SCET and Resummation

Frank Tackmann

Deutsches Elektronen-Synchrotron

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SCET and Resummation

Introduction.

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Consider an observable or measurement that resolves or restricts the phase-space of additional real emissions

- Say it only allows real emissions up to momentum scales of ~ p (i.e. only allows soft and/or collinear emissions but no hard emissions)
- Virtual emissions unconstrained and contribute up to the hard scale $\sim Q$
- As a result the cancellation between real and virtual IR singularities only works up to the scale of the real emission, leaving remnant logs of p/Q

$$\int_{0}^{p} \mathrm{d}k_{\mathrm{real}} \, rac{1}{k_{\mathrm{real}}} - \int_{0}^{Q} \mathrm{d}\ell_{\mathrm{virt}} \, rac{1}{\ell_{\mathrm{virt}}} = \ln rac{p}{Q}$$

Many examples of such IR-sensitive observables

- Low- p_T spectra ($p \simeq p_T$), production near threshold, jet substructure, ...
- More generally: When a problem/process involves several parametrically separate physical scales

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Resummation.

Soft-collinear singularities can cause up to two logarithms at each α_s order $d\sigma = 1 + \alpha_s [\ln^2 \tau + \ln \tau + 1 + \mathcal{O}(\tau)] + \alpha_s^2 [\ln^4 \tau + \ln^3 \tau + \ln^2 \tau + \ln \tau + 1 + \mathcal{O}(\tau)] + \vdots \vdots \vdots \vdots \cdots + \dots]$ $= d\sigma^{(0)} + \mathcal{O}(\tau)$

For $au\equiv p/Q\ll 1$ two related things happen

- Leading-power terms $d\sigma^{(0)}$ ("singular") dominate over $\mathcal{O}(\tau)$ power corrections ("nonsingular")
- As au decreases logarithms grow large deteriorating the $lpha_s$ expansion

Resummation

• Sums up most important terms $\alpha_s^n \ln^{2n} \tau$, $\alpha_s^n \ln^{2n-1} \tau$, $\alpha_s^n \ln^{2n-2} \tau$, ... in $d\sigma^{(0)}$ to all orders (for all *n*)

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To Resum or Not to Resum.



Very small au

- Fixed-order expansion breaks down
- Resummation is necessary to obtain meaningful predictions

To Resum or Not to Resum.



Moderately small $au \sim 0.1 - 0.2$

- Typical use case, often experimentally relevant region of interest
- Motivation is not that one must resum, but rather that doing so makes sense and improves predictions as long as power corrections are subdominant

To Resum or Not to Resum.



Large $au\gtrsim 0.5$

- Power expansion in τ becomes meaningless
- Often large cancellations between formally leading and subleading power terms
- Resummation would spoil these cancellations in which case it must not be done

Soft-Collinear Effective Theory (SCET).

SCET is the effective field theory (EFT) that arises from expanding full QCD in powers of τ at the Lagrangian and operator level

$$QCD = \underbrace{\text{SCET}^{(0)}}_{\text{leading-power}} + \underbrace{\text{SCET}^{(1)}}_{\text{next-to-leading power}} + \mathcal{O}(\tau^2)$$

By definition/construction SCET reproduces the small- τ limit of full QCD. Hence, calculating the cross section with SCET⁽⁰⁾ we exactly get the leading-power result

$$\mathrm{d}\sigma^{(0)} = \langle \mathsf{SCET}^{(0)}
angle$$

- Holds to all orders in α_s
 - Can use SCET to perform resummation with EFT methods
- Also holds nonperturbatively
 - E.g. PDFs defined in QCD and SCET are the same (in both cases the power expansion being the usual twist expansion)
 - Can also define and study other nonperturbative operator matrix elements, e.g. can include hadronization through nonperturbative soft functions

Soft-Collinear Effective Theory (SCET).



- Factorization properties of QCD in soft/collinear limit are manifest in SCET at Lagrangian level
- All pieces are defined through renormalized operator matrix elements
- Resummation is performed by deriving associated RGEs and using their solution to run from one scale to the next

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Simplest Case: Multiplicative RGE (Hard Function).

$$\mu \frac{\mathrm{d} H(Q,\mu)}{\mathrm{d} \mu} = \gamma_H(Q,\mu) H(Q,\mu)$$

Formal solution

$$egin{aligned} H(Q,\mu) &= H(Q,\mu_0) imes \expiggl[\int_{\mu_0}^\mu rac{\mathrm{d}\mu'}{\mu'} \gamma_H(Q,\mu') iggr] \ &\equiv H(Q,\mu_0) imes U_H(\mu_0,\mu) \end{aligned}$$

- Evolution factor $U_H(\mu_0,\mu)$ sums logarithms $\ln^n(\mu_0/\mu)$
 - A priori it only *shifts* logarithms from $\ln^n(Q/\mu_0)$ to $\ln^n(Q/\mu)$
- Resummation requires appropriate choice of $\mu_0=\mu_H\simeq Q$

$$H^{\mathrm{resum}}(Q,\mu) = \underbrace{H^{\mathrm{FO}}(Q,\mu_H \simeq Q)}_{1+lpha_s+\cdots} imes U_H(\mu_H \simeq Q,\mu)$$

- Boundary condition H(Q, μ_H) is free of logarithms (or we can neglect the effects of unresummed but now-small logarithms ln(Q/μ_H))
- Residual dependence on μ_H allows us to probe intrinsic resummation uncertainties by varying μ_H around its canonical value Q

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Resummation Structure in SCET_I.

For observables that constrain invariant mass p^2 (or $p^+ = p^0 - |ec{p}|)$

- e^+e^- event shapes: thrust, C-parameter
- mass-variables: jet mass, dijet invariant mass
- N-jettiness

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• Logs $\ln(p^2/Q^2)$ are resummed with canonical boundary scale choices

$$\mu_H = Q \,, \qquad \mu_J^2 = p^2 = \mu_H \mu_S \,, \qquad \mu_S = p^2/Q$$

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Resummation Structure in SCET_{II}.

For observables that constrain transverse momentum p_T

- Drell-Yan or Higgs q_T , jet- p_T
- N-jettiness with p_T-like measures (as used e.g. in N-subjettiness)



• Logs $\ln(p_T/Q)$ are resummed by canonical boundary scale choices

$$\mu_H = Q, \qquad \nu_B = Q, \quad \mu_B = p_T, \qquad \mu_S = \nu_S = p_T$$

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Multi-scale problems with several power-counting parameters/expansions

Can require sequence of EFTs

 $\mathsf{QCD} \xrightarrow{\tau_1} \mathsf{SCET}(\tau_1) \xrightarrow{\tau_2} \mathsf{SCET}(\tau_2)$

 Or more complicated SCET₊ setups accounting for additional intermediate scales



- Fundamentally, resummation order is (or should be) strictly defined by perturbative input (fixed-order expansions) for the anomalous dimensions and boundary conditions
 - Counting logarithms as α_s ln ~ 1 in the Sudakov exponent maps directly onto this strict definition (only) when the RGE is multiplicative and its solution a pure exponential

	Boundary conditions	Anomalous dimensions		FO matching
	(singular)	$\gamma_{H,B,S, u}$	$\Gamma_{ ext{cusp}},oldsymbol{eta}$	(nonsingular)
NLL	1	1-loop	2-loop	-
NLL'+NLO	$lpha_s$	1-loop	2-loop	$lpha_s$
NNLL+NLO	$lpha_s$	2-loop	3-loop	$lpha_s$
NNLL'+NNLO	$lpha_s^2$	2-loop	3-loop	$lpha_s^2$
N ³ LL+NNLO	$lpha_s^2$	3-loop	4-loop	$lpha_s^2$
N ³ LL'+N ³ LO	$lpha_s^3$	3-loop	4-loop	$lpha_s^3$

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Additional Remarks.

Analytical (RGE based) resummation

- System of RGEs encodes the all-order logarithmic structure
- Corresponding or equivalent evolution equations can be obtained directly by studying the soft-collinear limit of QCD
 - e.g. CSS formalism [Collins, Soper, Sterman '81-'85]
- Once the correct differential equations are known, resummation ultimately amounts to solving a coupled system of more-or-less complicated differential equations
- Different choices and approximations people make along the way should not be perceived as differences between SCET and "direct QCD"

Numerical (Monte-Carlo based) approaches

- Numerically sum up contributions from multiple soft-collinear emissions
- Parton showers matched to FO
 - Provide convenient lowest-order prediction across phase space
- Can also be applied at higher order for classes of observables [Banfi, Salam, Zanderighi '01-'04, Banfi, McAslan, Monni, Zanderighi '14]
- $(\rightarrow$ see talks by C. Bauer, Z. Nagy, D. Reichelt, ...)

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Some Recent Results.

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q_T Resummation at N³LL.

Required: 3-loop rapidity anomalous dimension [Li, Zhu, 1604.01404]

- Based on bootstrapping and new exponential regulator for rapidity divergences [Li, Neill, Zhu, 1604.00392]
- Result confirmed by [Vladimirov, 1610.05791, 1707.07606]
 - Maps rapidity divergences to UV divergences via conformal transformation, leading to all-order relation between soft and rapidity anomalous dimensions
 - Proof of factorization and renormalization of rapidity divergences



q_T Resummation in Momentum (Distribution) Space.

Essential nontrivial ingredient is the rapidity RGE [Chiu, Jain, Neill, Rothstein '12] $\nu \frac{\mathrm{d}S(\vec{p}_T, \mu, \nu)}{\mathrm{d}\nu} = \int \!\mathrm{d}^2 \vec{k}_T \, \gamma_\nu(\vec{k}_T, \mu) \, S(\vec{p}_T - \vec{k}_T, \mu, \nu)$

- Analogous evolution equation appears in all CSS or SCET based formulations
- Formal solution is easily obtained in either Fourier b-space or q_T -space

$$egin{aligned} S(ec{p}_{T},\mu,
u) &= \int\!rac{\mathrm{d}^{2}ec{b}_{T}}{(2\pi)^{2}}\,e^{iec{p}_{T}\cdotec{b}_{T}}\, ilde{S}(ec{b}_{T},\mu,
u_{0})\exp\!\left[\lnrac{
u}{
u_{0}} ilde{\gamma}_{
u}(ec{b}_{T},\mu)
ight] \ &= \int\!\mathrm{d}^{2}ec{k}_{T}\,S(ec{p}_{T}\!-\!ec{k}_{T},\mu,
u_{0})\!\left[\delta(ec{k}_{T})+\sum_{n=1}^{\infty}rac{1}{n!}\ln^{n}rac{
u}{
u_{0}}\,(\gamma_{
u}\otimes^{n})(ec{k}_{T},\mu)
ight] \end{aligned}$$

- Both are exactly equivalent and correctly *shift* logs from ν_0 to ν (at fixed μ)
- Complication lies in correct treatment of boundary condition
 - Easy in *b*-space: $\tilde{S}(\vec{b}_T, \mu = \nu_0 = 1/b) = 1 + \alpha_s + \cdots$
 - Resummation in b space, resums conjugate logarithms $\ln(Qb)$

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q_T Resummation in Momentum (Distribution) Space.

- Several previous attempts [Dokshitzer, Diakonov, Troian '78-'80; Ellis, Veseli '97; Frixione, Nason, Ridolfi '98; Kulesza, Stirling '99-'01]
 - Naive boundary choice v₀ = q_T leads to spurious divergences (causes mistreatment of energetic emissions that balance to give small q_T)
 - One cannot count logarithms $\ln^n(q_T/Q)$ (in the spectrum or cumulant)

Complete momentum-space solution [Ebert, FT, 1611.08610]

- Directly resums $[\ln^n(q_T^2/Q^2)/q_T^2]_+$ in physical q_T distribution
- Valid to any desired resummation order (strictly defined via anom. dim.)
- Requires new distributional scale setting
- Analogous nonpert. sensitivity in rapidity evolution kernel as in b-space

Similar results based on coherent branching formalism

[Bizon, Monni, Re, Rottoli, Torrielli, 1604.02191, 1705.09127] (→ see talk by P. Torrielli)

- Count and resum logs of k_T of hardest emission
- Differs from strict solution (by subleading logs and nonperturbative sensitivity), but allows for numerical implementation using Monte-Carlo methods

Joint q_T and Threshold Resummation Beyond NLL.

- Goal: simultaneously resum both q_T and threshold logs
 - Known before to NLL [Li '98; Banfi, Kulesza, Laenen, Sterman, Vogelsang '00-'04]

Extension to in principle any order [Lustermans, Waalewijn, Zeune, 1605.02740]

[see also Marzani, Theeuwes, 1612.01432; Muselli, Forte, Ridolfi, 1701.01464]

• Regime 1: $q_T/Q \ll 1-z \sim 1$: q_T resummation (SCET_{II})

• Regime 2: $q_T/Q \ll 1-z \ll 1$: joint resummation (SCET $_+$)

- Similar to previous multi-differential (joint) resummations
 [Bauer, FT, Walsh, Zuberi '11; Procura, Waaelwijn, Zeune '14; Larkoski, Moult, Neill '15]
- Regime 3: $q_T/Q \sim 1-z \ll 1$: threshold resummation (SCET_I)
- ⇒ All ingredients known for NNLL' resummation

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Form factor $H(q^2,\mu)$ contains $\ln^n[(-q^2-\mathrm{i}0)/\mu^2]$

[Altarelli, Ellis, Martinelli '79; Parisi '80; Sterman '87; Magnea, Sterman '90; Eynck, Laenen, Magnea '03]

- ullet For timelike $q^2\!>\!0$ and $\mu_{
 m FO}\simeq q^2$ contains large timelike logs $\ln^2(-1)\!=\!-\pi^2$
 - Can be resummed by evolving H from $\mu_H^2 \simeq -q^2$ to $\mu_{
 m FO}$
 - Applied to gg
 ightarrow H total cross section [Ahrens, Becher, Neubert, Yang '08]

- For inclusive cross sections, important to
 - ► consistently reexpand $H(\mu_H)R(\mu_{FO})$ to avoid spurious higher-order terms and to recover FO result in the limit where $\mu_H \rightarrow \mu_{FO}$
 - check that there are no large cancellations between *H* and *R* that might be spoiled by resummation

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[Ebert, Michel, FT, 1702.00794]

 Significantly improved perturbative convergence, yields uncertainties that are smaller (about factor of two) and more reliable

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Top Mass from Boosted Top Jets.

Boosted top production: $pp ightarrow t ar{t}$ at $p_T \gg m_t$

- Top quarks are boosted enough so they can be reconstructed as a single (fat) top jet
- Jet mass of the top-jet is directly sensitive to top quark mass
- Starting to get measured by experiments
 - $(\rightarrow \text{ see talks by F. Stober, A. Buckley})$

 $m_t = 170.8 \pm 6.0$ (stat) ± 2.8 (syst) ± 4.6 (model) ± 4.0 (theo) GeV

Top Mass from Boosted Top Jets.

Factorization-based resummed (so far NLL) hadron-level predictions

- Light soft-drop grooming [Larkoski, Marzani, Soyez, Thaler '14]
 - Reduces MPI/UE effects while retaining explicit control of top mass scheme
- Can measure well-defined m_t by fitting to hadron-level theory predictions
- And/or calibrate Monte-Carlo mass by fitting MC to theory predictions
 - Pythia8 mass parameter is not the same as the pole mass
 - ▶ Consistent with earlier findings from $e^+e^- \rightarrow t\bar{t}$ [Butenschoen, Dehnadi, Hoang, Mateu, Preisser, Stewart, 1608.01318]

Resummation for groomed jet mass for light jets [Frye, Larkoski, Schwartz, Yan, 1603.09338, Marzani, Schunk, Soyez, 1704.02210]

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Analytic Predictions for Groomed Jet Substructure.

0.8

0.2

Groomed D₂ Spectrum PYTHIA

m_J ∈ [30, 40] GeV, p_{TJ} > 500 GeV

Pert+MPI Pert+MPI+Hadr

−− Pert ⊗ F

 $R = 1.0, z_{cut} = 0.1$

[Larkoski, Moult, Neill, 1708.06760]

Resummed hadron-level predictions for Relative Probability groomed 2-prong jet substructure variable D_2

- Based on multi-scale SCET₊ setup
- Grooming removes MPI effects
- Hadronization effects taken into account via nonperturbative shape function

N-Jettiness Subtractions.

[Boughezal, Focke, Liu, Petriello '15; Gaunt, Stahlhofen, FT, Walsh '15]

 Subtractions correspond to leading-power terms, which are obtained using N-jettiness factorization theorem in SCET [Stewart, FT, Waaelwijn '09, '10]

$$\sigma^{
m sub}(au_{
m cut}) = \sigma^{(0)}(au_{
m cut}) \left[1 + \mathcal{O}(au_{
m cut})
ight]$$

Power Corrections for N-Jettiness Subtractions.

[Moult, Rothen, Stewart, FT, Zhu, 1612.00450, 1709.sooon]

SCET provides the tools to systematically study power corrections

- Explicitly constructed to maintain manifest power counting
- Provides organization of different sources of power corrections
 - Insertions of subleading SCET Lagrangian
 - Subleading hard-scattering operators [Feige, Kolodrubetz, Moult, Stewart, Vita '17]
 - Subleading corrections to the measurement/observable

Alternative: Analytically expand NLO V+j calculation [Boughezal, Liu, Petriello '16]

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 - Subleading corrections to the measurement/observable
- It is crucial to use the right N-jettiness definition, otherwise power corrections can grow exponentially with rapidity

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Summary.

Active field with many applications, really only gave you a glimpse

- Resummation is important whenever measurements are sensitive (directly or indirectly) to infrared QCD dynamics (happens more often than not)
 - Even when fixed-order predictions work, resummation can often add perturbative information and improve predictions (smaller and better understood uncertainties)
- SCET applies powerful EFT toolset to study infrared/soft-collinear regime of QCD
 - Includes systematic control of power corrections and nonperturbative effects
- Standard applications of resummation being pushed to higher precision
- Moving toward tackling multi-scale problems
 - ► Jet substructure, jet-radius logarithms, quark mass effects, ...
- Many things I could not cover (apologies ...)
 - ▶ Threshold resummation (→ talks by A. Kulesza, A. Broggio, C. Schwinn)
 - ▶ heavy-ion, e⁺e⁻ collisions (→ talk by Z. Tulipánt)
 - Forward scattering, factorization breaking effects, nonglobal logs

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