Event Generators

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Theoretical calculations can be performed in three different limits of field theory

Fixed perturbation theory	$\alpha_s \to 0$
Logarithmic resummation	$\alpha_s \to 0,\ \alpha_s L^2$ fixed
Kinematic expansion (parton shower)	$\theta_{ij} \to 0$

Each expansion important in different regions

Theoretical calculations can be performed in three different limits of field theory

Fixed order perturbation theory

Best precision for inclusive observables
(only one relevant scale in problem)

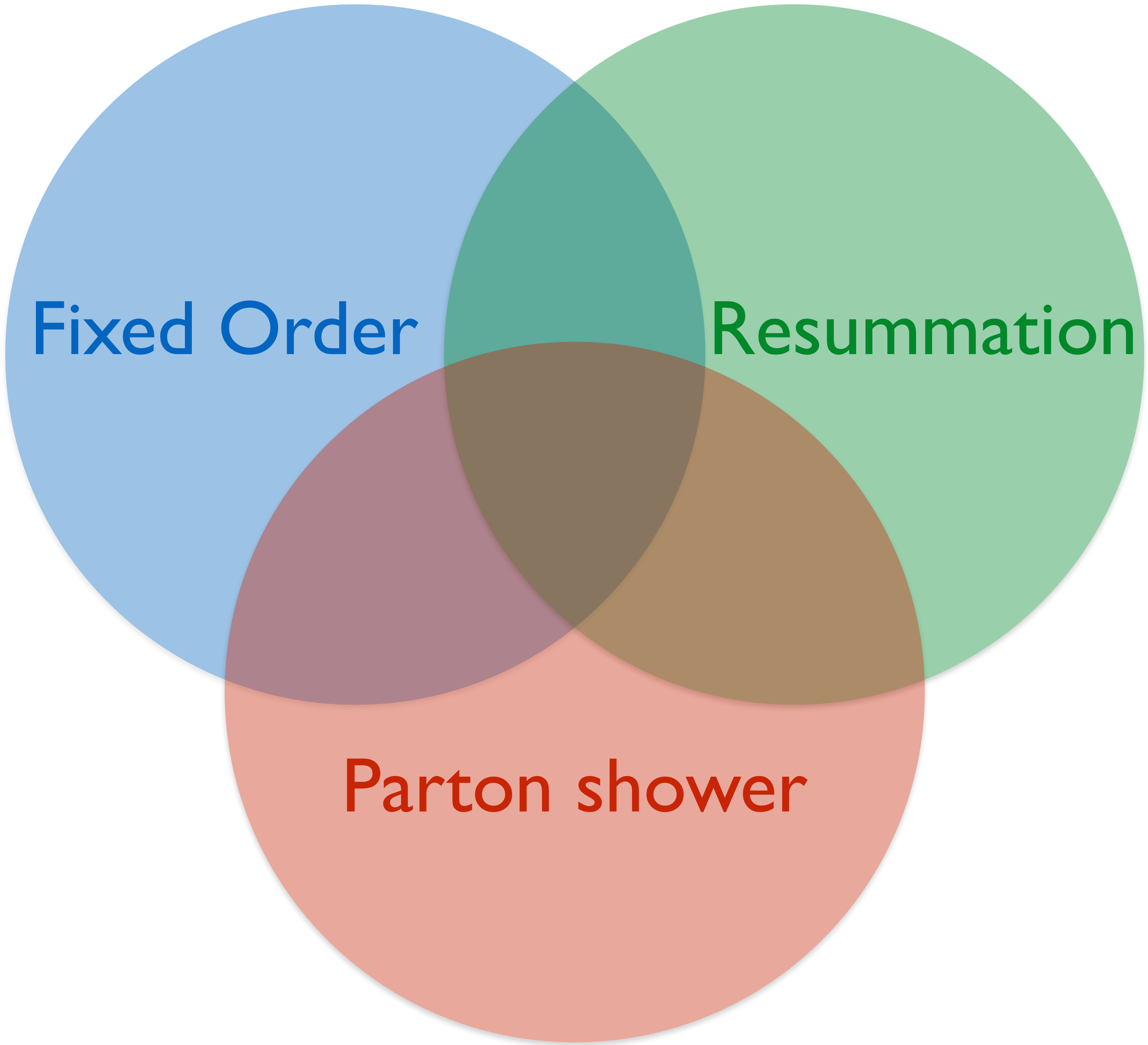
Logarithmic resummation

Best precision for semi-inclusive observables (large ratio(s)
of scale in problem)

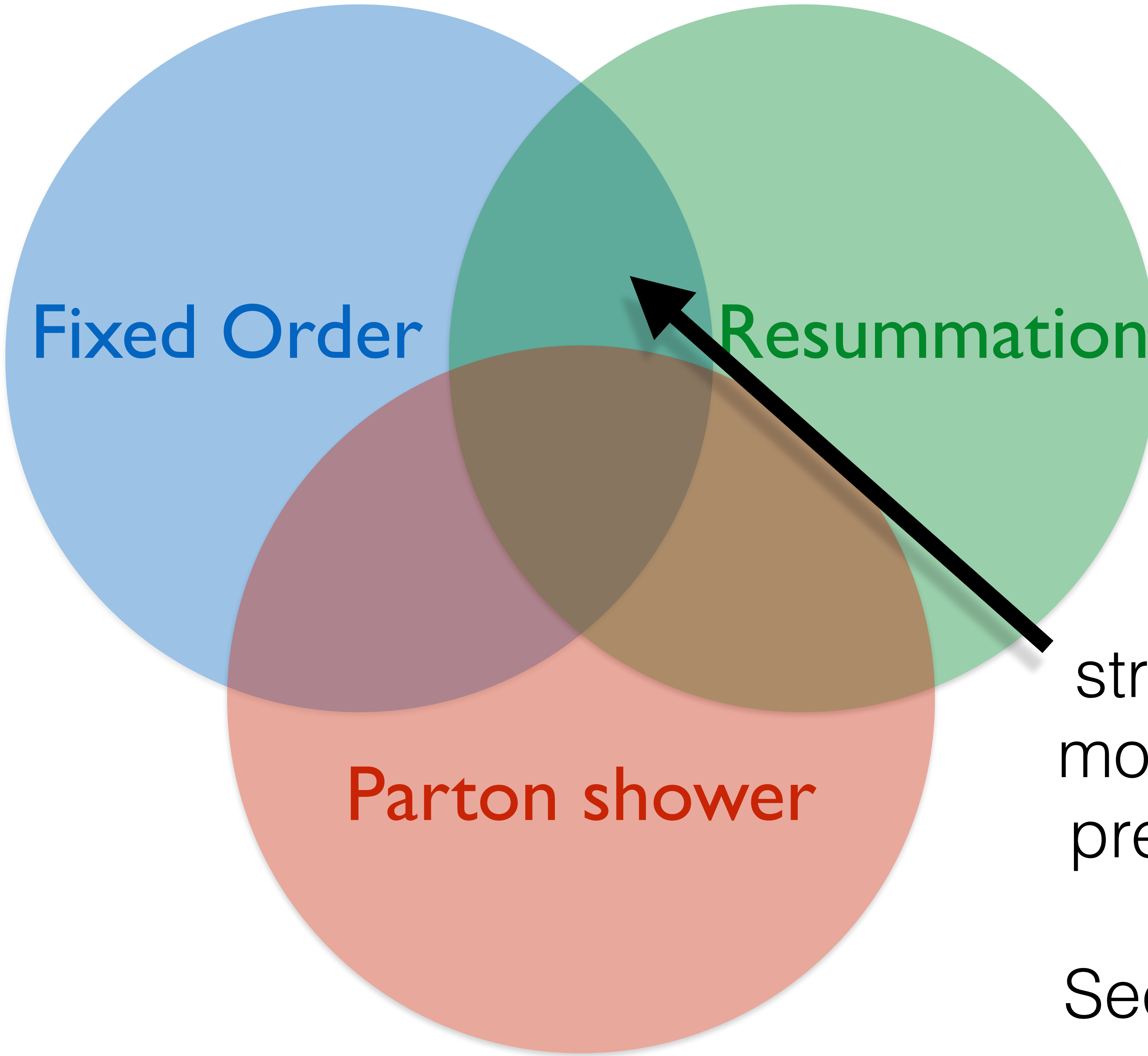
Parton shower

Only tool for events with arbitrary multiplicity
(event simulation)

Lots of efforts to combine the various limits

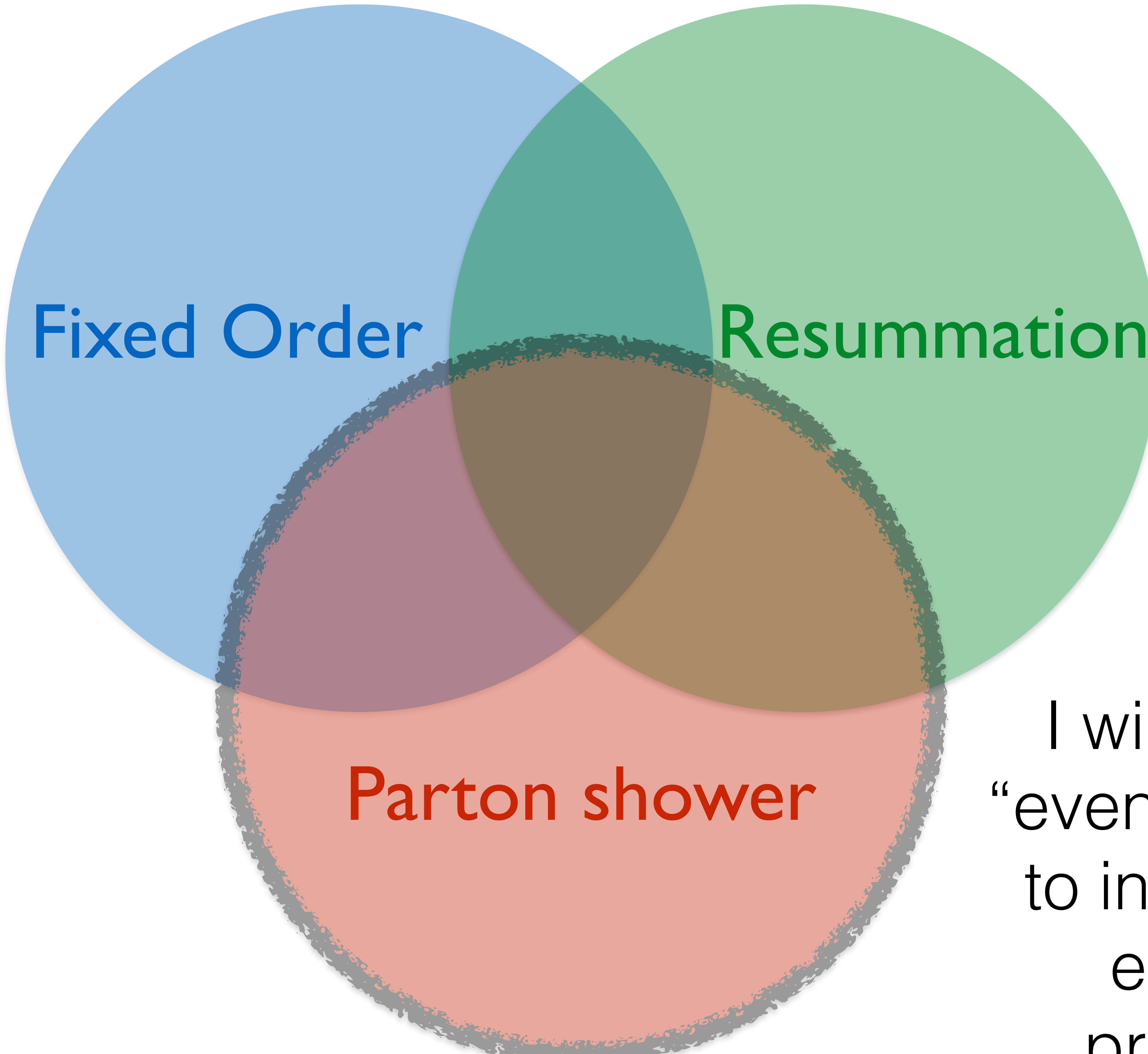


Lots of efforts to combine the various limits

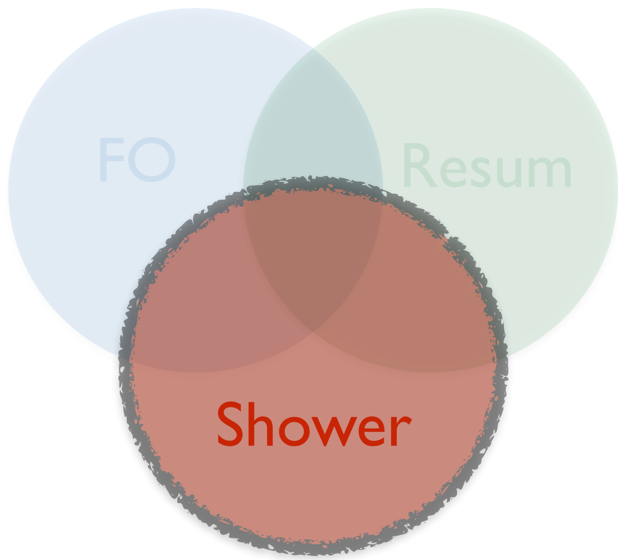


Relatively straighforward, most resummed predictions use this.
See Tackmanns talk

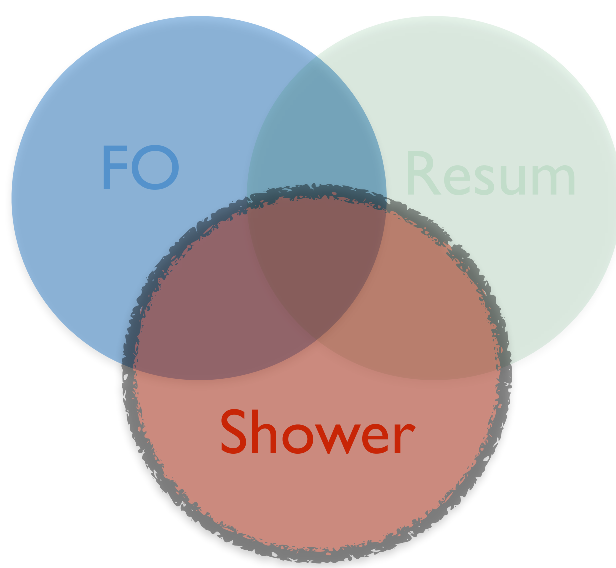
Lots of efforts to combine the various limits



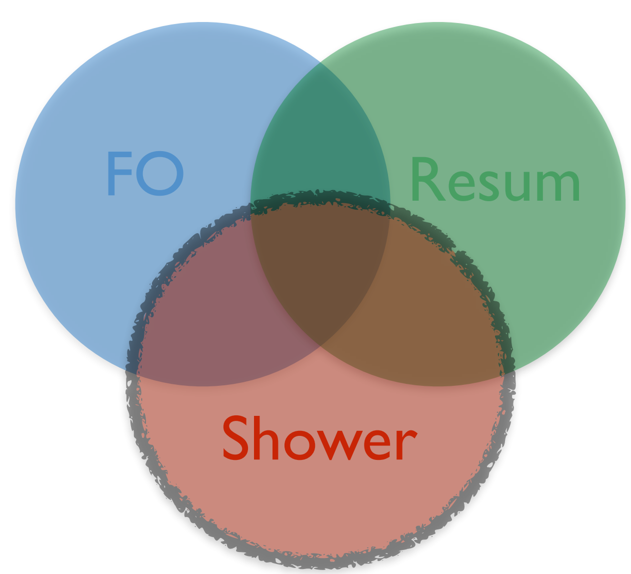
I will use term "event generator" to indicate fully exclusive predictions



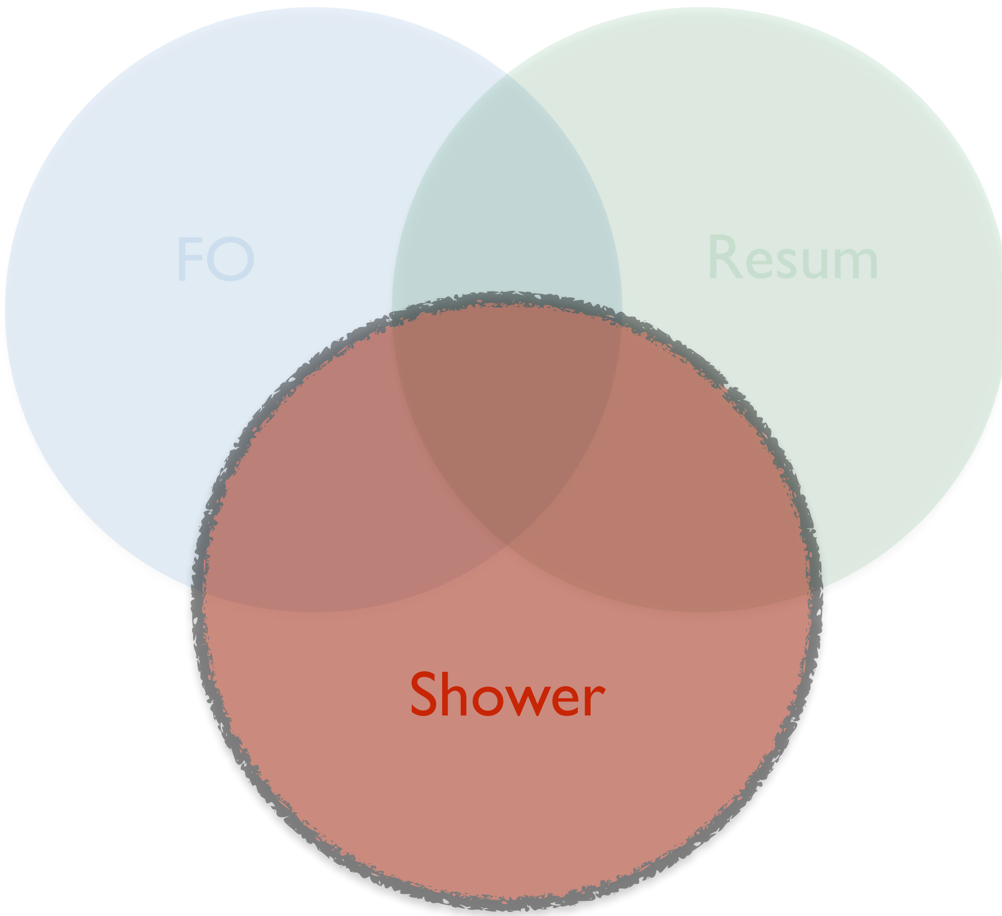
Recent developments in parton showers



Combining FO with showers

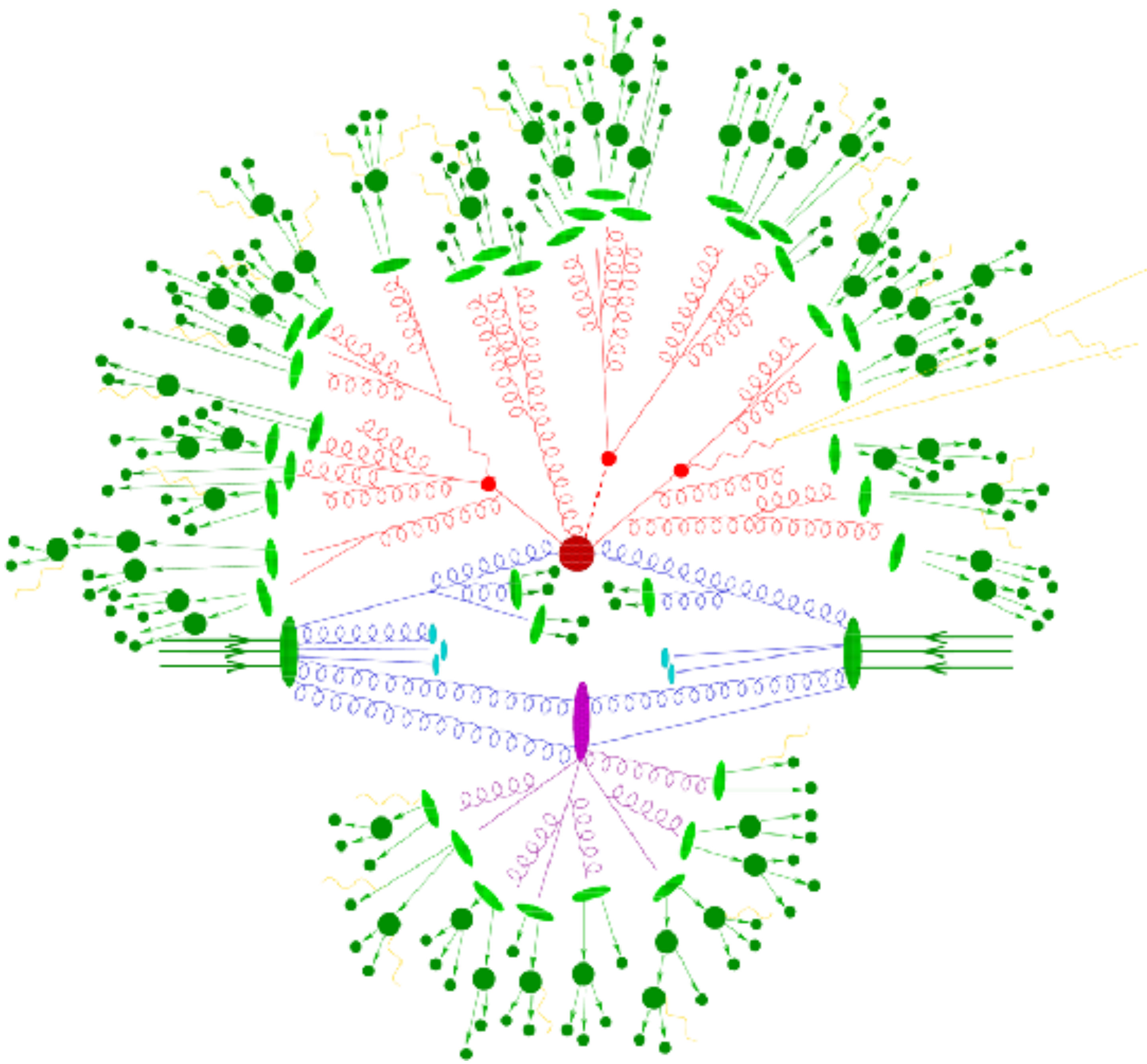


Combining all three types of calculations



Recent developments in parton showers

Parton showers need to describe many physics effects



- Initial hard interaction
- Radiation of additional partons
- Multi-parton interactions
- Hadronization of resulting partons
- Decay of unstable hadrons

Three main multipurpose parton showers:
Herwig, Pythia, Sherpa

Initial hard interactions

Hard interaction described by differential cross section

$$\mathrm{d}\sigma/\mathrm{d}\Phi_N$$

For simple 2 → 2 interactions at LO, cross section given by very simple formula, and are known for essentially all processes of interest

When producing unstable short-lived particles (top, W, Z) care needs to be taken about the decays of these particles

Initial hard interactions

Combination with more complex hard interactions
will be discussed later

Merging
Use hard interactions with various multiplicities

Matching
Use higher order hard interactions

Three main multipurpose generators have taken different
approaches to this topic

Initial hard interactions

Herwig	Pythia	Sherpa
Recent developments towards combining hard interaction with shower “in house”	Maintained focus on parton shower, with hard interactions provided by others	Original focus of Sherpa was combination of hard interaction and shower
New framework in Herwig 7.1 built to perform matching / merging directly in Herwig	Provides tools (UserHooks) to allow interface through standards such LHE	Has both internal matrix element generators, as well as interface to external codes

Radiation of additional partons

Parton shower creates additional radiation

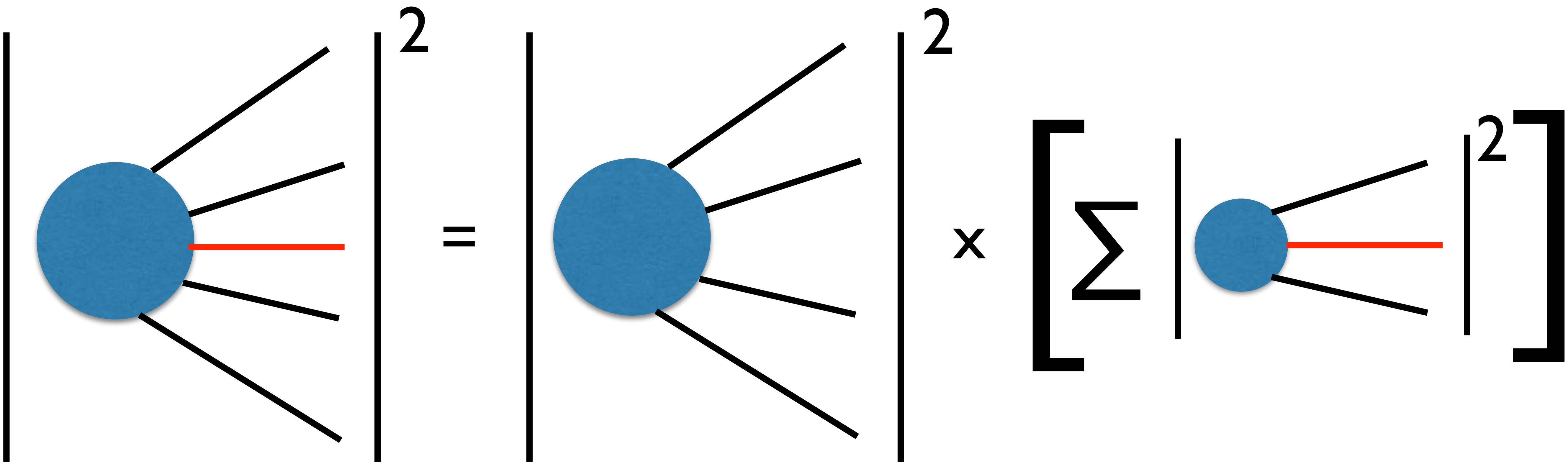
All showers rely on collinear / soft limit of QCD, combined with Unitarity (conservation of inclusive cross section)

In this collinear / soft limit, emissions can be treated in a probabilistic manner, allowing for Markov process

Arbitrary number of emissions can be implemented using simple and efficient algorithms

In the collinear / soft limit QCD cross sections simplify

One can easily show that in collinear / soft limit



$$\sigma_N = \sigma_{N-1} \times \sum SP(t,z)$$

Higher multiplicities built recursively from lower multiplicities

Unitarity is a simple statement about inclusive cross sections

Shower 2 → 2 process with cross section σ_2

Unitarity states that the inclusive cross section after the shower should be unchanged

$$\sigma_2 = \sigma_2^{PS} + \sigma_3^{PS} + \sigma_4^{PS} + \ldots$$

σ_2^{PS} : σ_2 x P(no emission)

σ_3^{PS}: σ_2 x P(1 emission)

σ_4^{PS}: σ_2 x P(2 emissions)

…

Unitarity can be interpreted as “conservation of probability”, namely

P(no emission) + P(≥1 emissions) = 1

Combining this, one arrives at the general formula

The basic equation underlying a parton shower is

$$\langle O \rangle = G_N(Q_N, O)$$

<O>: expectation value of observable
G_N: Shower generating functional
N: Multiplicity of hard interaction

Generating functional can symbolically be written as

$$G_N(t,O) = \frac{\mathrm{d}\sigma}{\mathrm{d}\Phi_N}\left[\Pi_N(t,t_c)\langle O\rangle_{\Phi_N} + \int_{t_c}^{t}\mathrm{d}t'\,\Pi_N(t,t')\,\mathrm{SP}(t')\,G_{N+1}(t',O)\right]$$

This gives recursive definition (with t_c) being shower cutoff

The basic idea of a parton shower

$$G_N(t,O) = \frac{\mathrm{d}\sigma}{\mathrm{d}\Phi_N}\left[\Pi_N(t,t_c)\langle O\rangle_{\Phi_N} + \int_{t_c}^{t}\mathrm{d}t'\,\Pi_N(t,t')\,\mathrm{SP}(t')\,G_{N+1}(t',O)\right]$$

Expand recursive definitions to a few orders (with N=2)

$$\langle O\rangle = \frac{\mathrm{d}\sigma}{\mathrm{d}\Phi_2}\Bigg[\Pi_2(Q,t_c)\langle O\rangle_{\Phi_2} + \int_{t_c}^{Q}\mathrm{d}t_1\,\Pi_2(Q,t_1)\,\mathrm{SP}(t_1)\,\Pi_3(t_1,t_c)\langle O\rangle_{\Phi_3}$$

$$+ \int_{t_c}^{Q}\mathrm{d}t_2\,\int_{t_c}^{t_1}\mathrm{d}t_2\,\Pi_2(Q,t_1)\,\mathrm{SP}(t_1)\,\Pi_3(t_1,t_2)\,\mathrm{SP}(t_2)\Pi_4(t_2,t_c)\langle O\rangle_{\Phi_4}$$

$$+ \ldots\Bigg]$$

A parton shower is probabilistic description that relies on

$\Pi_N(t,t_c)$: probability that N-body system does not change between t_1 and

$\mathrm{SP}(t)$: probability of one emission at scale t

Probabilistic evolution requires unitarity

The two main building blocks of a parton shower are

$\Pi_N(t, t_c)$: probability that N-body system does not change between t_1 and

$\mathrm{SP}(t)$: probability of one emission at scale t

Probability conservation (unitarity) requires

$$P_{\text{no branch}} = 1 - P_{\text{branch}}$$

This gives a relation between the splitting function and no-branching probability

$$\Pi_N(t, t_c) = \exp\left\{ - \int_{t_c}^{t} \mathrm{d}t' \sum_{i=1}^{N} \mathrm{SP}_i(t') \right\}$$

Radiation of additional partons

All showers rely on this basic recursive formula

$$G_N(t,O) = \frac{\mathrm{d}\sigma}{\mathrm{d}\Phi_N}\left[\Pi_N(t,t_c)\langle O\rangle_{\Phi_N} + \int_{t_c}^{t}\mathrm{d}t'\,\Pi_N(t,t')\,\mathrm{SP}(t')\,G_{N+1}(t',O)\right]$$

This gives predictions with the following accuracy

- Only correct in the large N_C limit
- Only correct for collinear / soft radiation
- Leading logarithmic resummation of logs

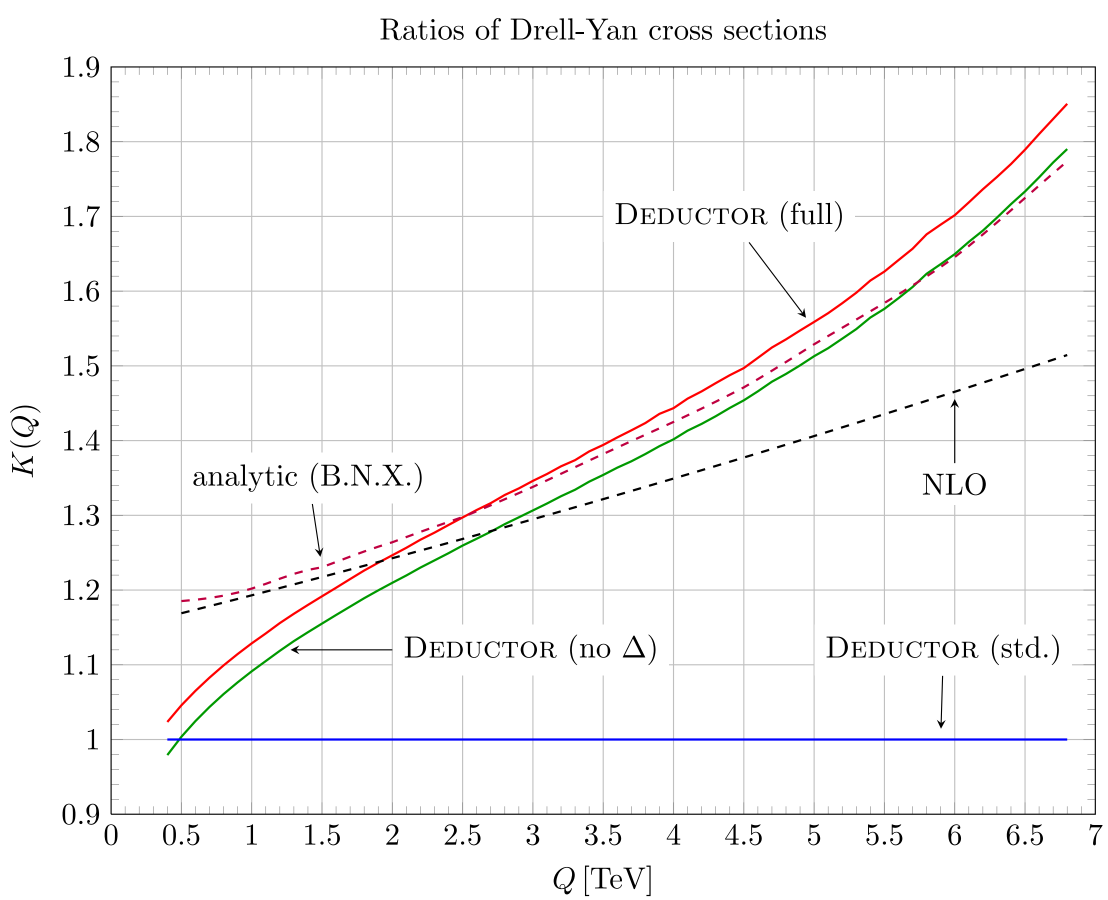
Different shower algorithms make different choices about details of splitting probabilities P(t,z)

Differences between perturbative showers should all be beyond the accuracy of the shower

Many ideas of how to go beyond the standard parton shower picture discussed here. Two examples:

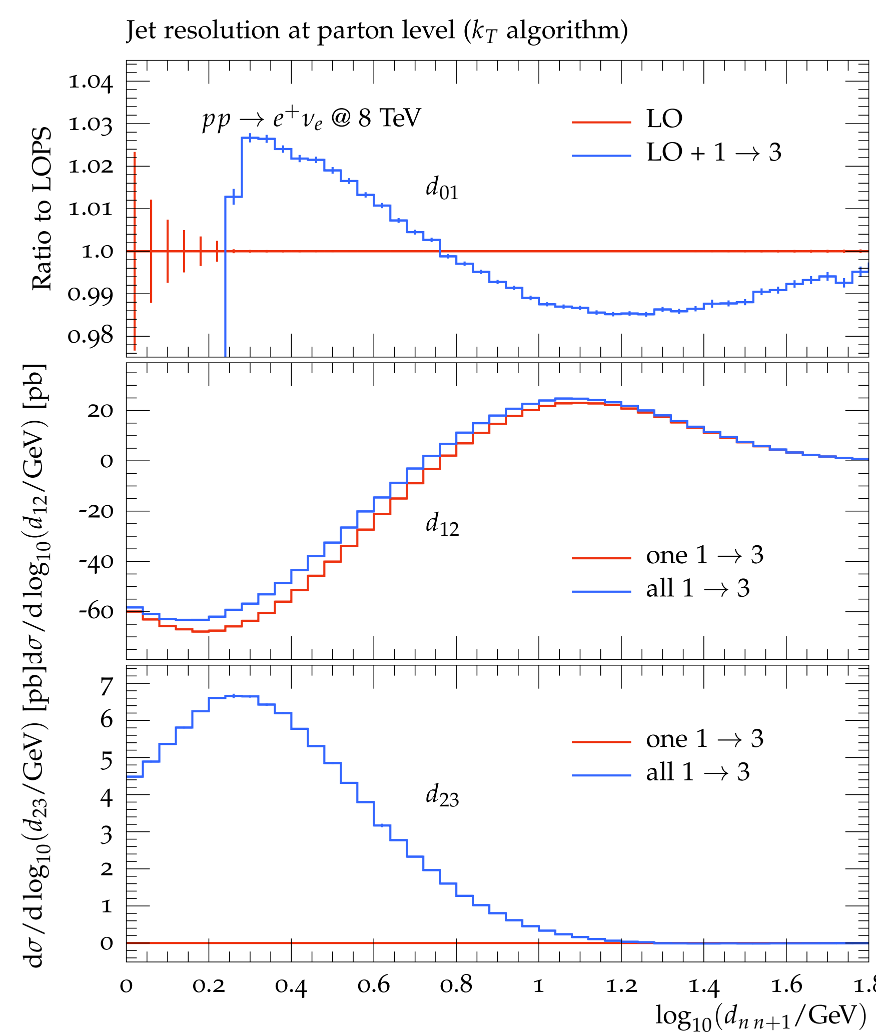
Implement threshold resummation

Nagy, Soper ('16)

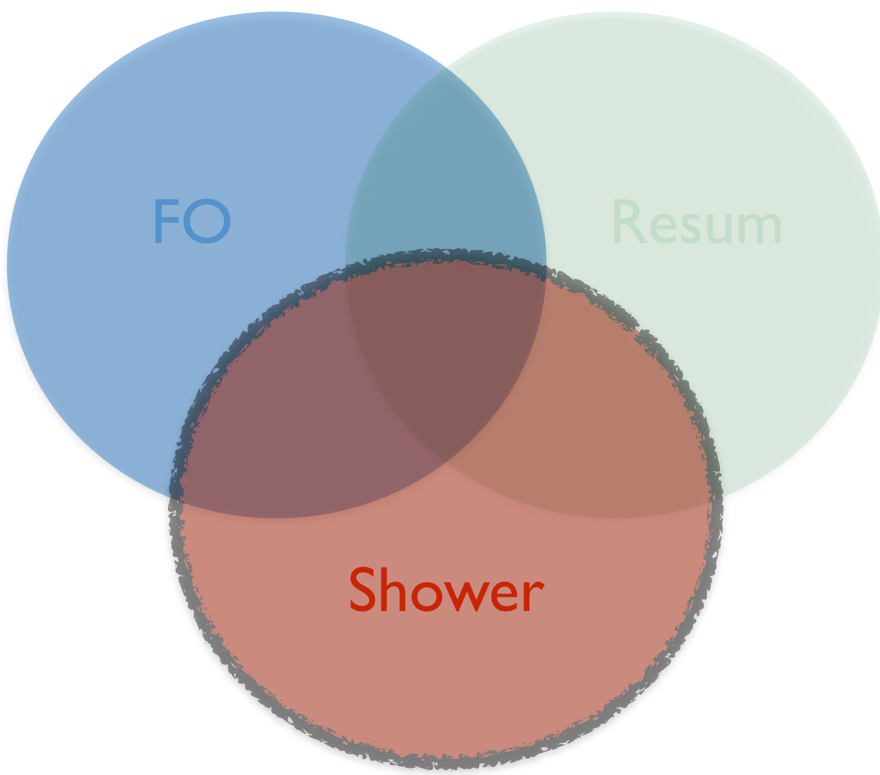


Include higher order splitting functions

Hoeche, Prestel ('17)



So far, these ideas have not increased the overall formal accuracy of showers, but will probably see more ideas soon



Combining FO with showers

Extending the validity of parton showers

Goal of combination of FO and parton showers is

- For given hard multiplicity, results correct to given FO accuracy
- Higher mult only correct in large N_C limit
- Higher mult only correct for collinear / soft radiation
- Leading logarithmic resummation of logs

LO match	**LO Merge**	**NLO Match**	**…**
Lowest multiplicity correct to LO	Several multiplicities correct to LO	Lowest multiplicity correct to NLO	…

LO matching included in essentially every shower

Start again from the expanded parton shower expression

$$\langle O \rangle = \frac{\mathrm{d}\sigma}{\mathrm{d}\Phi_2}\left[\Pi_2(Q,t_c)\langle O\rangle_{\Phi_2} + \int_{t_c}^{Q}\mathrm{d}t_1\,\Pi_2(Q,t_1)\,\mathrm{SP}(t_1)\,\Pi_3(t_1,t_c)\langle O\rangle_{\Phi_3} + \ldots\right]$$

LO matching included in essentially every shower

Start again from the expanded parton shower expression

$$\langle O\rangle = \boxed{\frac{\mathrm{d}\sigma}{\mathrm{d}\Phi_2}}\left[\Pi_2(Q,t_c)\langle O\rangle_{\Phi_2} + \int_{t_c}^{Q}\mathrm{d}t_1\,\Pi_2(Q,t_1)\boxed{\mathrm{SP}(t_1)}\Pi_3(t_1,t_c)\langle O\rangle_{\Phi_3} + \ldots\right]$$

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Separate the fixed order pieces from the resummed pieces

LO matching included in essentially every shower

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Separate the fixed order pieces from the resummed pieces

Φ_2	Φ_3
$\frac{\mathrm{d}\sigma_2}{\mathrm{d}\Phi_2}$	$\frac{\mathrm{d}\sigma_2}{\mathrm{d}\Phi_2}\mathrm{SP}(t_1)$
$\Pi_2(Q, t_c)$	$\Pi_2(Q, t_1)\,\Pi_3(t_1, t_c)$

LO matching included in essentially every shower

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Separate the fixed order pieces from the resummed pieces

Φ_2	Φ_3
$\frac{\mathrm{d}\sigma_2}{\mathrm{d}\Phi_2}$	$\frac{\mathrm{d}\sigma_2}{\mathrm{d}\Phi_2}\mathrm{SP}(t_1)$
$\Pi_2(Q,t_c)$	$\Pi_2(Q,t_1)\,\Pi_3(t_1,t_c)$

Lowest multiplicity always correct at LO

LO merging requires combination of FO with LL resummation

Catani, Krauss, Kuhn, Webber ('01)

Change the fixed order pieces in the original expression

Φ_2	Φ_3
$\frac{d\sigma_2}{d\Phi_2}$	$\frac{d\sigma_2}{d\Phi_2}\mathrm{SP}(t_1)$
$\Pi_2(Q,t_c)$	$\Pi_2(Q,t_1)\,\Pi_3(t_1,t_c)$

to something that has LO correct for both Φ_2 and Φ_3

Φ_2	Φ_3
$\frac{d\sigma_2}{d\Phi_2}$	$\frac{d\sigma_3}{d\Phi_3}\theta(t_1 > t_M) + \frac{d\sigma_2}{d\Phi_2}\mathrm{SP}(t_1)\theta(t_1 < t_M)$
$\Pi_2(Q,t_c)$	$\Pi_2(Q,t_1)\,\Pi_3(t_1,t_c)$

both multiplicities correct at LO (at large t_1)

NLO matching can be obtained by a “simple replacement” in the original formula

Frixione, Webber (’02)
Nason (’04)

For NLO matching

Φ_2	Φ_3
$\frac{d\sigma_2}{d\Phi_2}$	$\frac{d\sigma_2}{d\Phi_2}\mathrm{SP}(t_1)$
$\Pi_2(Q,t_c)$	$\Pi_2(Q,t_1)\,\Pi_3(t_1,t_c)$

need to change both fixed order and resummed pieces

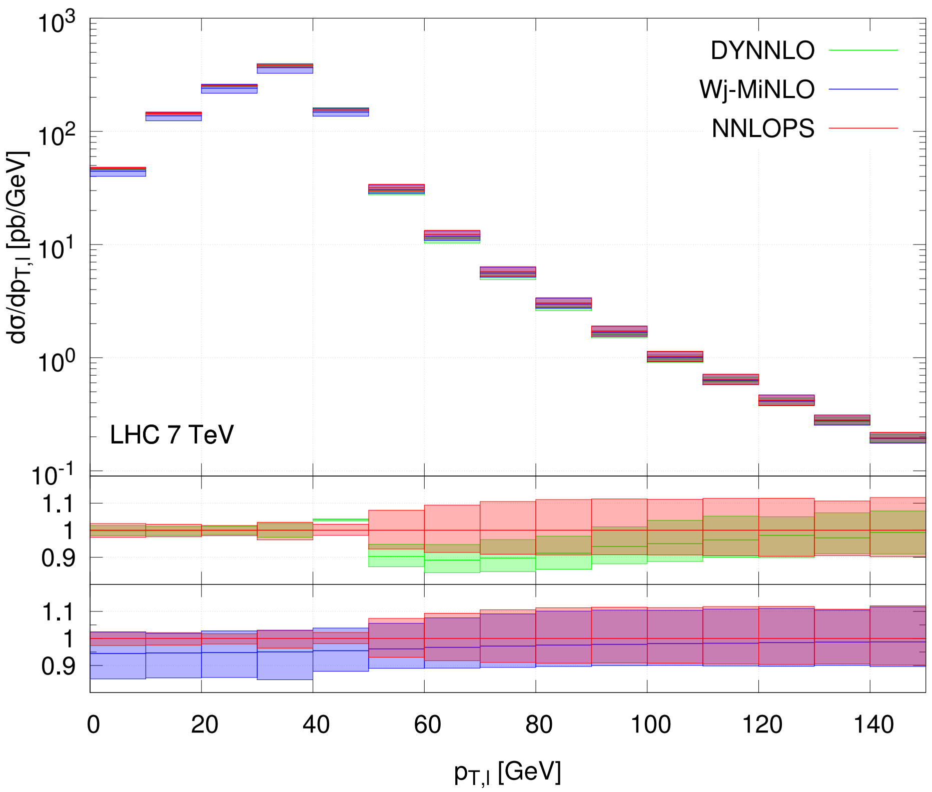
Φ_2	Φ_3
$\frac{d\sigma_2^{\mathrm{NLO}}}{d\Phi_2}$	$\frac{d\sigma_2^{\mathrm{NLO}}}{d\Phi_2}\left[\frac{d\sigma_3}{d\Phi_3}\Big/\frac{d\sigma_2}{d\Phi_2}\right]$
$\Pi_2^R(Q,t_c)$	$\Pi_2^R(Q,t_1)\Pi_3(t_1,t_c)$

Inclusive 2-jet at NLO, 3-jet at LO

Another recent development is NNLO + PS

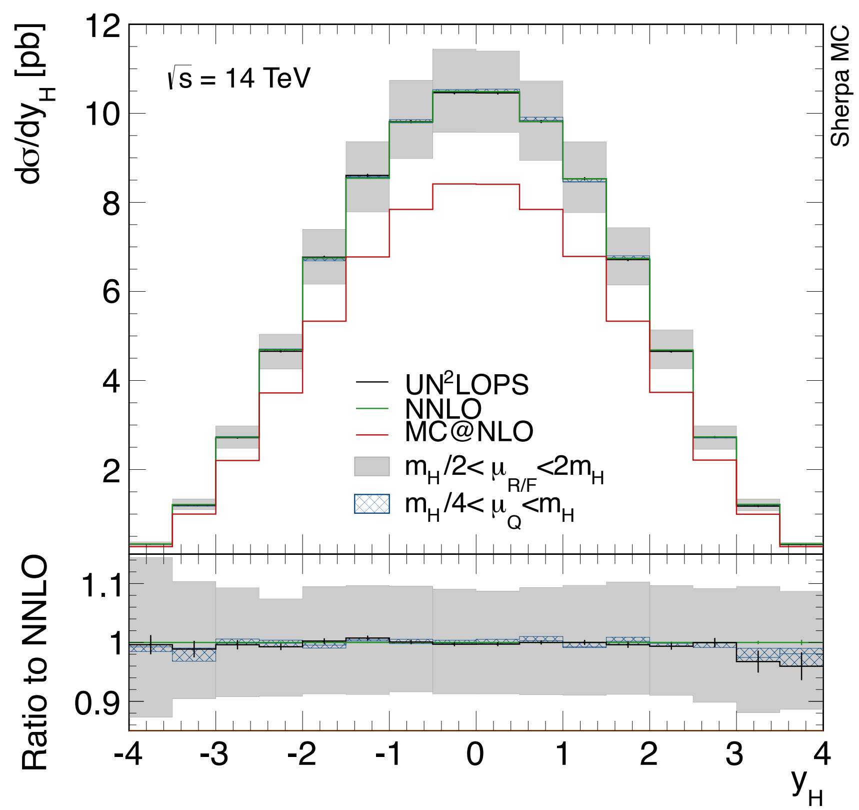
There are three main methods available at this point

MINLO-NNLOPS



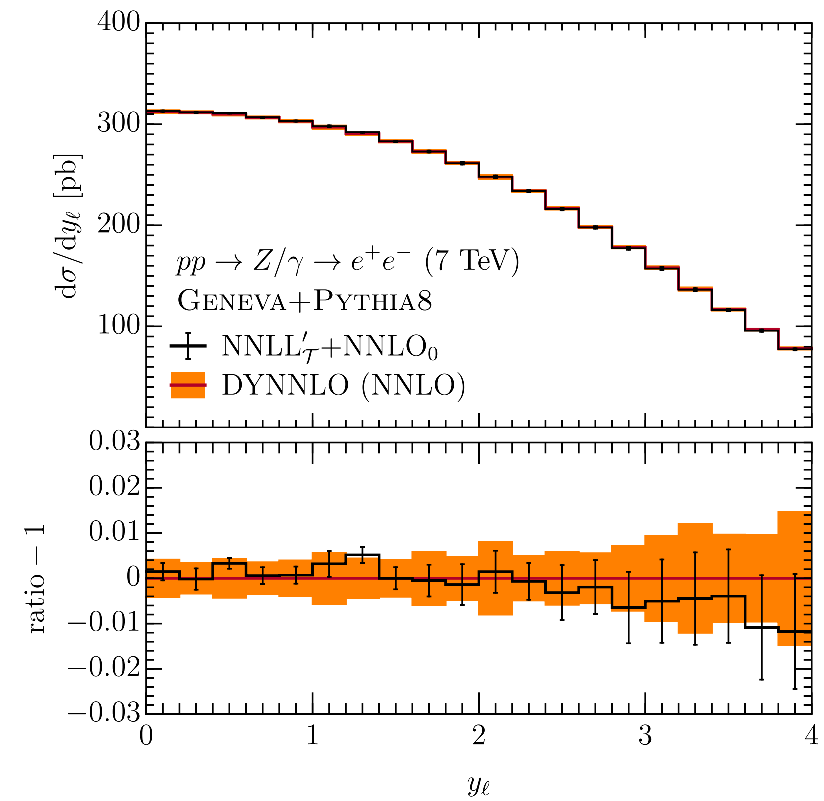
Hamilton, et al ’13 - ‘16

UNNLOPS



Hamilton, et al ’14 - ‘16

Geneva



Alioli, CWB et al, et al ’13 - ‘16

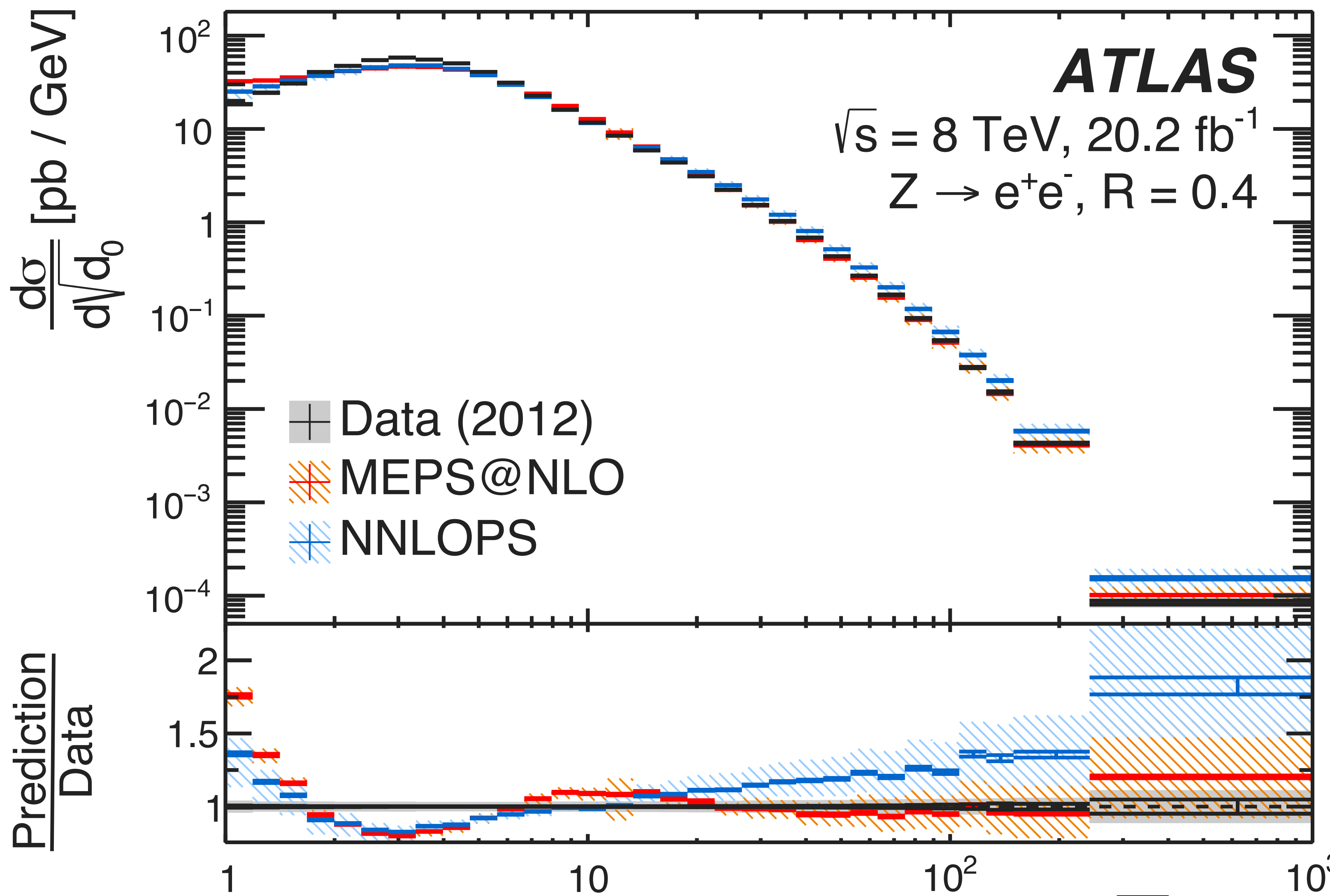
MINLO improved NLO reweighted to NNLO

N-jettiness slicing and Unitarity

N-jettiness slicing and NNLL’ resummation

Another recent development is NNLO + PS

NNLO + PS predictions are already being used by experimental collaborations



ATLAS (1704.01530)

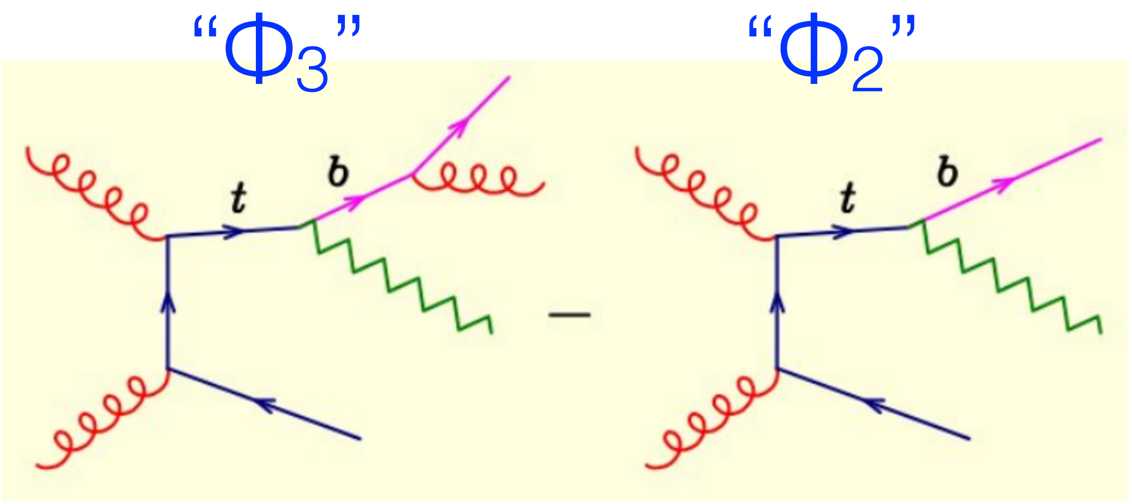
Some questions as to why NNLOPS and MEPS@NLO disagree for $\sqrt{d_0}$

Recently resonant aware NLO matching schemes have been developed

Jezo, Lindert, Nason, Oleari, Pozzorini ('16)

The problem can easily by understood from the previous table

Φ_2	Φ_3
$\dfrac{\mathrm{d}\sigma_2^{\mathrm{NLO}}}{\mathrm{d}\Phi_2}$	$\left[\dfrac{\mathrm{d}\sigma_3}{\mathrm{d}\Phi_3} \Big/ \dfrac{\mathrm{d}\sigma_2}{\mathrm{d}\Phi_2}\right]$
$\Pi_2^R(Q, t_c)$	$\Pi_2^R(Q, t_1)\Pi_3(t_1, t_c)$



Unless one is careful, mapping from "Φ_3" onto "Φ_2" does not maintain resonance, such that [$\mathrm{d}\sigma_3/\mathrm{d}\Phi_3$ / $\mathrm{d}\sigma_3/\mathrm{d}\Phi_3$] can be large away from collinear / soft limit

With resonance aware mapping, this problem is avoided

Recently resonant aware NLO matching schemes have been developed

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Φ_2	Φ_3
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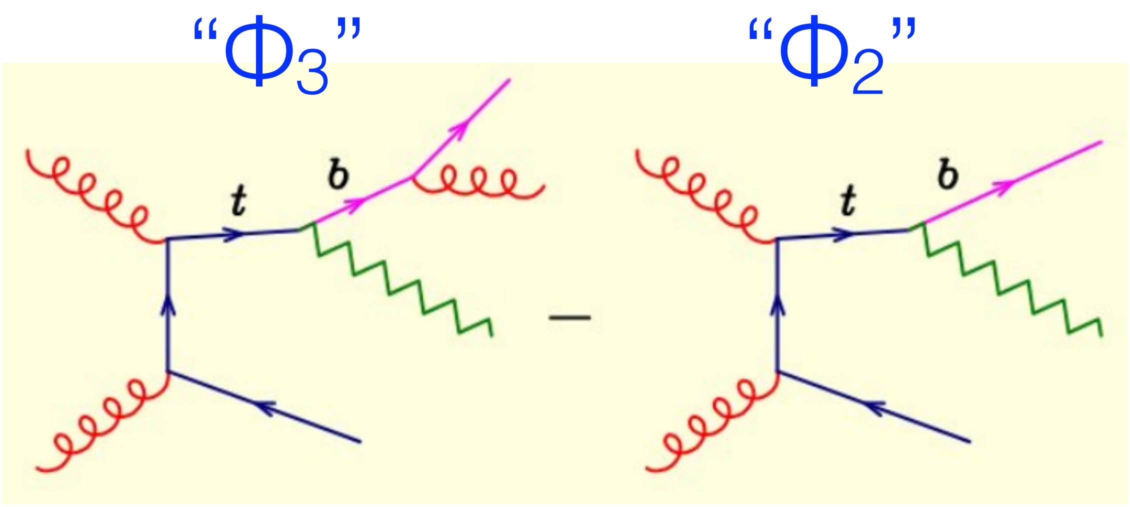
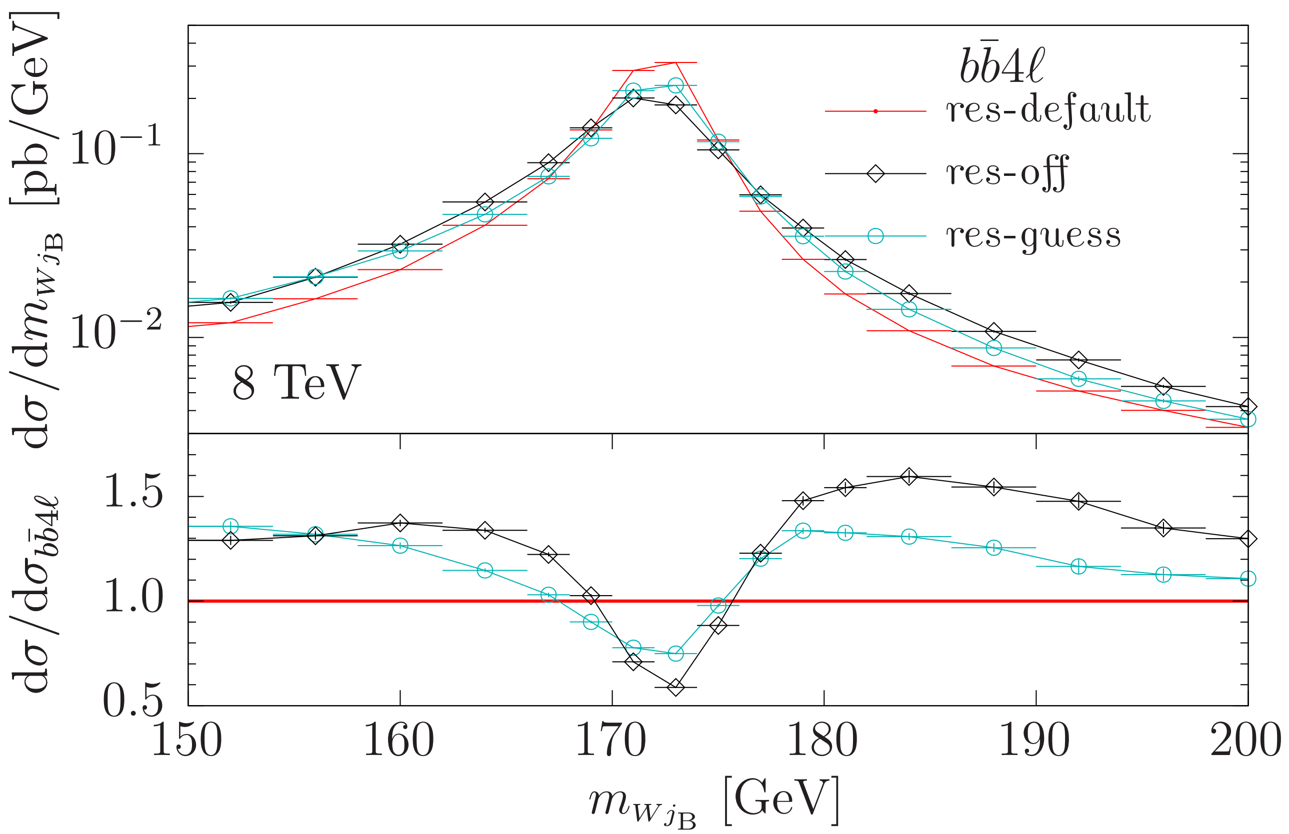
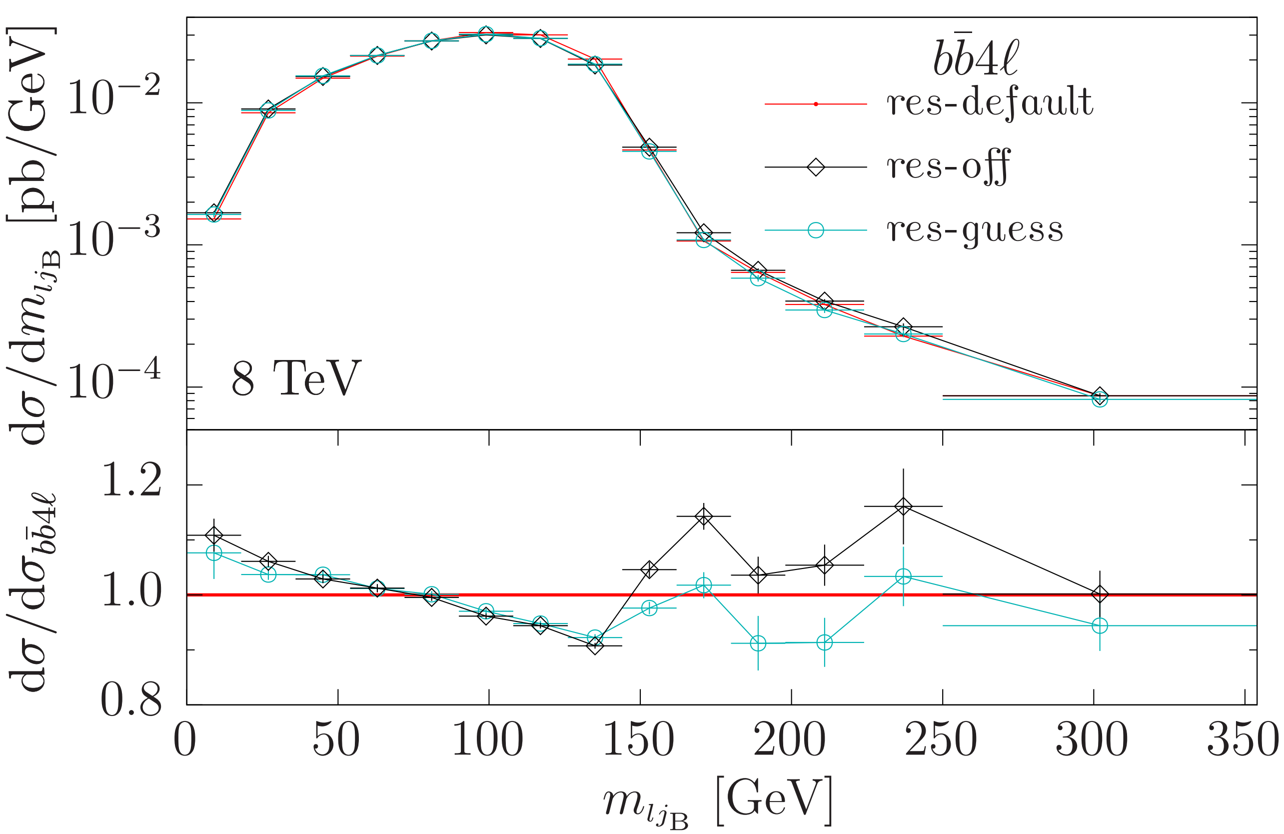
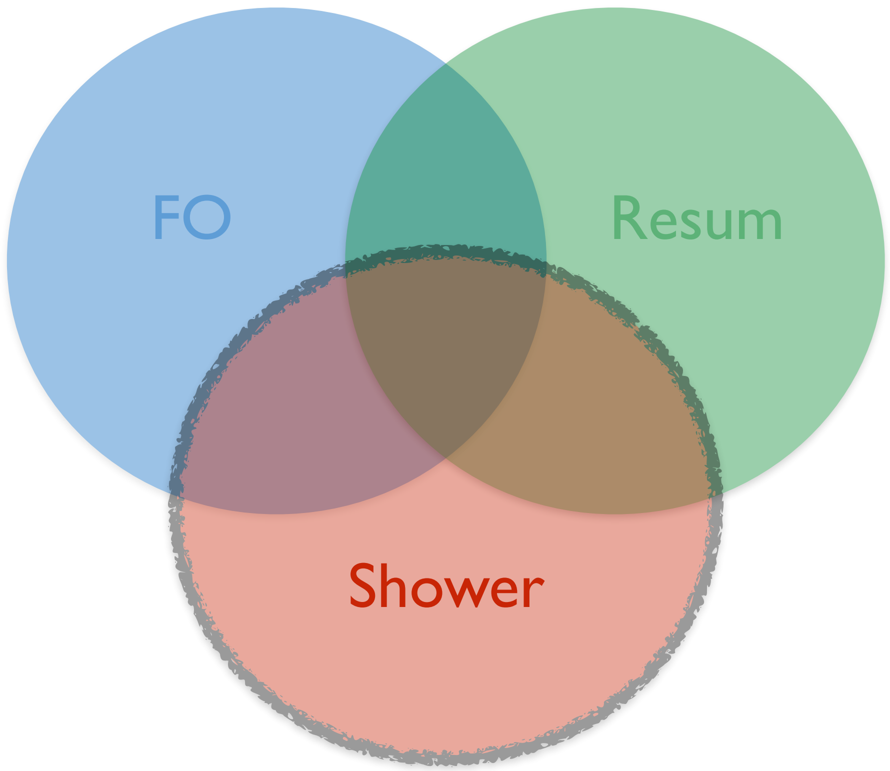
$\Pi_2^R(Q,t_c)$	$\Pi_2^R(Q,t_1)\Pi_3(t_1,t_c)$









Combining all three calculations

Merging higher logarithmic resummation with parton showers has received less attention

Alioli, CWB, Tackmann ('14-'17)

For several observables both higher fixed order as well as higher resummed order is important for precise predictions and reduced theoretical uncertainties

Geneva event generator combines NNLO calculations with NNLL' resummation and a parton shower

One calculates jet cross sections at high order in perturbation theory and then lets a parton shower fill jets with radiation

Higher logarithmic resummation means that expressions can no longer be expressed as FO x Sudakov

Merging higher logarithmic resummation with parton showers has received less attention

Alioli, CWB, Tackmann ('14-'17)

To get combined higher fixed and resummed orders

Φ_2	Φ_3
$\frac{\mathrm{d}\sigma_2}{\mathrm{d}\Phi_2}$	$\frac{\mathrm{d}\sigma_2}{\mathrm{d}\Phi_2}\mathrm{SP}(t_1)$
$\Pi_2(Q, t_c)$	$\Pi_2(Q, t_1)\,\Pi_3(t_1, t_c)$

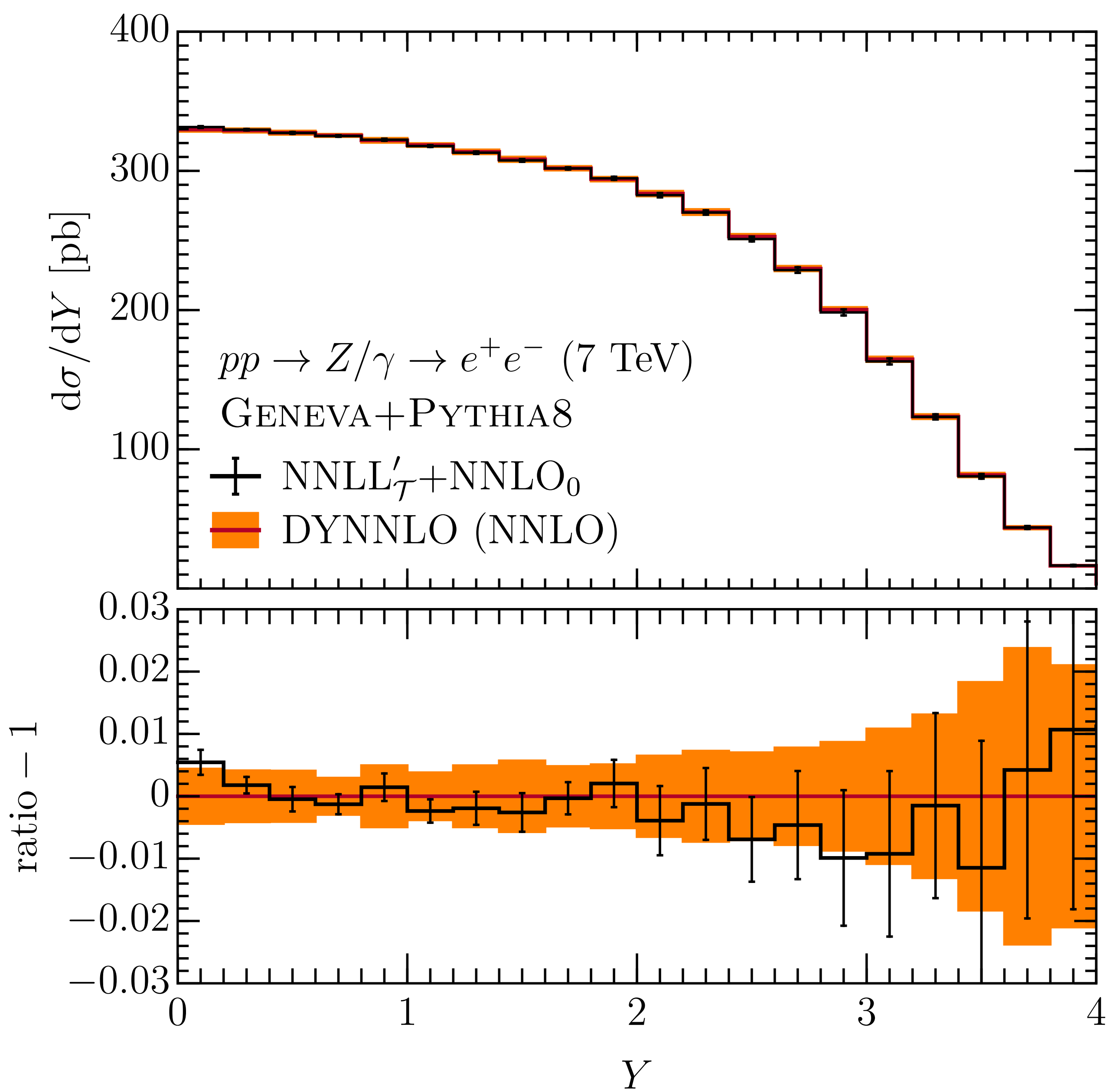
can not separate the two pieces any longer

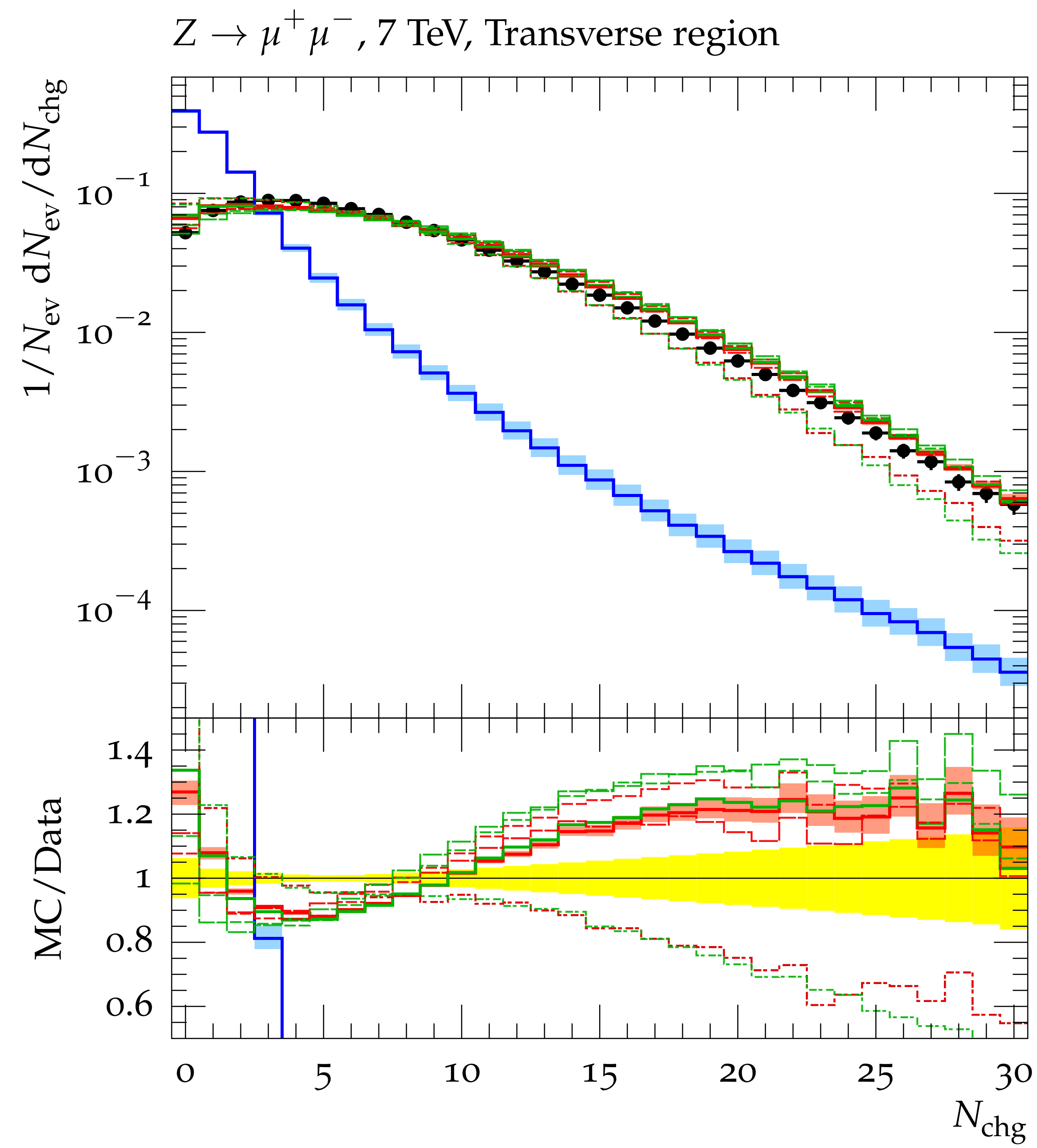
Φ_2	Φ_3
$\frac{\mathrm{d}\sigma_2^{\mathrm{NNLL}'}}{\mathrm{d}\Phi_2}$	$\frac{\mathrm{d}\sigma_3^{\mathrm{NNLL}'}}{\mathrm{d}\Phi_3}$

Combining FO with resummed accuracy gives the right result

Merging higher logarithmic resummation with parton showers has received less attention

Alioli, CWB, Tackmann ('14-'17)





Completely inclusive observables correct to NNLO

Observables sensitive to underlying event still well described

In conclusion, the development of event generators is a very active field of research

Many ideas of including new effects in parton showers

Merging with fixed order calculations is becoming ever more sophisticated

Combination of all three types of approximations is becoming a reality

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STAY TUNED!