Parton shower and top-quark mass effects in Higgs pair production

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MAX-PLANCK-GESELLSCHAFT

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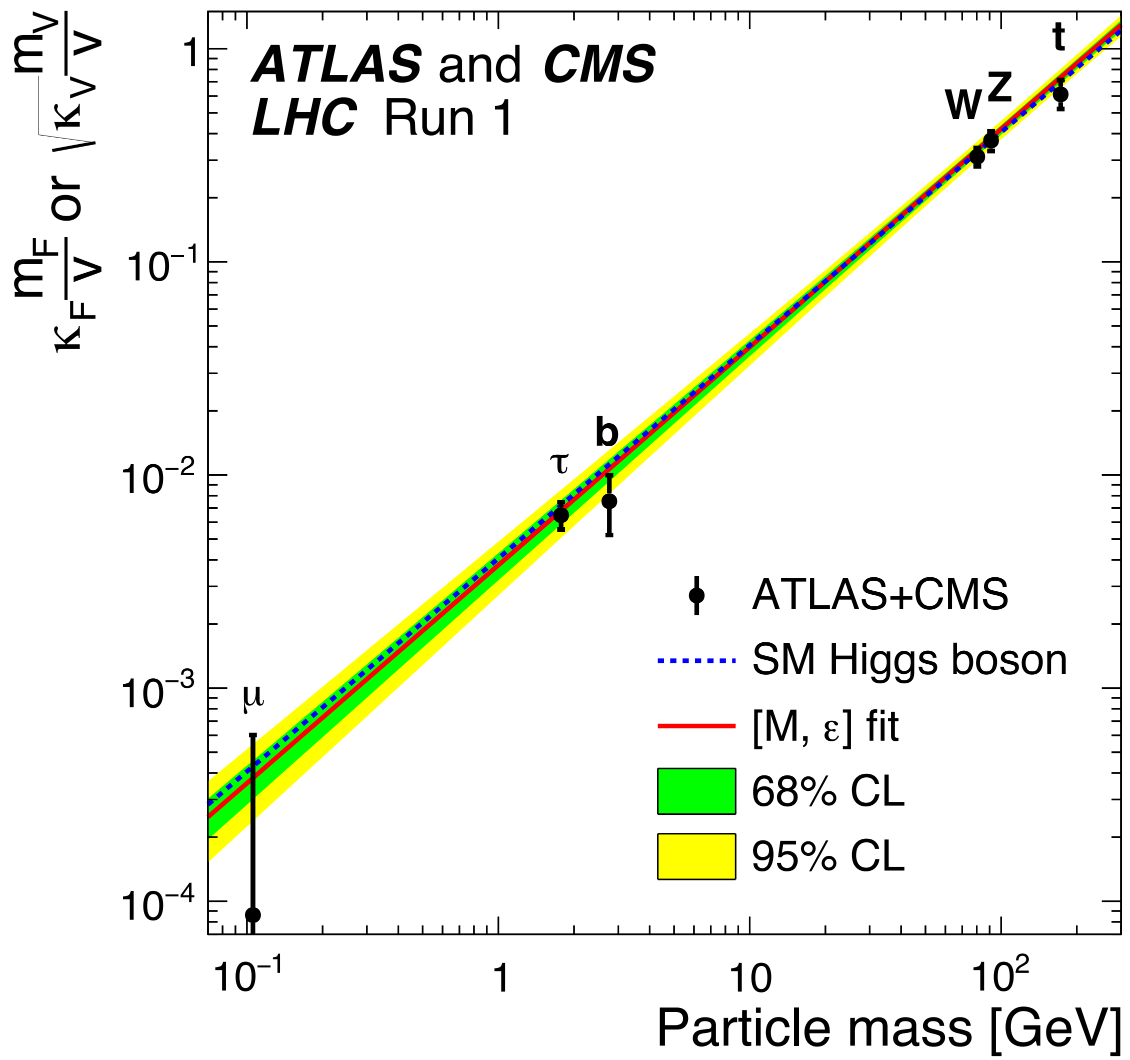
In collaboration with
Borowka, Greiner, Heinrich, Jones, Luisoni, Schlenk, Schubert, Vryonidou, Zirke

JHEP 1708 (2017) 088 [1703.09252]
JHEP 1610 (2016) 107 [1608.04798]
PRL 117 (2016) 012001, Erratum 079901 [1604.06447]

Motivation



$$V(\Phi) = \frac{1}{2}\mu^2\Phi^2 + \frac{1}{4}\lambda\Phi^4$$

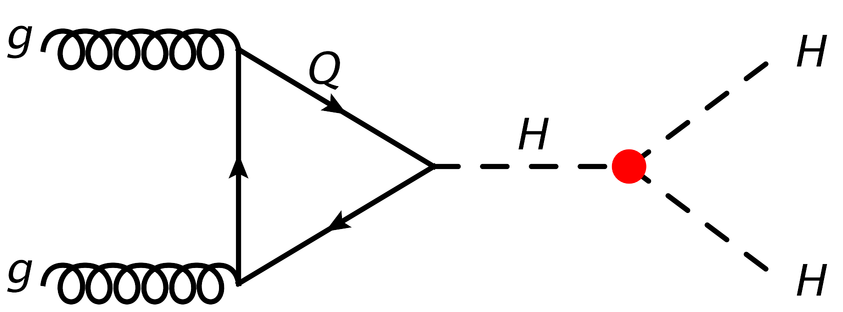
↓ EW symmetry breaking

$$\frac{m_H^2}{2}H^2 + \frac{m_H^2}{2v}H^3 + \frac{m_H^2}{8v^2}H^4$$

Measurements of Higgs couplings agree with SM predictions, but

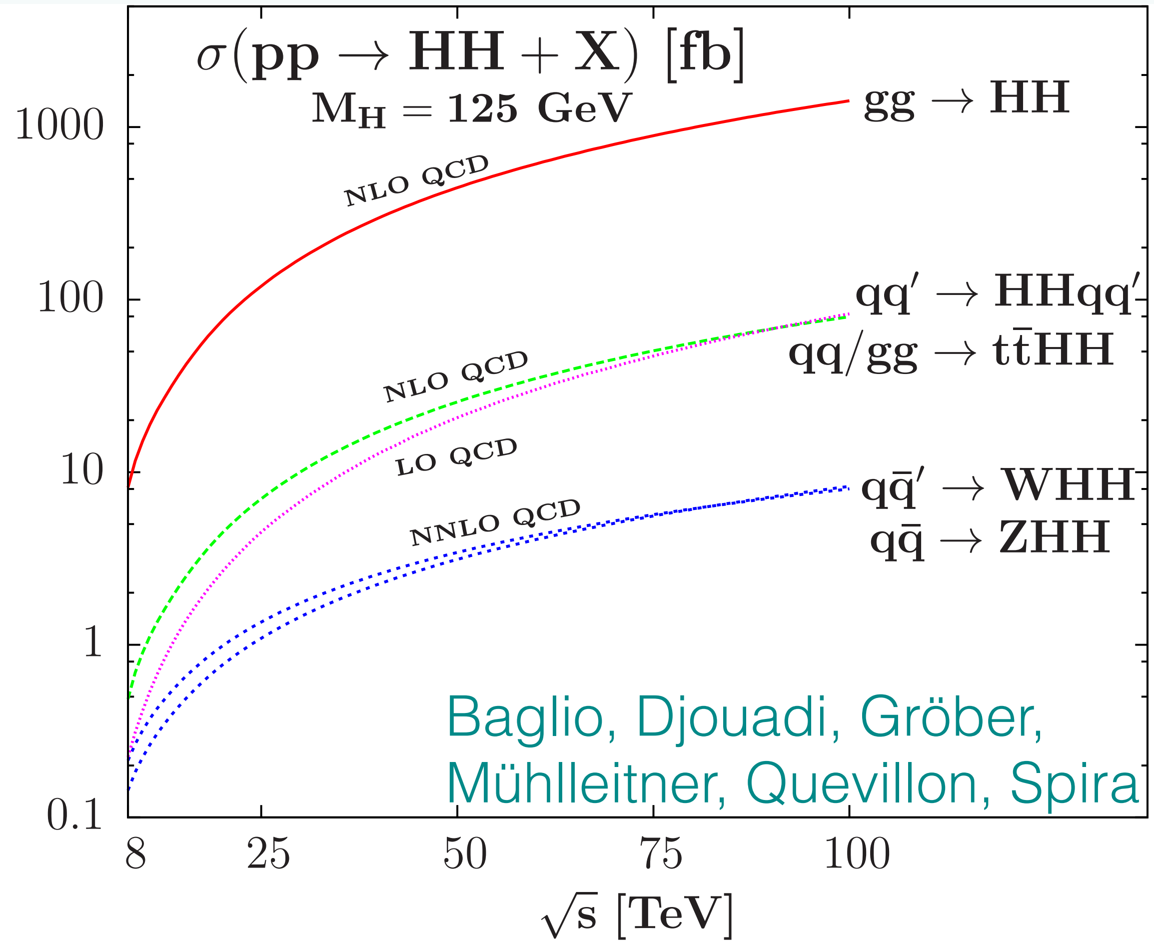
triple-Higgs coupling not established yet

$\rightarrow$ Higgs pair production



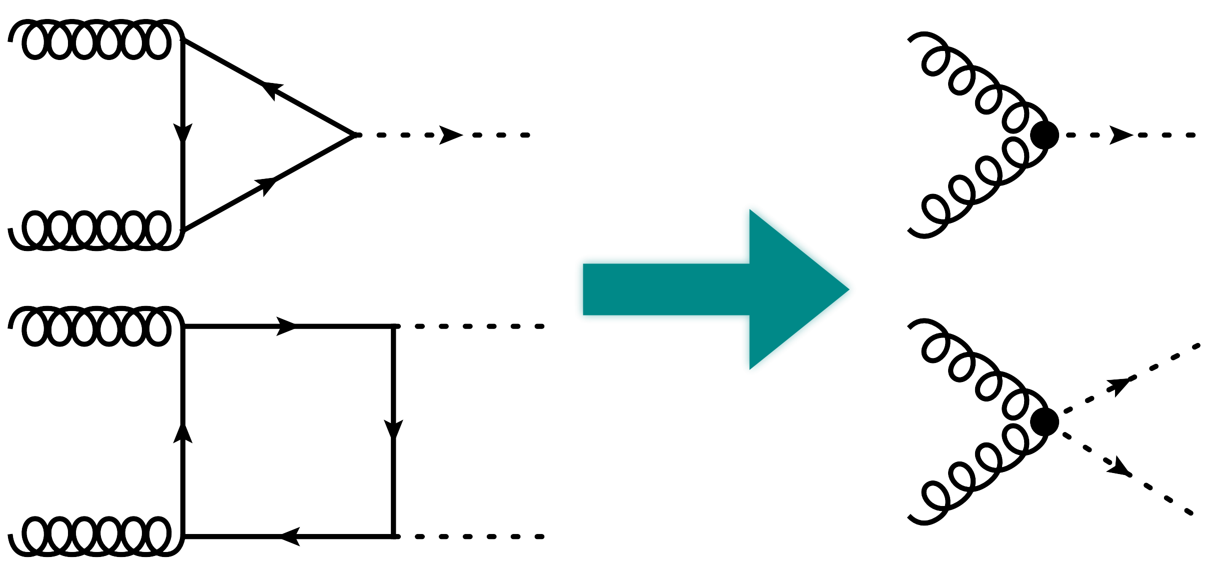
Test of Higgs potential & EW symmetry breaking

Motivation



gluon fusion is dominant production mechanism

Most calculations are done in $m_t \to \infty$ limit (Higgs EFT)



HEFT valid for $\sqrt{s} \ll 2\,m_T$

Higgs pair production: $2\,m_H < \sqrt{s}$

$\to$ only small validity range of HEFT approximation

full top quark mass dependence required for accurate predictions

Overview

- Motivation
- Higgs pair production @NLO with full m_t dependence
  - details of the calculation
  - results
- Parton shower effects
  - interface of virtual amplitude via grid
  - results
- Outlook

gg→HH calculations

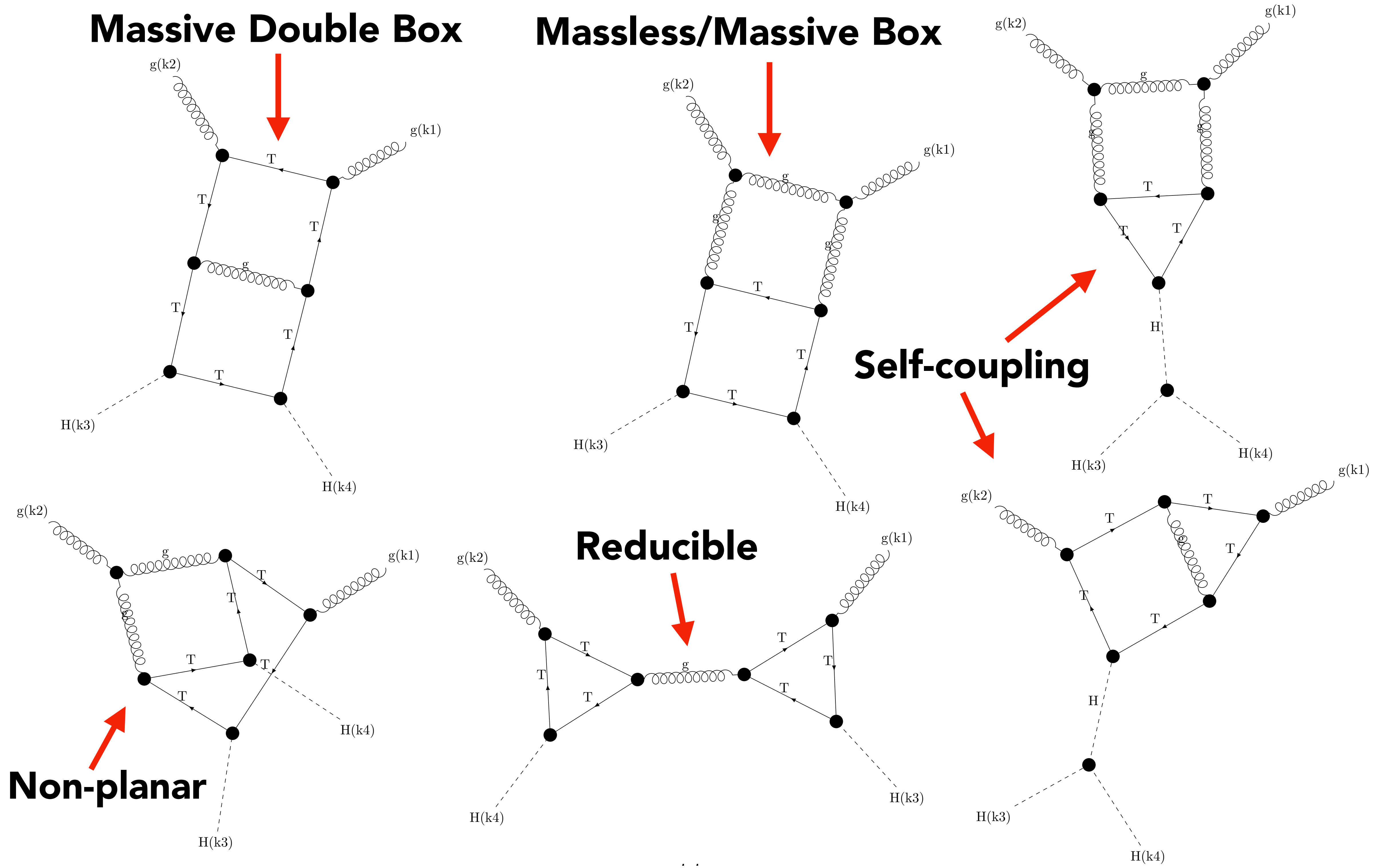
1. LO, including full m_T dependence
   Glover, van der Bij `88

2. NLO, (Born-improved) HEFT **K ≈ 2**
   Dawson, Dittmaier, Spira `98
   - including full m_T dependence in real radiation (FT approx.) **-10%**
     Maltoni, Vryonidou, Zaro `14
   - including $1/m_T$ expansion **±10%**
     Grigo, Hoff, Melnikov, Steinhauser `13;
     Grigo, Hoff, Steinhauser `15
     Degrassi, Giardino, Gröber `16

3. NLO, including full m_T dependence
   Borowka, Greiner, Heinrich, Jones, MK, Schlenk, Schubert, Zirke `16
   - NLO matched to parton shower
     Heinrich, Jones, Luisoni, MK, Vryonidou `17 **this talk**
   - transverse momentum NLL+NLO
     Ferrera, Pires `16

4. NNLO (HEFT) **+20%**
   de Florian, Mazzitelli `13
   - including all matching coefficients
     Grigo, Melnikov, Steinhauser `14
   - including $1/m_T$ expansion
     Grigo, Hoff, Steinhauser `15
   - NNLL soft gluon resummation
     Shao, Li, Li, Wang `13
   - NNLL + NNLO matching
     de Florian, Mazzitelli `15
   - fully differential
     de Florian, Grazzini, Hanga, Kallweit, Lindert, Maierhöfer, Mazzitelli, Rathlev `16

Two Loop Diagrams



most complicated integrals not known analytically
→ numeric calculation required

Loop Integrals

numerical evaluation of loop integrals using

SecDec 3 [Borowka, Heinrich, Jones, MK, Schlenk, Zirke]

- sector decomposition of loop integrals [Binoth, Heinrich]
  → resolves overlapping singularities
- expansion in ε
- contour deformation [Nagy, Soper]

→ Feynman parameter integrals finite for each order in ε

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new version: pySecDec

[Borowka, Heinrich, Jahn, Jones, MK, Schlenk, Zirke]

available at github.com/mppmu/secdec

- new implementation using python and FORM [Kuipers, Ueda, Vermaseren]
- modular structure
- generates library that can be linked to e.g. amplitude code
- many improvements:
  - improved code optimization
  - improved symmetry finder
  - improved treatment of numerators
  - …

Amplitude evaluation

Form factor decomposition of amplitude

Integral reduction using Reduze [von Manteuffel, Studerus]

- quite challenging, simplification: fix m_T, m_H
- didn't achieve reduction of non-planar integrals → evaluated directly
- planar integrals: use finite basis [von Manteuffel, Panzer, Schabinger]

SecDec

Numerical integration

- using Quasi-Monte-Carlo (QMC) integration $\mathcal{O}(n^{-1})$ scaling of integration error
- split each integral into sectors
- dynamically set n for each integral, minimizing

$$T = \sum_{\text{integral } i} t_i + \lambda \left(\sigma^2 - \sum_i \sigma_i^2 \right) \qquad \sigma_i = c_i \cdot t_i^{-e}$$

$\sigma_i = \text{error estimate (including coefficients in amplitude)}$

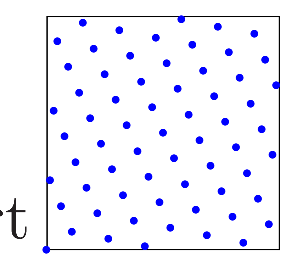
$\lambda = \text{Lagrange multiplier} \qquad \sigma = \text{precision goal}$

- avoid reevaluation of integrals for different orders in ε and form factors
- parallelization on gpu

QMC rank-1 lattice rule

$$I = \int \mathrm{d}\vec{x} f(\vec{x}) \approx I_k = \frac{1}{n} \sum_{i=1}^{n} f(\vec{x}_{i,k})$$

$$\vec{x}_{i,k} = \left\{ \frac{i \cdot \vec{g}}{n} + \vec{\Delta}_k \right\}$$



$\{\dots\} = \text{fractional part}$

$\vec{g} = \text{generating vector}$

$\vec{\Delta}_k = \text{randomized shift}$

m different estimates $I_1 \dots I_m$

$\rightarrow$ error estimate

Li, Wang, Yan, Zhao `15
Review: Dick, Kuo, Sloan

Phase Space Integration and Real Radiation

Fixed order calculation

- Phase space integration based on unweighted Born events
  → close to perfect sampling

  virtual amplitude evaluated
  at ~1000 phase-space points

- Real radiation amplitudes:
  GoSam
  Cullen, van Deurzen, Greiner, Heinrich, Luisoni, Mastrolia, Mirabella, Ossola, Peraro, Schlenk, von Soden-Fraunhofen, Tramontano

- Dipole subtraction
  Catani, Seymour

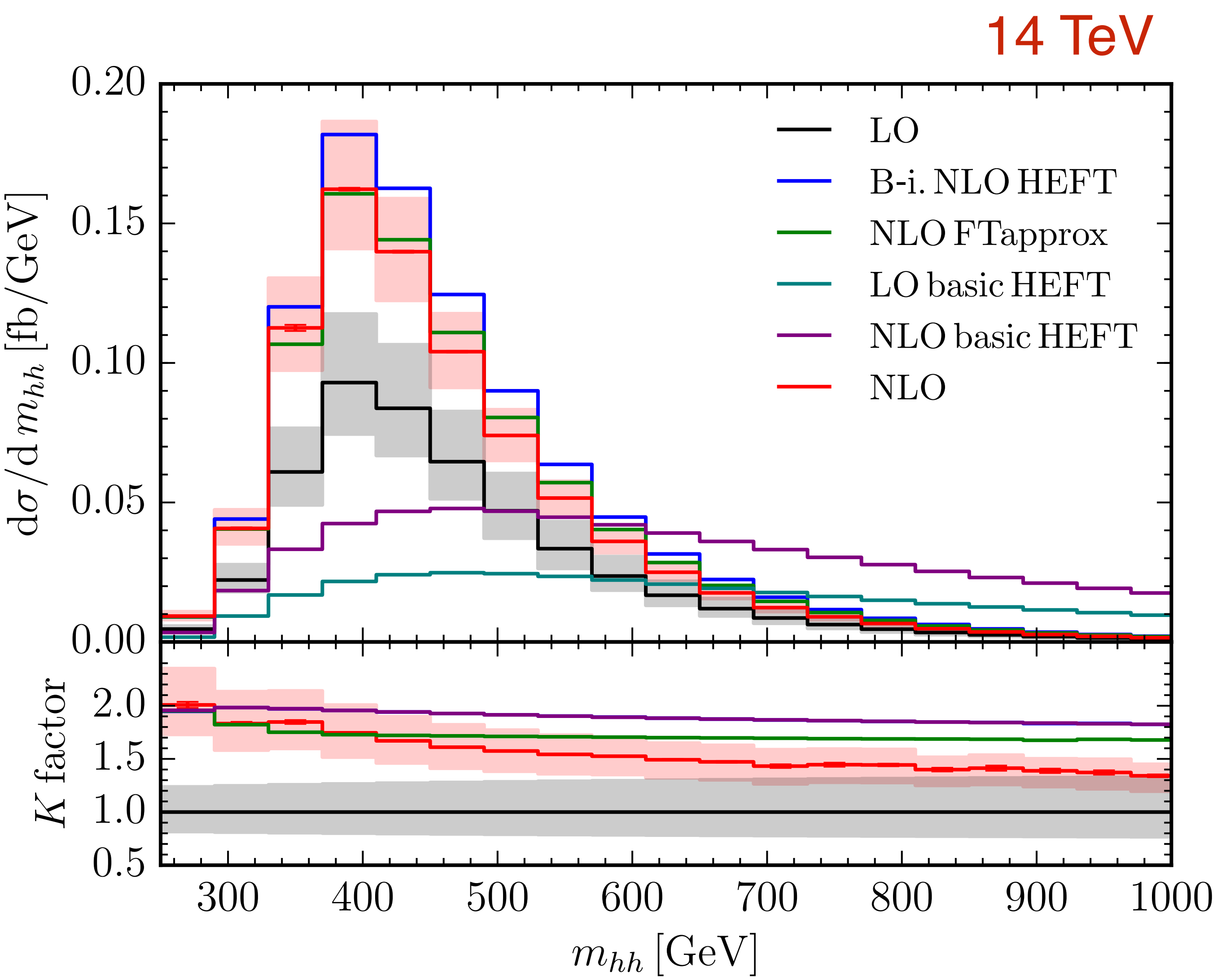
Combination with parton shower

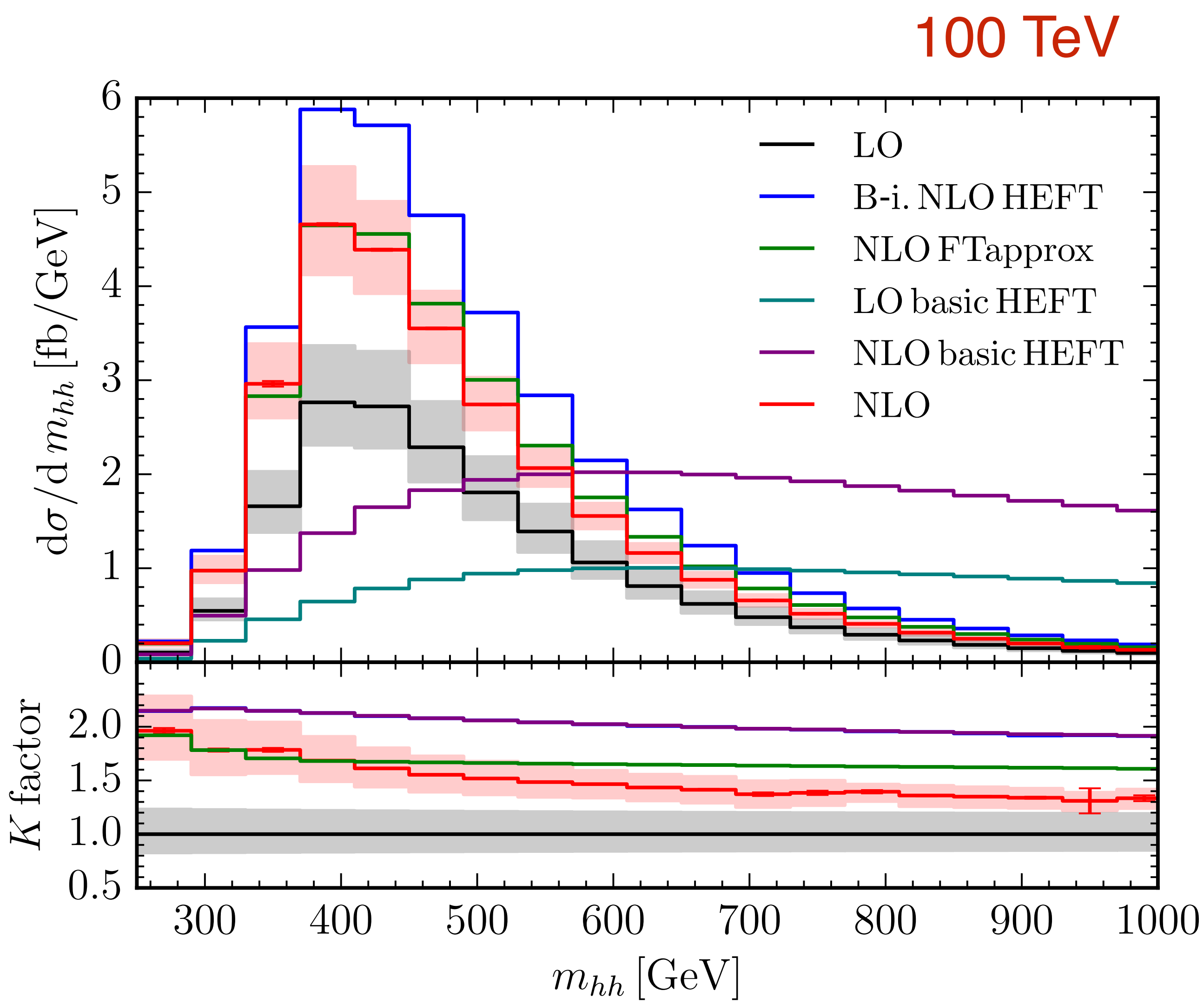
- Interface of virtual amplitude via 2-dimensional grid in s and t

- Parton Shower frameworks:
  - POWHEG-BOX
    using GoSam amplitudes in real radiation
  - MadGraph5_aMC@NLO

- Pythia 8 parton shower

NLO Results — Invariant Mass

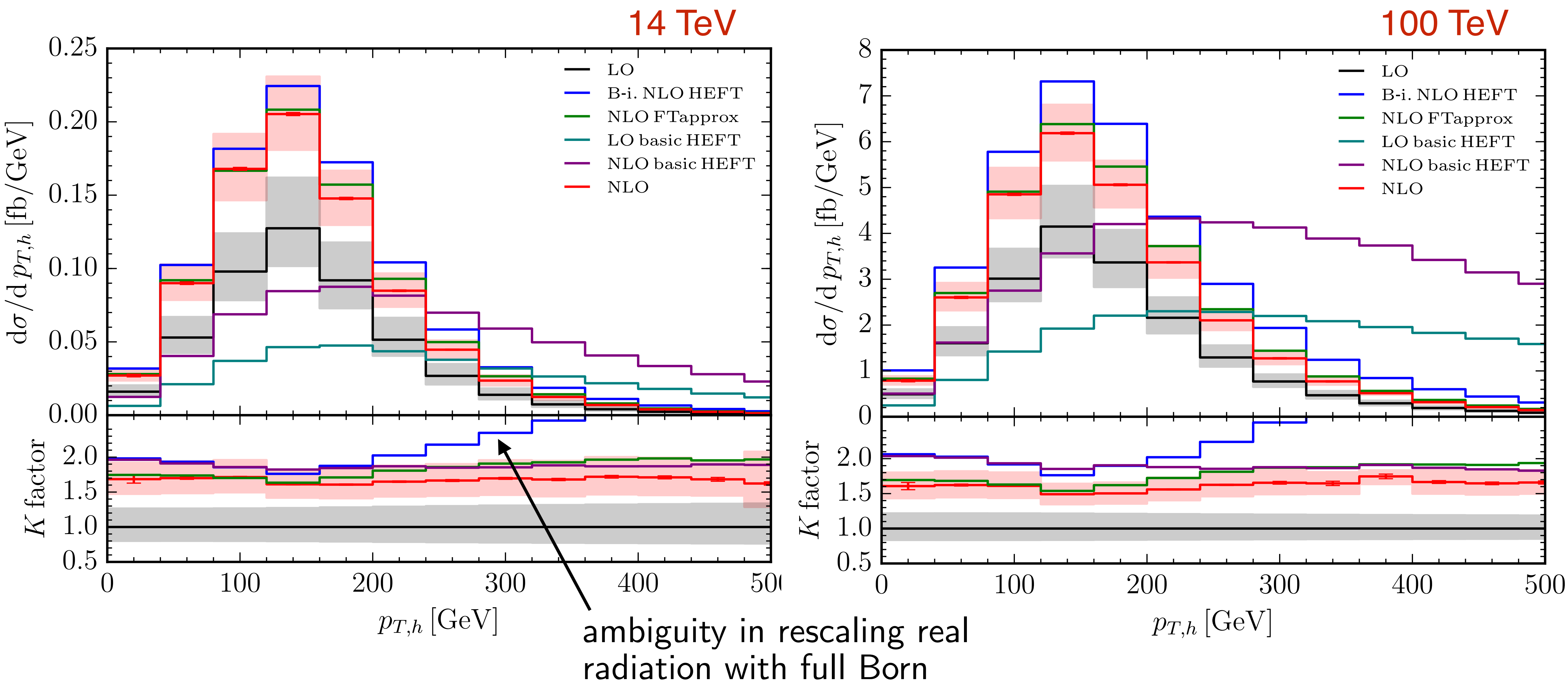




- basic HEFT leads to wrong shape
- B.I. HEFT overestimates by 16% / 30%
- FT approx closer to full result
  (difference increasing with m_{hh})

	14 TeV	100 TeV
LO	$19.85^{+27.6\%}_{-20.5\%}$	$731.3^{+20.9\%}_{-15.9\%}$
B.i. HEFT	$38.32^{+18.1\%}_{-14.9\%}$	$1511^{+16.0\%}_{-13.0\%}$
FT approx	$34.26^{+14.7\%}_{-13.2\%}$	$1220^{+11.9\%}_{-10.7\%}$
NLO full	$32.91^{+13.6\%}_{-12.6\%}$	$1149^{+10.8\%}_{-10.0\%}$

NLO Results — Higgs Momentum



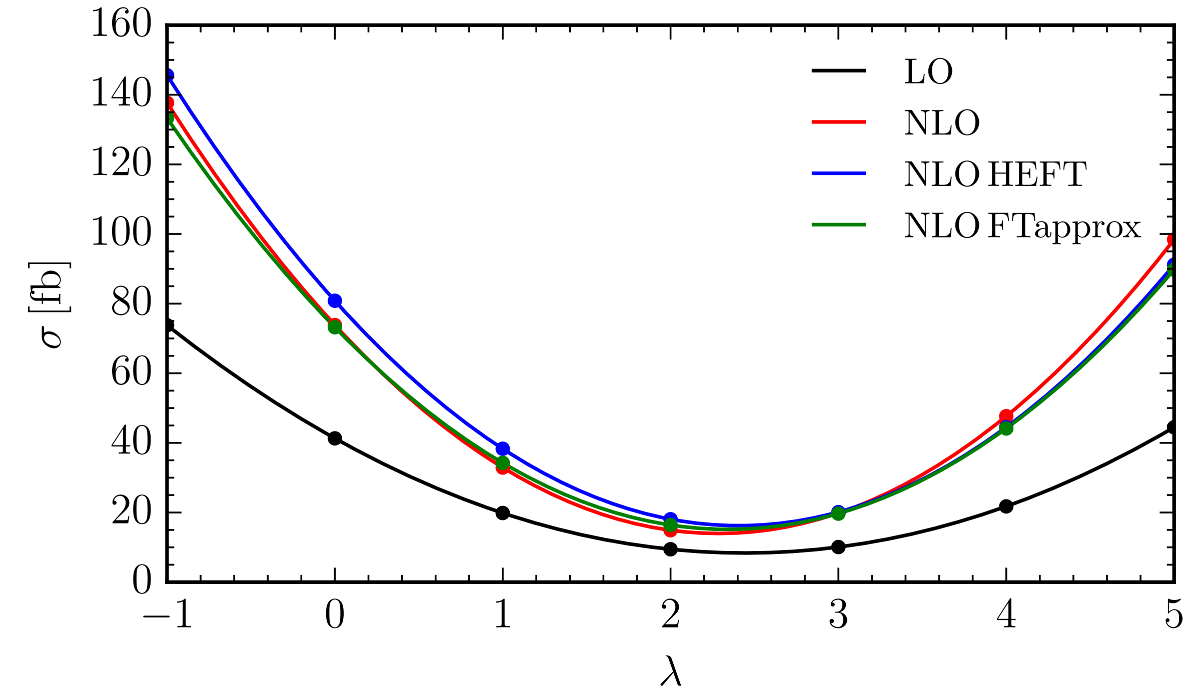
- basic HEFT leads to wrong shape
- B.I. HEFT overestimates by 16% / 30%
- FT approx closer to full result
  (difference increasing with $p_{T,h}$)

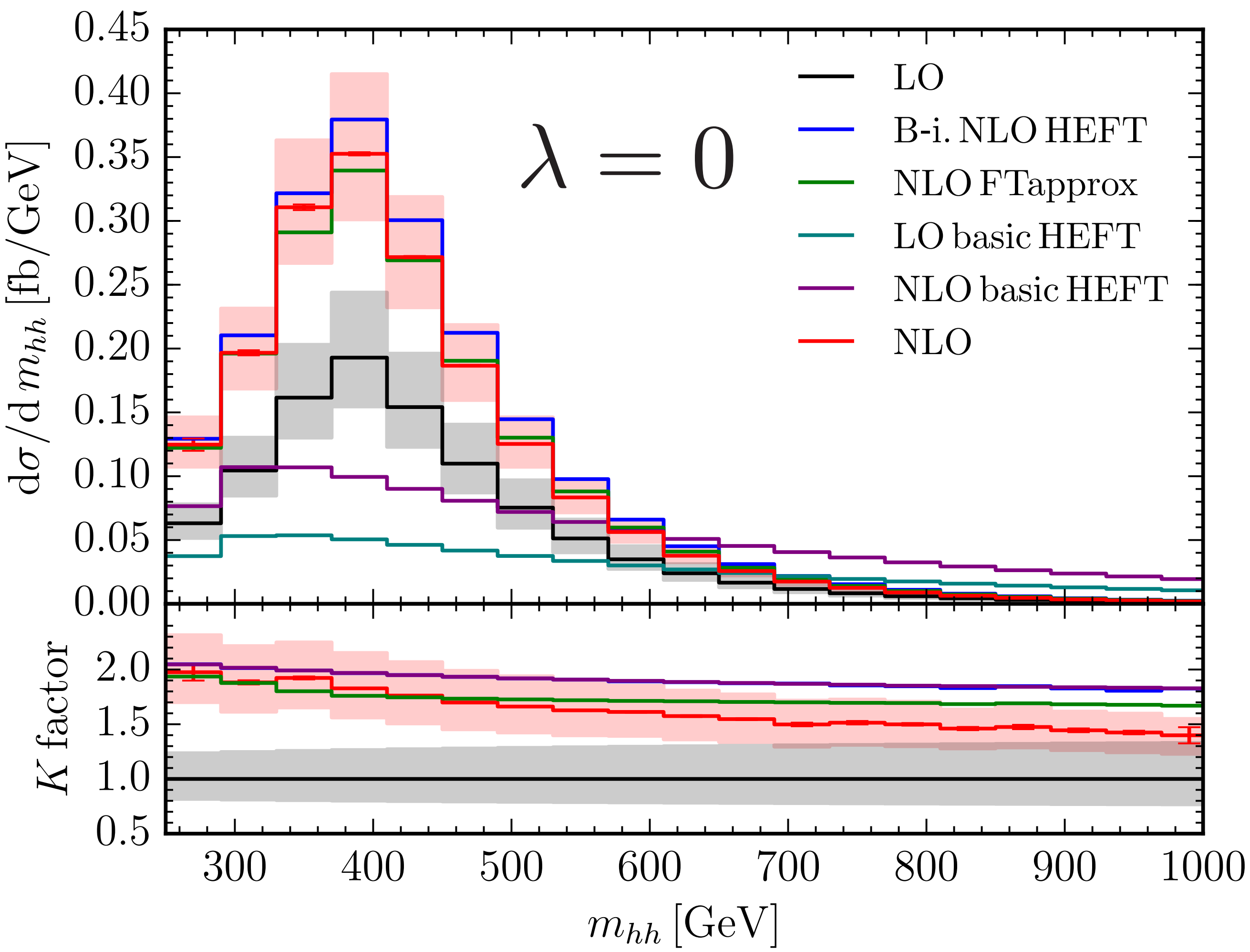
top mass effects important, in particular at $\sqrt{s} = 100\,\mathrm{TeV}$

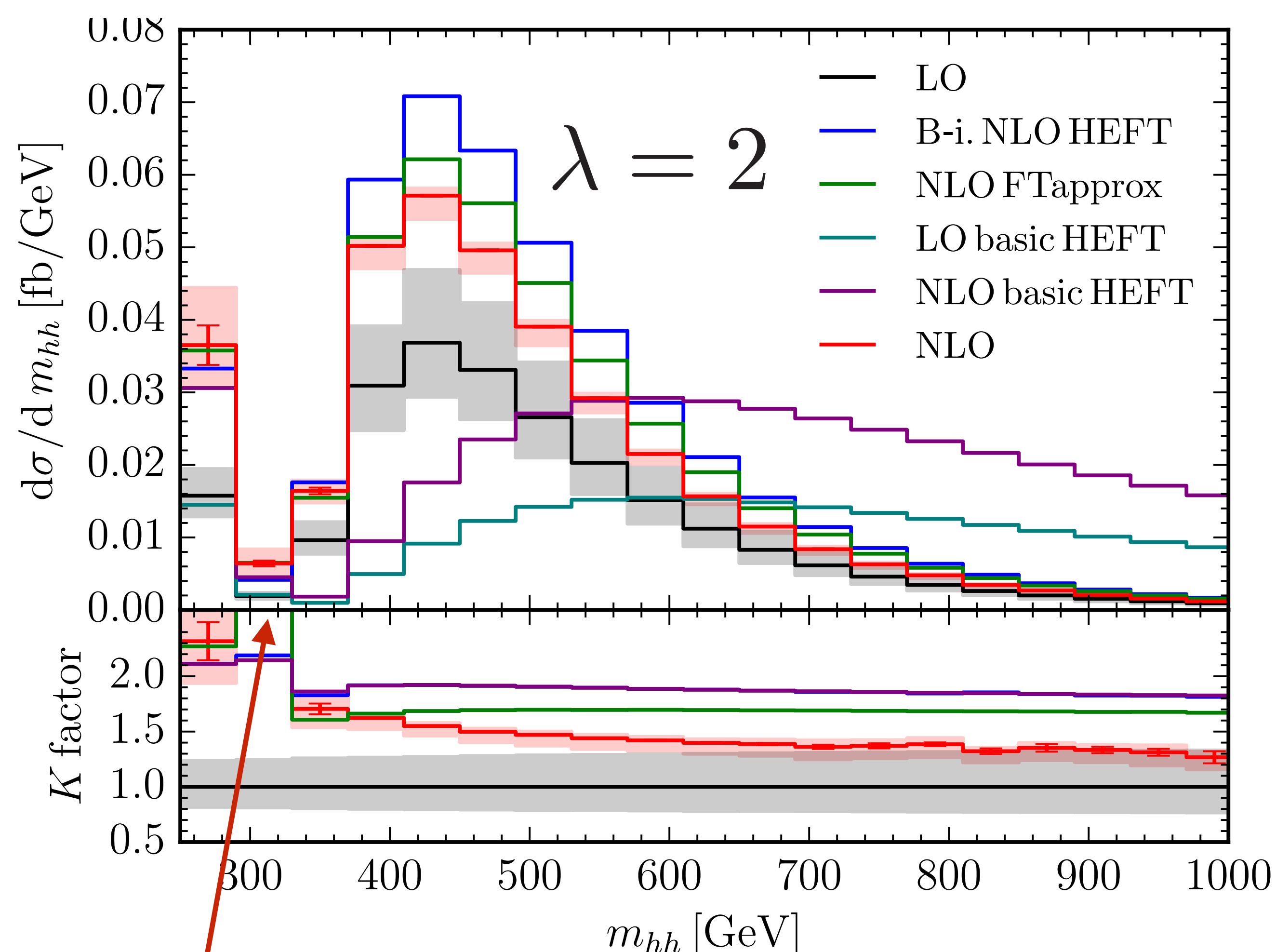
NLO Results — Modified Coupling

modified Higgs self-interaction:

$$g_{hhh} = \lambda \cdot g_{hhh}^{SM}$$

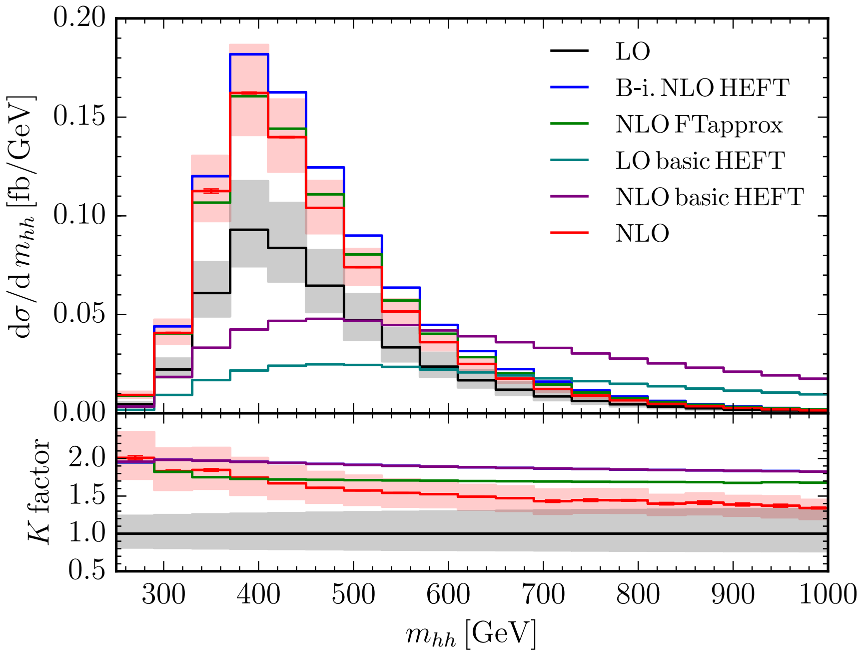


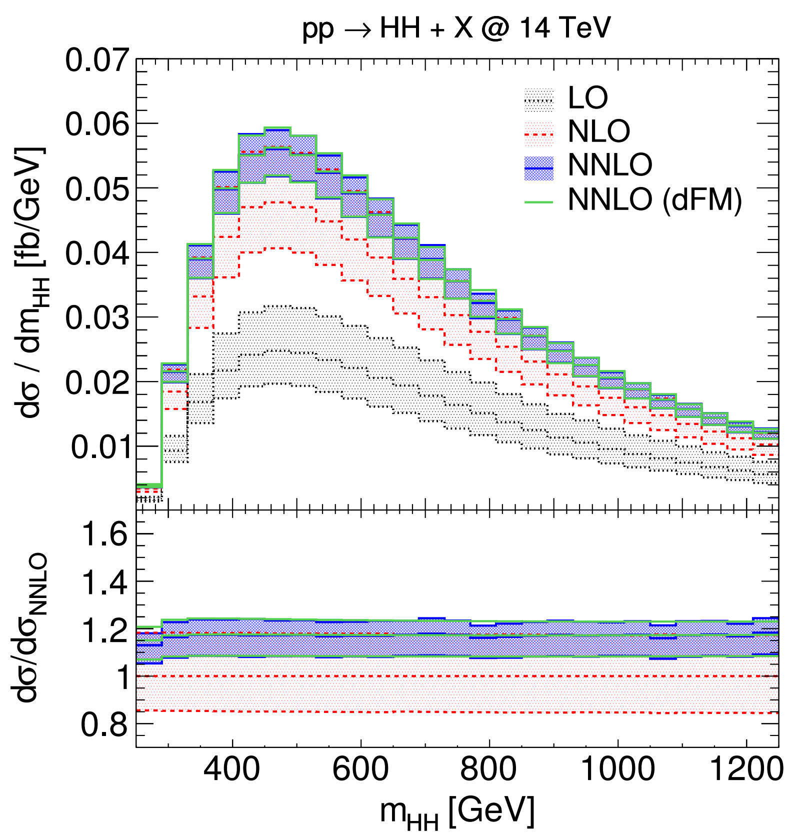




destructive interferenceof △ and □ contributions

Results - Combination with NNLO$_{\text{HEFT}}$





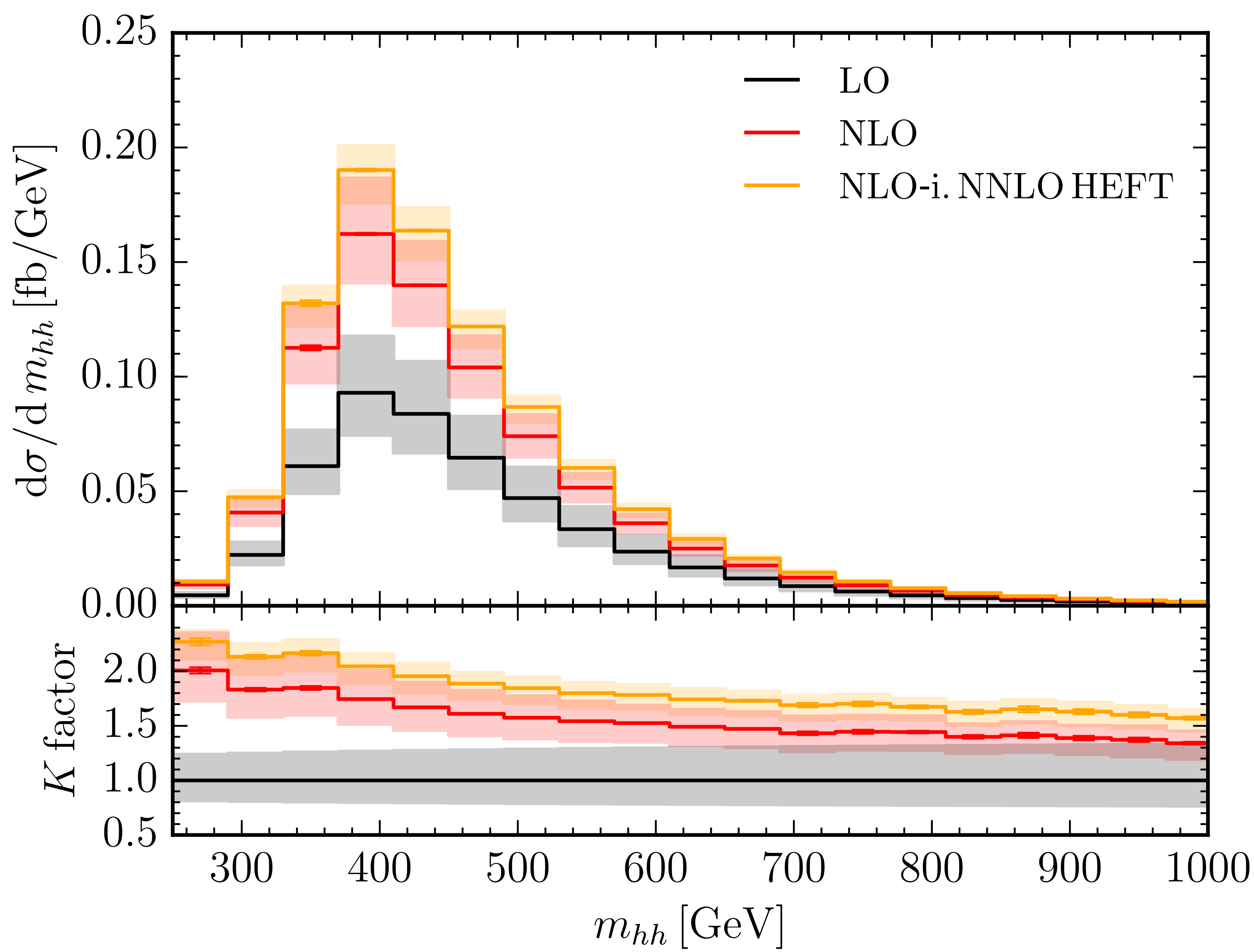
de Florian, Grazzini, Hanga, Kallweit, Lindert, Maierhöfer, Mazzitelli, Rathlev `16

combination of NLO$_{\text{full}}$ with NNLO$_{\text{HEFT}}$

NLO-improved NNLO HEFT:

$$d\sigma^{\text{NLO-i. NNLO HEFT}} = d\sigma^{\text{NLO}} \frac{d\sigma^{\text{NNLO basic HEFT}}}{d\sigma^{\text{NLO basic HEFT}}}$$

$$\sigma^{\text{NLO-i. NNLO HEFT}} = 38.67^{+5.2\%}_{-7.6\%}$$



Parton Shower Interface

2-loop amplitude too slow (median 2h on gpu) for direct interface to PS
$\rightarrow$ construct grid for interpolation of virtual amplitude

- included additional points in large m_{HH} region (total of 3741 2-loop results used)
- input parameters ($\hat{s}$, $\hat{t}$) transformed to

$$x = f(\beta(\hat{s})), \quad c_\theta = |\cos\theta| = \left| \frac{\hat{s} + 2\hat{t} - 2m_H^2}{\hat{s}\beta(\hat{s})} \right|, \quad \beta = \left(1 - \frac{4m_H^2}{\hat{s}}\right)^{\frac{1}{2}}$$

$\rightarrow$ nearly uniform distribution of phase space points in $(x, c_\theta) \in [0,1]^2$ if $f(\beta)$ chosen according to cumulative distribution of points in original calculation

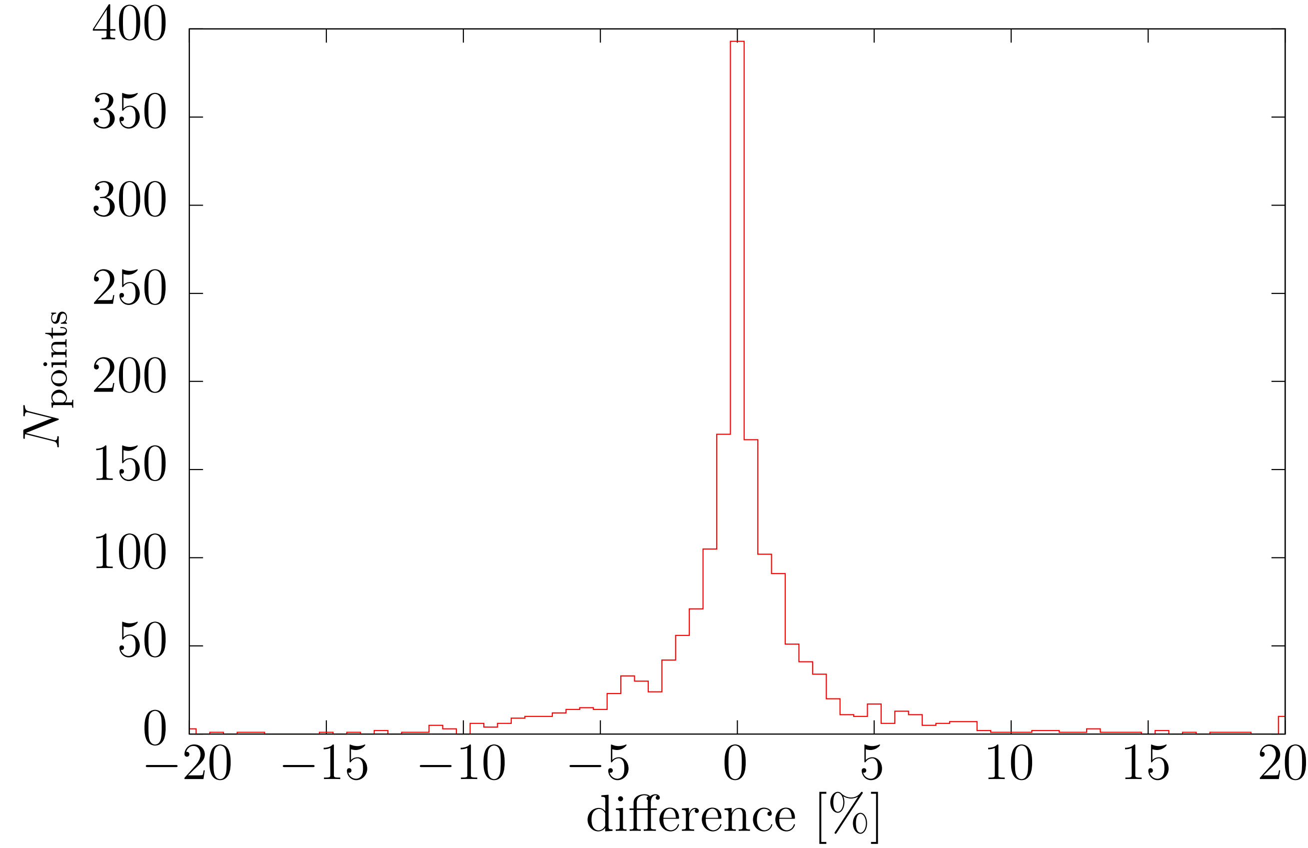
- interpolation done in 2 steps:
  1. choose equidistant grid points, estimate result at each grid point with linear interpolation of amplitude results in vicinity
  2. Clough-Tocher interpolation (as implemented in SciPy) to estimate amplitude at arbitrary sampling points

  $\rightarrow$ reduces sensitivity to uncertainties of input-data points
- available at github.com/mppmu/hhgrid

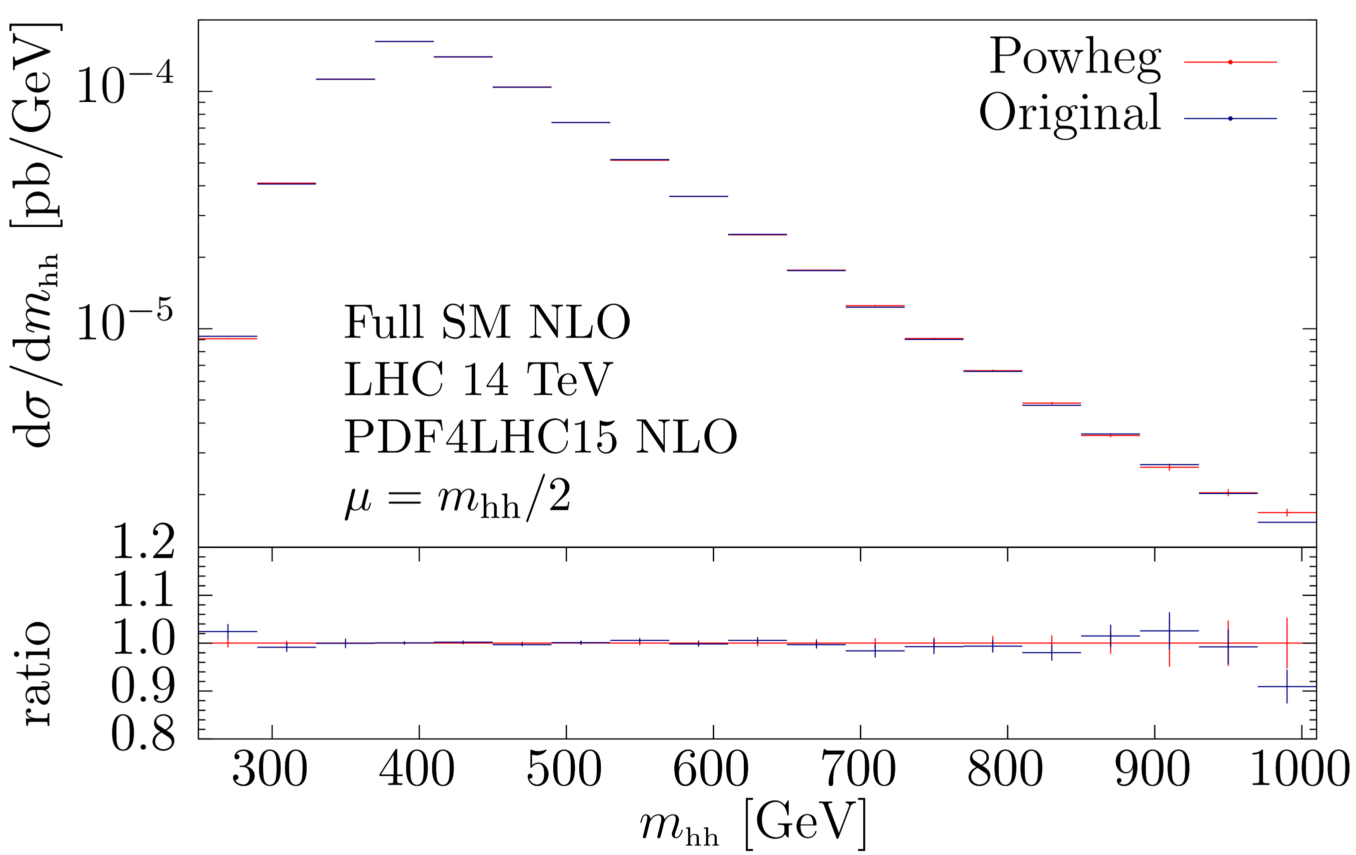
Grid Validation

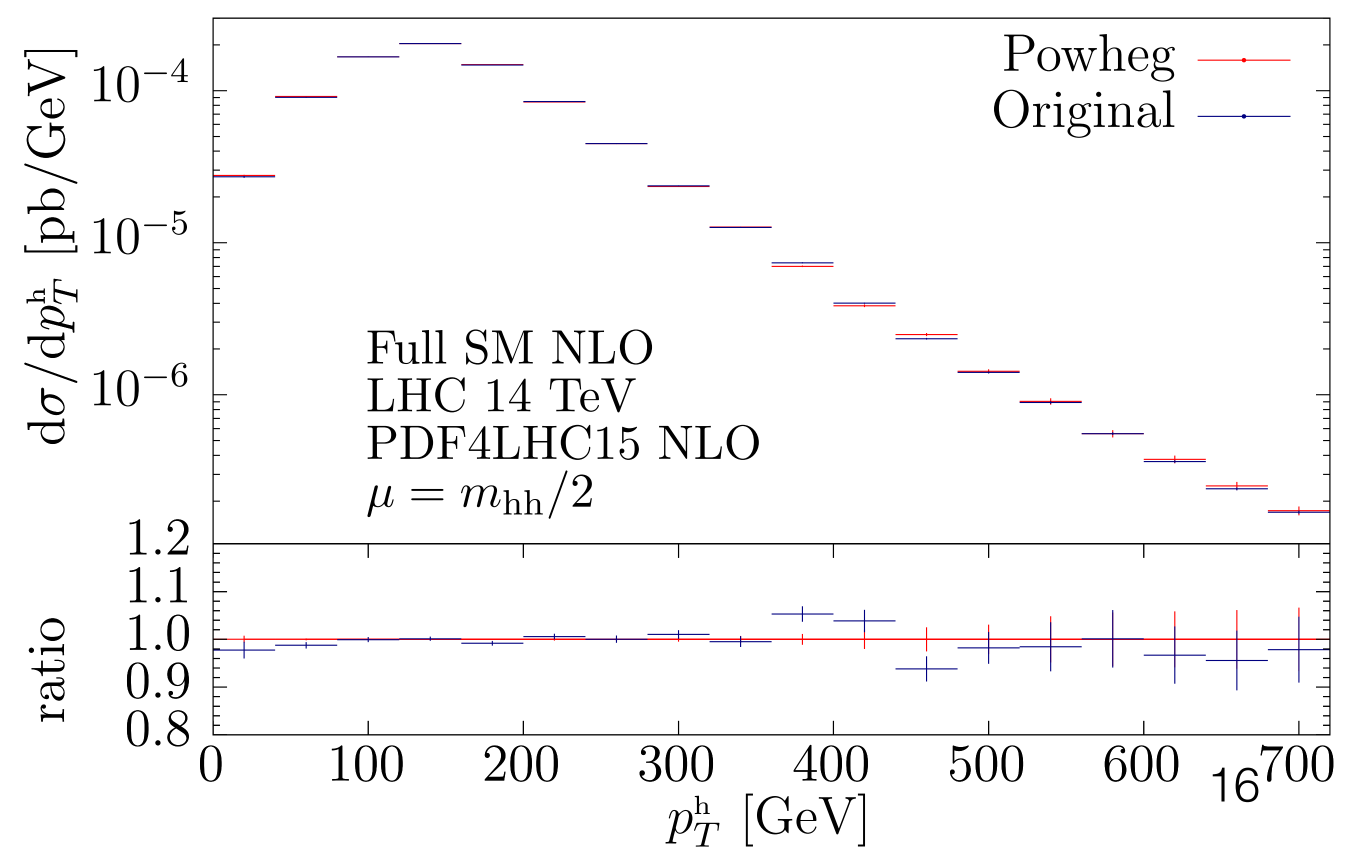
<u>closure test</u>

difference of grid results after removing 50% of input data points



agreement with fixed order calculation:





LHE Events in HEFT & comparison with NNLO

Les Houches Event Level:

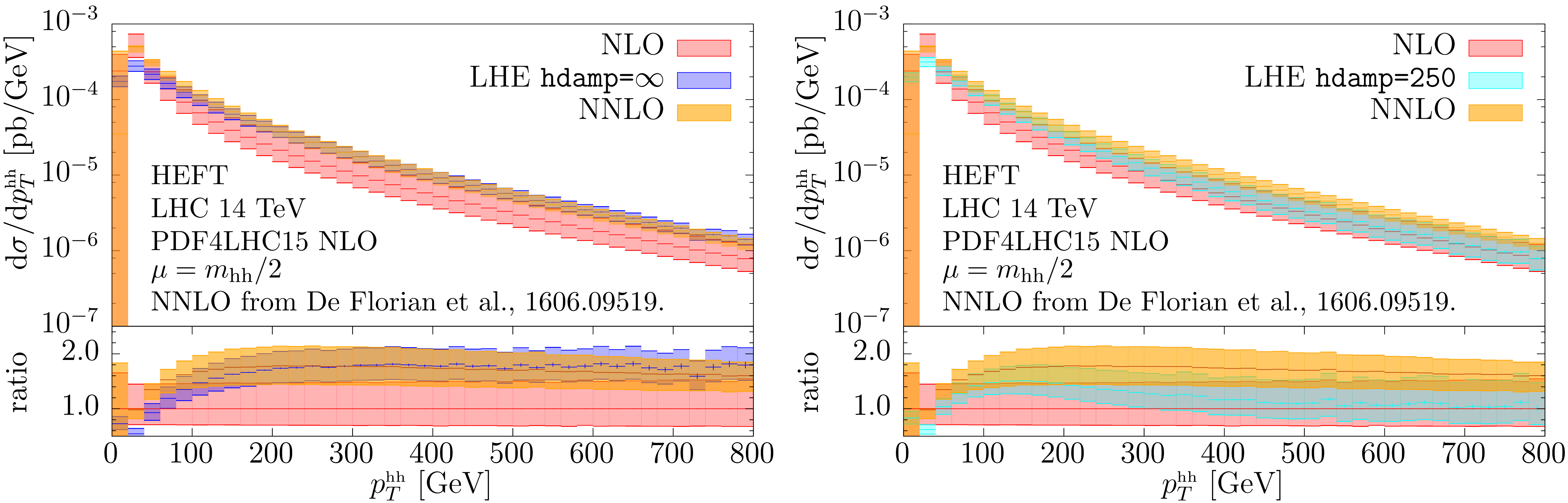
Sudakov factor included, but no parton shower

Powheg allows to split real radiation into (exponentiated) singular and regular part

$$R_{\text{sing}} = R \times F$$

$$R_{\text{reg}} = R \times (1 - F)$$

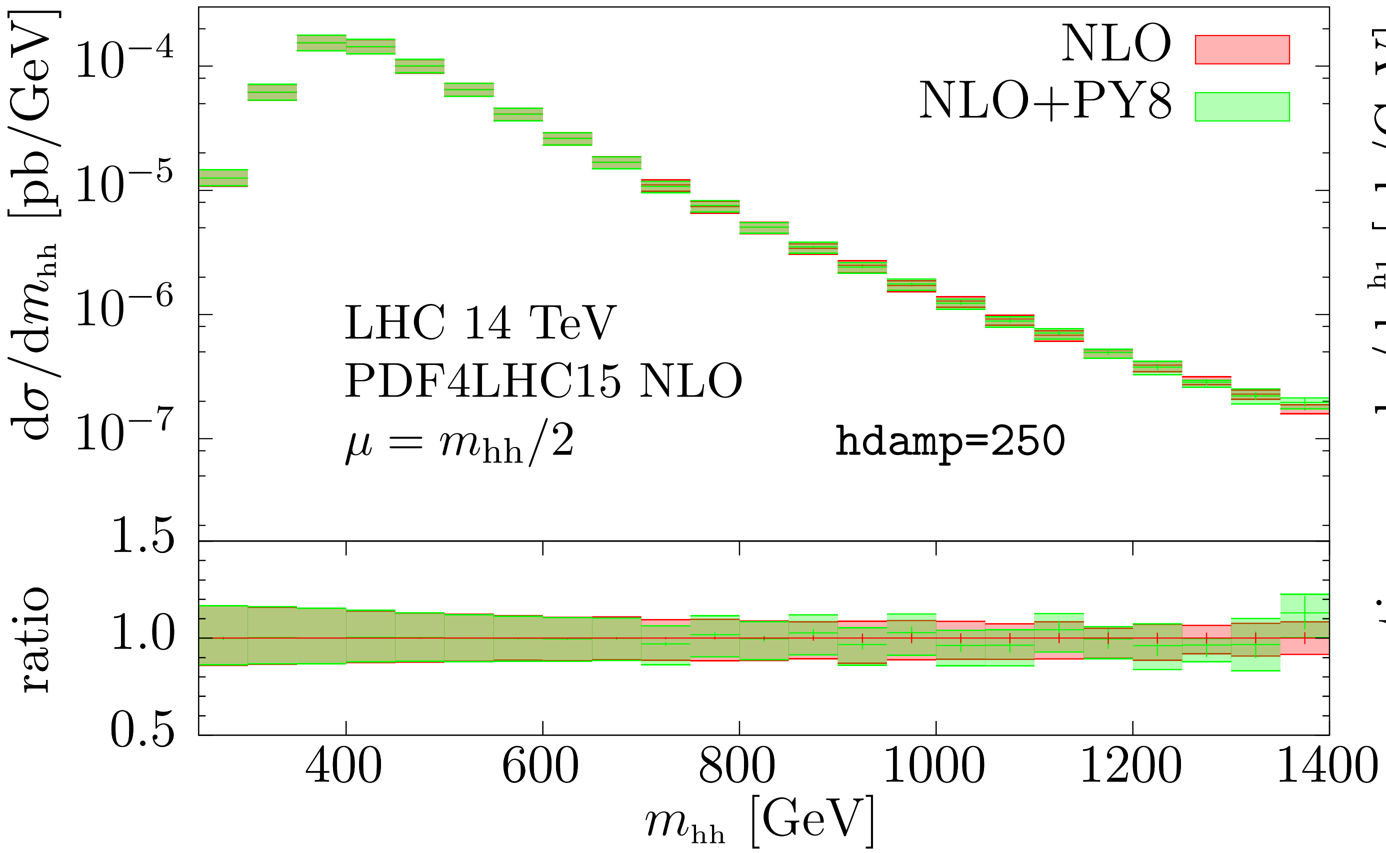
with $F = \dfrac{h^2}{(p_T^{\text{hh}})^2 + h^2}$

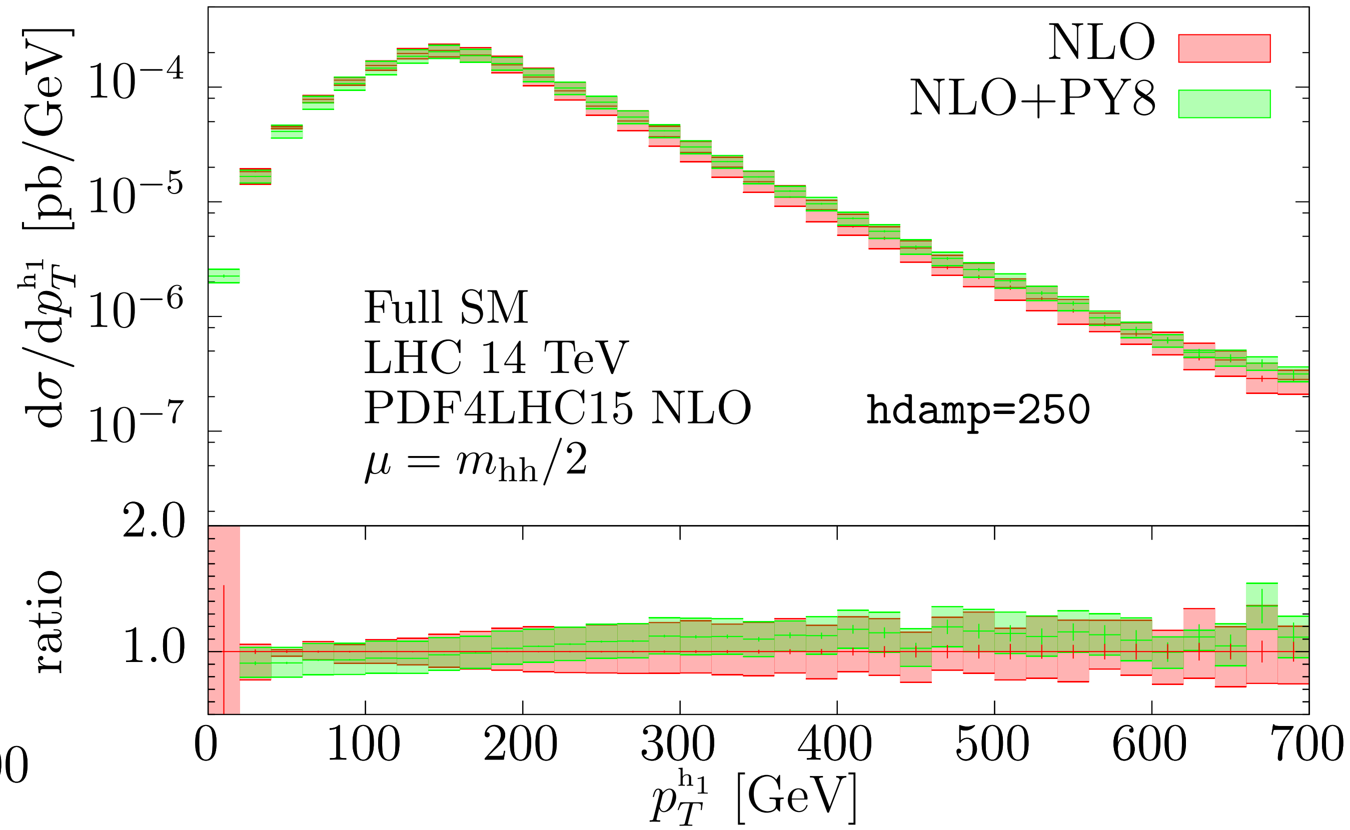


- $h = \infty$: LHE level results close to NNLO
- $h = 250$: LHE level approaches NLO in tail of p_T^{hh} distribution

Results including Parton Shower

Powheg + Pythia8





no effect on invariant mass

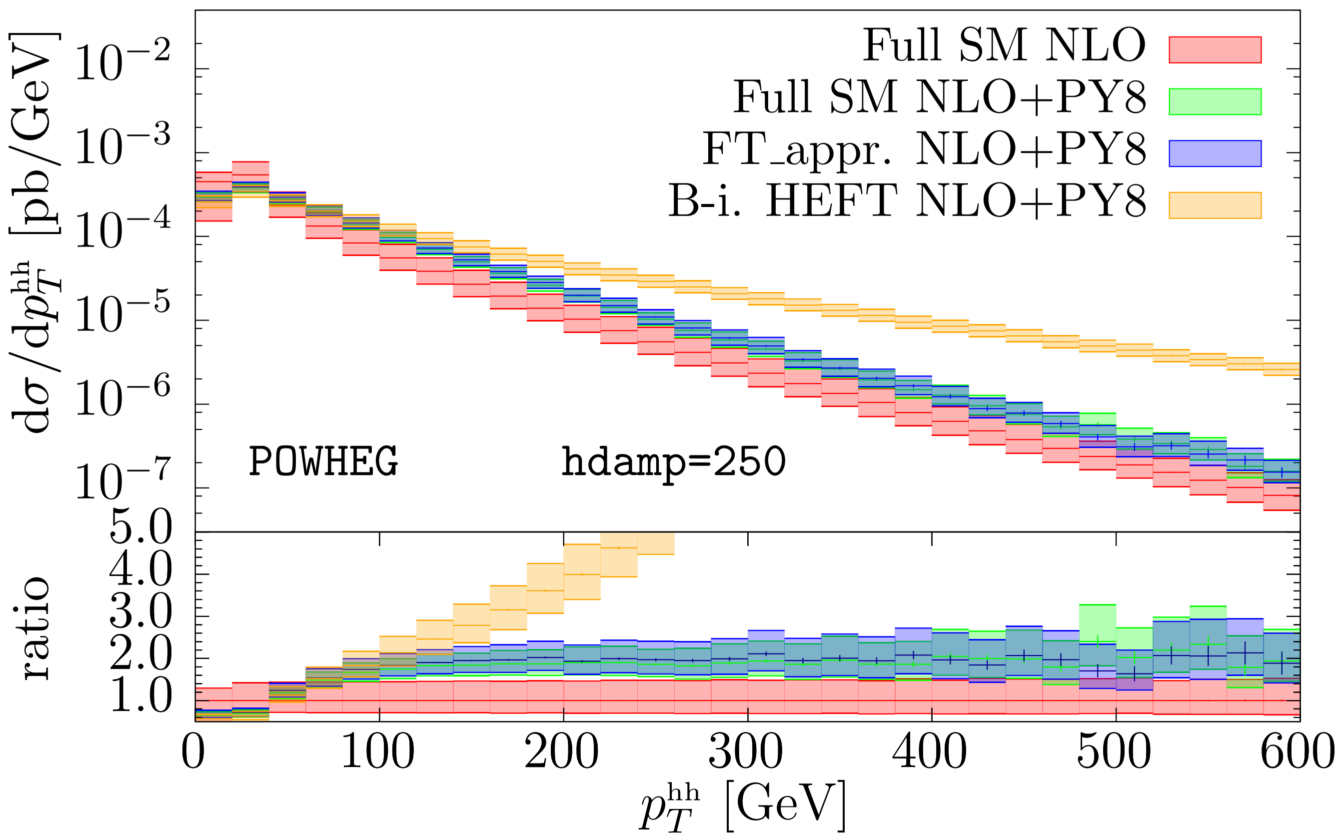
parton shower enhances tails of p_T distributions

only small parton shower effects on NLO accurate observables

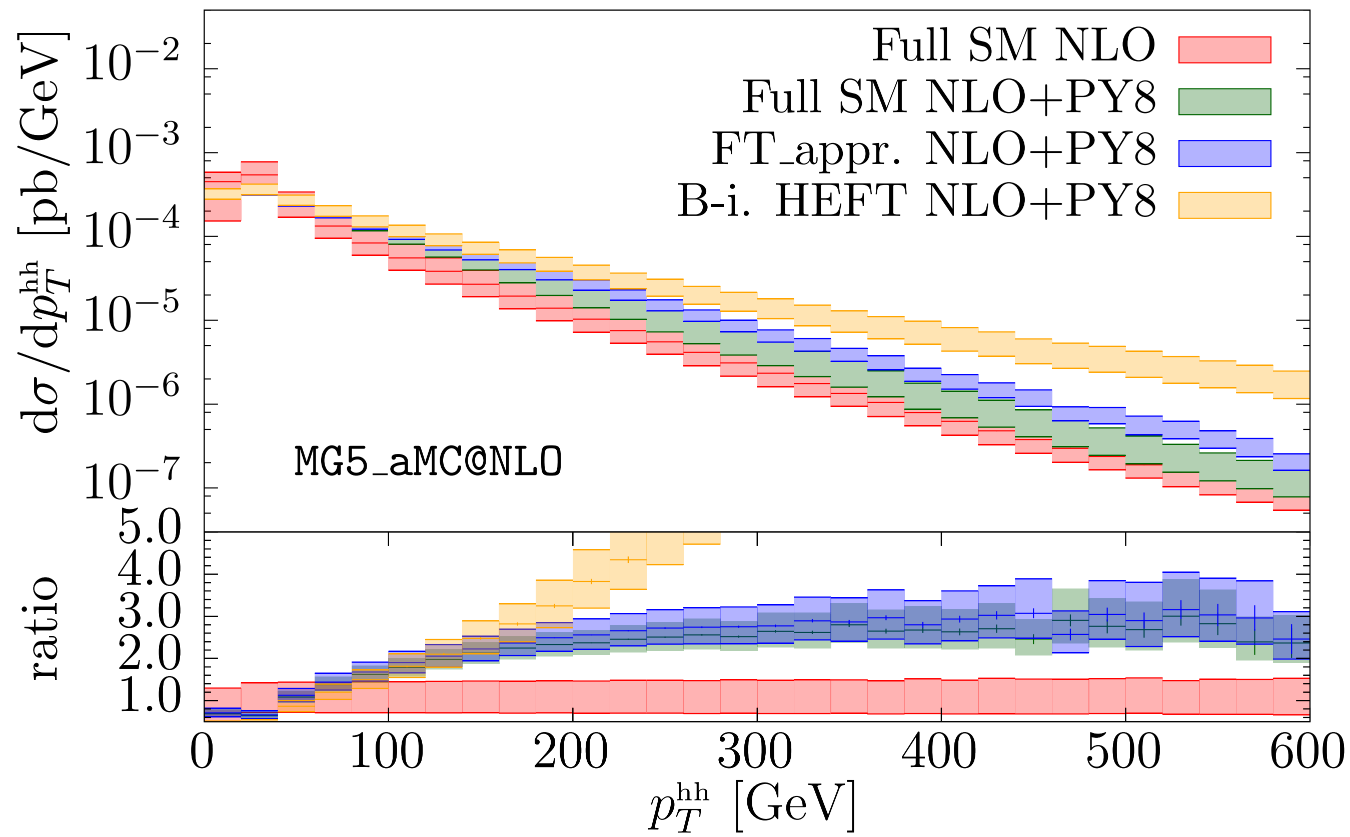
Results including Parton Shower

Parton shower effects large for observables sensitive to real radiation, e.g. p_T^{hh}

Powheg

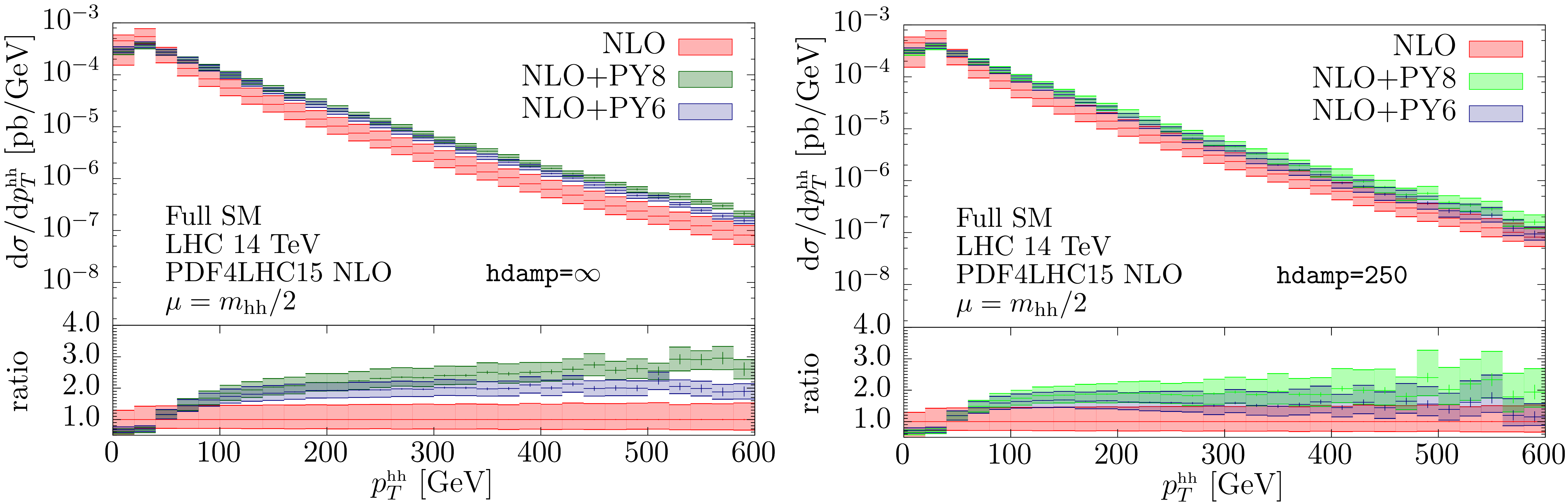


MadGraph5_aMC@NLO



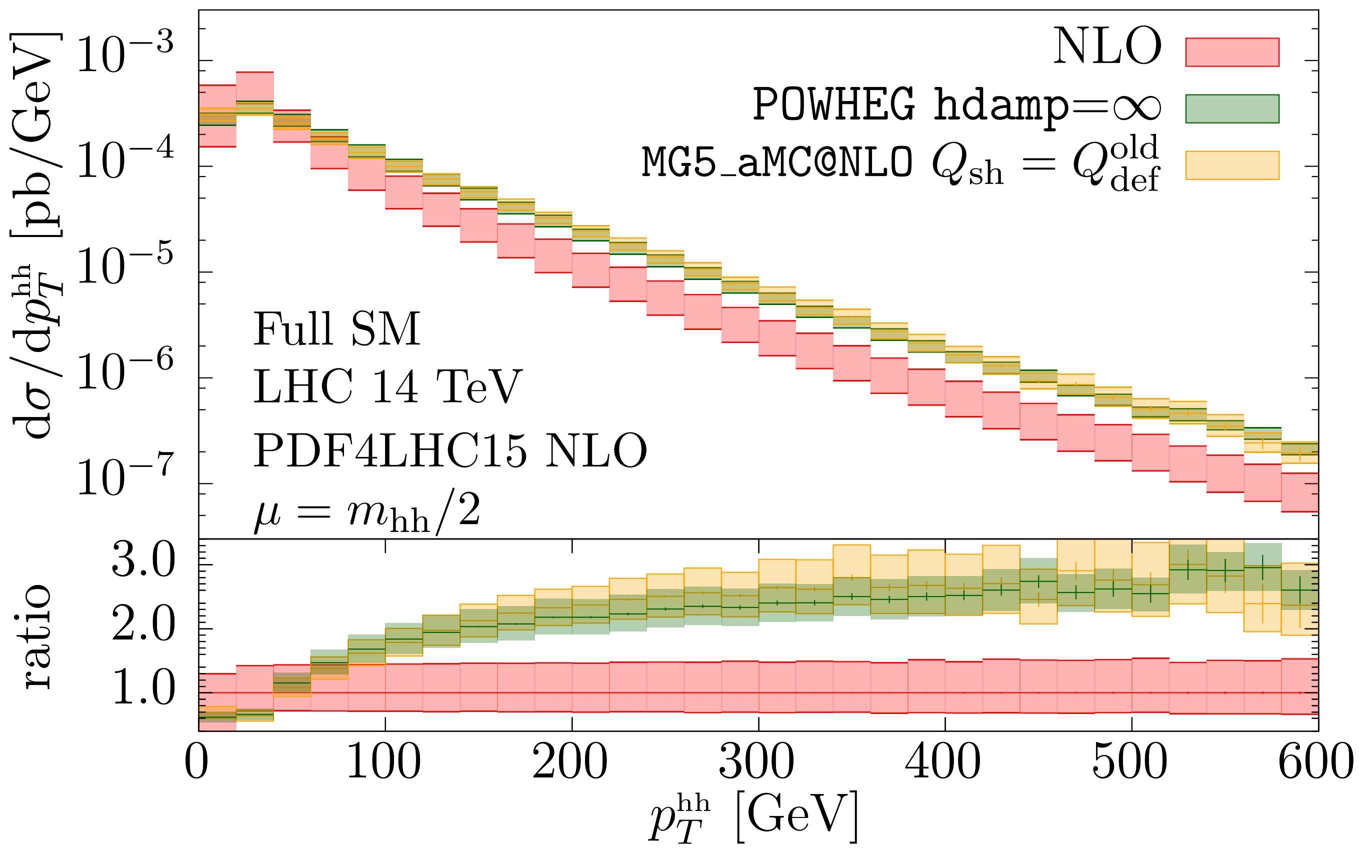
- parton shower enhances tail of p_T^{hh} distribution by factor of ~2
- small difference between full NLO and FT approx. result
- different behavior of Born-improved HEFT result
  → top-mass effects in real radiation important

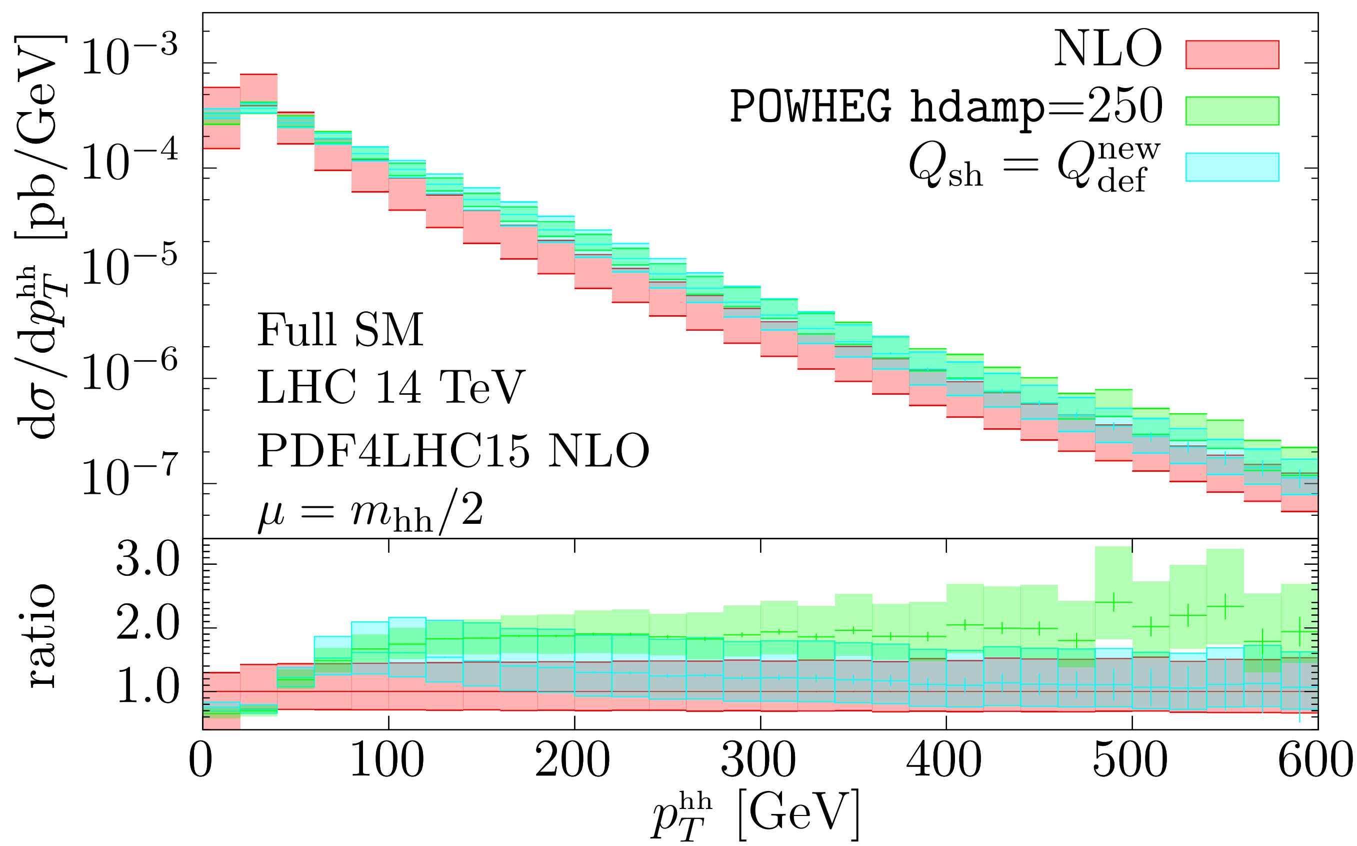
$p_{T,HH}$ with Pythia 6 and Pythia 8



larger enhancement of tails with Pythia 8, in particular for hdamp=∞

Change of shower starting scale in MG5





Shower starting scale changed in MG5_aMC@NLO version 2.5.3

$Q_{def}^{old} \in [0.1\sqrt{\hat{s}},\ \sqrt{\hat{s}}]$

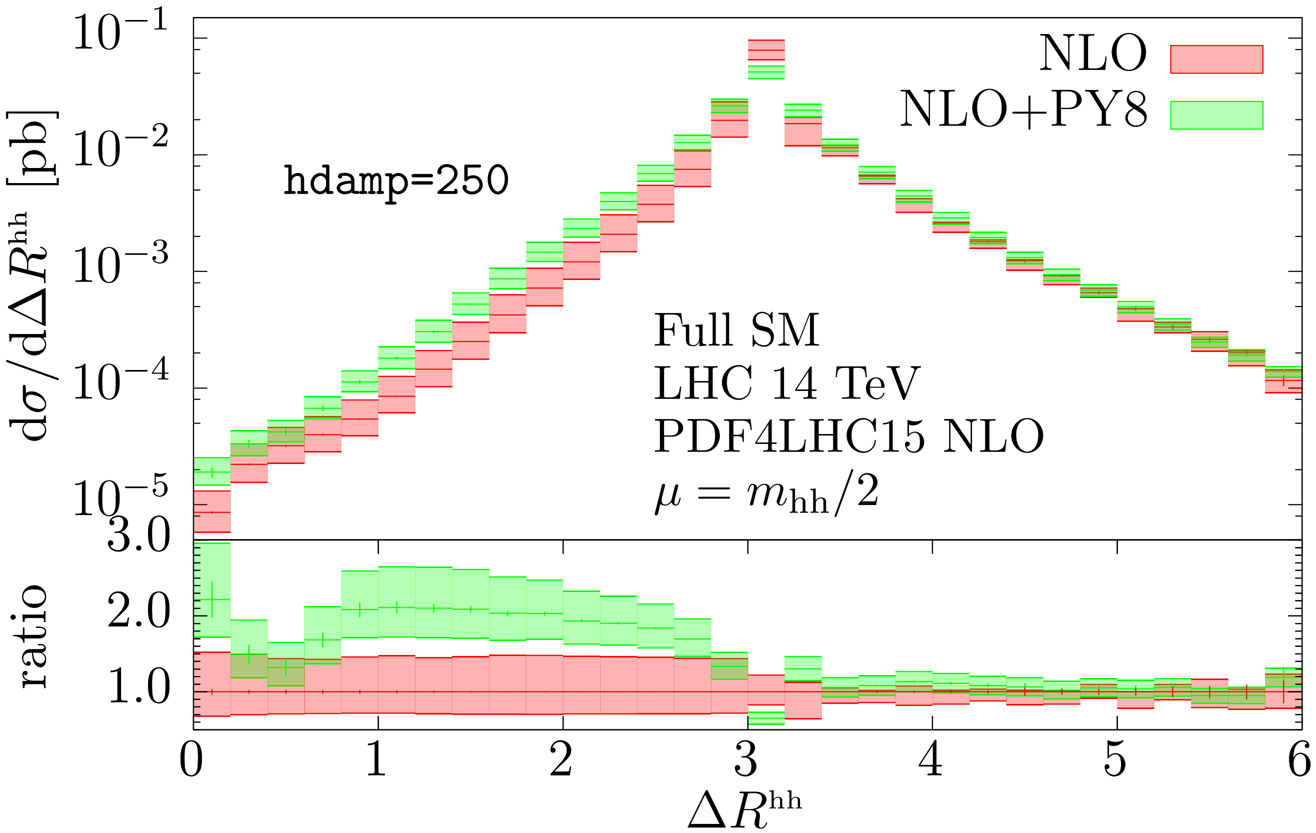
$Q_{def}^{new} \in [0.1 H_T/2,\ H_T/2]$

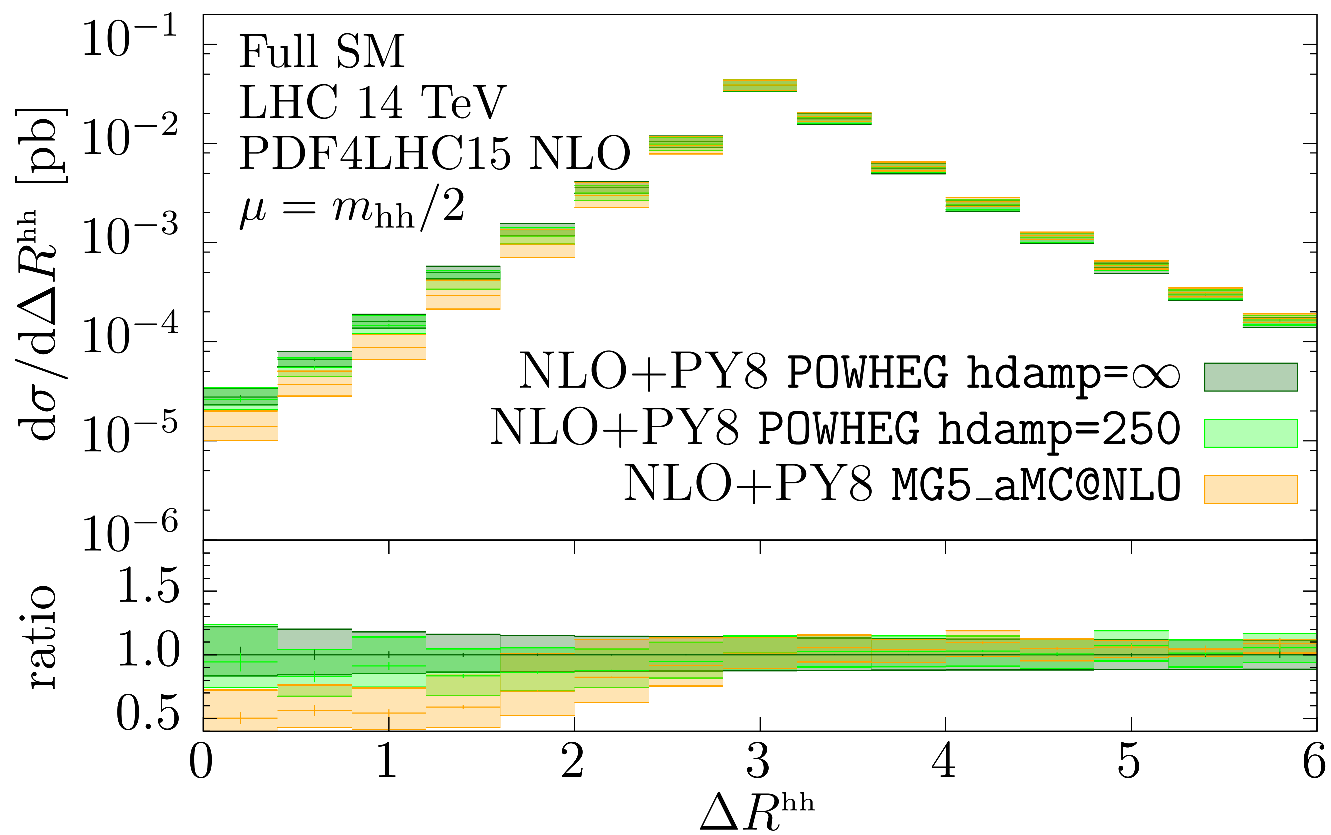
parton shower effects up to factor of ~3

parton shower results agree with NLO at large $p_{T,HH}$

Results including Parton Shower

Parton shower effects large for observables sensitive to real radiation, e.g. ΔR^{hh}





$\Delta R^{hh} < \pi$

- filled by real radiation
  only LO accurate
- large parton shower corrections
- differences due to matching method visible

$\Delta R^{hh} > \pi$

- NLO accurate
- small dependence on parton shower / matching

Summary & Outlook

Higgs pair production at NLO

- virtual corrections computed numerically
- grid for virtual amplitude publicly available
- top-quark mass effects important at large m_{HH}

Parton shower effects for HH production

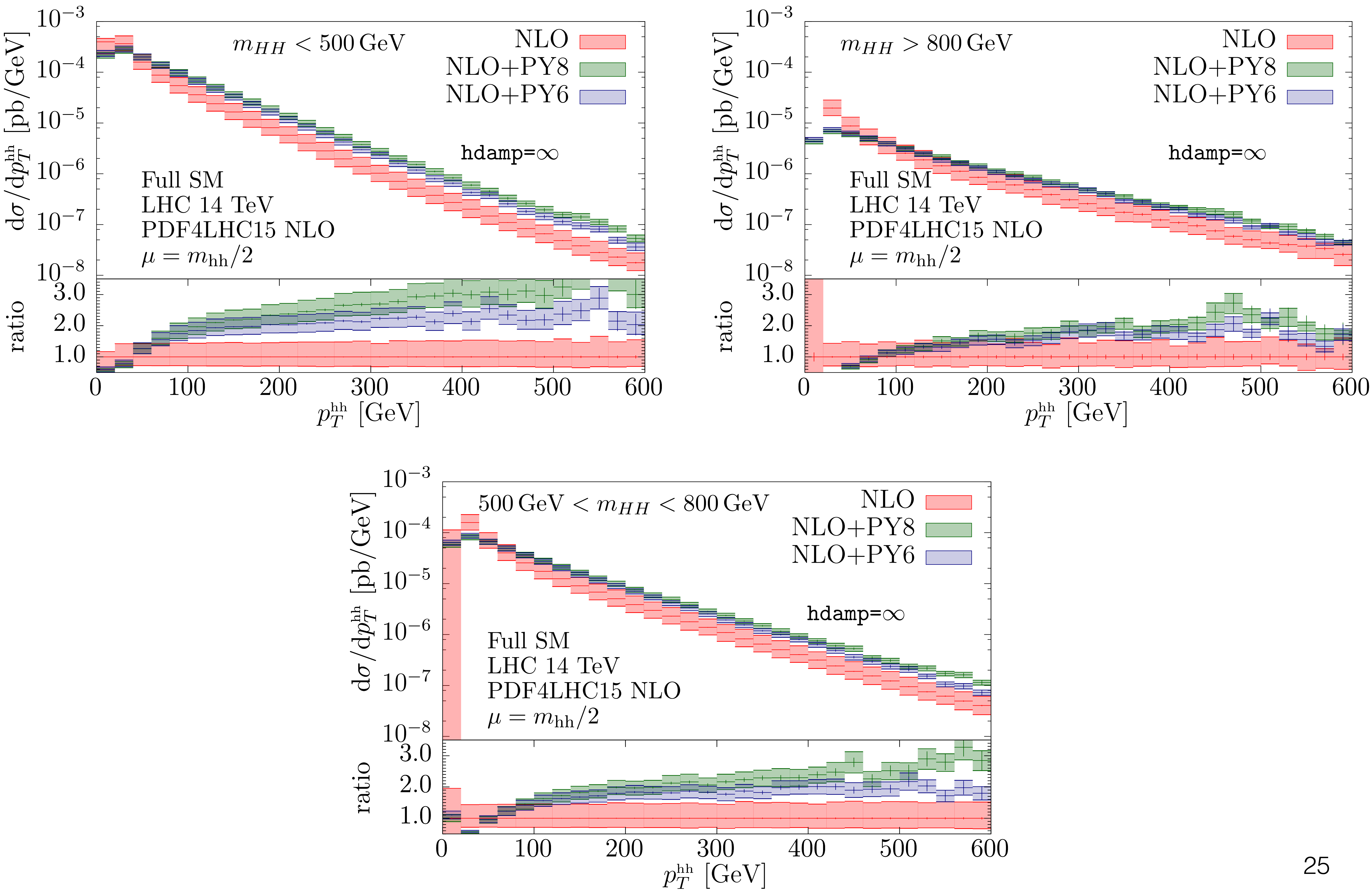
- using POWHEG-BOX and MadGraph5_aMC@NLO frameworks
- for observables sensitive to real radiation:
  - large parton shower effects
  - large dependence on matching procedure

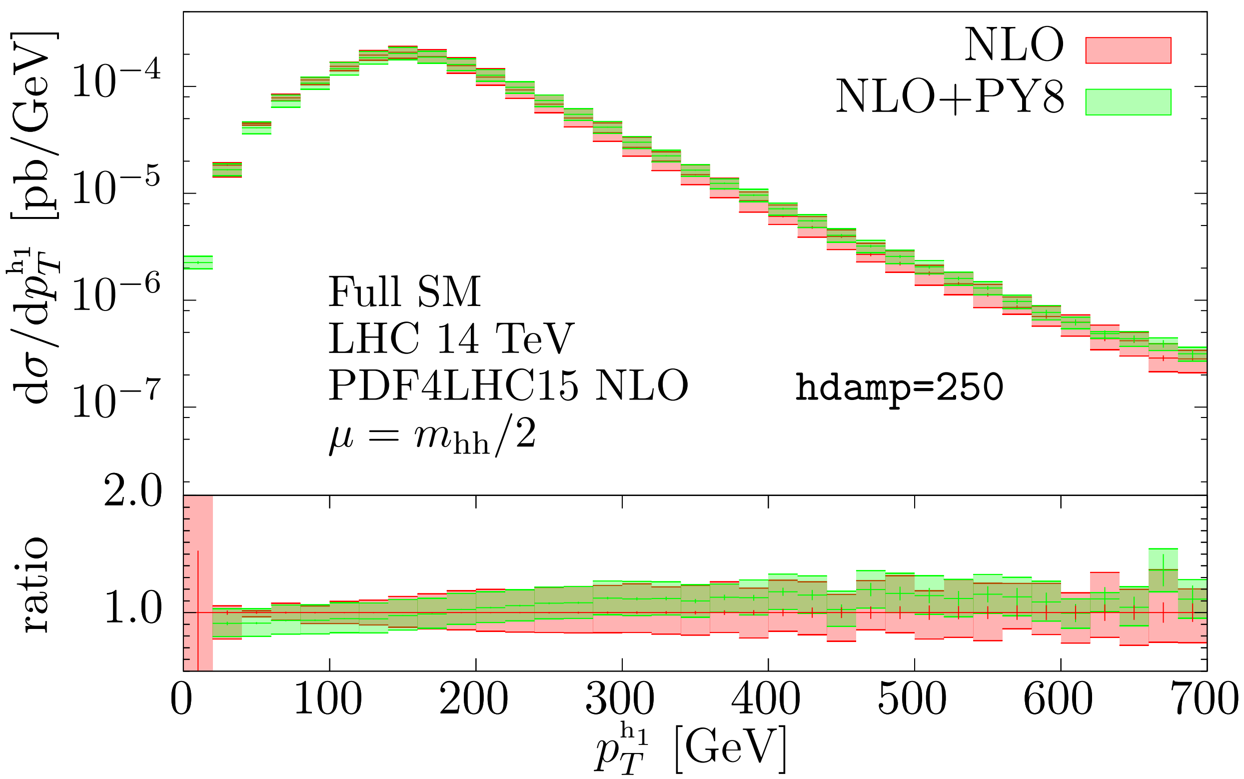
Ongoing work

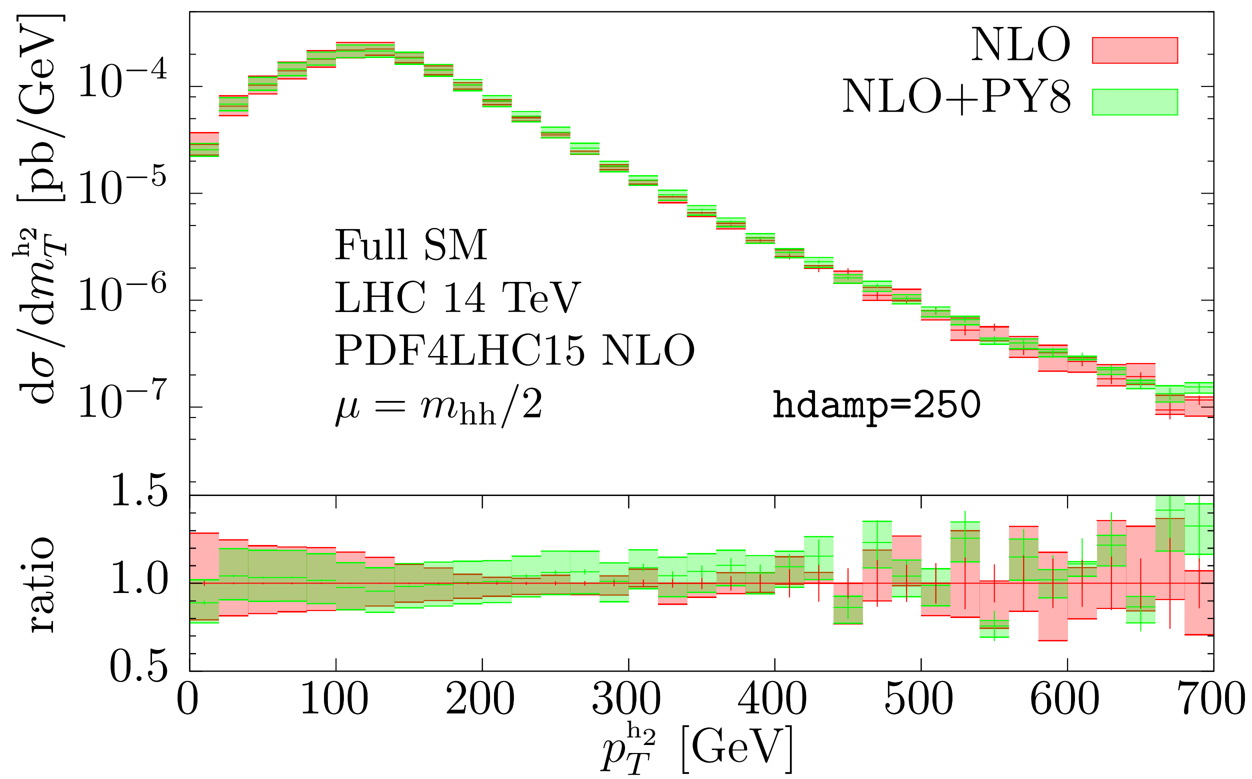
- comparison with Herwig and Sherpa parton shower
- improve combination with NNLO HEFT
- apply methods to other processes

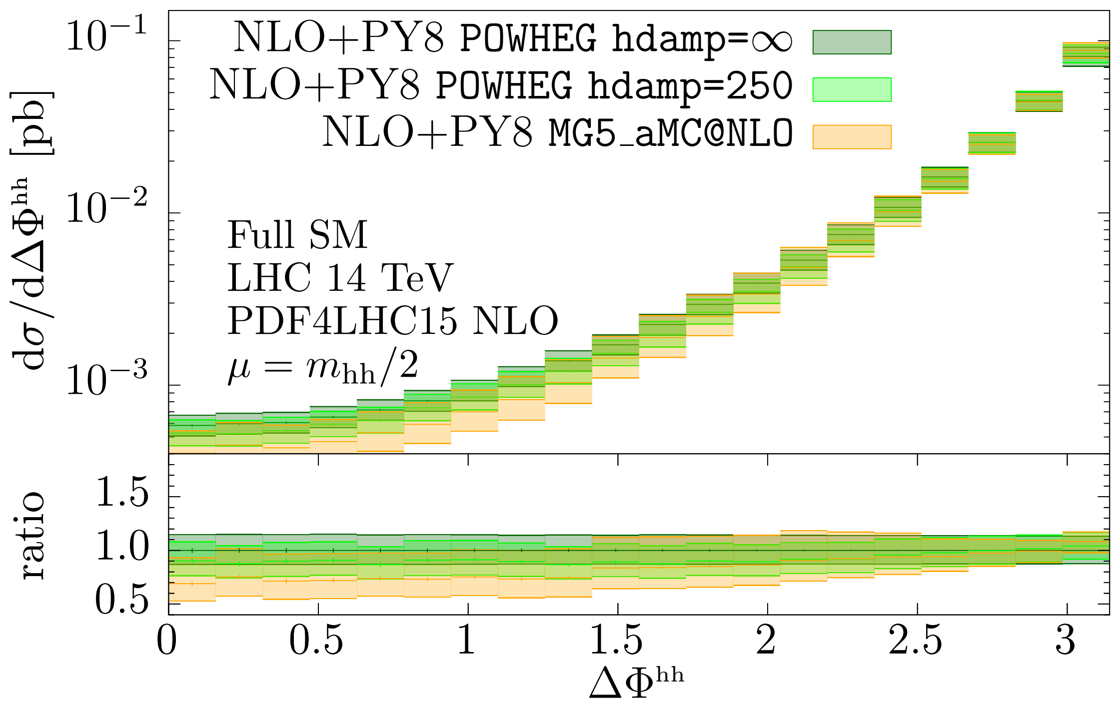
Backup

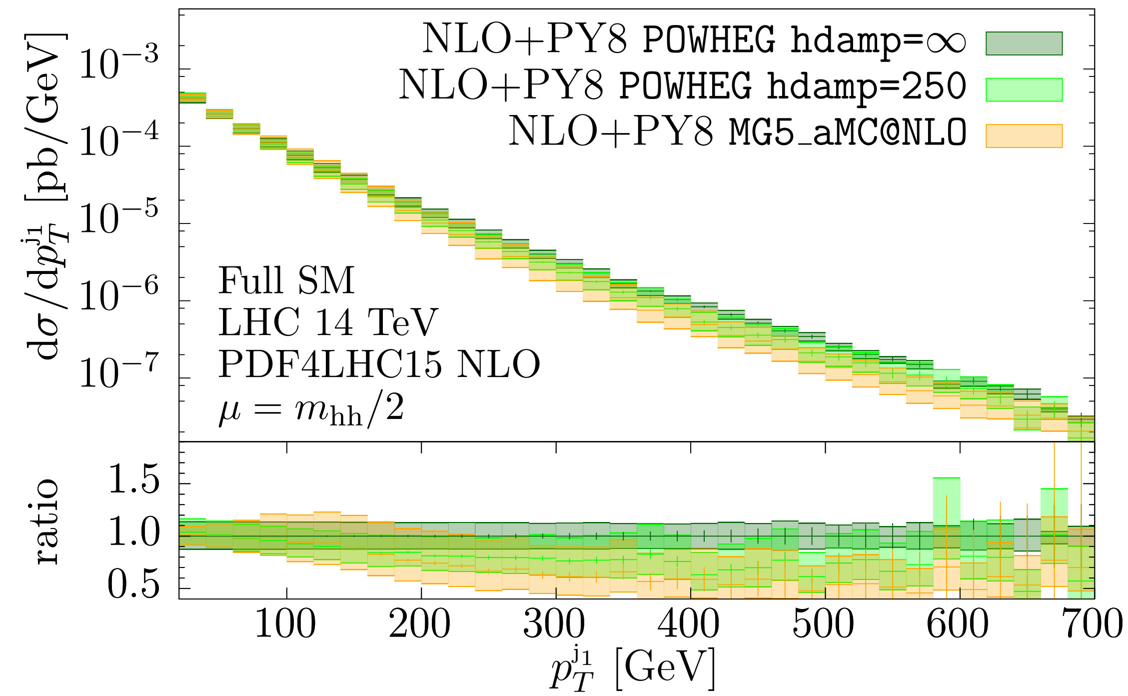
$p_{T,HH}$ in m_{HH} bins




NLO
NLO+PY8
Full SM
LHC 14 TeV
PDF4LHC15 NLO
$\mu = m_{hh}/2$
hdamp=250
$d\sigma/dp_T^{h_1}$ [pb/GeV]
ratio
$p_T^{h_1}$ [GeV]


NLO
NLO+PY8
Full SM
LHC 14 TeV
PDF4LHC15 NLO
$\mu = m_{hh}/2$
hdamp=250
$d\sigma/dm_T^{h_2}$ [pb/GeV]
ratio
$p_T^{h_2}$ [GeV]


NLO+PY8 POWHEG hdamp=∞
NLO+PY8 POWHEG hdamp=250
NLO+PY8 MG5_aMC@NLO
Full SM
LHC 14 TeV
PDF4LHC15 NLO
$\mu = m_{hh}/2$
$d\sigma/d\Delta\Phi^{hh}$ [pb]
ratio
$\Delta\Phi^{hh}$


NLO+PY8 POWHEG hdamp=∞
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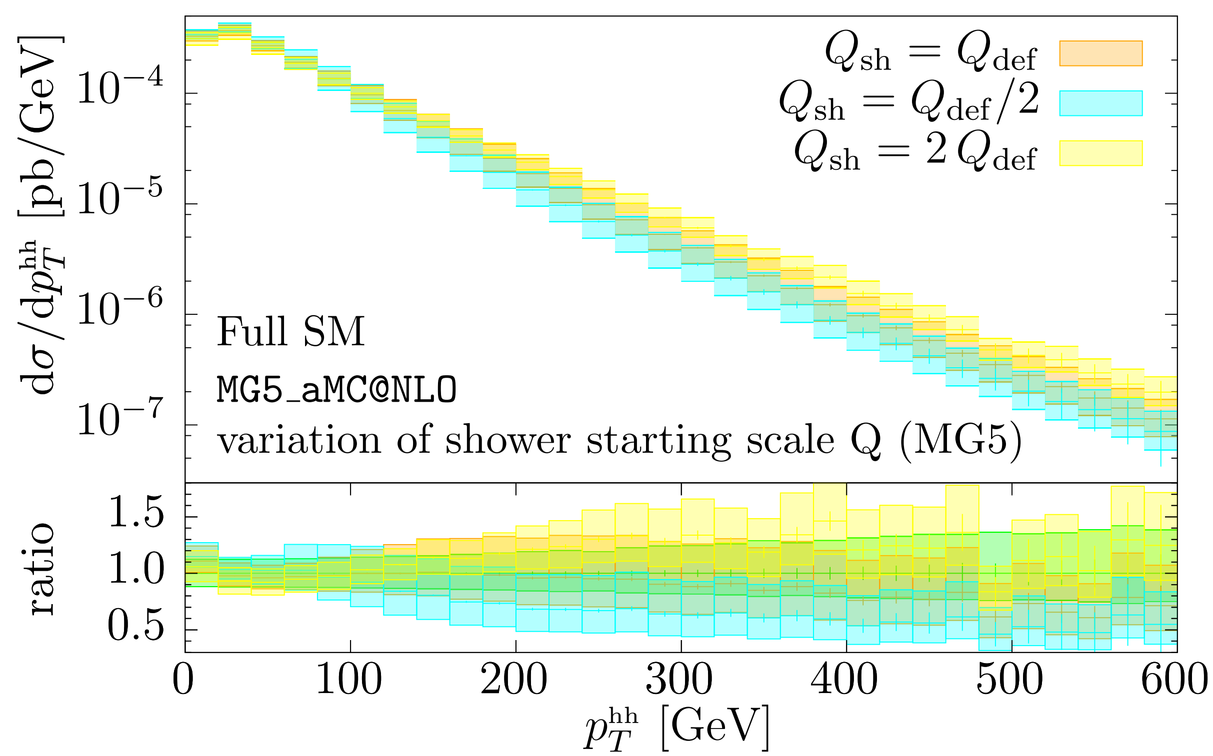
$\mu = m_{hh}/2$
$d\sigma/dp_T^{j_1}$ [pb/GeV]
ratio
$p_T^{j_1}$ [GeV]


$Q_{sh} = Q_{def}$
$Q_{sh} = Q_{def}/2$
$Q_{sh} = 2\,Q_{def}$
Full SM
MG5_aMC@NLO
variation of shower starting scale Q (MG5)
$d\sigma/dp_T^{hh}$ [pb/GeV]
ratio
p_T^{hh} [GeV]

Two Loop Amplitude

- tensor structure Glover, van der Bij `88

$$\mathcal{M} = \epsilon_\mu(p_1, n_1)\epsilon_\nu(p_2, n_2)\, \mathcal{M}^{\mu\nu}$$

$$\mathcal{M}^{\mu\nu} = A_1(s, t, m_H^2, m_t^2, D)\, T_1^{\mu\nu} + A_2(s, t, m_H^2, m_t^2, D)\, T_2^{\mu\nu}$$

with

$$\mathcal{M}^{++} = \mathcal{M}^{--} = -A_1$$
$$\mathcal{M}^{+-} = \mathcal{M}^{-+} = -A_2$$

triangle diagrams gg→H→HH only contribute to A_1

$$T_1^{\mu\nu} = g^{\mu\nu} - \frac{p_1^\nu\, p_2^\mu}{p_1 \cdot p_2}$$

$$T_2^{\mu\nu} = g^{\mu\nu} + \frac{1}{p_T^2\,(p_1 \cdot p_2)} \left\{ m_H^2 p_1^\nu\, p_2^\mu - 2\,(p_1 \cdot p_3)\, p_3^\nu\, p_2^\mu - 2\,(p_2 \cdot p_3)\, p_3^\nu\, p_1^\mu + 2\,(p_1 \cdot p_2)\, p_3^\nu\, p_3^\mu \right\}$$

- projectors

construct $P_i^{\mu\nu} = \sum_j c_{ij} T_j^{\mu\nu}$ such that

$$P_1^{\mu\nu}\mathcal{M}_{\mu\nu} = A_1(s, t, m_H^2, m_t^2, D)$$
$$P_2^{\mu\nu}\mathcal{M}_{\mu\nu} = A_2(s, t, m_H^2, m_t^2, D)$$

Amplitude Structure

rewrite loop integrals with r propagators and s inverse propagators as

$$I_{r,s}(s,t,m_h^2,m_t^2) = (M^2)^{-L\epsilon}(M^2)^{2L-r+s} I_{r,s}\left(\frac{s}{M^2},\frac{t}{M^2},\frac{m_h^2}{M^2},\frac{m_t^2}{M^2}\right)$$

arbitrary scale

and write renormalized form factors as

$$F^{\text{virt}} = aF^{(1)} + a^2(\frac{n_g}{2}\,\delta Z_A + \delta Z_a)F^{(1)} + a^2\delta m_t^2 F^{ct,(1)} + a^2F^{(2)} + \mathcal{O}(a^3)$$

$$F^{(1)} = \left(\frac{\mu_R^2}{M^2}\right)^{\varepsilon}\left[b_0^{(1)} + b_1^{(1)}\varepsilon + b_2^{(1)}\varepsilon^2 + \mathcal{O}(\varepsilon^3)\right], \qquad \text{(1-loop)}$$

$$F^{ct,(1)} = \left(\frac{\mu_R^2}{M^2}\right)^{\varepsilon}\left[c_0^{(1)} + c_1^{(1)}\varepsilon + \mathcal{O}(\varepsilon^2)\right], \qquad \text{(mass counter-term)}$$

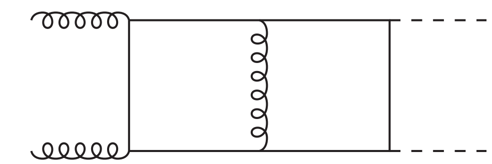
$$F^{(2)} = \left(\frac{\mu_R^2}{M^2}\right)^{2\varepsilon}\left[\frac{b_{-2}^{(2)}}{\varepsilon^2} + \frac{b_{-1}^{(2)}}{\varepsilon} + b_0^{(2)} + \mathcal{O}(\varepsilon)\right], \qquad \text{(2-loop)}$$

$\rightarrow$ scale variations do not require re-computation of $b_i^{(n)}$, $c_i^{(n)}$

Amplitude Evaluation — Example

$\sqrt{s} = 327.25\,\text{GeV},\ \sqrt{-t} = 170.05\,\text{GeV},\ M^2 = s/4$

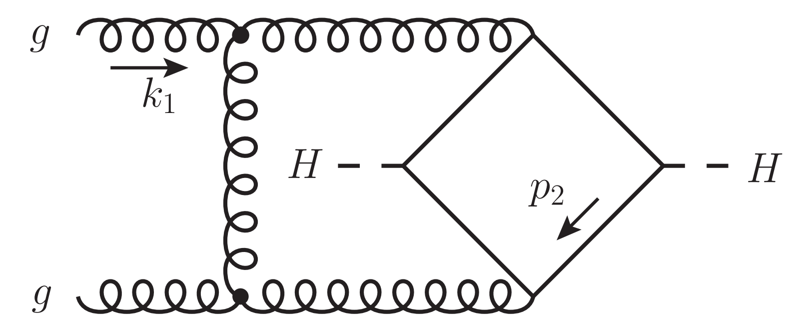
contributing integrals:



integral	value	error	time [s]
...			
F1_011111110_ord0	(0.484, 4.96e-05)	(4.40e-05, 4.23e-05)	11.8459
...			
N3_111111100_k1p2k2p2_ord0	(0.0929, -0.224)	(6.32e-05, 5.93e-05)	235.412
N3_111111100_1_ord0	(-0.0282, 0.179)	(8.01e-05, 9.18e-05)	265.896
N3_111111100_k1p2k1p2_ord0	(0.0245, 0.0888)	(5.06e-05, 5.31e-05)	282.794
N3_111111100_k1p2_ord0	(-0.00692, -0.108)	(3.05e-05, 3.05e-05)	433.342

≈ 700 integrals

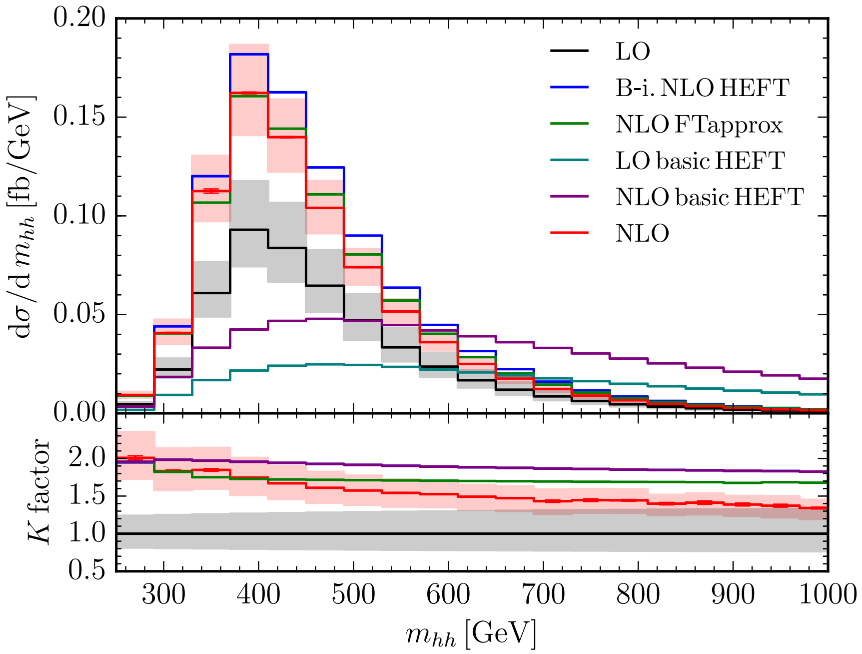
$$I(s,t,m_t^2,m_h^2) = -\left(\frac{\mu^2}{M^2}\right)^{2\varepsilon} \Gamma(3+2\epsilon)M^{-4}\left(\frac{A_{-2}}{\epsilon^2} + \frac{A_{-1}}{\epsilon^1} + A_0 + \mathcal{O}(\epsilon)\right)$$

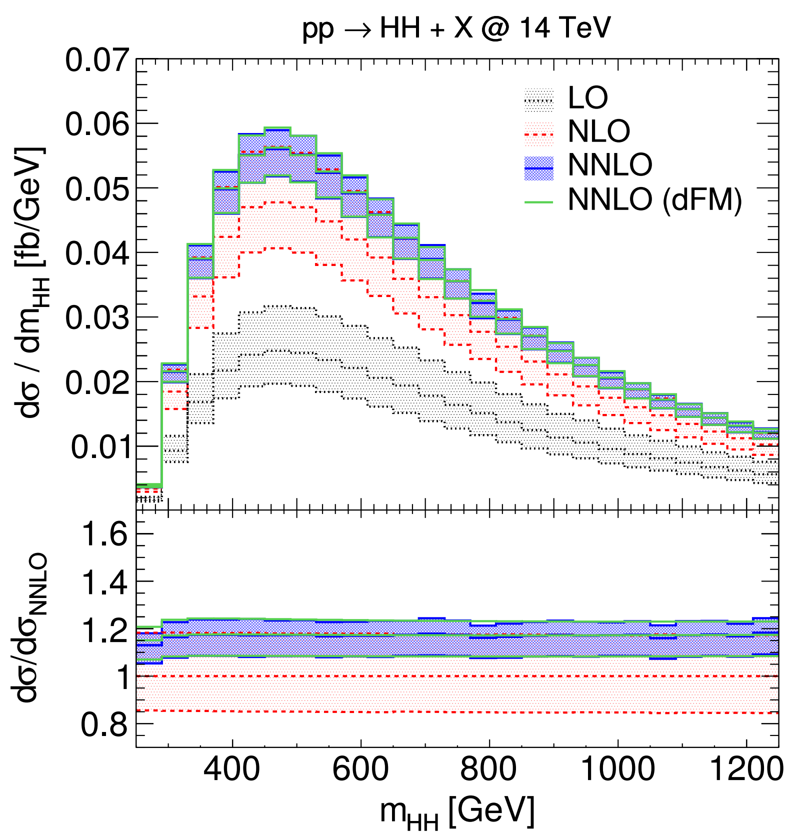


sector decomposition

sector	integral value	error	time [s]	#points
5	(-1.34e-03, 2.00e-07)	(2.38e-07, 2.69e-07)	0.255	1310420
6	(-1.58e-03, -9.23e-05)	(7.44e-07, 5.34e-07)	0.266	1310420
...				
41	(0.179, -0.856)	(1.10e-05, 1.22e-05)	29.484	79952820
42	(0.359, -1.308)	(1.40e-06, 1.58e-06)	80.24	211436900
44	(0.0752, -1.185)	(5.44e-07, 6.76e-07)	99.301	282904860

Results - Combination with NNLO$_{\text{HEFT}}$



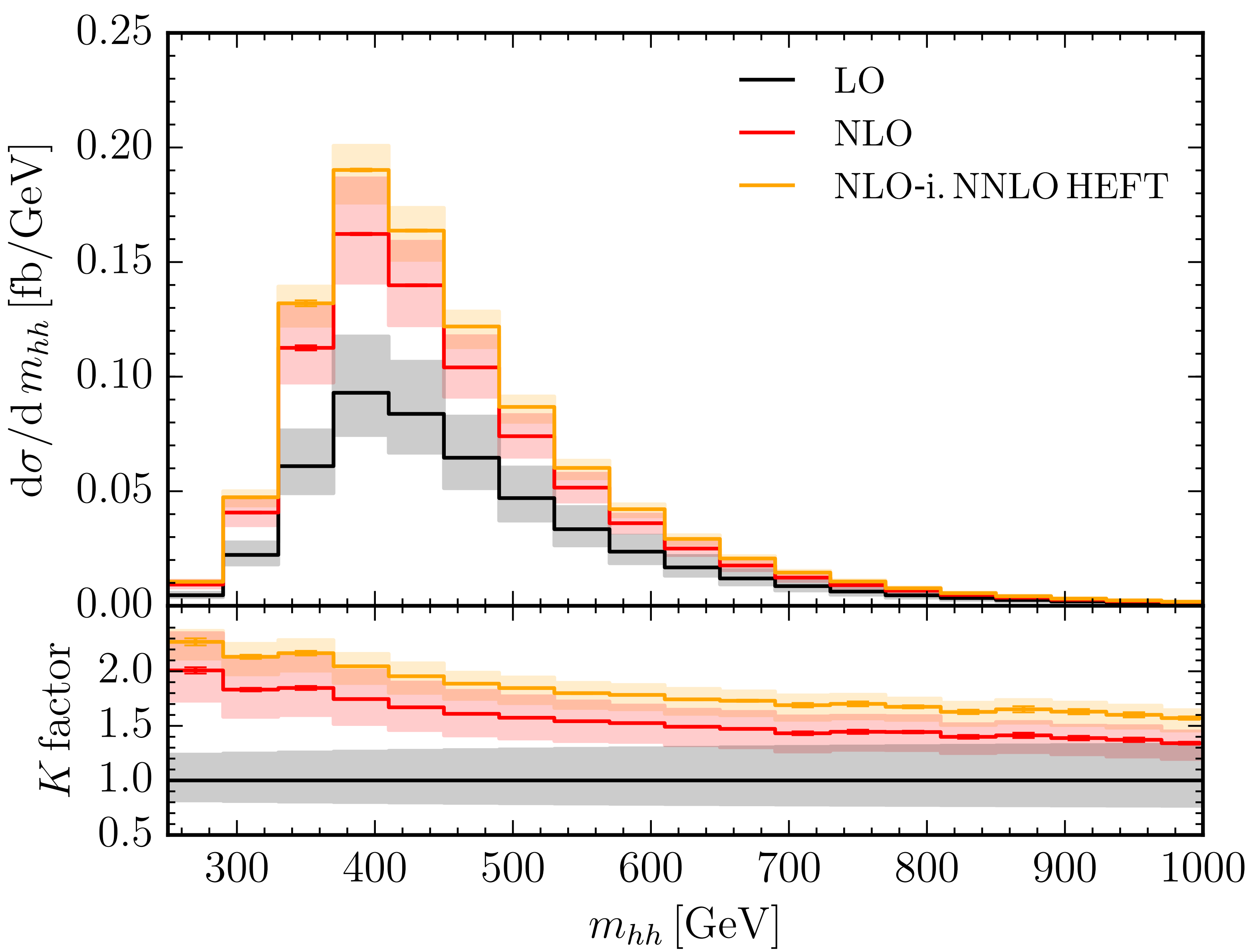


de Florian, Grazzini, Hanga, Kallweit, Lindert, Maierhöfer, Mazzitelli, Rathlev `16

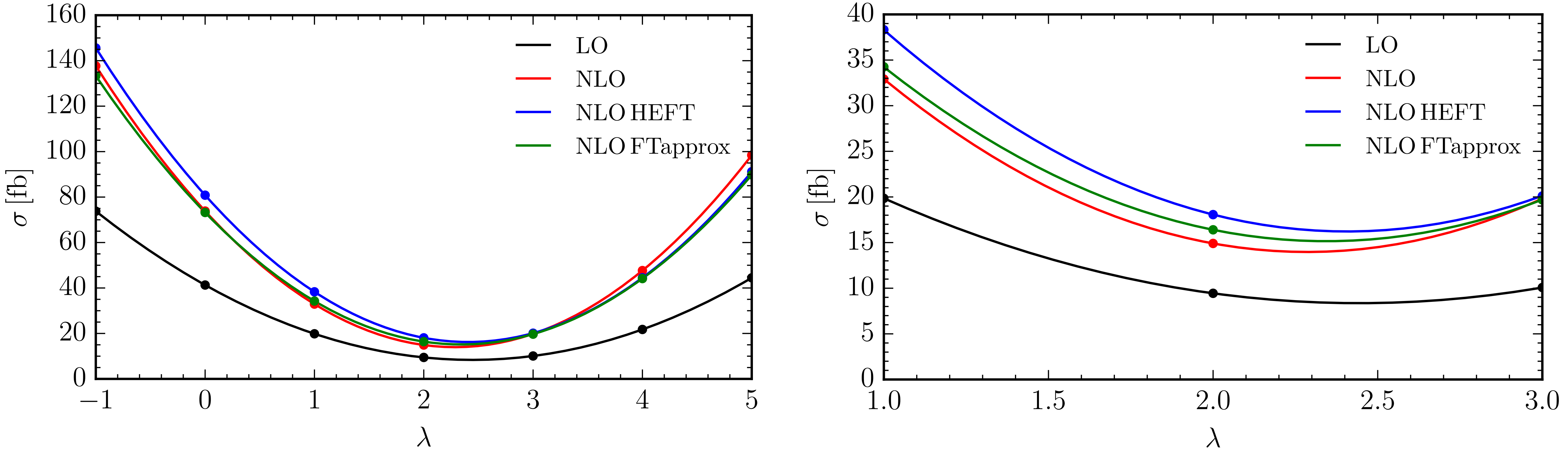
first attempt to combine NLO$_{\text{full}}$ with NNLO$_{\text{HEFT}}$

NLO-improved NNLO HEFT:

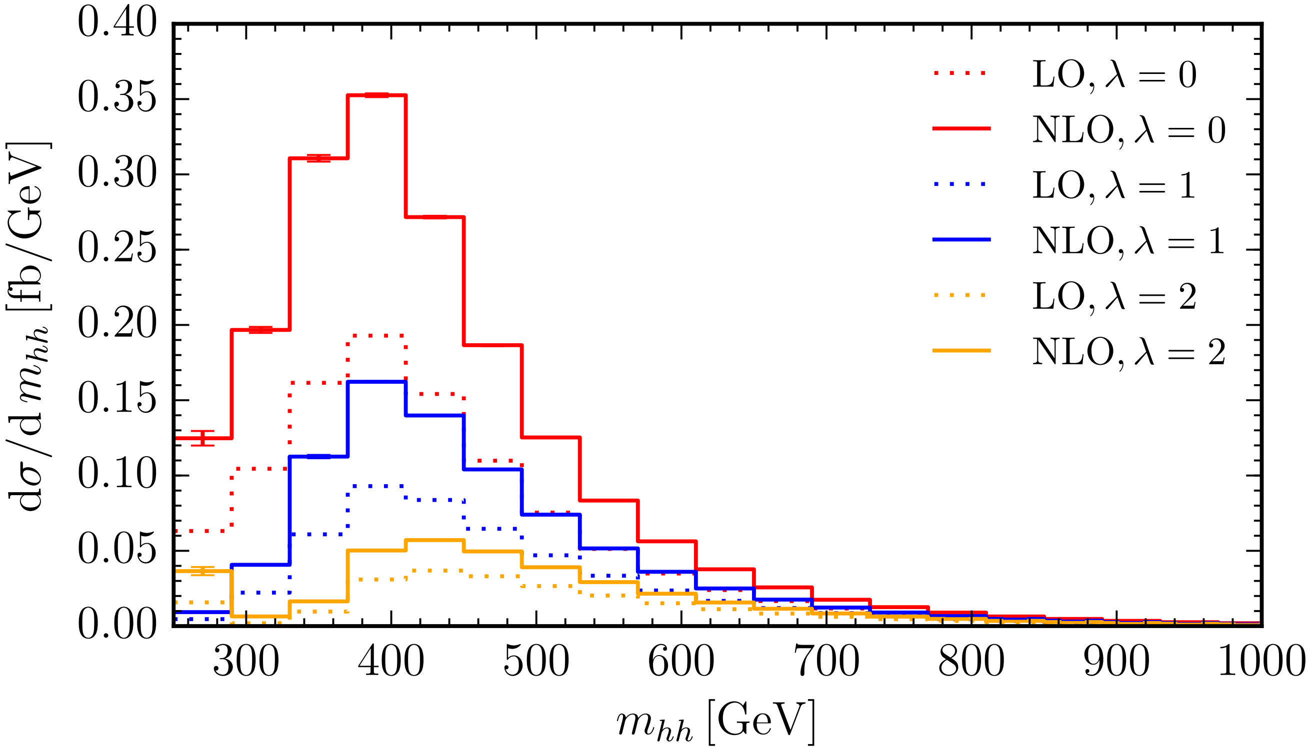
$$\frac{\mathrm{d}\sigma^{\text{full}}_{\text{NLO}}}{\mathrm{d}m_{hh}} \cdot \frac{\mathrm{d}\sigma^{\text{HEFT}}_{\text{NNLO}}/\mathrm{d}m_{hh}}{\mathrm{d}\sigma^{\text{HEFT}}_{\text{NLO}}/\mathrm{d}m_{hh}}$$

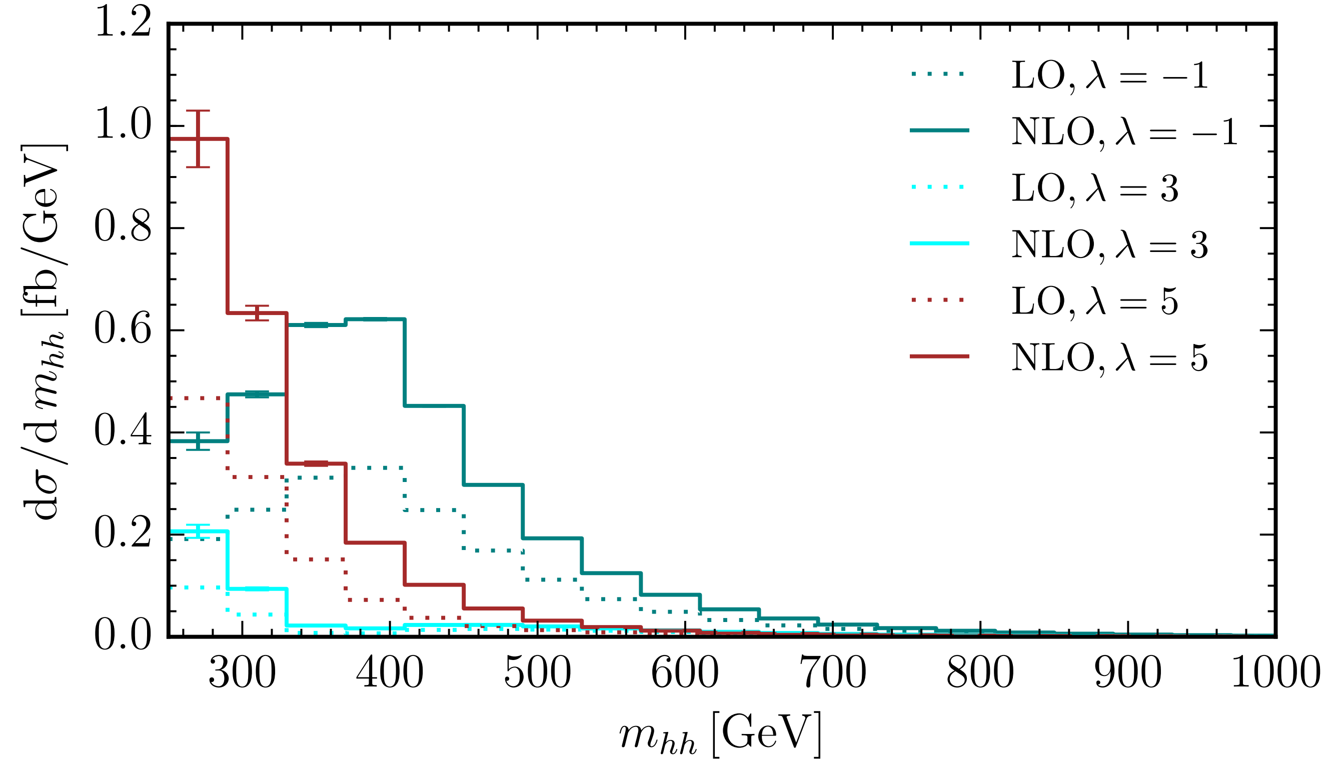


modified Higgs self-interactions



modified Higgs self-interactions





Calculation of σ^V

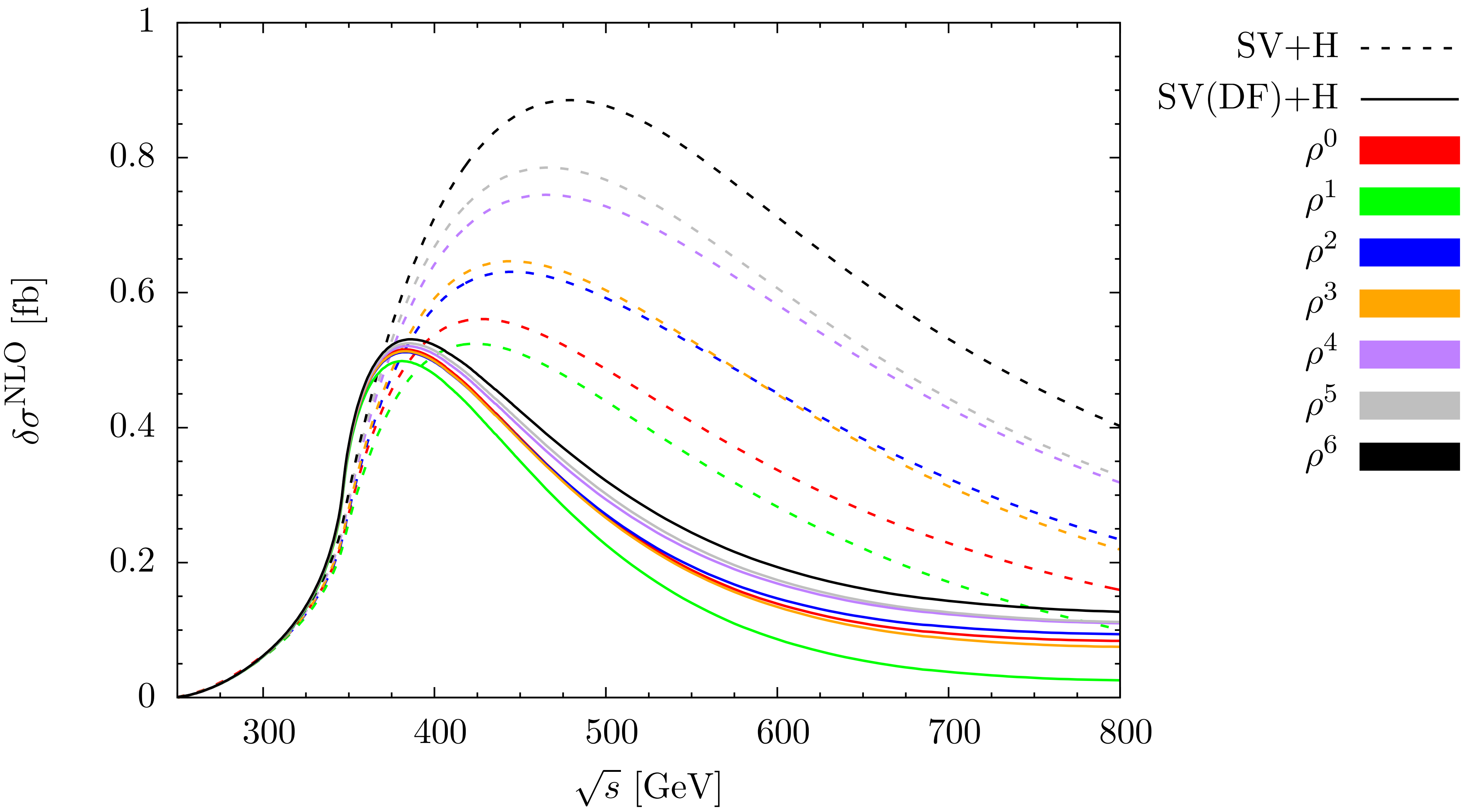
- Importance sampling:

  LO calculation
  ↓
  unweighted events
  ↓
  sampling points virtual amplitude

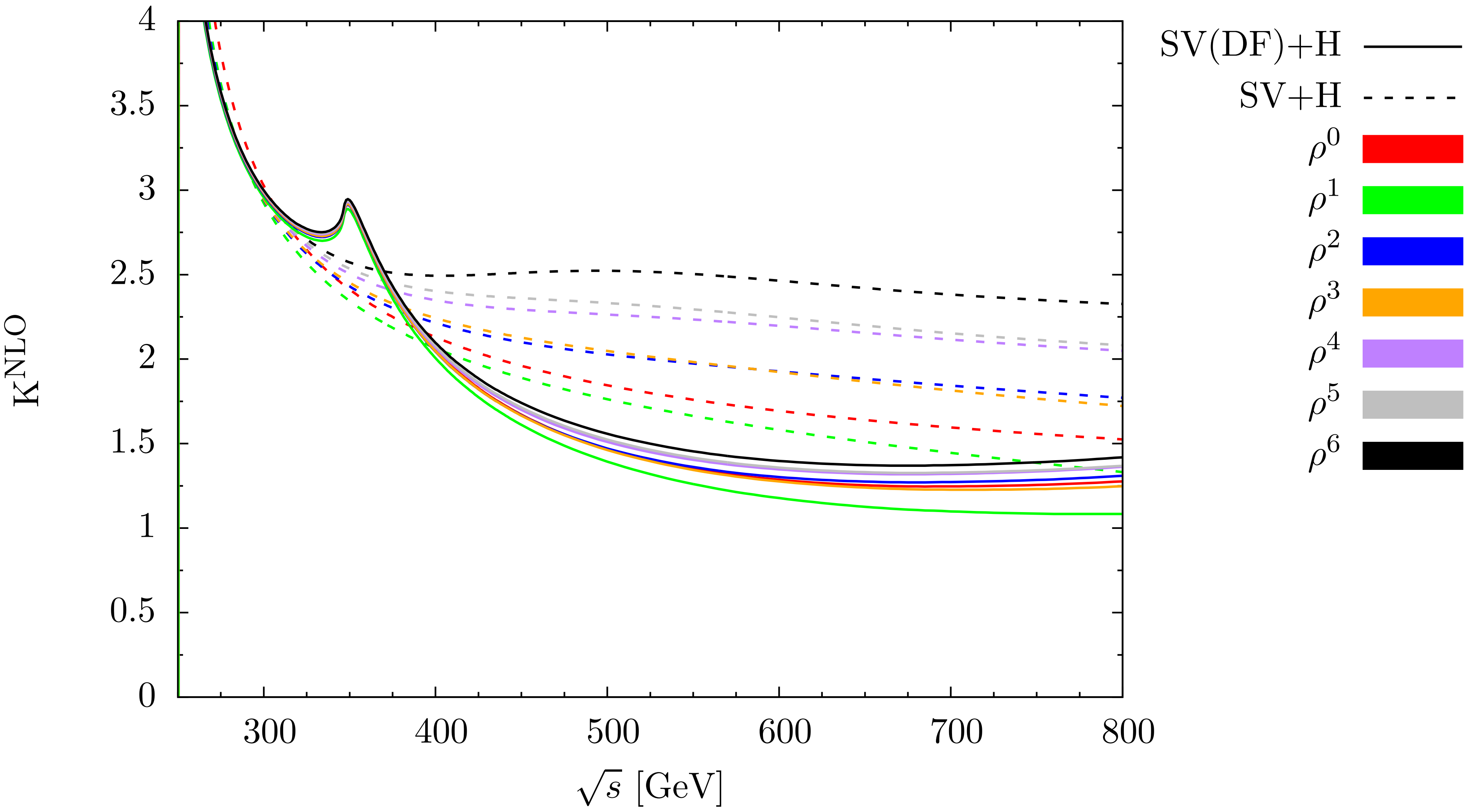
  } σ^V with 2.5% accuracy using ~1000 phase-space points

- Accuracy goal:
  - 3% for form factor F_1
  - 5-20% for form factor F_2 (depending on F_2/F_1)

- Run time: (gpu time)
  - 80 min - 2 d (≙wall-clock limit)
  - median: 2h

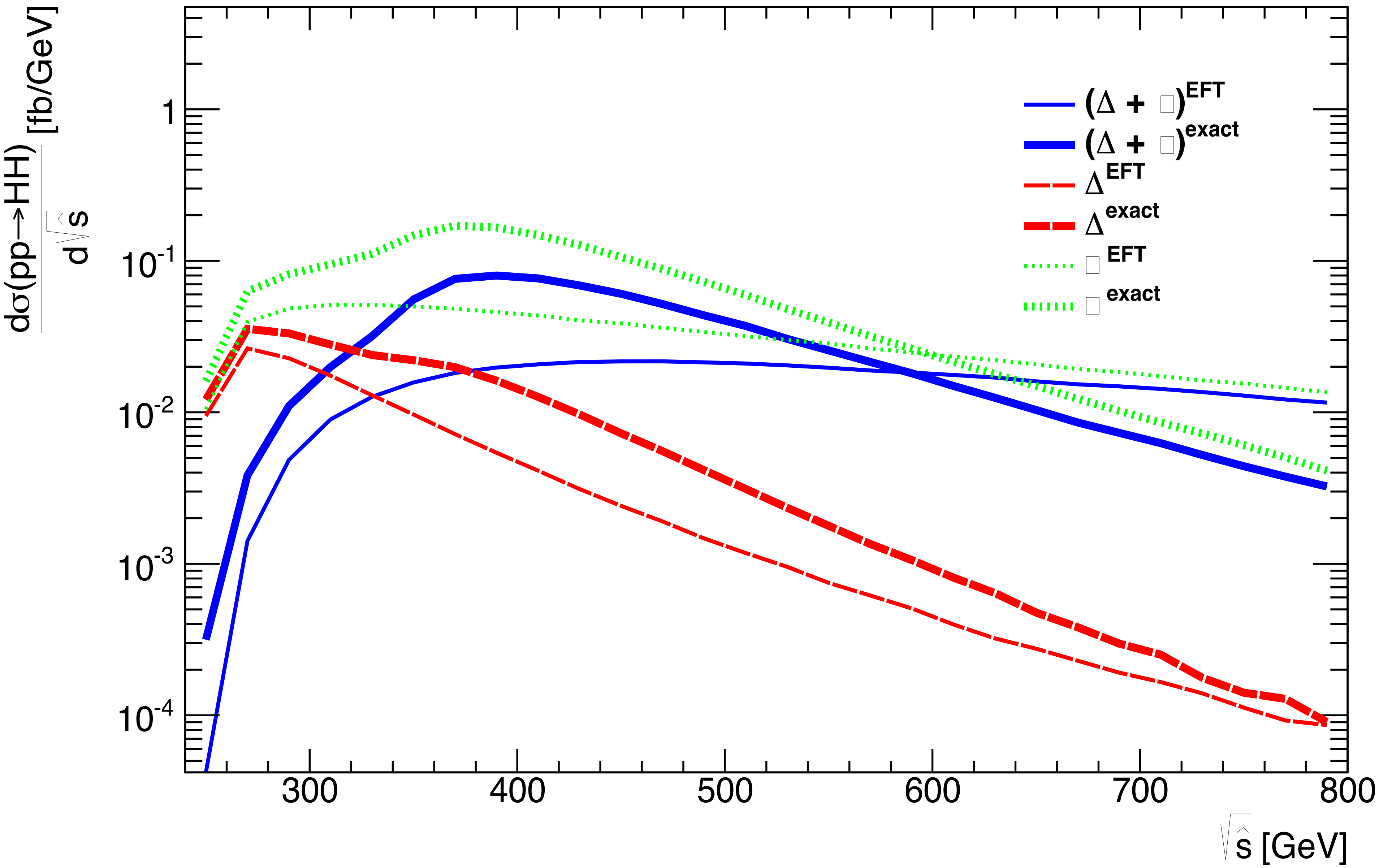


Grigo, Hoff, Steinhauser `15



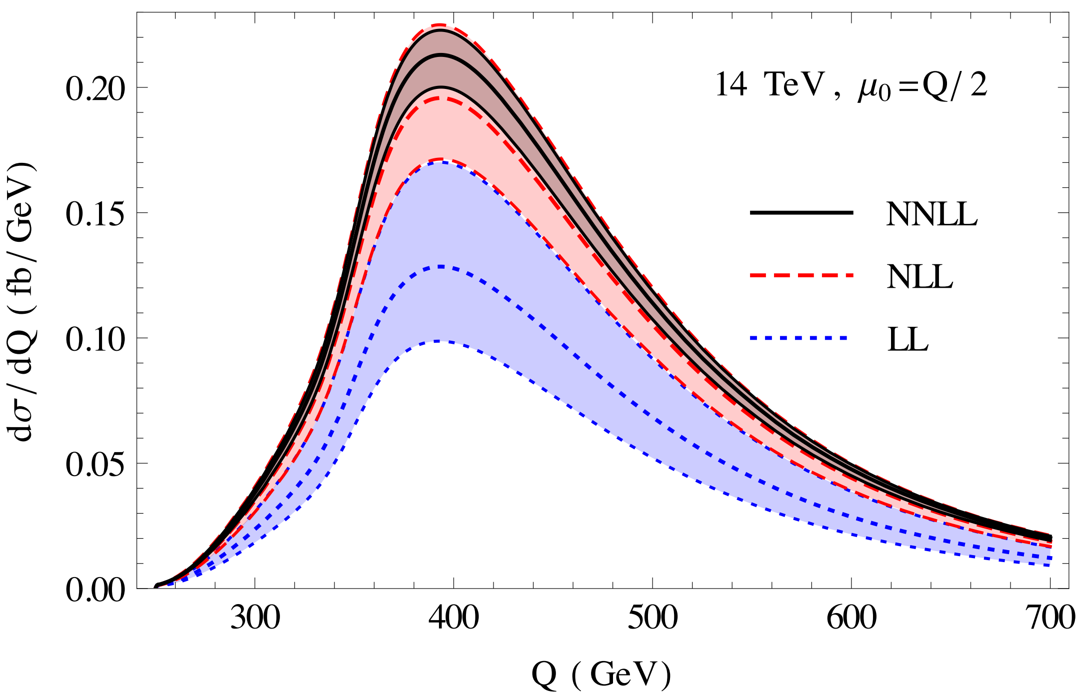
Grigo, Hoff, Steinhauser `15

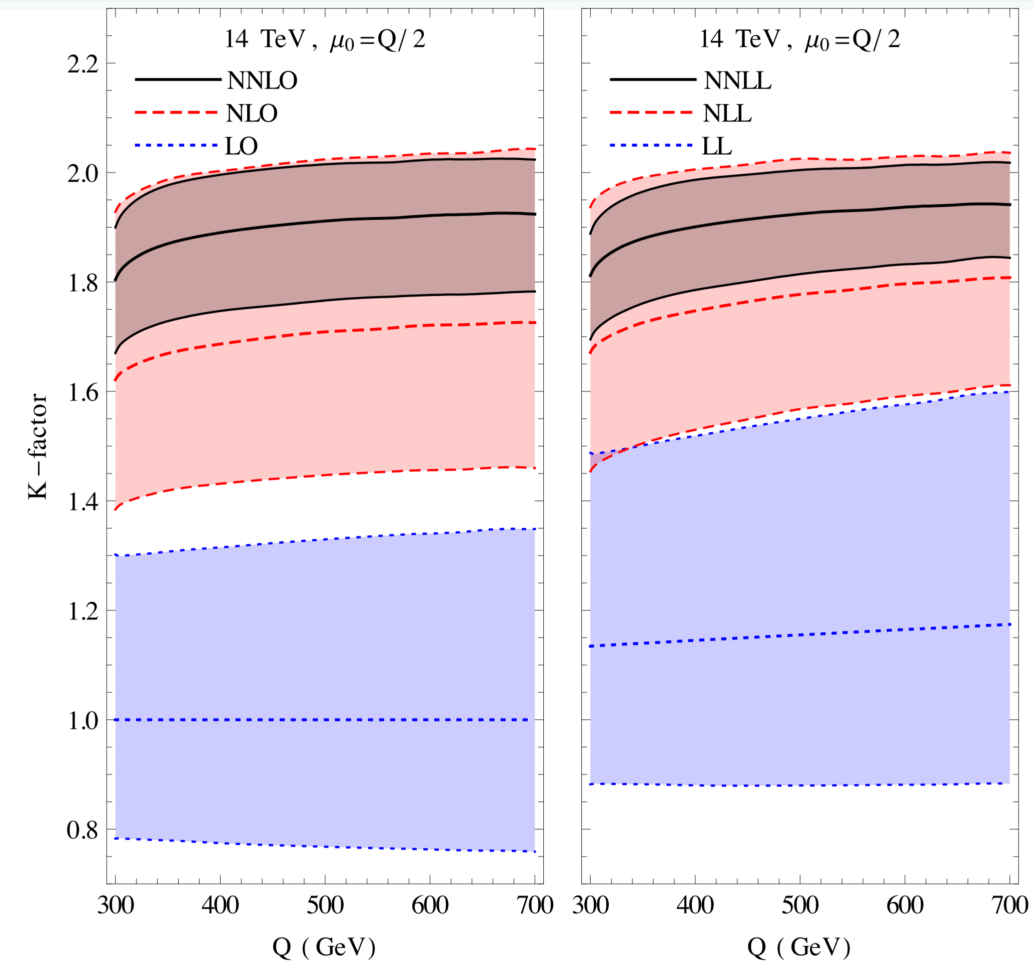
Differential Cross Section



Slawinska, van den Wollenberg,
van Eijk, Bentvelsen `14

NNLO and NNLL results



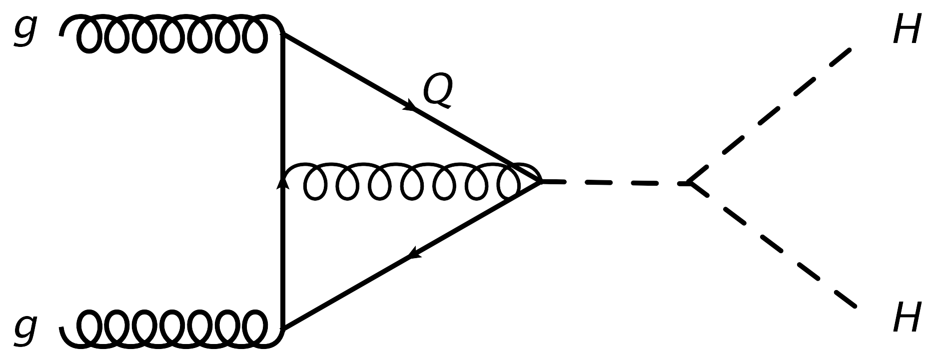


de Florian, Mazzitelli `15

$\mu_0 = Q$	NNLO (fb)	scale unc. (%)	NNLL (fb)	scale unc. (%)	PDF unc. (%)	PDF+α_S unc. (%)
8 TeV	9.92	$+9.3-10$	10.8	$+5.4-5.9$	$+5.6-6.0$	$+9.3-9.2$
13 TeV	34.3	$+8.3-8.9$	36.8	$+5.1-6.0$	$+4.0-4.3$	$+7.7-7.5$
14 TeV	40.9	$+8.2-8.8$	43.7	$+5.1-6.0$	$+3.8-4.0$	$+7.5-7.3$
33 TeV	247	$+7.1-7.4$	259	$+5.0-6.1$	$+2.2-2.8$	$+6.1-6.1$
100 TeV	1660	$+6.8-7.1$	1723	$+5.2-6.1$	$+2.1-3.0$	$+5.7-5.8$
$\mu_0 = Q/2$	NNLO (fb)	scale unc. (%)	NNLL (fb)	scale unc. (%)	PDF unc. (%)	PDF+α_S unc. (%)
8 TeV	10.8	$+5.7-8.5$	11.0	$+4.0-5.6$	$+5.8-6.1$	$+9.6-9.3$
13 TeV	37.2	$+5.5-7.6$	37.4	$+4.2-5.8$	$+4.1-4.3$	$+7.8-7.6$
14 TeV	44.2	$+5.5-7.6$	44.5	$+4.2-5.9$	$+3.9-4.1$	$+7.6-7.4$
33 TeV	264	$+5.3-6.6$	265	$+4.6-6.1$	$+2.4-2.7$	$+6.3-6.1$
100 TeV	1760	$+5.3-6.7$	1762	$+4.9-6.4$	$+2.2-3.1$	$+6.2-7.0$

Analytically known integrals

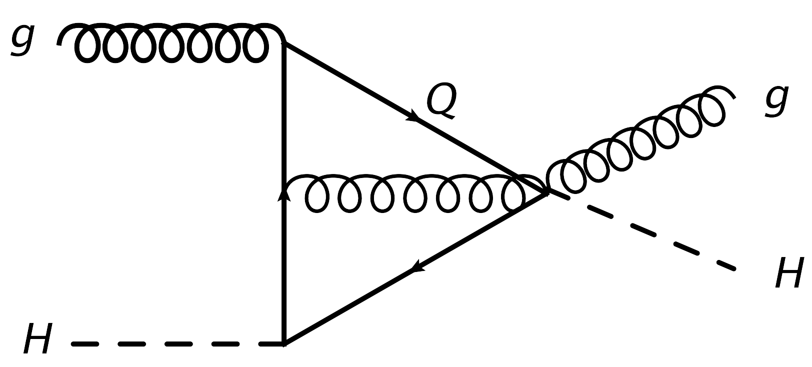
3-point, 1 off-shell leg



Spira, Djouadi et al. `93, `95
Bonciani, Mastrolia `03, `04
Anastasiou, Beerli et al. `06

→HPLs

3-point, 2 off-shell leg



Gehrmann, Guns, Kara `15

→ generalized HPLs,
12 letters

Amplitude Structure

Form factors are sums of rational functions multiplied by integrals that depend on ratios of the scales s, t, m_h^2, m_t^2 and the arbitrary scale M^2

$$F^{(L)} = \sum_i \left[\left(\sum_j C_{i,j}^{(L)} \epsilon^j \right) \cdot \left(\sum_k I_{i,k}^{(L)} \epsilon^k \right) \right]$$
$$= \quad \epsilon^{-2} \left[C_{1,-2}^{(L)} \cdot I_{1,0}^{(L)} + C_{1,-1}^{(L)} \cdot I_{1,-1}^{(L)} + \ldots \right]$$
$$+ \epsilon^{-1} \left[C_{1,-1}^{(L)} \cdot I_{1,0}^{(L)} + \ldots \right] + \ldots$$

compute only once

Additionally, all L-loop form factors are computed simultaneously without re-evaluating common integrals

Note: $gg \to HH$ is a loop induced process, real subtraction and mass factorisation contained in $\mathbf{I}, \mathbf{P}, \mathbf{K}$ operators (not discussed here)

Catani, Seymour 96

Slide: Stephen Jones — L&L 2016

Phase-Space Sampling

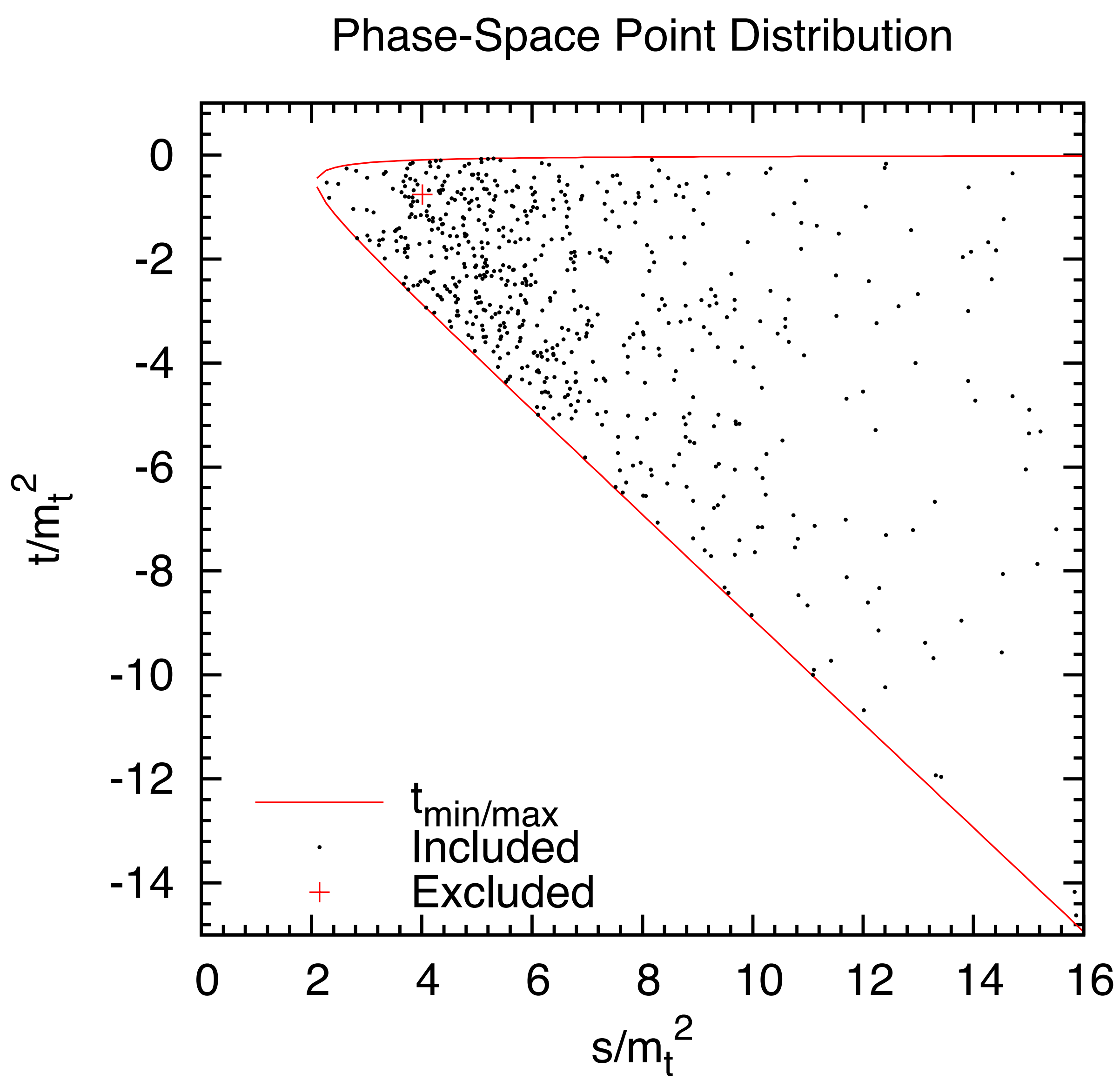
Phase-space implemented by hand
limited to 2-3 w/ 2 massive particles

Events for virtual:

1) VEGAS algorithm applied to LO matrix element $\mathcal{O}(100k)$ events computed

2) Using LO events unweighted events generated using accept/reject method $\mathcal{O}(30k)$ events remain

3) Randomly select 666 Events (woops), compute at NLO, exclude 1



Note: No grids used either for integrals or phase-space

Slide: Stephen Jones — L&L 2016