max})
  (important check)

EEC at NNLO



Higher order predictions improve agreement with data for medium angles

Sizable differences remain due to hadronization and resummation corrections

In the forward ($\chi = 180^\circ$) and back-to-back ($\chi = 0^\circ$) regions fixed-order calculations diverge due to multiple soft emissions

Resummation

EEC resummation is known in the back-to-back region up to NNLL, [*D. de Florian, M. Grazzini*, (2005)]

$$\left[\frac{1}{\sigma_t}\frac{d\Sigma}{d\cos\chi}\right]_{(\text{res.})} = \frac{Q^2}{8} H(\alpha_S) \int_0^\infty db\, b\, J_0(b\, Q\sqrt{y})\, S(Q,b)$$

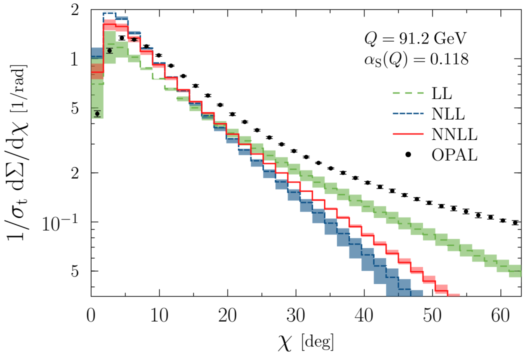
where $y = \sin^2\left(\frac{\chi}{2}\right)$ and the Sudakov form factor collects all log-enhanced terms

$$S(Q,b) = \exp\left\{ -\int_{b_0^2/b^2}^{Q^2} \frac{dq^2}{q^2} \left[A(\alpha_S(q^2))\, \ln\frac{Q^2}{q^2} \;+\; B(\alpha_S(q^2))\right]\right\}$$

The functions $A(\alpha_S)$, $B(\alpha_S)$ and $H(\alpha_S)$ can be computed perturbatively

$$A(\alpha_S) = \sum_{n=1}^{\infty}\left(\frac{\alpha_S}{4\pi}\right)^n A^{(n)}, \quad B(\alpha_S) = \sum_{n=1}^{\infty}\left(\frac{\alpha_S}{4\pi}\right)^n B^{(n)}, \quad H(\alpha_S) = 1+\sum_{n=1}^{\infty}\left(\frac{\alpha_S}{4\pi}\right)^n H^{(n)}$$

Resummation



The pure resummed results capture the general behavior of the data for small angles

Differences become sizable even for moderate values of χ

R matching (naive)

Resummed and fixed-order calculations are complementary to each other

One way of combining the two is naive R matching

$$\frac{1}{\sigma_t}\frac{d\Sigma}{d\cos\chi} = \left[\frac{1}{\sigma_t}\frac{d\Sigma}{d\cos\chi}\right]_{\text{(res.)}} + \left[\frac{1}{\sigma_t}\frac{d\Sigma}{d\cos\chi}\right]_{\text{f.o.}} - \left[\frac{1}{\sigma_t}\frac{d\Sigma}{d\cos\chi}\right]_{\text{(res.)}}\Bigg|_{\text{f.o.}}$$

The fixed-order expansion of the NNLL does not contain all logarithms of the NNLO result

The naively R-matched NNLL+NNLO distribution contains non-expanentiated subleading logarithmic terms

The naively R-matched NNLL+NNLO is unphysical (divergent) in the back-to-back region

log-R matching

In this scheme we consider the cumulative event shape distribution

$$R(y) = \frac{1}{\sigma_t} \int_0^y dy' \, \frac{d\sigma}{dy'}$$

This has the following fixed-order exansion

$$R_{\text{f.o.}} = 1 + \frac{\alpha_{\mathrm{S}}}{2\pi} \bar{\mathcal{A}} + \left(\frac{\alpha_{\mathrm{S}}}{2\pi}\right)^2 \bar{\mathcal{B}} + \left(\frac{\alpha_{\mathrm{S}}}{2\pi}\right)^3 \bar{\mathcal{C}} + \mathcal{O}(\alpha_{\mathrm{S}}^4)$$

The formulae in the literature pertain to observables that can be resummed in a completely exponentiated form

$$R_{(\text{res.})} = (1 + \alpha_{\mathrm{S}} C_1 + \alpha_{\mathrm{S}}^2 C_2 + \ldots) e^{L g_1(\alpha_{\mathrm{S}} L) + g_2(\alpha_{\mathrm{S}} L) + \alpha_{\mathrm{S}} g_3(\alpha_{\mathrm{S}} L) + \ldots} + \mathcal{O}(\alpha_{\mathrm{S}} y)$$

The function g_n can be expanded in powers of α_{S} and $L = \log y$

$$g_n(\alpha_{\mathrm{S}} L) = \sum_{i=1}^{\infty} G_{i,i+2-n} \left(\frac{\alpha_{\mathrm{S}}}{2\pi}\right)^i L^{i+2-n}$$

log-R matching

In the log-R scheme we take

$$\ln R_{(\text{res.})} = Lg_1(\alpha_S L) + g_2(\alpha_S L) + \alpha_S g_3(\alpha_S L) + \alpha_S C_1 + \alpha_S^2 \left(C_2 - \frac{1}{2} C_1 \right) + \alpha_S^3 \left(C_3 - C_2 C_1 + \frac{1}{3} C_1^3 \right) + \mathcal{O}(\alpha_S^4)$$

and replace the terms up to $\mathcal{O}(\alpha_S^3)$ with those of the fixed-order

$$\begin{aligned} \ln R = {} & Lg_1(\alpha_S L) + g_2(\alpha_S L) + \alpha_S g_3(\alpha_S L) \\ & + \frac{\alpha_S}{2\pi} \left(\bar{\mathcal{A}} - G_{11} L - G_{12} L^2 \right) \\ & + \left(\frac{\alpha_S}{2\pi} \right)^2 \left(\bar{\mathcal{B}} - \frac{1}{2} \bar{\mathcal{A}}^2 - G_{21} L - G_{22} L^2 - G_{23} L^3 \right) \\ & + \left(\frac{\alpha_S}{2\pi} \right)^3 \left(\bar{\mathcal{C}} - \bar{\mathcal{B}} \bar{\mathcal{A}} + \frac{1}{3} \bar{\mathcal{A}}^3 - G_{32} L^2 - G_{33} L^3 - G_{34} L^4 \right) \end{aligned}$$

The C_n do not appear since constant terms of the form $C_n \alpha_S^n$ must be factorized with respect to the form factor and should not be exponentiated

log-R matching

In our case, there are two difficulties with this method:

- the fixed-order expansion of the event shape diverges for both small and large angles, so the cumulants cannot be determined reliably
- the resummed distribution is not in a completely exponentiated form

To solve the first issue, we consider a linear combination of moments:

$$\frac{1}{\sigma_t}\tilde{\Sigma}(\chi) \equiv \frac{1}{\sigma_t}\int_0^{\chi} d\chi'(1+\cos\chi')\,\frac{d\Sigma}{d\chi'}$$

The singularity of the differential distribution at $\chi = \pi$ is suppressed

$$\frac{1}{\sigma_t}\tilde{\Sigma}(\pi) = \frac{1}{\sigma_t}\int\sum_{i,j}\frac{E_i E_j}{Q^2}(1-\cos\theta_{ij})\,d\sigma_{e^+e^-\to ij+X} = 1 \quad \text{(in massless QCD)}$$

This condition fixes the integration constants in the fixed-order coefficients $\bar{\mathcal{A}}$, $\bar{\mathcal{B}}$ and $\bar{\mathcal{C}}$

log-R matching

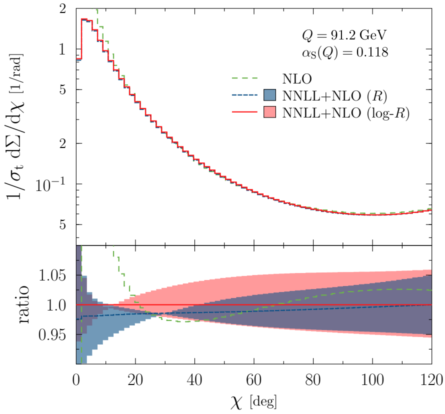
Second issue: the formulae do not translate to our case exactly but we can repeat the constructions

Non-logarithmically enhanced constant terms from $H(\alpha_S)$ must not be exponentiated and thus should not appear in the formula for the matched expression

We compute

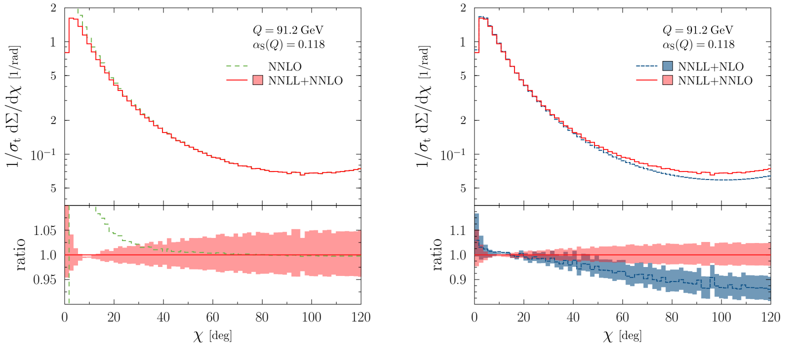
$$\ln\left[\frac{1}{\sigma_t}\tilde{\Sigma}\right] = \ln\left\{\frac{1}{H(\alpha_S)}\left[\frac{1}{\sigma_t}\tilde{\Sigma}\right]_{\text{(res.)}}\right\} - \ln\left\{\frac{1}{H(\alpha_S)}\left[\frac{1}{\sigma_t}\tilde{\Sigma}\right]_{\text{(res.)}}\right\}\Bigg|_{\text{f.o.}}$$
$$+ \frac{\alpha_S}{2\pi}\bar{\mathcal{A}} + \left(\frac{\alpha_S}{2\pi}\right)^2\left(\bar{\mathcal{B}} - \frac{1}{2}\bar{\mathcal{A}}^2\right) + \left(\frac{\alpha_S}{2\pi}\right)^3\left(\bar{\mathcal{C}} - \bar{\mathcal{B}}\bar{\mathcal{A}} + \frac{1}{3}\bar{\mathcal{A}}^3\right) + \mathcal{O}(\alpha_S^4)$$

NNLL+NLO, R vs log-R



The difference of the two matched distributions is $\sim 2\%$ for small angles and $< 1\%$ for the bulk of the region

NNLL+NNLO, log-R



Sizable difference between NNLL+NLO and NNLL+NNLO for $\chi > 40^\circ$

Reduced uncertainty band from scale variation at NNLL+NNLO (not apparent on plot due to normalization)

Comparison to data

- Predictions compared to OPAL and SLD data
- We use χ^2 analysis
- Virtually no information available on the correlation of uncertainties in measurements
- The uncertainties are determined by adding statistical and systematic uncertainties in quadrature and treating them as uncorrelated between all data points
- Theoretical uncertainties are obtained by varying the renormalization scale in the region $[Q/2, 2Q]$ and repeating the fits
- Two ways of treating non-perturbative corrections:
  - omitting entirely
  - using analytic model

Fit to data: no NP corrections

Fit to OPAL and SLD data, no hadronization corrections

Fit ranges chosen as in [*D. de Florian, M. Grazzini*, (2005)]

Fit range	NNLL+NLO (R)		NNLL+NLO (log-R)		NNLL+NNLO (log-R)	
	$\alpha_S(M_Z)$	χ^2/d.o.f.	$\alpha_S(M_Z)$	χ^2/d.o.f.	$\alpha_S(M_Z)$	χ^2/d.o.f.
$0^\circ < \chi < 63^\circ$	0.133 ± 0.001	1.96	0.131 ± 0.003	1.21	0.129 ± 0.003	4.13
$15^\circ < \chi < 63^\circ$	0.132 ± 0.001	0.59	0.131 ± 0.003	0.54	0.128 ± 0.003	1.58
$15^\circ < \chi < 120^\circ$	0.135 ± 0.002	3.96	0.134 ± 0.004	5.12	0.127 ± 0.003	1.12

Taking NNLO corrections into account, the values for $\alpha_S(M_Z)$ get closer to the world average 0.1181 ± 0.0011 (Particle Data Group)

NNLO is especially relevant for describing data at intermediate χ values

Fit to data: analytic NP correction

We have repeated the analysis by taking hadronization into account through an analytic model

Multiply the Sudakov with

$$S_{NP} = e^{-\frac{1}{2}a_1 b^2}(1 - 2a_2 b)$$

Fit for the parameters a_1 and a_2 [*Y. L. Dokshitzer, G. Marchesini, B. R. Webber*, (1999)]

Using R-matching at NNLL+NLO the best fit is

$$\alpha_{\mathrm{S}}(M_Z) = 0.134^{+0.001}_{-0.009}\,, \quad a_1 = 1.55^{+4.26}_{-1.54}\ \mathrm{GeV}^2\,, \quad a_2 = -0.13^{+0.50}_{-0.05}\ \mathrm{GeV}$$

with $\chi^2/\mathrm{d.o.f.} = 38.7/48 = 0.81$

Fit to data: analytic NP correction

Using log-R matching:

- NNLL+NLO:

$$\alpha_S(M_Z) = 0.128^{+0.002}_{-0.006}\,, \quad a_1 = 1.17^{+1.46}_{-0.29}\ \mathrm{GeV}^2\,, \quad a_2 = 0.13^{+0.14}_{-0.09}\ \mathrm{GeV}$$

with $\chi^2/\mathrm{d.o.f.} = 40.8/48 = 0.85$

- NNLL+NNLO:

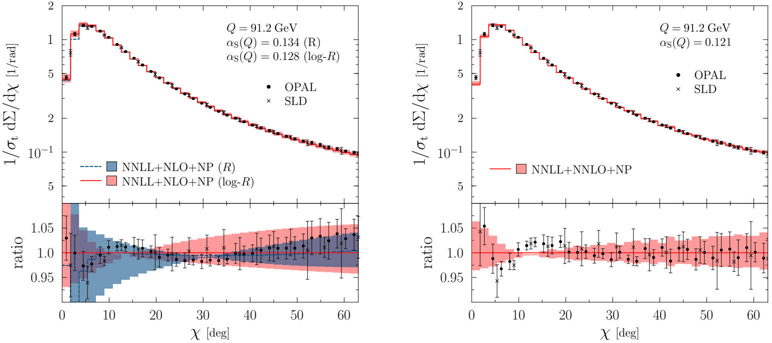
$$\alpha_S(M_Z) = 0.121^{+0.001}_{-0.003}\,, \quad a_1 = 2.47^{+0.48}_{-2.38}\ \mathrm{GeV}^2\,, \quad a_2 = 0.31^{+0.27}_{-0.05}\ \mathrm{GeV}$$

with $\chi^2/\mathrm{d.o.f.} = 56.7/48 = 1.18$

$\alpha_S(M_Z)$: closer to the world average and its uncertainty is reduced

$a_1,\ a_2$: not determined very well, strong anticorrelation between a_2 and α_S

Fit to data: analytic NP correction



The shape is better modelled by the NNLL+NNLO prediction

The renormalization scale band becomes narrower for NNLL+NNLO implying smaller theoretical uncertainty

Summary

Theoretical predictions for EEC in e^+e^- collisions at NNLL+NNLO accuracy were presented

Log-R matching was used to obtain physical predictions over a wide kinematic range

We performed a comparison to data and extracted $\alpha_S(M_Z)$

Using an analytic hadronization model the best fit is $\alpha_S(M_Z) = 0.121^{+0.001}_{-0.003}$

Impact of NNLO corrections:

- better modelling of the shape of the distribution, better fit quality
- theoretical uncertainties are reduced
- the extracted value of $\alpha_S(M_Z)$ is closer to the world average

$\Longrightarrow$ NNLO must be included in a precise measurement of α_S from EEC

Acknowledgements

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Correlations

NNLL+NLO (R):

$$\alpha_S(M_Z) = 0.134^{+0.001}_{-0.009}\,, \quad a_1 = 1.55^{+4.26}_{-1.54}\ \mathrm{GeV}^2\,, \quad a_2 = -0.13^{+0.50}_{-0.05}\ \mathrm{GeV}$$

$$\mathrm{corr}(\alpha_S, a_1, a_2) = \begin{pmatrix} 1 & 0.04 & -0.70 \\ 0.04 & 1 & -0.03 \\ -0.70 & -0.03 & 1 \end{pmatrix}$$

NNLL+NLO (log-R):

$$\alpha_S(M_Z) = 0.128^{+0.002}_{-0.006}\,, \quad a_1 = 1.17^{+1.46}_{-0.29}\ \mathrm{GeV}^2\,, \quad a_2 = 0.13^{+0.14}_{-0.09}\ \mathrm{GeV}$$

$$\mathrm{corr}(\alpha_S, a_1, a_2) = \begin{pmatrix} 1 & -0.17 & -0.98 \\ -0.17 & 1 & 0.08 \\ -0.98 & 0.08 & 1 \end{pmatrix}$$

NNLL+NNLO (log-R):

$$\alpha_S(M_Z) = 0.121^{+0.001}_{-0.003}\,, \quad a_1 = 2.47^{+0.48}_{-2.38}\ \mathrm{GeV}^2\,, \quad a_2 = 0.31^{+0.27}_{-0.05}\ \mathrm{GeV}$$

$$\mathrm{corr}(\alpha_S, a_1, a_2) = \begin{pmatrix} 1 & 0.05 & -0.97 \\ 0.05 & 1 & -0.07 \\ -0.97 & -0.07 & 1 \end{pmatrix}$$