

Status of QCD corrections for BSM Higgs Physics

Emanuele A. Bagnaschi (DESY Hamburg)



29 August 2017
QCD@LHC2017
Debrecen, Hungary

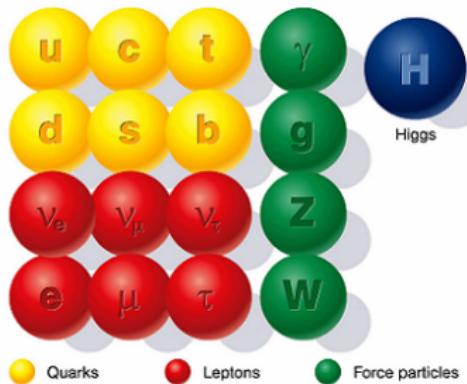
Talk structure

1. Introduction
2. Production processes
3. Decays
4. The mass(es)
5. Conclusions

Introduction

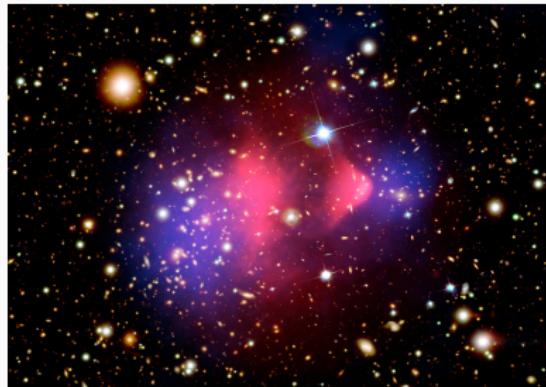
Limits of the Standard Model

Standard particles



Experimental issues

- Lack of a cold DM candidate
- No neutrino masses
- Unable to explain the baryon asymmetry of the universe
- $(g - 2)_\mu$ anomaly



Theoretical flaws

- Hierarchy problem
- No gauge couplings unification
- No consistent inclusion of gravity
- Strong CP problem

Going beyond the SM

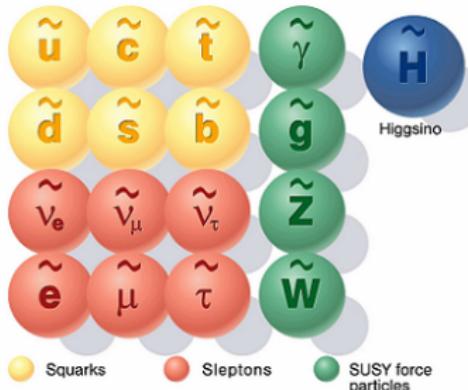
Supersymmetry

- Assume that SUSY is a softly broken symmetry of Nature
- In the minimal extension, the MSSM, the Higgs sector is a type-II 2HDM
- Beyond minimal extensions presents even more complex signatures (e.g. more Higgses)
- We will focus on the MSSM, since it is well studied and it is a good representative of a complex BSM model

Composite Higgs models

- Higgs as a pseudo-Goldstone boson.

SUSY particles



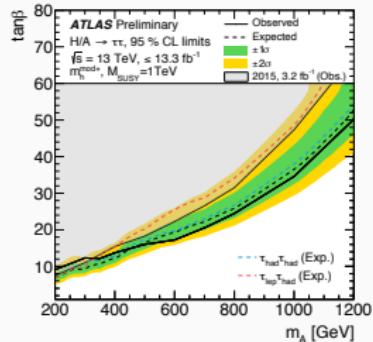
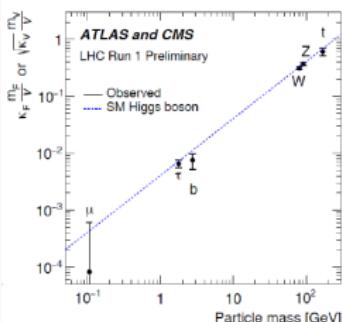
Other extensions

- Add singlet (e.g. SM+singlet)
- Add doublets (e.g. 2HDM)
- Add triplets (e.g. SM+triplet)
- ...

BSM physics in the Higgs sector

Characterization of the boson at 125 GeV

- Deviation of the Yukawa couplings from the SM
- Deviation of the couplings to the EW gauge bosons
- BSM intermediates states in the production processes (e.g. new particles in the gluon fusion loops)



[ATLAS-CONF-2016-085]

New Higgs states

- Extended Higgs sector?
- Simplest extension: Two Higgs Doublet Model (2HDM).
- MSSM has a type-II 2HDM.
- Non-SM like Yukawas can alter the hierarchy between different production processes.

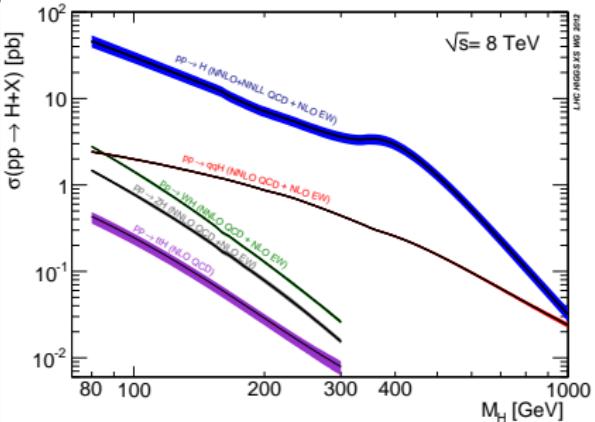
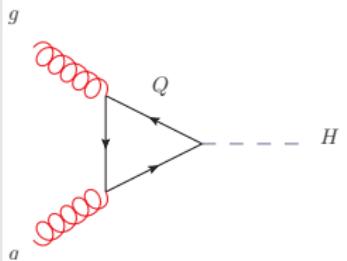
The Minimal Supersymmetric Standard Model

Chiral supermultiplets				
Name	Symbol	spin 0	spin 1/2	$(SU(3)_C, SU(2)_L, U(1)_Y)$
squarks,quarks ($\times 3$ families)	Q \bar{u} \bar{d}	$(\tilde{u}_L, \tilde{d}_L)$ \tilde{u}_R^* \tilde{d}_R^*	(u_L, d_L) u_R^\dagger d_R^\dagger	$(\mathbf{3}, \mathbf{2}, \frac{1}{6})$ $(\bar{\mathbf{3}}, \mathbf{1}, -\frac{2}{3})$ $(\bar{\mathbf{3}}, \mathbf{1}, \frac{1}{3})$
sleptons,leptons ($\times 3$ families)	L \bar{e}	$(\tilde{\nu}, \tilde{e}_L)$ \tilde{e}_R^*	(ν, e_L) e_R^\dagger	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$ $(\mathbf{1}, \mathbf{1}, 1)$
Higgses, Higgsinos	H_u H_d	(H_u^+, H_u^0) (H_d^0, H_d^-)	$(\tilde{H}_u^+, \tilde{H}_u^0)$ $(\tilde{H}_d^0, \tilde{H}_d^-)$	$(\mathbf{1}, \mathbf{2}, \frac{1}{2})$ $(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$
Gauge supermultiplets				
Name		spin 1/2	spin 1	$(SU(3)_C, SU(2)_L, U(1)_Y)$
gluino,gluon		\tilde{g}	g	$(\mathbf{8}, \mathbf{1}, 0)$
winos, W bosons		\tilde{W}^\pm	W^\pm	$(\mathbf{1}, \mathbf{3}, 0)$
bino, B boson		\tilde{B}^0	B^0	$(\mathbf{1}, \mathbf{1}, 0)$

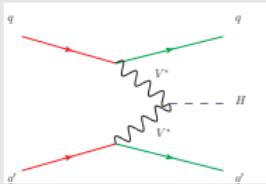
Production processes

Higgs production channels at the LHC

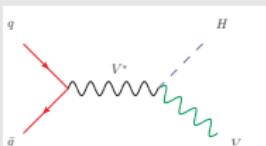
Gluon Fusion



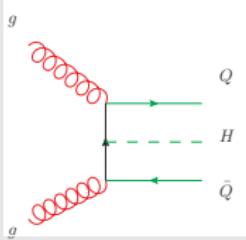
Vector Boson Fusion



Higgs Strahlung



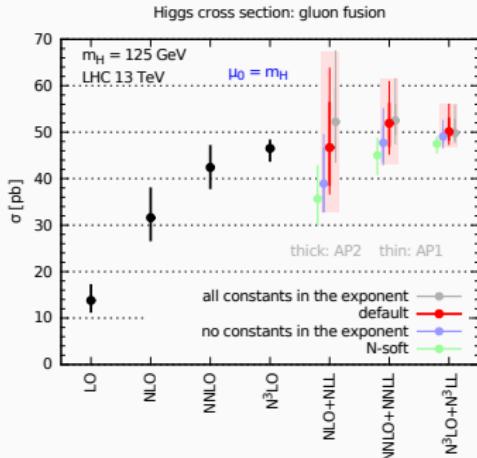
Quark associated production



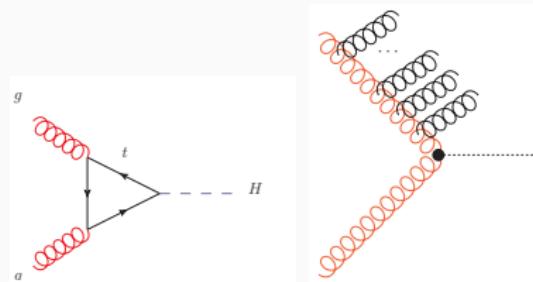
Gluon fusion

Baseline: the SM

- Inclusive cross section known in the SM @ $N^3\text{LO}$ QCD in the HQEFT ($m_t \rightarrow \infty$) [Anastasiou et al, ...] [1].
- Differential results for the inclusive process available up to NNLO, p_t resummation [Catani et al '03, ...]
- SM process known up to NLO; higher order terms in the expansion $1/m_t$ known [Marzani et al '08, Harlander et al '10, ...] [2].
- Soft-resummation available up to $N^3\text{LL}$ ([De Florian et al '14, ...],[3]) in the HQEFT.
- $H + j$ known up to NNLO in the HQEFT [Boughezal et al, ...].
- Merged/matched MCs [NNLOPS, POWHEG, MG5_aMC@NLO, SHERPA, ...]
- See Marius's talk for a in depth review of the SM



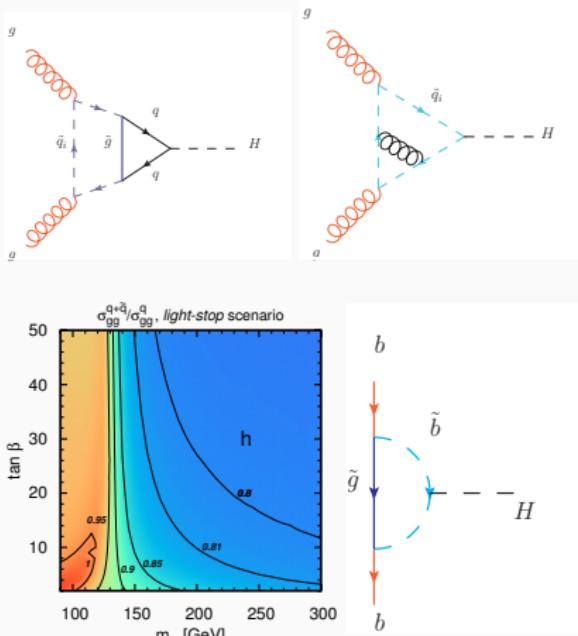
[YR4]



Gluon fusion

SUSY

- Squark diagrams known up to NLO [4, 5, 6, 7].
- Complete NLO results for the squark-gluino contribution known only from semi-numeric computation or using Taylor/asymptotic expansions publicly available [Harlander et al '03, Degrassi et al '08,'10,'11,'12] [8].
- Need resummation of $\tan \beta$ enhanced contribution proportional to the bottom Yukawa [9]



[EB et al '14]

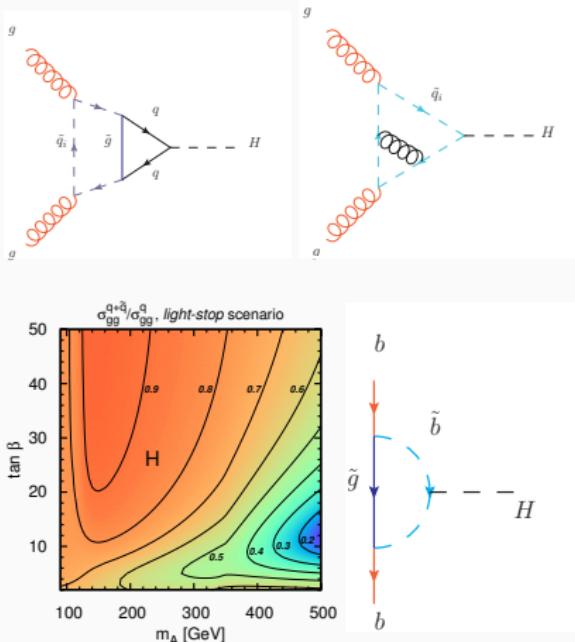
$$\begin{aligned}\widetilde{Y}_b^h &= \frac{Y_b^h}{1 + \Delta_b} \left(1 - \Delta_b \frac{\cot \alpha}{\tan \beta} \right) \\ \widetilde{Y}_b^H &= \frac{Y_b^H}{1 + \Delta_b} \left(1 + \Delta_b \frac{\tan \alpha}{\tan \beta} \right) \\ \widetilde{Y}_b^A &= \frac{Y_b^A}{1 + \Delta_b} \left(1 - \Delta_b \cot^2 \beta \right)\end{aligned}$$

- Resummation of non-abelian $\log(m_b/m_h)$ still missing. Resummation of Abelian logs computed [Melnikov et al '16].
- Codes: SusHi, MoRe-SusHi, POWHEG-BOX/gg_H_MSSM, aMC_SusHi

Gluon fusion

SUSY

- Squark diagrams known up to NLO [4, 5, 6, 7].
- Complete NLO results for the squark-gluino contribution known only from semi-numeric computation or using Taylor/asymptotic expansions publicly available [Harlander et al '03, Degrassi et al '08,'10,'11,'12] [8].
- Need resummation of $\tan \beta$ enhanced contribution proportional to the bottom Yukawa [9]



[EB et al '14]

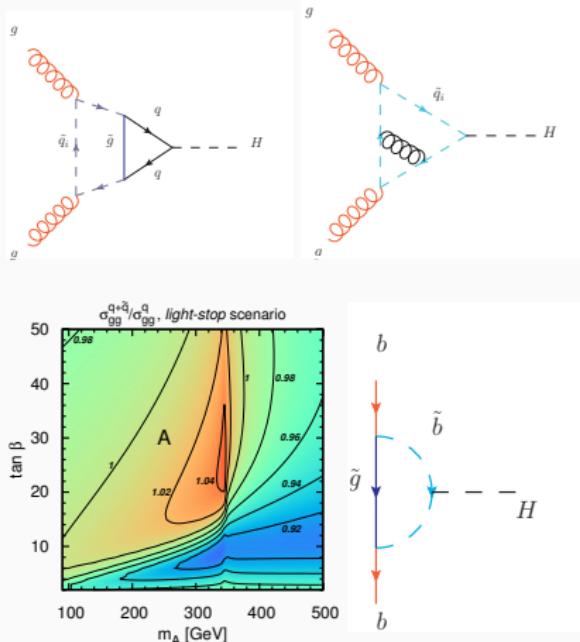
$$\begin{aligned}\widetilde{Y}_b^h &= \frac{Y_b^h}{1 + \Delta_b} \left(1 - \Delta_b \frac{\cot \alpha}{\tan \beta} \right) \\ \widetilde{Y}_b^H &= \frac{Y_b^H}{1 + \Delta_b} \left(1 + \Delta_b \frac{\tan \alpha}{\tan \beta} \right) \\ \widetilde{Y}_b^A &= \frac{Y_b^A}{1 + \Delta_b} \left(1 - \Delta_b \cot^2 \beta \right)\end{aligned}$$

- Resummation of non-abelian $\log(m_b/m_h)$ still missing. Resummation of Abelian logs computed [Melnikov et al '16].
- Codes: SusHi, MoRe-SusHi, POWHEG-BOX/gg_H_MSSM, aMC_SusHi

Gluon fusion

SUSY

- Squark diagrams known up to NLO [4, 5, 6, 7].
- Complete NLO results for the squark-gluino contribution known only from semi-numeric computation or using Taylor/asymptotic expansions publicly available [Harlander et al '03, Degrassi et al '08,'10,'11,'12] [8].
- Need resummation of $\tan \beta$ enhanced contribution proportional to the bottom Yukawa [9]

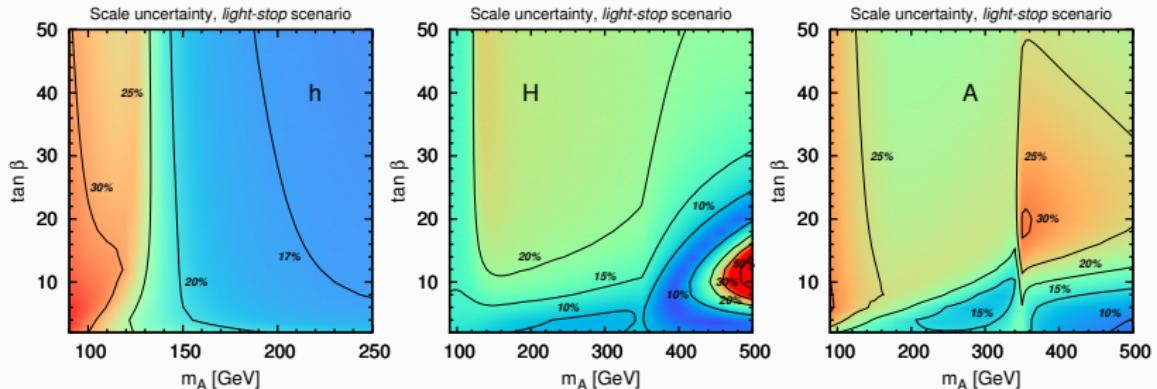


[EB et al '14]

$$\begin{aligned}\widetilde{Y}_b^h &= \frac{Y_b^h}{1 + \Delta_b} \left(1 - \Delta_b \frac{\cot \alpha}{\tan \beta} \right) \\ \widetilde{Y}_b^H &= \frac{Y_b^H}{1 + \Delta_b} \left(1 + \Delta_b \frac{\tan \alpha}{\tan \beta} \right) \\ \widetilde{Y}_b^A &= \frac{Y_b^A}{1 + \Delta_b} \left(1 - \Delta_b \cot^2 \beta \right)\end{aligned}$$

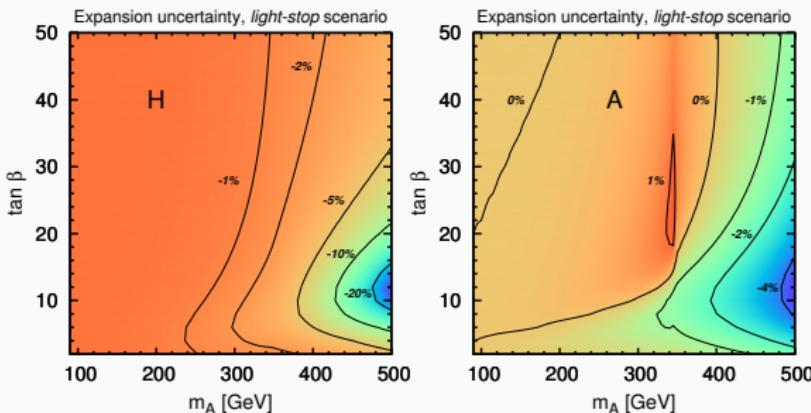
- Resummation of non-abelian $\log(m_b/m_h)$ still missing. Resummation of Abelian logs computed [Melnikov et al '16].
- Codes: SusHi, MoRe-SusHi, POWHEG-BOX/gg_H_MSSM, aMC_SusHi

Scale uncertainty for gluon fusion



- We take the set $C_\mu \equiv \{(\mu_R, \mu_F)\}$ of combinations of renormalization and factorization scales, where $\mu_R = \mu_F = \{m_\phi/4, m_\phi/2, m_\phi\}$, with the constraint $1/2 \leq \mu_r/\mu_f \leq 2$.
- $\sigma^- \equiv \min_{(\mu_R, \mu_F) \in C_\mu} \{\sigma(\mu_R, \mu_F)\}, \quad \sigma^+ \equiv \max_{(\mu_R, \mu_F) \in C_\mu} \{\sigma(\mu_R, \mu_F)\}$
- $\Delta_\mu^\pm \equiv \frac{\sigma^\pm - \sigma(\bar{\mu}_R, \bar{\mu}_F)}{\sigma(\bar{\mu}_R, \bar{\mu}_F)}, \quad \text{We plot } \Delta_\mu \equiv \Delta_\mu^+ - \Delta_\mu^-$
- Non trivial dependence on the parameters. For h up to 35%, for H up to 50% and for A up to 30%.

Validity of the SUSY expansions at NLO

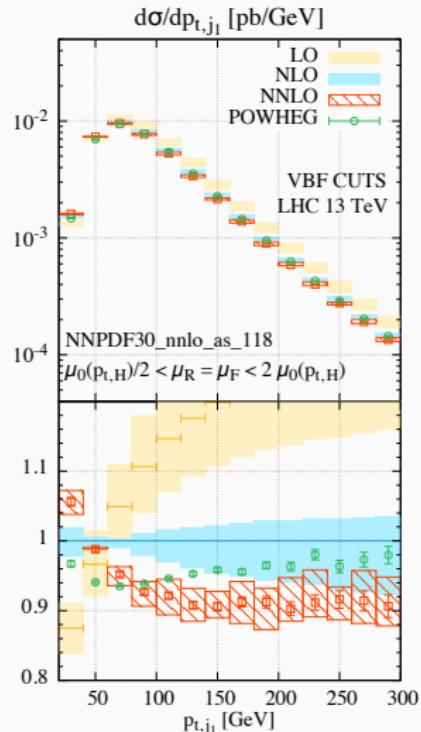
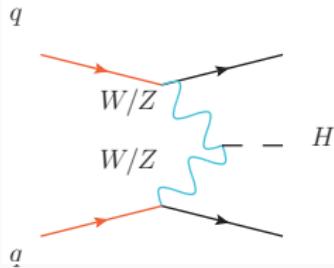


- Contributions from diagrams with quark-squark-gluino are available only as an expansion (in the limit $m_h \rightarrow 0$ for h and as an expansion in the inverse SUSY-particle masses for H and A).
- We compute a test factor $T = A_{\tilde{q}}^{1l}/A_{\tilde{q}}^{1l,\exp}$, where $\tilde{q}_i = \{\tilde{b}, \tilde{t}\}$ and $A_{\tilde{q}}^{1l-\exp}$ is the result by keeping just the leading $\mathcal{O}(m_{\tilde{q}}^{-2})$.
- We multiply the 2-loop stop and sbottom approximate contributions by T and take the resulting value for the cross section as a probe of the uncertainty in the expansion.

Vector Boson Fusion

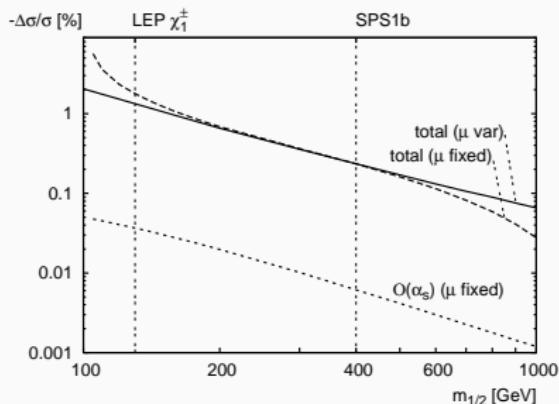
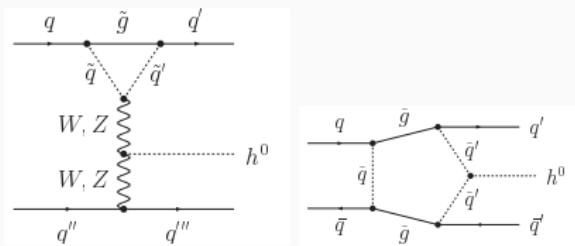
Baseline: the SM

- Know up to NNLO-QCD differentially using a structure function approach [Bolzoni et al '10, ...] [10].
- Non-factorizing contribution shown to be small [Harlander et al '08] [11, 10, 12].
- Codes: HAWK, POWHEG, MG5_aMC@NLO, VBF@NNLO, proVBFH, VBFNLO.



[Cacciari et al, 1506.02660, proVBF]

Vector Boson Fusion



[Plots from Hollik et al '08]

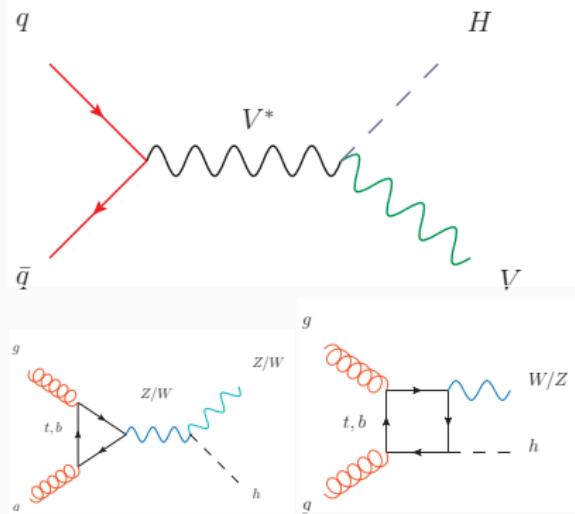
The MSSM

- Known up to NLO in SUSY-QCD (also NLO-EW available) [Hollik et al '08', Figy et al '10']
- Small impact on the total cross section.
- In the decoupling limit, h is SM-like and the H couples only weakly with vector bosons. No AVV coupling at tree level.

Higgs Strahlung

Baseline: the SM

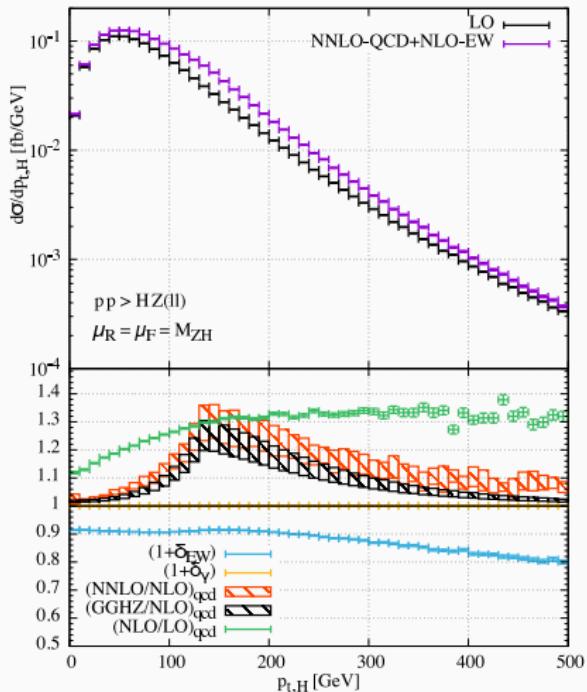
- NLO-QCD corrections as DY (30% of the total cross section) [13].
- QCD corrections known up to NNLO [14].
- New channel, $gg \rightarrow ZH$ opens up NNLO and yields a large contribution (20% of the total cross section); now known at HQEFT-NNLO [Altenkamp et al '13] [15].
- Codes: HVNNLO, MCFM, VHNNLO, VH@NNLO, NNLOPS, POWHEG, MG5_aMC@NLO



Higgs Strahlung

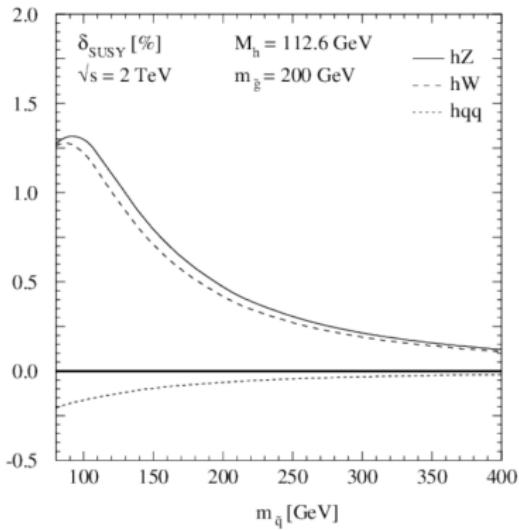
Baseline: the SM

- NLO-QCD corrections as DY (30% of the total cross section) [13].
- QCD corrections known up to NNLO [14].
- New channel, $gg \rightarrow ZH$ opens up NNLO and yields a large contribution (20% of the total cross section); now known at HQEFT-NNLO [Altenkamp et al '13] [15].
- Codes: HVNNLO, MCFM, VHNNLO, VH@NNLO, NNLOPS, POWHEG, MG5_aMC@NLO

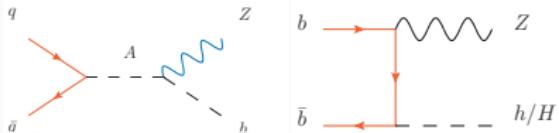


[Spira '16]

Higgs Strahlung



[Djouadi Spira '99]



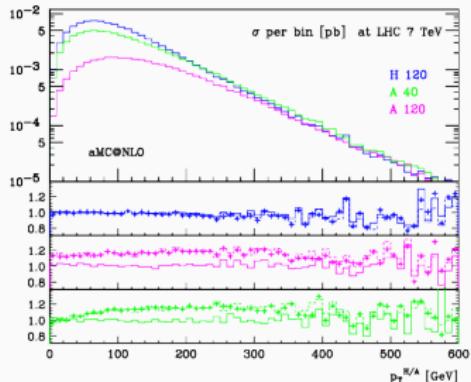
The MSSM

- As for VBF, in the decoupling limit only sensible production rate is for h .
- No pseudoscalar production tree level.
- Relative NLO-QCD corrections are the same as the SM.
- At NNLO, the $gg \rightarrow ZH$ contribution will be different due to the different top/bottom Yukawa.
- SUSY-QCD small [16].

Quark associated production

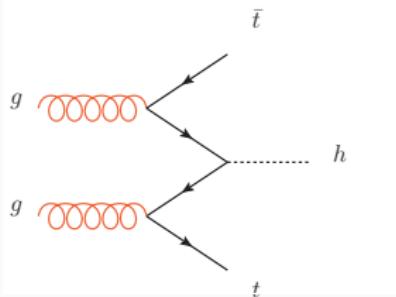
Baseline: the SM

- ttH: known up to NLO-QCD [Beenakker et al '01, ...] [17, 18, 19] ($\mathcal{O}(20\%)$), SCET [Kulesza et al '16] [20], resummation, EW corrections.
- ttH: available @ NLO+PS in POWHEG-BOX, MG5_aMC@NLO, SHERPA.
- bbH-4FS: up to NLO-QCD.
- bbH-5FS: up to NNLO-QCD (bbh@nnlo) [Harlander et al '03] [21].
- bbH, scheme matching: Santander, FONLL, SCET.

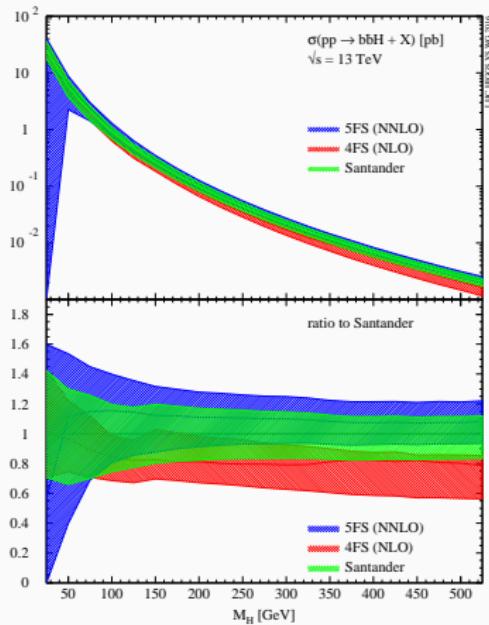


dashed – ratio over LO; solid – ratio over NLO; crosses – LO+PS

[Frederix et al 11]



Quark associated production

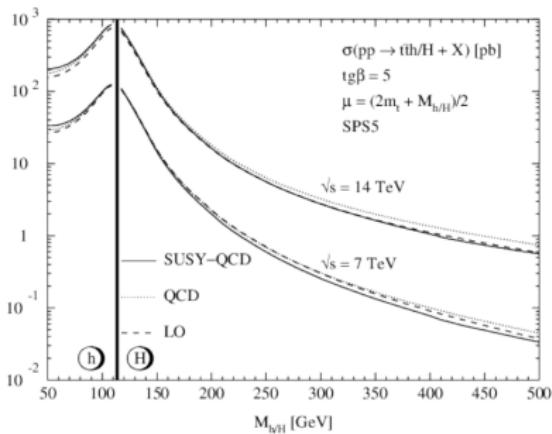


the MSSM

- $t\bar{t}H$ suppressed for $\tan \beta > 1$.
- 4FS- $t\bar{t}A$: NLO-QCD corrections for $t\bar{t}A$ known [19].
- 5FS-bbh: replace the bottom Yukawa with the resummed one.
- 4FS-(bb/tt)(A/H): Full NLO SUSY-QCD corrections recently computed [Dittmaier et al '14] [22, 23]
- Can be re-adsorbed with good approximations in the resummed bottom Yukawa coupling [Dittmaier et al '14].
- 5FS-code: SusHi (NNLO-QCD).

[YR4]

Quark associated production

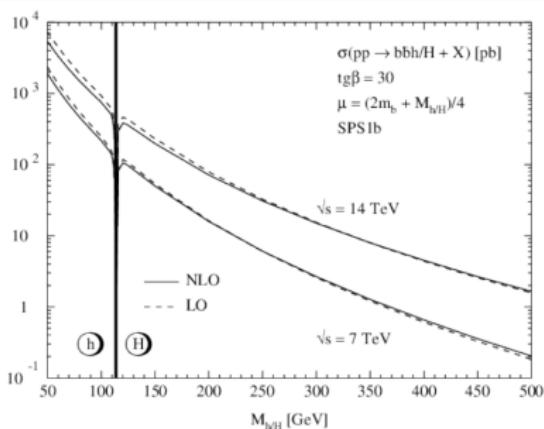


[Dittmaier et al '14]

the MSSM

- $t\bar{t}H$ suppressed for $\tan\beta > 1$.
- 4FS- $t\bar{t}A$:NLO-QCD corrections for $t\bar{t}A$ known [19].
- 5FS-bbh: replace the bottom Yukawa with the resummed one.
- 4FS-(bb/tt)(A/H): Full NLO SUSY-QCD corrections recently computed [Dittmaier et al '14] [22, 23]
- Can be re-adsorbed with good approximations in the resummed bottom Yukawa coupling [Dittmaier et al '14].
- 5FS-code: SusHi (NNLO-QCD).

Quark associated production

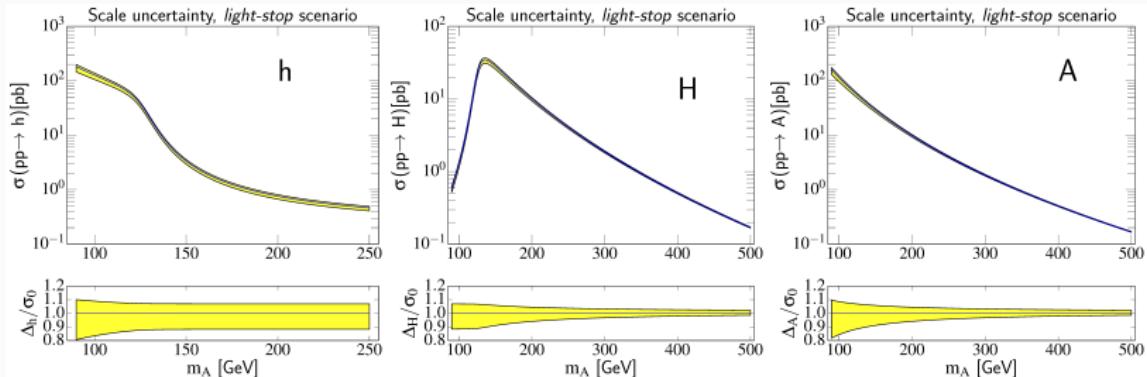


[Dittmaier et al '14]

the MSSM

- $t\bar{t}H$ suppressed for $\tan\beta > 1$.
- 4FS- $t\bar{t}A$:NLO-QCD corrections for $t\bar{t}A$ known [19].
- 5FS-bbh: replace the bottom Yukawa with the resummed one.
- 4FS-(bb/tt)(A/H): Full NLO SUSY-QCD corrections recently computed [Dittmaier et al '14] [22, 23]
- Can be re-adsorbed with good approximations in the resummed bottom Yukawa coupling [Dittmaier et al '14].
- 5FS-code: SusHi (NNLO-QCD).

Scale uncertainty for bottom annihilation

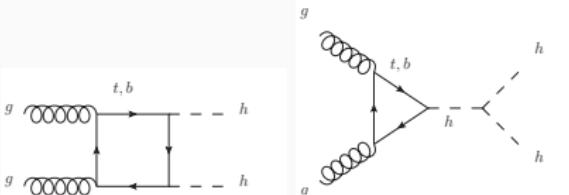
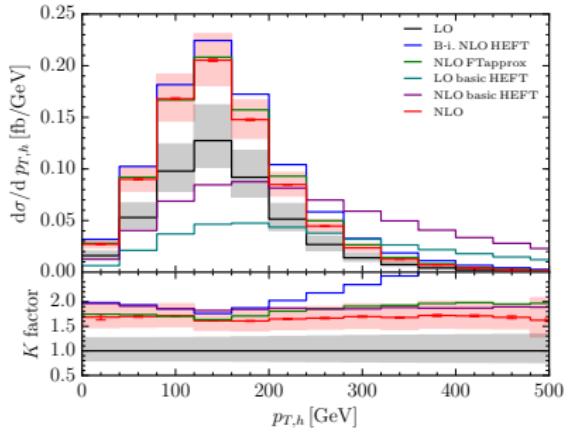


- We take the set $C_\mu \equiv \{(\mu_R, \mu_F)\}$ of combinations of renormalization and factorization scales, where $\mu_R = \{m_\phi/2, m_\phi, 2m_\phi\}$ and $\mu_F = \{m_\phi/8, m_\phi/4, m_\phi/2\}$, with the constraint $2 \leq \mu_r/\mu_f \leq 8$
- Approximately independent of $\tan \beta$
- $\mathcal{O}(20\%)$ for h , from 20% to a few % for H and A

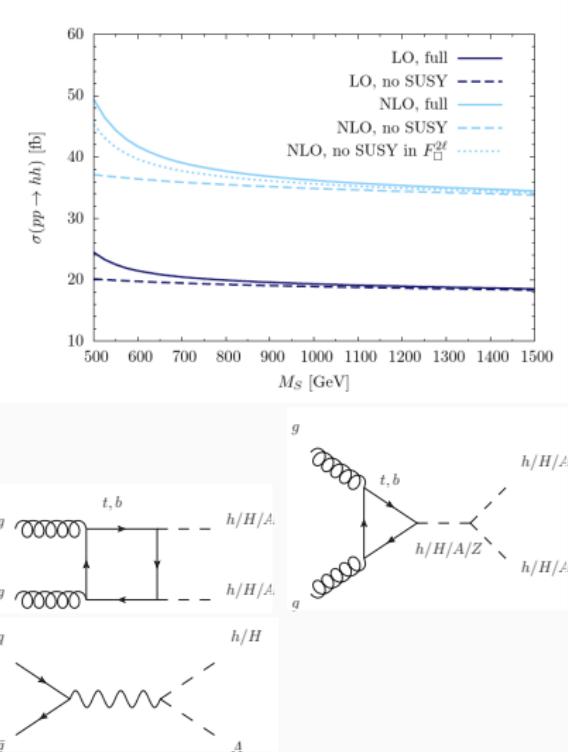
Double Higgs production

Baseline: the SM

- Dominated by gluon fusion.
- Other channels (VBF, double Higgs strahlung, double quark-associated production) subdominant.
- Gluon fusion known up to NLO-QCD in SM via a numerical computation [Borowska et al '17].
- Known up to NNLO-QCD in the HQEFT [de Florian et al '13, ...] [24].
- NNLL soft and collinear resummation available [Shao et al '13, ...] [25].



Double Higgs production



The MSSM

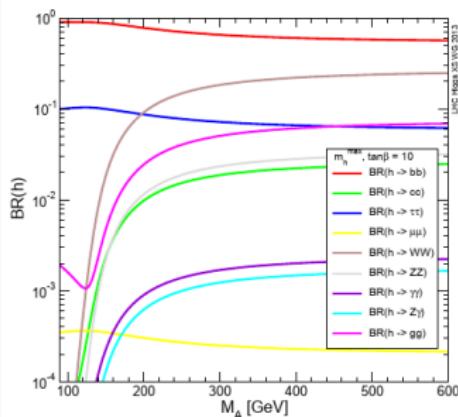
- Final states: hh , hH , hA , HH , HA , AA
- hA , HA dominated by DY like process
- The new SM results important, because bottom quark may dominate in the MSSM.
- Recent progress: gluon fusion SUSY corrections known at NLO-QCD in the limit of vanishing external momenta [Degrassi et al '16].
- Code: HPAIR

Decays

Higgs decays

For the light SM-like Higgs, BSM physics can enter as

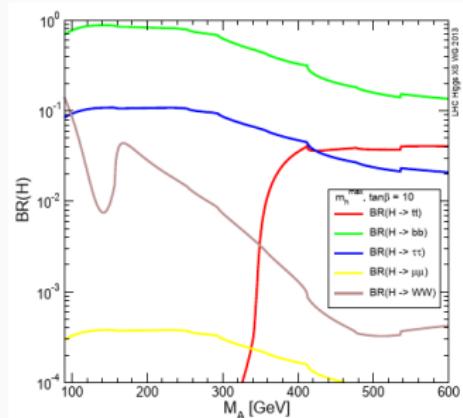
- Modified couplings.
- New intermediate state particles.
- New final states.



[YR3]

In an extended Higgs sector, we need to consider the decays of the new resonances

- Decay of CP-even neutral resonances.
- Decay of CP-odd neutral resonances.
- Decay of charged scalar resonances.



[YR3]

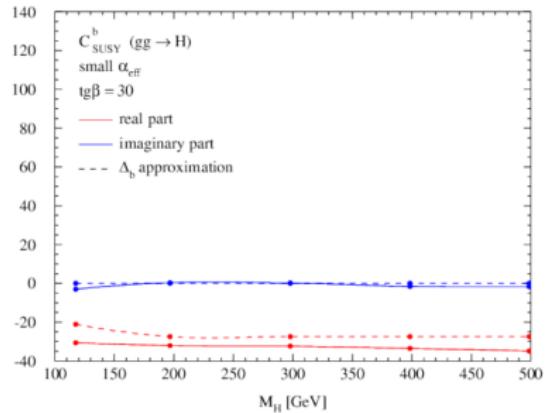
Higgs decays to gluons

Baseline: the SM

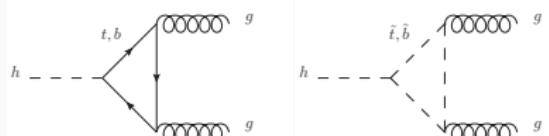
- Loop induced decay, known at NLO-QCD in the SM, N³LO in the HQEFT [J.R. Ellis et al '76, ...][26, 4, 27, 28, 29, 30].

The MSSM

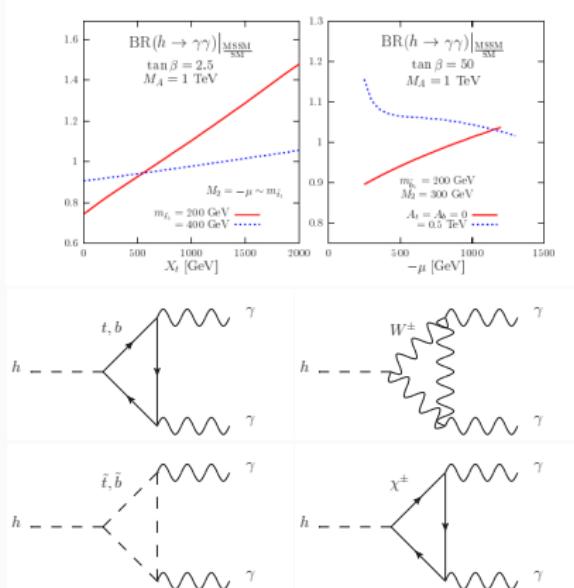
- HQEFT not applicable, couplings to b-quark strongly enhanced in the large $\tan \beta$ regime
- Squarks loop known up to NLO with full mass dependence [Bonciani et al '07, Mühlleitner et al '08] [6].
- Full QCD corrections computed either through expansions or numerically [Harlander et al '03, Degrassi et al '08, ...] [8].
- Sizable effect but can be decently described by the Δ_b approximation



[Spira et al '16]



Higgs decays to photons



Baseline: the SM

- Full two-loop (NLO-QCD) corrections to quarks loop available [Spira et al '95] [4, 31, 5]
- In the HQEFT, QCD corrections known up to $N^3\text{LO}$ [26, 32, 4, 33].
- Perturbative behavior improved if expressed in terms of the running top mass @ $Q = M_H/2$
- Resummation of $\log(m_b/m_h)$ [34]

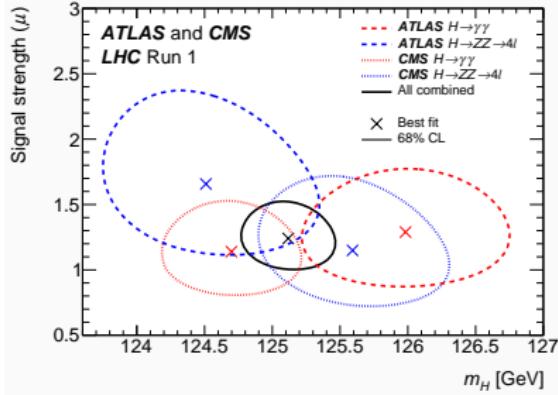
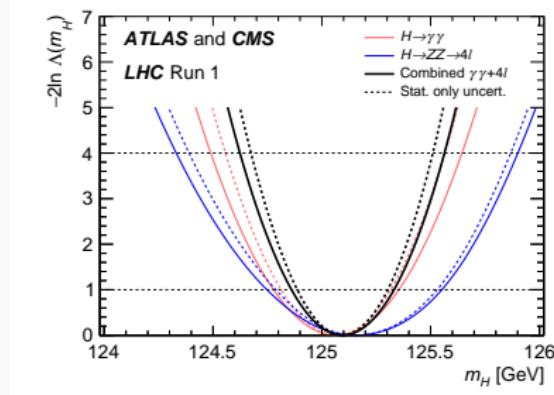
The MSSM

- NLO-QCD corrections for squark loops known [4, 6, 31, 5, 35].
- Pseudoscalar amplitudes show a Coulomb singularity at the $t\bar{t}$ threshold (regularized by the top and squark widths)
- Important phenomenological aspects, interplay of stops (and stau) to enhance/suppress $pp \rightarrow H \rightarrow \gamma\gamma$.

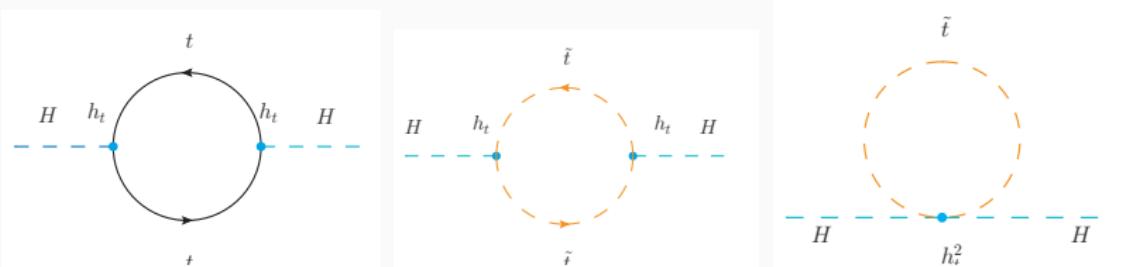
The mass(es)

The Higgs mass

- Mass measured with high-accuracy (e.g. [\[hep-ex/1503.07589\]](#)).
- BSM models, such as the MSSM, differently from the SM, can provide a *prediction* for the Higgs mass.



Higher order corrections to the Higgs mass



Considering radiative corrections to the self-energies then all MSSM particles contributes.

- $\hat{\Sigma}_{ij}(q^2) = \hat{\Sigma}_{ij}^1(q^2) + \hat{\Sigma}_{ij}^2(q^2) + \dots$

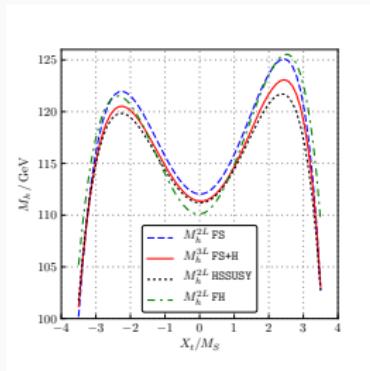
Structure of radiative corrections

Only stop-top sector for simplicity

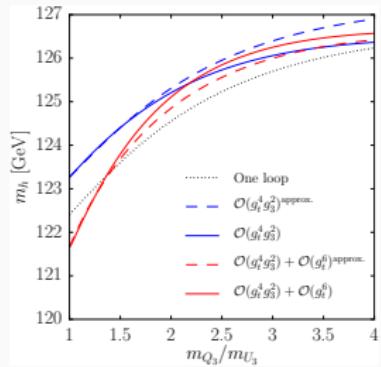
1. At one loop: $\Delta(M_h^{(1)})^2 = m_t^4 [L + C^{(1)}]$ with $L = \log\left(\frac{m_{\tilde{t}}}{m_t}\right)$
2. At two loop: $\Delta(M_h^{(2)})^2 = m_t^2 [m_t^2 \alpha_s (L^2 + L + C^{(2)}) + m_t^4 (L^2 + L + D^{(2)})]$

QCD corrections enter at two loop and can be sizable compared with the experimental precision of the measurement, $\mathcal{O}(1 - 10)\text{GeV}$.

QCD corrections to the Higgs mass



[Harlander et al '17]



[EB et al '17]

Status QCD corrections in the MSSM

- Corrections of $\mathcal{O}(g_3^2 g_t^4)$, $\mathcal{O}(g_3^2 g_b^4)$ both as corrections in complete MSSM computation or as threshold for the EFT ones [36, 37, 38, 39, 40, 41, 42, 43, 44]
- Corrections of $\mathcal{O}(\alpha g_t^2 g_e^2)$ also computed [Martin, Di Vita et al].
- Three-loop $\mathcal{O}(g_3^4 g_t^4)$ also computed the MSSM [Harlander et al '08, ...] [45, 46].
- Codes: 2-loop results in FeynHiggs [47], FlexibleSUSY [48], SPHENO [49, 50], Suspect [51], SoftSUSY [52, 53].
- Codes: 3-loop results in H3M [Kant et al '10] and HIMALAYA [Harlander et al '17].

Conclusions

Summary

- SUSY-QCD corrections in the MSSM have been extensively studied in the past and enters in a significant way a variety of observables.
- See M. Spira review, [\[1612.07651\]](#), for a detailed account of QCD corrections to production and decay in the MSSM.
- Several items were left out of the talk, including other decay modes and production channels (charged Higgs).
- QCD corrections were also computed and included in the calculation of the pole masses of the sparticles and in the determination of the running parameters of the MSSM (e.g. to be used in spectrum generators).

Backup slides

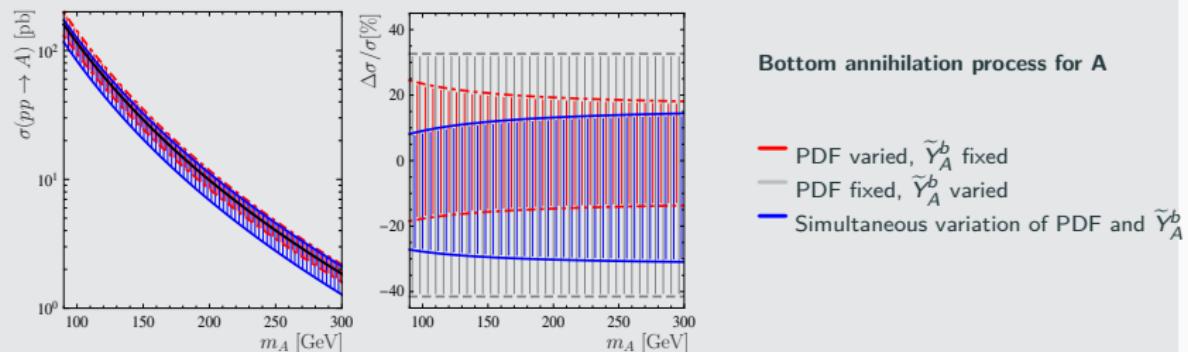
PDF related uncertainties

PDF+ α_s (PDF4LHC)

- SM-like values for gluon fusion
- $\pm 6/6/8/21\%$ for $m_\phi = 124/300/500/1000$ GeV for bottom annihilation

m_b dependence in the PDF

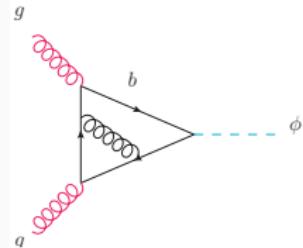
- We use seven MSTW2008 sets extracted with $M_b = 4.75 \pm 0.75$ GeV
- In gluon fusion negligible effect; Relevant effect in bottom annihilation



We decided to use the red band computed in the range 4.75 ± 0.25 GeV as our prescription. We found an uncertainty of the order of $\pm 6\%$.

Bottom Yukawa renormalization scheme

- Great sensitivity for non-standard Higgs bosons on the choice of the bottom Yukawa (e.g. $M_b \simeq 4.93$ GeV, $m_b(m_b) = 4.16$ GeV and $m_b(m_\phi) = 2.92$ for $m_h = 125$ GeV).
- NLO correction to the bottom gluon fusion contribution are enhanced by $\log(m_b^2/m_h^2)$ terms



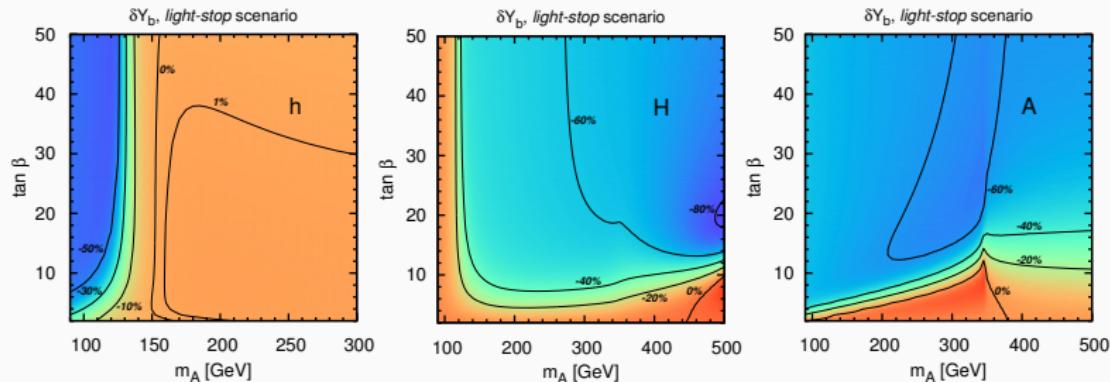
- Spira et al ('95): relating the Bottom yukawa to the pole mass makes the NLO corrections small
- Reason: if one uses M_b , the term proportional to $\mathcal{F}_{1/2}^{1\ell}$ does not appear and the term proportional to C_F roughly cancels out against the one proportional to C_A in

$$\mathcal{A}_b^{2\ell}(\tau) \propto C_F \left[\mathcal{F}_{C_F}(\tau) + \mathcal{F}_{1/2}^{1\ell}(\tau) \left(1 - \frac{3}{4} \ln \frac{m_b^2}{\mu_b^2} \right) \right] + C_A \mathcal{F}_{C_A}(\tau)$$

$$\text{where } \tau_b = 4m_b^2/m_\phi^2$$

- The cancellation is purely accidental → we investigate numerically the impact of the different choices

Bottom Yukawa renormalization scheme



- We compute the cross section either by extracting the bottom Yukawa from M_b (the pole mass) or from the $\overline{\text{MS}}$ -mass $m_b(\mu_R = m_\phi/2)$
- δY_b is the relative variation of the cross section with $\overline{\text{MS}}$ over the standard one with the pole mass
- Great sensitivity in some region (uncertainty up to 80%), though these are the same parameter space points where the bottom annihilation process is dominant

References i

- [1] S. Catani, D. de Florian and M. Grazzini, JHEP **0105** (2001) 025; R. V. Harlander and W. B. Kilgore, Phys. Rev. D **64** (2001) 013015 and Phys. Rev. Lett. **88** (2002) 201801; C. Anastasiou and K. Melnikov, Nucl. Phys. B **646** (2002) 220; V. Ravindran, J. Smith and W. L. van Neerven, Nucl. Phys. B **665** (2003) 325; S. Marzani, R. D. Ball, V. Del Duca, S. Forte and A. Vicini, Nucl. Phys. B **800** (2008) 127; T. Gehrmann, M. Jaquier, E. W. N. Glover and A. Koukoutsakis, JHEP **1202** (2012) 056; C. Anastasiou, C. Duhr, F. Dulat and B. Mistlberger, JHEP **1307** (2013) 003; C. Anastasiou, C. Duhr, F. Dulat, F. Herzog and B. Mistlberger, JHEP **1312** (2013) 088; W. B. Kilgore, Phys. Rev. D **89** (2014) 7, 073008; Y. Li, A. von Manteuffel, R. M. Schabinger and H. X. Zhu, Phys. Rev. D **90** (2014) 5, 053006; C. Anastasiou, C. Duhr, F. Dulat, E. Furlan, T. Gehrmann, F. Herzog and B. Mistlberger, JHEP **1503** (2015) 091; C. Anastasiou, C. Duhr, F. Dulat, F. Herzog and B. Mistlberger, Phys. Rev. Lett. **114** (2015) 21, 212001; C. Anastasiou, C. Duhr, F. Dulat, E. Furlan, T. Gehrmann, F. Herzog, A. Lazopoulos and B. Mistlberger, JHEP **1605** (2016) 058.
- [2] R. V. Harlander and K. J. Ozeren, Phys. Lett. B **679** (2009) 467 and JHEP **0911** (2009) 088; A. Pak, M. Rogal and M. Steinhauser, Phys. Lett. B **679** (2009) 473 and JHEP **1002** (2010) 025.
- [3] D. de Florian, J. Mazzitelli, S. Moch and A. Vogt, JHEP **1410** (2014) 176; M. Bonvini and L. Rottoli, Phys. Rev. D **91** (2015) 5, 051301. S. Catani, L. Cieri, D. de Florian, G. Ferrera and M. Grazzini, Nucl. Phys. B **888** (2014) 75.
- [4] M. Spira, A. Djouadi, D. Graudenz and P. M. Zerwas, Nucl. Phys. B **453** (1995) 17.

References ii

- [5] R. Harlander and P. Kant, JHEP **0512** (2005) 015; C. Anastasiou, S. Beerli, S. Bucherer, A. Daleo and Z. Kunszt, JHEP **0701** (2007) 082; U. Aglietti, R. Bonciani, G. Degrassi and A. Vicini, JHEP **0701** (2007) 021.
- [6] M. Mühlleitner and M. Spira, Nucl. Phys. B **790** (2008) 1; R. Bonciani, G. Degrassi and A. Vicini, JHEP **0711** (2007) 095.
- [7] M. Spira, A. Djouadi, D. Graudenz and P. M. Zerwas, Phys. Lett. B **318** (1993) 347.
- [8] R. V. Harlander and M. Steinhauser, Phys. Lett. B **574** (2003) 258 and JHEP **0409** (2004) 066; G. Degrassi and P. Slavich, Nucl. Phys. B **805** (2008) 267; G. Degrassi, S. Di Vita and P. Slavich, Eur. Phys. J. C **72** (2012) 2032; R. V. Harlander and F. Hofmann, JHEP **0603** (2006) 050; R. V. Harlander, F. Hofmann and H. Mantler, JHEP **1102** (2011) 055; G. Degrassi, S. Di Vita and P. Slavich, JHEP **1108** (2011) 128;
- [9] L. J. Hall, R. Rattazzi and U. Sarid, Phys. Rev. D **50**, 7048 (1994); R. Hempfling, Phys. Rev. D **49**, 6168 (1994); M. Carena, M. Olechowski, S. Pokorski and C. E. M. Wagner, Nucl. Phys. B **426** (1994) 269; D. M. Pierce, J. A. Bagger, K. T. Matchev and R.-J. Zhang, Nucl. Phys. B **491**, 3 (1997); J. Guasch, W. Hollik and S. Peñaranda, Phys. Lett. B **515** (2001) 367; G. D'Ambrosio, G. F. Giudice, G. Isidori and A. Strumia, Nucl. Phys. B **645**, 155 (2002); A. J. Buras, P. H. Chankowski, J. Rosiek and L. Slawianowska, Nucl. Phys. B **659**, 3 (2003); V. Barger, H. E. Logan and G. Shaughnessy, Phys. Rev. D **79**, 115018 (2009); N. D. Christensen, T. Han and S. Su, Phys. Rev. D **85**, 115018 (2012).

References iii

- [10] P. Bolzoni, F. Maltoni, S. O. Moch and M. Zaro, Phys. Rev. Lett. **105** (2010) 011801 and Phys. Rev. D **85** (2012) 035002; M. Cacciari, F. A. Dreyer, A. Karlberg, G. P. Salam and G. Zanderighi, Phys. Rev. Lett. **115** (2015) no.8, 082002.
- [11] M. Ciccolini, A. Denner and S. Dittmaier, Phys. Rev. Lett. **99** (2007) 161803 and Phys. Rev. D **77** (2008) 013002.
- [12] R. V. Harlander, J. Vollinga and M. M. Weber, Phys. Rev. D **77**, 053010 (2008); J. R. Andersen, T. Binoth, G. Heinrich and J. M. Smillie, JHEP **0802** (2008) 057; A. Bredenstein, K. Hagiwara and B. Jäger, Phys. Rev. D **77** (2008) 073004.
- [13] T. Han and S. Willenbrock, Phys. Lett. B **273** (1991) 167.
- [14] O. Brein, A. Djouadi and R. Harlander, Phys. Lett. B **579** (2004) 149.
- [15] L. Altenkamp, S. Dittmaier, R. V. Harlander, H. Rzehak and T. J. E. Zirke, JHEP **1302** (2013) 078; A. Hasselhuhn, T. Luthe and M. Steinhauser, arXiv:1611.05881 [hep-ph].
- [16] A. Djouadi and M. Spira, Phys. Rev. D **62** (2000) 014004.
- [17] W. Beenakker et al., Phys. Rev. Lett. **87** (2001) 201805.

References iv

- [18] W. Beenakker et al., Nucl. Phys. **B653** (2003) 151–203; L. Reina and S. Dawson, Phys. Rev. Lett. **87** (2001) 201804; S. Dawson, L. H. Orr, L. Reina, and D. Wackerlo, Phys. Rev. **D67** (2003) 071503.
- [19] R. Frederix, S. Frixione, V. Hirschi, F. Maltoni, R. Pittau and P. Torrielli, Phys. Lett. B **701** (2011) 427.
- [20] A. Kulesza, L. Motyka, T. Stebel and V. Theeuwes, JHEP **1603** (2016) 065.
- [21] R. V. Harlander and W. B. Kilgore, Phys. Rev. D **68** (2003) 013001.
- [22] S. Dittmaier, P. Häfliger, M. Krämer, M. Spira and M. Walser, Phys. Rev. D **90** (2014) no.3, 035010.
- [23] P. Wu, W. G. Ma, H. S. Hou, R. Y. Zhang, L. Han and Y. Jiang, Phys. Lett. B **618** (2005) 209.
- [24] D. de Florian and J. Mazzitelli, Phys. Lett. B **724** (2013) 306 and Phys. Rev. Lett. **111** (2013) 201801; J. Grigo, K. Melnikov and M. Steinhauser, Nucl. Phys. B **888** (2014) 17.
- [25] D. Y. Shao, C. S. Li, H. T. Li and J. Wang, JHEP **1307** (2013) 169; D. de Florian and J. Mazzitelli, JHEP **1509** (2015) 053.
- [26] J. R. Ellis, M. K. Gaillard and D. V. Nanopoulos, Nucl. Phys. B **106** (1976) 292;

References v

- [27] T. Inami, T. Kubota and Y. Okada, Z. Phys. C **18** (1983) 69.
- [28] A. Djouadi, M. Spira and P. M. Zerwas, Phys. Lett. B **264** (1991) 440.
- [29] K. G. Chetyrkin, B. A. Kniehl and M. Steinhauser, Phys. Rev. Lett. **79** (1997) 353.
- [30] P. A. Baikov and K. G. Chetyrkin, Phys. Rev. Lett. **97** (2006) 061803.
- [31] H.-Q. Zheng and D.-D. Wu, Phys. Rev. D **42** (1990) 3760; A. Djouadi, M. Spira, J. J. van der Bij and P. M. Zerwas, Phys. Lett. B **257** (1991) 187; S. Dawson and R. P. Kauffman, Phys. Rev. D **47** (1993) 1264; A. Djouadi, M. Spira and P. M. Zerwas, Phys. Lett. B **311** (1993) 255; K. Melnikov and O. I. Yakovlev, Phys. Lett. B **312** (1993) 179; M. Inoue, R. Najima, T. Oka and J. Saito, Mod. Phys. Lett. A **9** (1994) 1189; J. Fleischer, O. V. Tarasov and V. O. Tarasov, Phys. Lett. B **584** (2004) 294.
- [32] B. A. Kniehl and M. Spira, Z. Phys. C **69** (1995) 77.
- [33] M. A. Shifman, A. I. Vainshtein, M. B. Voloshin and V. I. Zakharov, Sov. J. Nucl. Phys. **30** (1979) 711 [Yad. Fiz. **30** (1979) 1368].
- [34] M. I. Kotsky and O. I. Yakovlev, Phys. Lett. B **418** (1998) 335; R. Akhoury, H. Wang and O. I. Yakovlev, Phys. Rev. D **64** (2001) 113008.

References vi

- [35] A. Djouadi, V. Driesen, W. Hollik and J. I. Illana, Eur. Phys. J. C **1** (1998) 149.
- [36] S. Heinemeyer, W. Hollik and G. Weiglein, Phys. Rev. D **58** (1998) 091701 doi:10.1103/PhysRevD.58.091701 [hep-ph/9803277].
- [37] G. Degrassi, P. Slavich and F. Zwirner, Nucl. Phys. B **611** (2001) 403 doi:10.1016/S0550-3213(01)00343-1 [hep-ph/0105096].
- [38] A. Brignole, G. Degrassi, P. Slavich and F. Zwirner, Nucl. Phys. B **643** (2002) 79 doi:10.1016/S0550-3213(02)00748-4 [hep-ph/0206101].
- [39] S. Borowka, T. Hahn, S. Heinemeyer, G. Heinrich and W. Hollik, Eur. Phys. J. C **74** (2014) no.8, 2994 doi:10.1140/epjc/s10052-014-2994-0 [arXiv:1404.7074 [hep-ph]].
- [40] A. Dedes and P. Slavich, Nucl. Phys. B **657** (2003) 333 doi:10.1016/S0550-3213(03)00173-1 [hep-ph/0212132].
- [41] E. Bagnaschi, G. F. Giudice, P. Slavich and A. Strumia, JHEP **1409** (2014) 092 doi:10.1007/JHEP09(2014)092 [arXiv:1407.4081 [hep-ph]].

References vii

- [42] E. Bagnaschi, J. Pardo Vega and P. Slavich, Eur. Phys. J. C **77**, no. 5, 334 (2017) doi:10.1140/epjc/s10052-017-4885-7 [arXiv:1703.08166 [hep-ph]].
- [43] S. P. Martin, Phys. Rev. D **66** (2002) 096001 doi:10.1103/PhysRevD.66.096001 [hep-ph/0206136].
- [44] S. P. Martin, Phys. Rev. D **67** (2003) 095012 doi:10.1103/PhysRevD.67.095012 [hep-ph/0211366].
- [45] R. V. Harlander, P. Kant, L. Mihaila and M. Steinhauser, Phys. Rev. Lett. **100** (2008) 191602 [Phys. Rev. Lett. **101** (2008) 039901] doi:10.1103/PhysRevLett.101.039901, 10.1103/PhysRevLett.100.191602 [arXiv:0803.0672 [hep-ph]].
- [46] P. Kant, R. V. Harlander, L. Mihaila and M. Steinhauser, JHEP **1008** (2010) 104 doi:10.1007/JHEP08(2010)104 [arXiv:1005.5709 [hep-ph]].
- [47] S. Heinemeyer, W. Hollik and G. Weiglein, Comput. Phys. Commun. **124** (2000) 76 doi:10.1016/S0010-4655(99)00364-1 [hep-ph/9812320].
- [48] P. Athron, J. h. Park, D. Stöckinger and A. Voigt, Comput. Phys. Commun. **190** (2015) 139 doi:10.1016/j.cpc.2014.12.020 [arXiv:1406.2319 [hep-ph]].
- [49] W. Porod, Comput. Phys. Commun. **153** (2003) 275 doi:10.1016/S0010-4655(03)00222-4 [hep-ph/0301101].

References viii

- [50] W. Porod and F. Staub, *Comput. Phys. Commun.* **183** (2012) 2458 doi:10.1016/j.cpc.2012.05.021 [arXiv:1104.1573 [hep-ph]].
- [51] A. Djouadi, J. L. Kneur and G. Moultaka, *Comput. Phys. Commun.* **176** (2007) 426 doi:10.1016/j.cpc.2006.11.009 [hep-ph/0211331].
- [52] B. C. Allanach, *Comput. Phys. Commun.* **143** (2002) 305 doi:10.1016/S0010-4655(01)00460-X [hep-ph/0104145].
- [53] B. C. Allanach, A. Bednyakov and R. Ruiz de Austri, *Comput. Phys. Commun.* **189** (2015) 192 doi:10.1016/j.cpc.2014.12.006 [arXiv:1407.6130 [hep-ph]].