

# Fixed Order QCD corrections

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## QCD@LHC 2017



Debrecen, August 2017

# Outline

-  Introduction
-  NLO
-  NNLO
-   $N^3\text{LO}$
-  TH Uncertainties
-  Conclusions

# Conclusion



EXP

We measure XXX and the observable is in agreement with the Standard Model predictions

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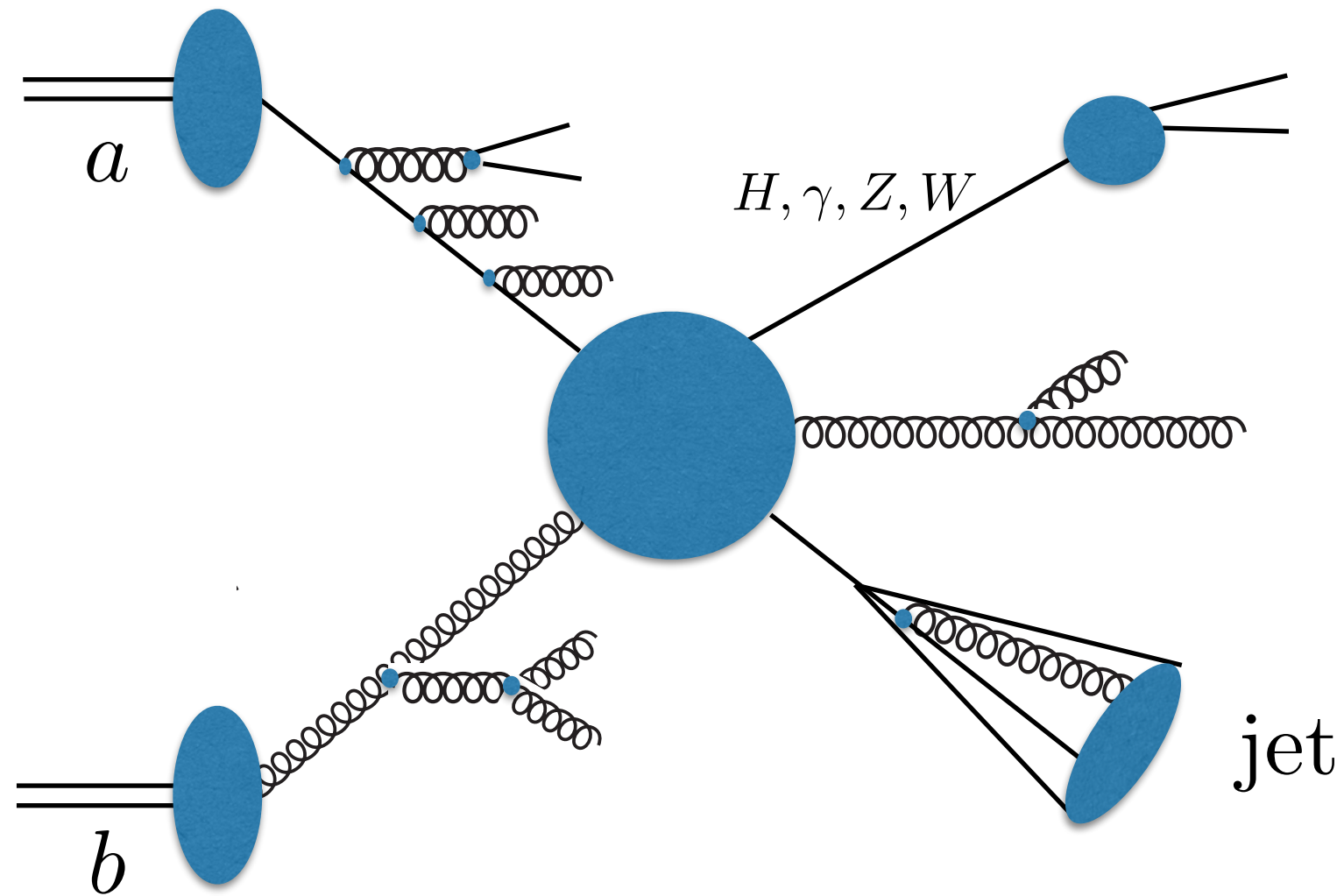


TH

We compute XXX at **N<sup>i</sup>LO** and find a considerable reduction in scale dependence and a better description of the data



- ▶ In the LHC era, QCD is everywhere!



non-perturbative parton distributions

$$d\sigma = \sum_{ab} \int dx_a \int dx_b f_a(x_a, \mu_F^2) f_b(x_b, \mu_F^2) \times d\hat{\sigma}_{ab}(x_a, x_b, Q^2, \alpha_s(\mu_R^2)) + \mathcal{O}\left(\left(\frac{\Lambda}{Q}\right)^m\right)$$

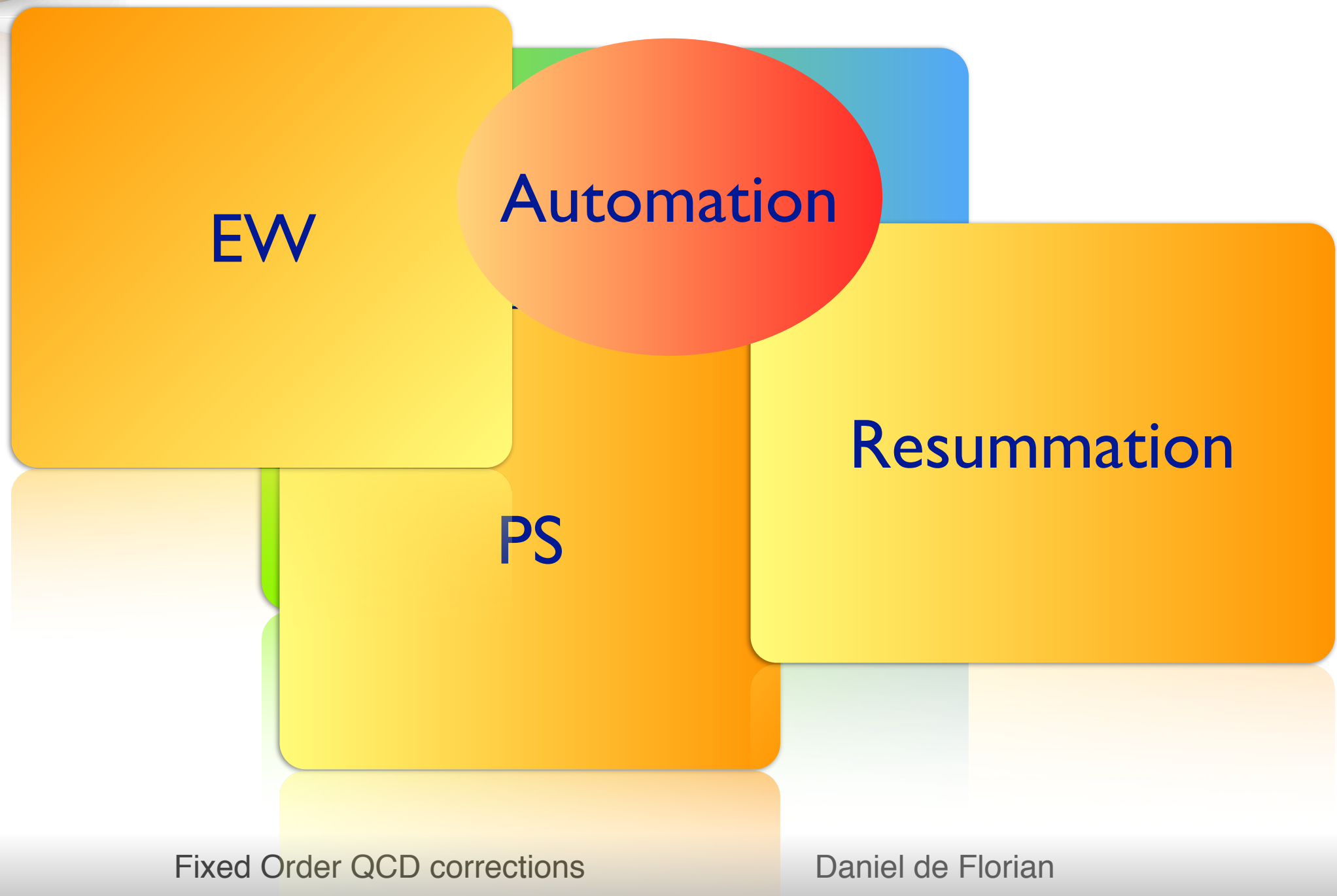
perturbative partonic cross-section

- ▶ Require precision for perturbative and non-perturbative contribution

# The perturbative toolkit for precision at colliders



Everything starts with a fixed order calculation



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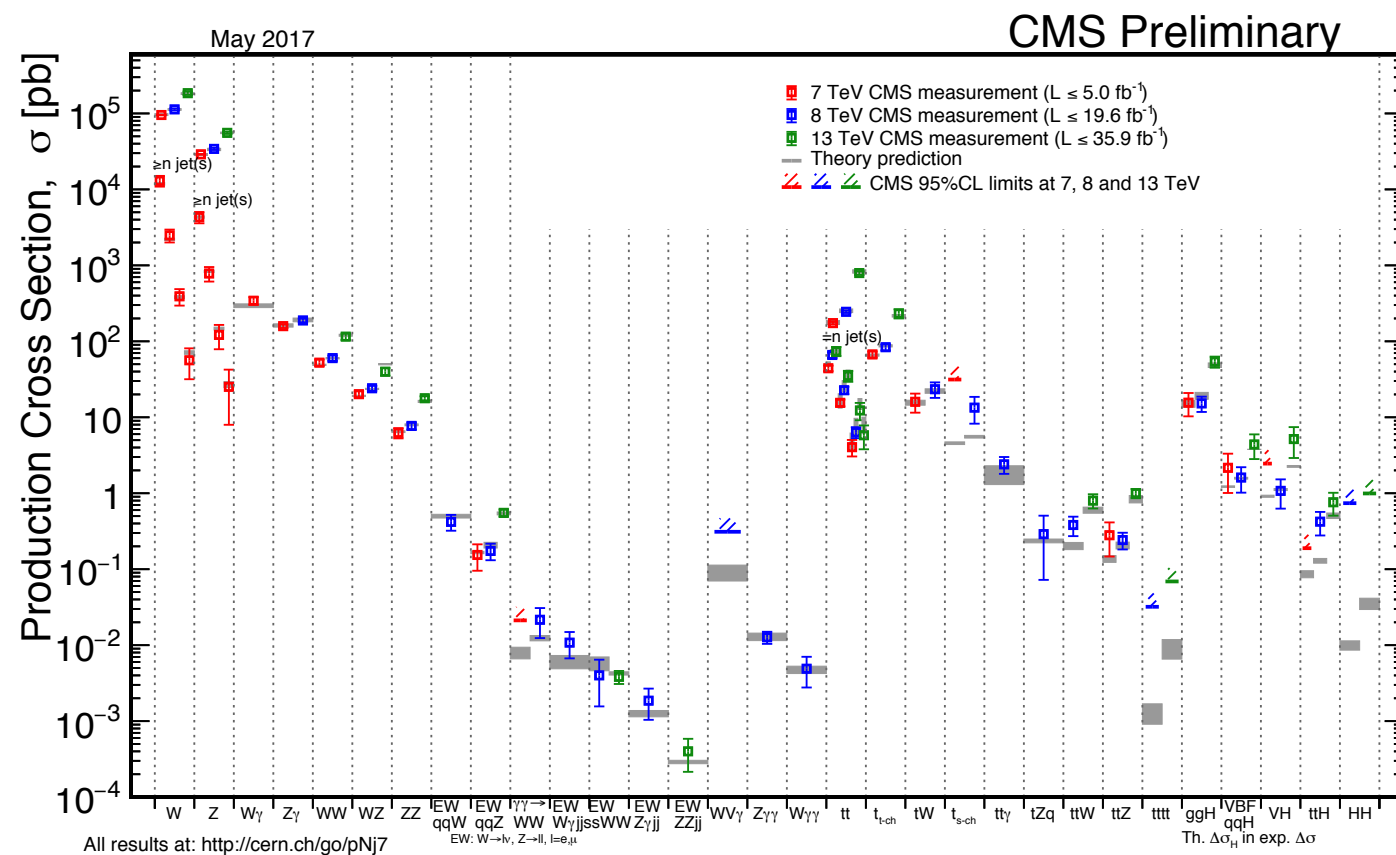
Everything starts with a fixed order calculation

- Partonic cross-section: expansion in  $\alpha_s(\mu_R^2) \ll 1$

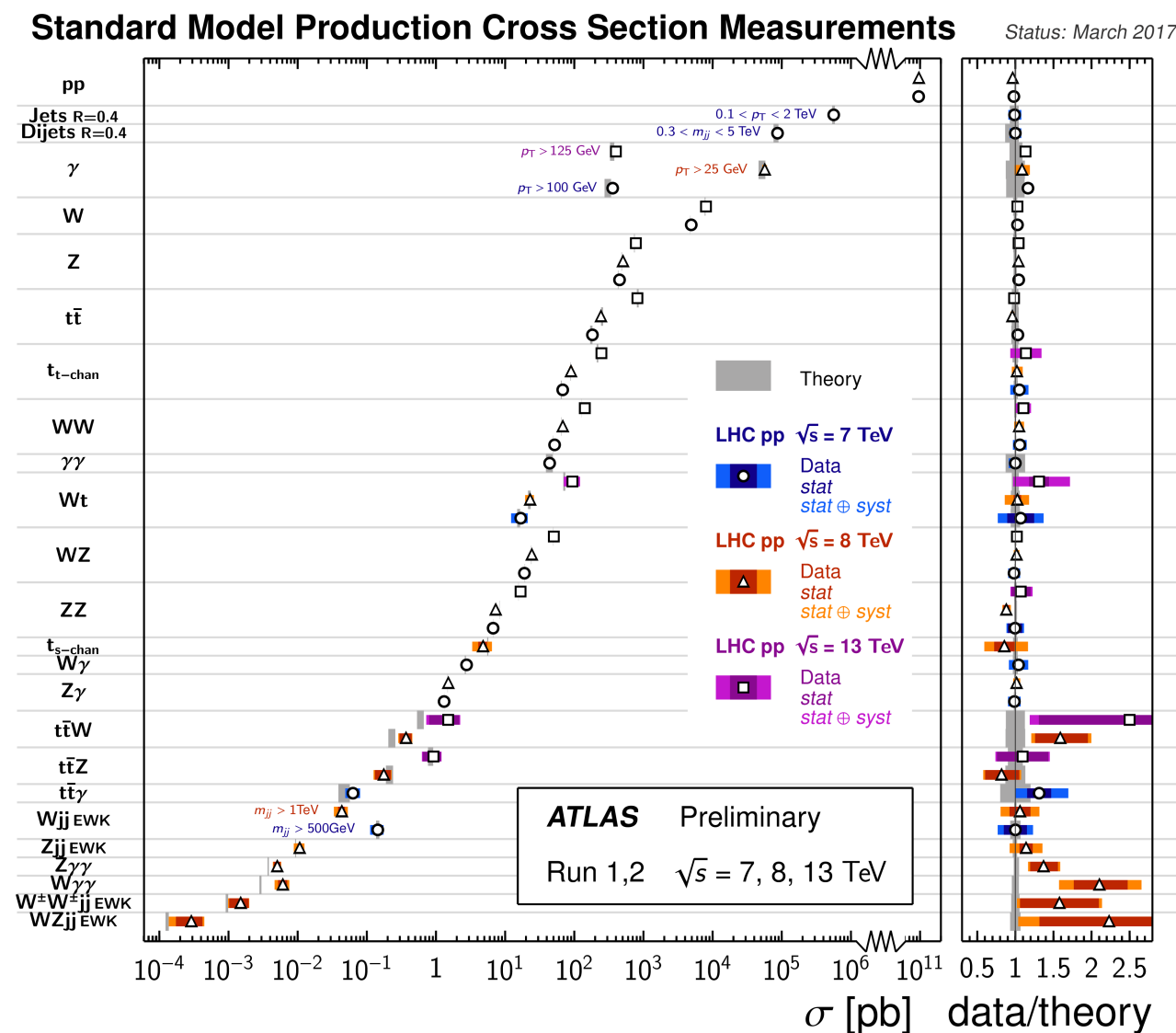
$$d\hat{\sigma} = \alpha_s^n d\hat{\sigma}^{(0)} + \alpha_s^{n+1} d\hat{\sigma}^{(1)} + \dots$$



► LHC incredibly successful at 7 , 8 & 13 TeV (Runs I and II)



► Everything SM like (including Higgs)





# Why higher order corrections?

- ▶ Large Corrections : check PT shape and normalization

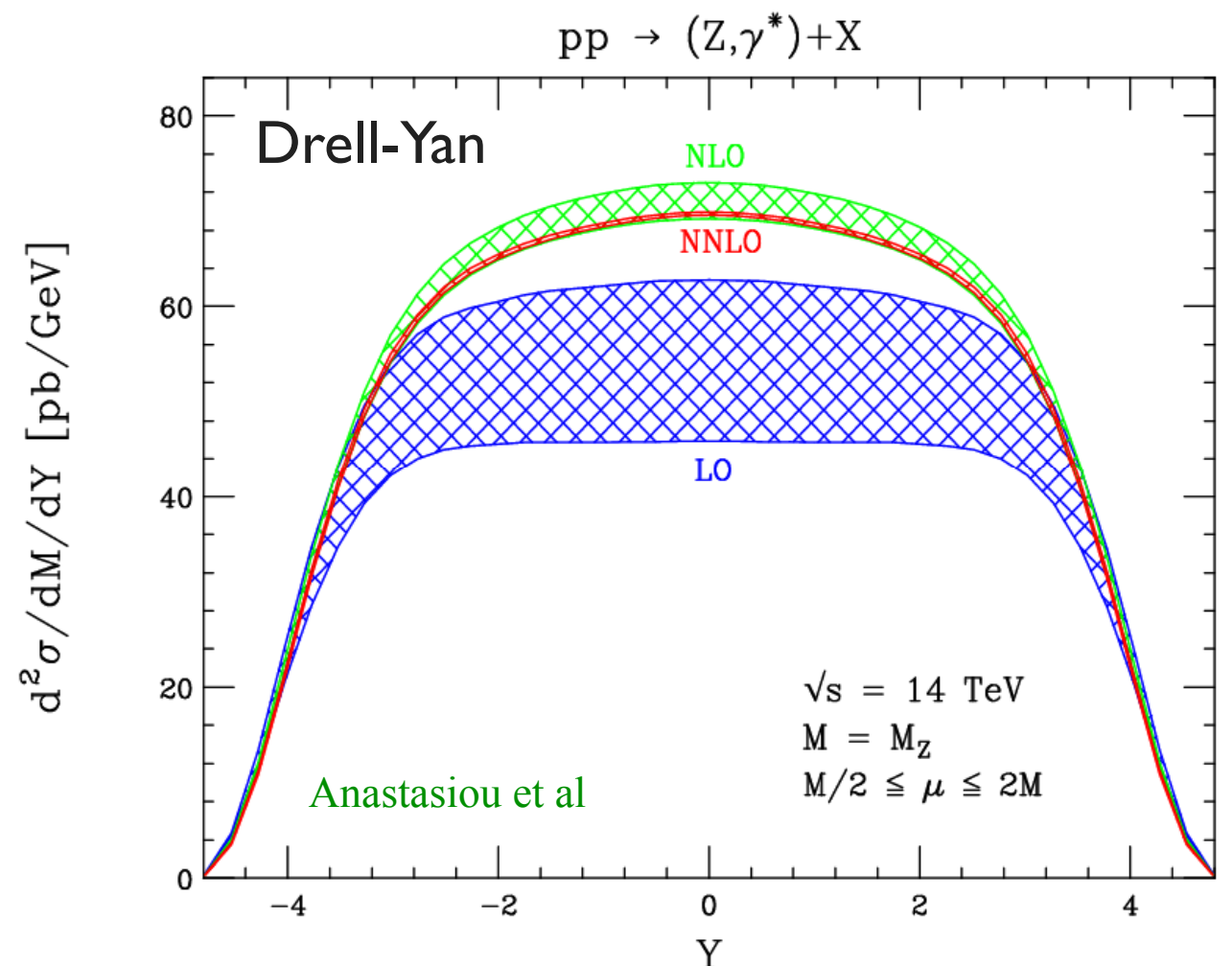
$\alpha_s \sim 0.1$   $\longrightarrow$  slow convergence

- ▶ Accurate Theoretical Predictions

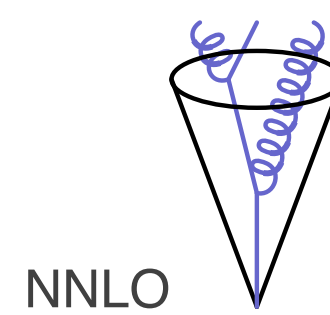
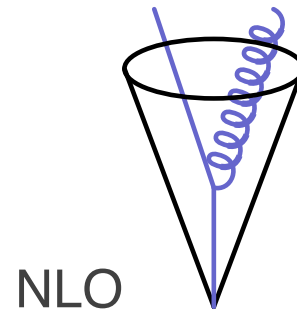
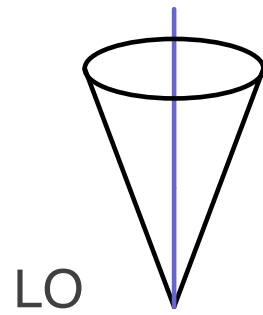
$$\sigma(p_1, p_2) = \sum_{a,b} \int_0^1 dx_1 \int_0^1 dx_2 f_{a/h_1}(x_1, \mu_F^2) f_{b/h_2}(x_2, \mu_F^2) \times \hat{\sigma}_{ab}(x_1 p_1, x_2 p_2, \alpha_s(\mu_R^2), \mu_R^2, \mu_F^2)$$

Scale dependence considerably reduced at higher orders

$\longrightarrow$  TH uncertainty



- ▶ Extra radiation : more partons result in better TH/EXP matching

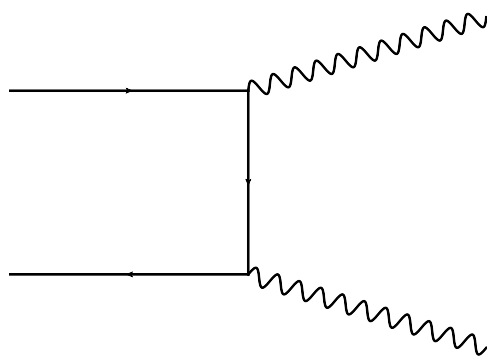


Description of jets, transverse momentum, etc

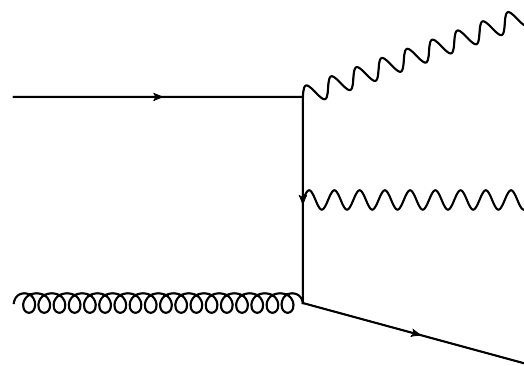
- ▶ Opening of new channels

Sometimes new channels at higher order provide large corrections due to parton luminosity (pdf, non-perturbative-perturbative interplay)

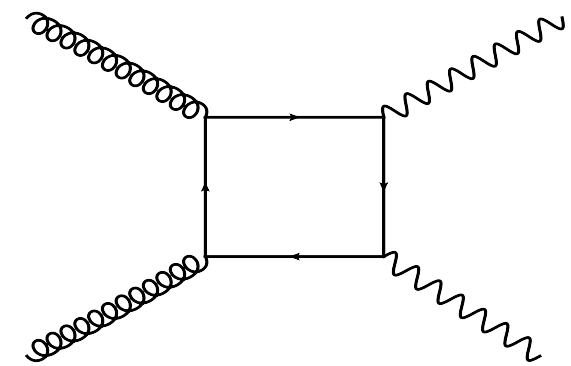
## ● Diboson production



$\mathcal{O}(\alpha_s^0)$  but  $q\bar{q}$  Luminosity



$\mathcal{O}(\alpha_s)$  but  $qg$  Luminosity



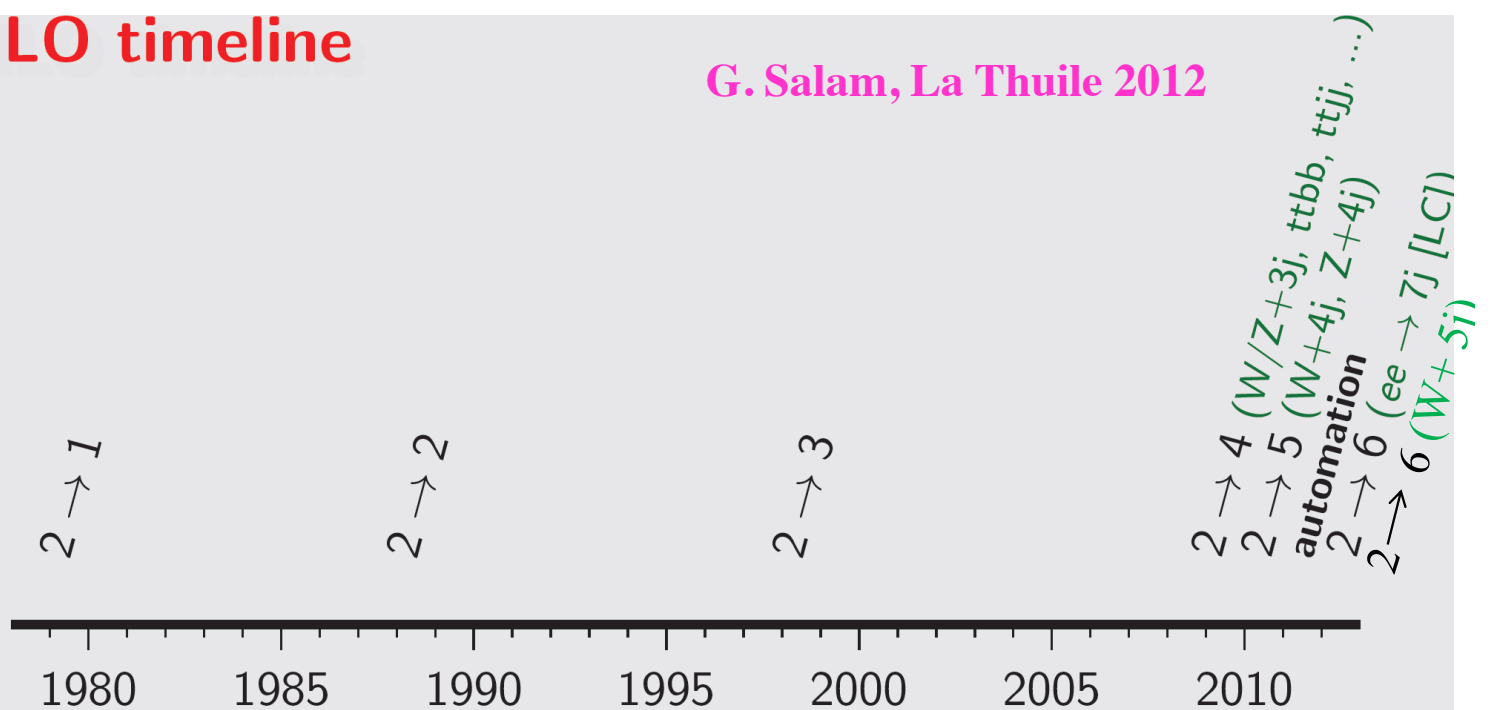
$\mathcal{O}(\alpha_s^2)$  but  $gg$  Luminosity

# NLO

## The NLO revolution

### NLO timeline

G. Salam, La Thuile 2012



# Revolution in calculation of 1-loop amplitudes

- ▶ Bottleneck was in the virtual contribution : large multiplicities

$$\text{Sun diagram} = \sum_i d_i \text{Box} + \sum_i c_i \text{Triangle} + \sum_i b_i \text{Bubble} + \sum_i a_i \text{ tadpole} + \frac{x}{y}$$

## Feynmanian approach

Improvements in decomposition and reduction

Denner, Dittmaier; Pozzorini; Binoth, Guillet, Heinrich, Pilon, Schubert + many others

## Unitarian approach

Use multi-particle cuts from generalized unitarity

Bern, Dixon, Dunbar, Kosower; Britto, Cachazo, Feng; Mastrolia; Forde; Badger; Ellis, Giele, Kunszt, Melnikov + many others

OPP Ossola, Papadopoulos, Pittau

decomposition at the integrand level

J. Henn QCD@LHC17



► Final goal: Really automatic NLO calculations

zero cost for humans

- Specify the process (input card)
- Input parameters
- Define final cuts

► Automatic NLO calculation “conceptually” solved

- in a few years a number of codes

HELAC-NLO, Rocket, BlackHat+SHERPA, GoSam+SHERPA/MADGRAPH, NJet+SHERPA, Madgraph5-aMC@NLO, RECOLA, OpenLoops+SHERPA

- compete on precision, flexibility, speed, stability, ...
- many features : uncertainties, Parton shower, ...

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## How easy is NLO these days?

$pp \rightarrow tt + j$

```
import model loop_sm-no_b_mass
define p = g u u~ c c~ d d~ s s~ b b~
define j = g u u~ c c~ d d~ s s~ b b~
generate p p > t~ t j [QCD]
output my_pp_ttj
calculate_xs NLO
```

e.g. MadGraph5\_aMC@NLO v2.1.1  
[Alwall et al. 1405.0301]

generation time ~ 5 mins  
total cross section ~ 30 mins (20 cores)

# ► Not everything solved at NLO yet... but constant progress

## ● EW corrections

A.Vicini QCD@LHC17

MADGRAPH5\_AMC@NLO

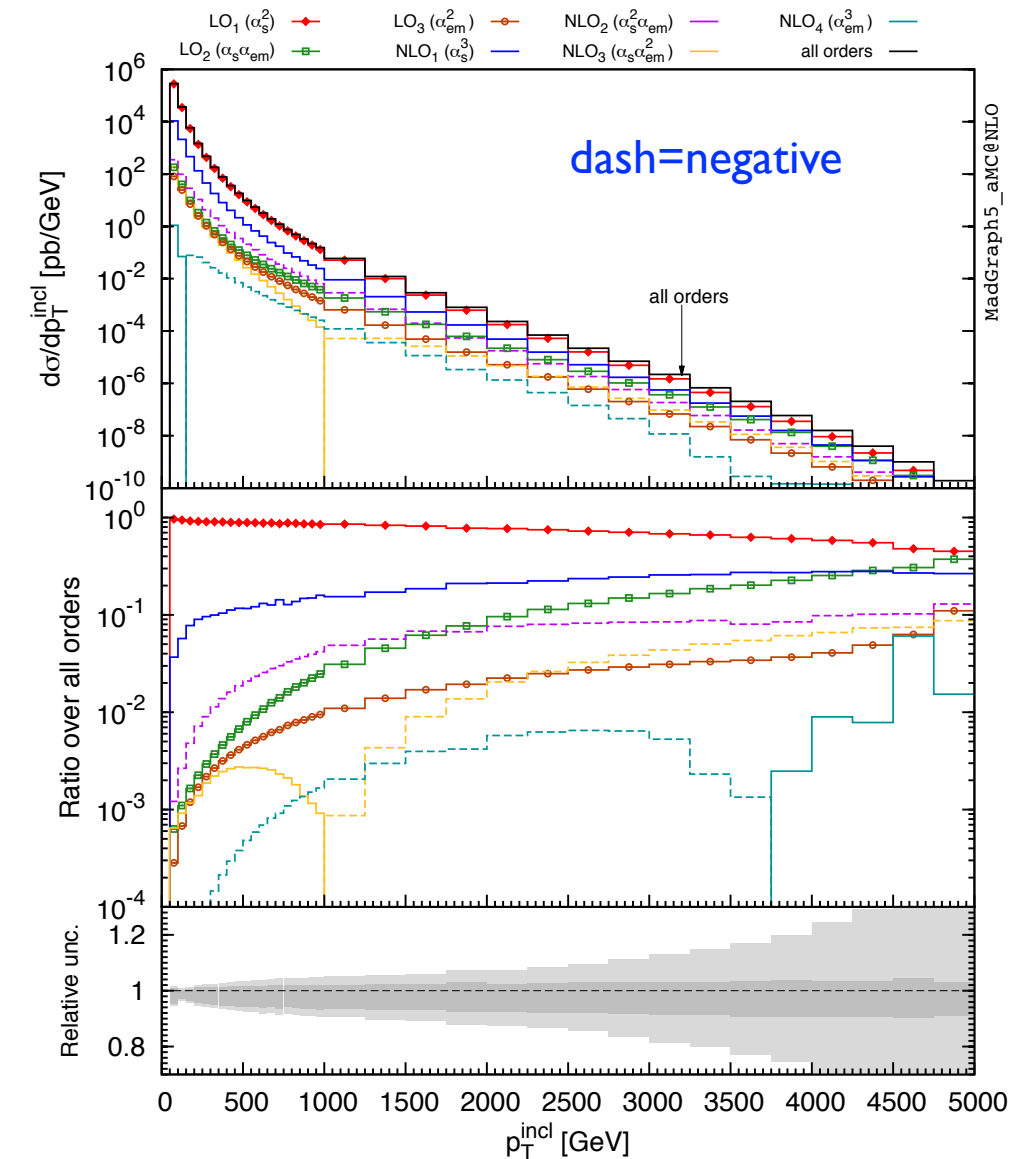
- QCD dominant (except very large pT)
- Coupling hierarchy ~ respected
- Large cancellations in EW contributions
- No HB radiation

Sherpa+Recola

B. Biedermann QCD@LHC17

## Dijet production

Frederix, Frixione, Hirschi, Pagani, Shao, Zaro (2017)



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## ● Off-shell effects

e.g., ttj Bevilacqua et al (2015)

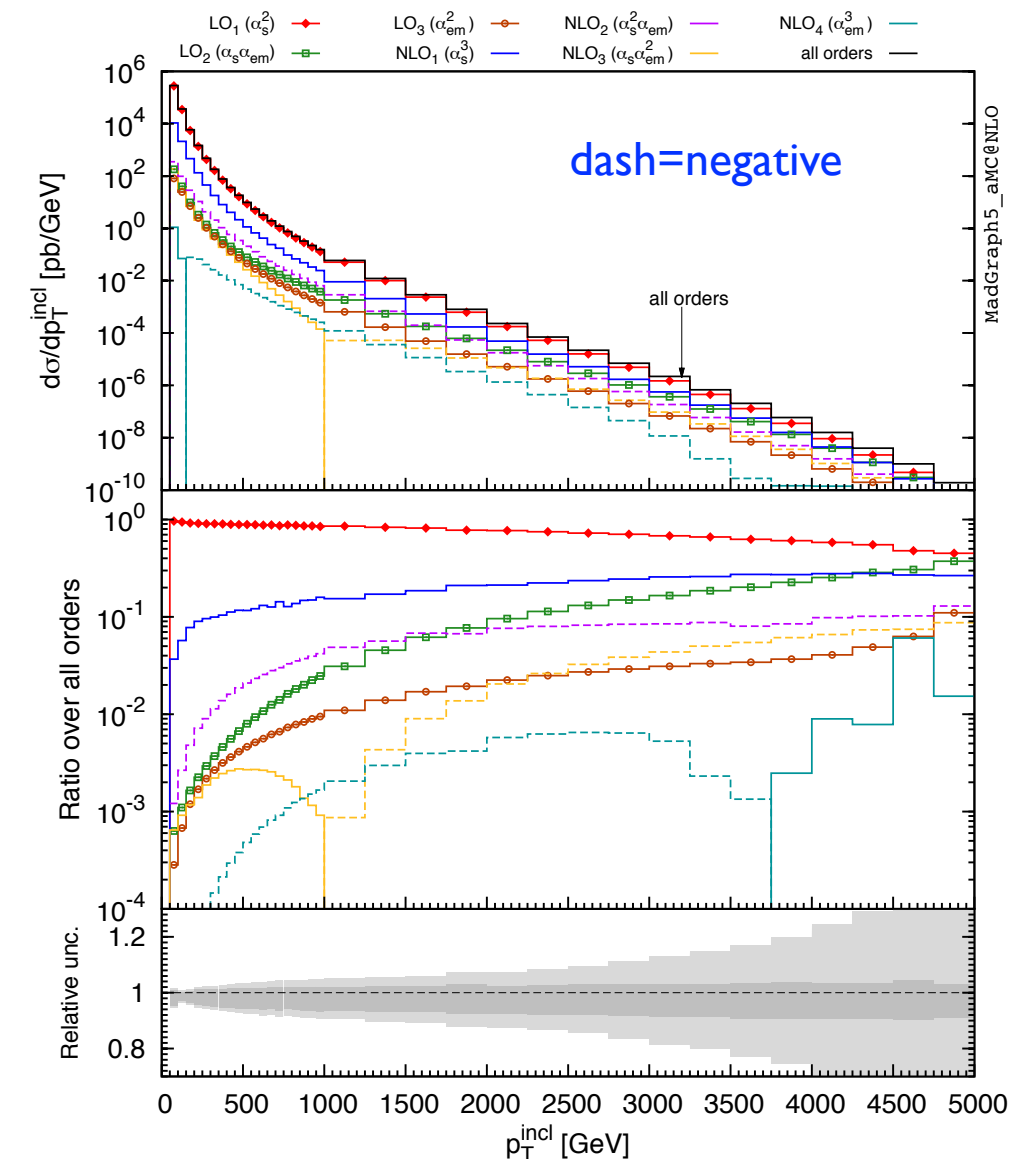
- Large corrections in kinematical edges

H. Hartanto QCD@LHC17

## ● BSM (arbitrary, higher dimensional operators, etc)

## Dijet production

Frederix, Frixione, Hirschi, Pagani, Shao, Zaro (2017)



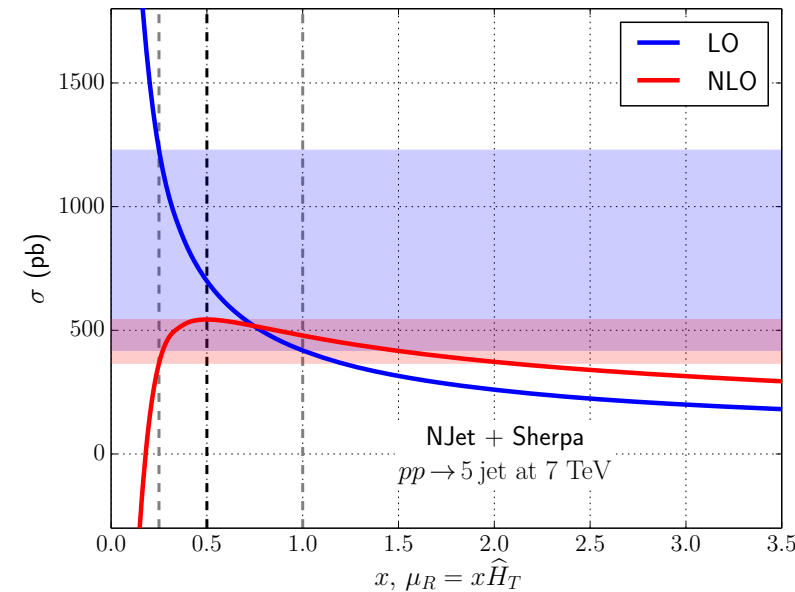
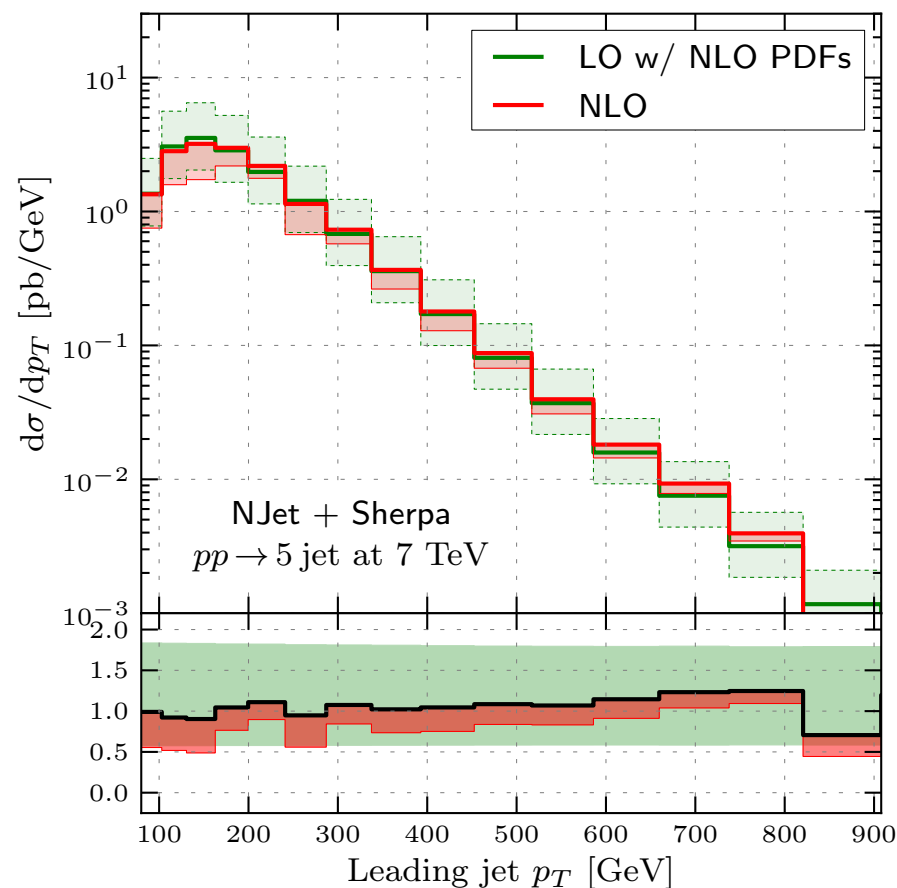


- Still limitations in numerical accuracy for processes with many particles (>4) in final state

# Multi-jet production

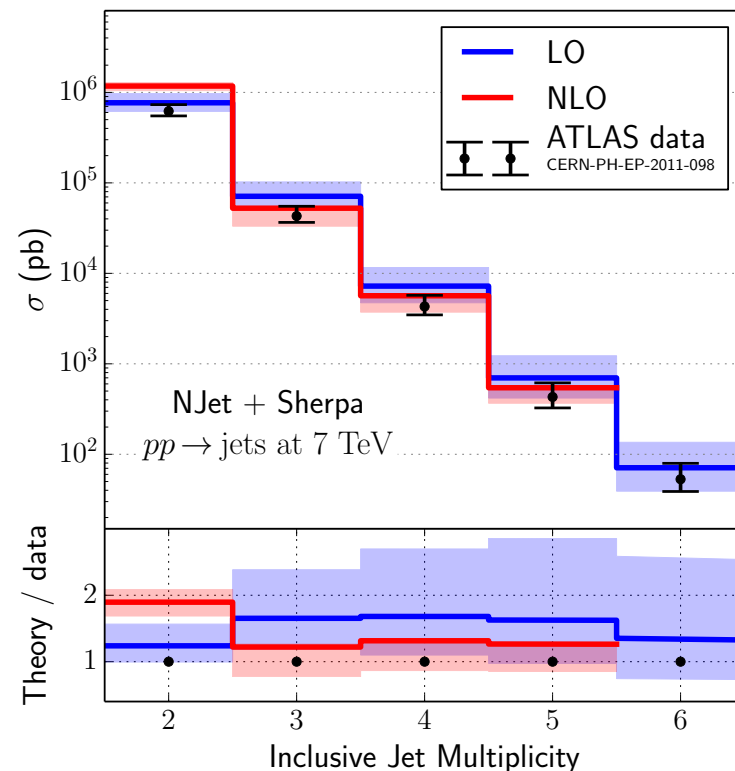
NJet+Sherpa (Badger, Biedermann, Uwer, Yundin)

$pp \rightarrow 5 \text{ jets at NLO}$



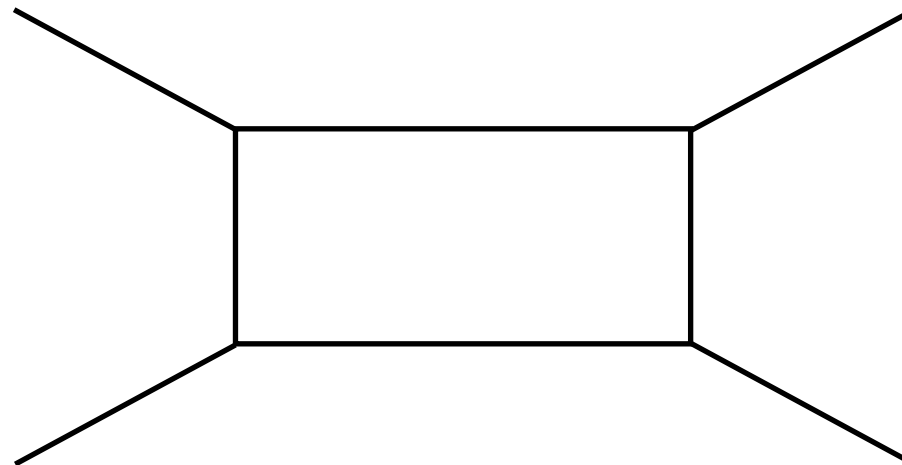
$$\hat{H}_T = \sum_{i=1}^{N_{\text{parton}}} p_{T,i}^{\text{parton}}$$

- Better stability
- NLO in very good agreement with data!



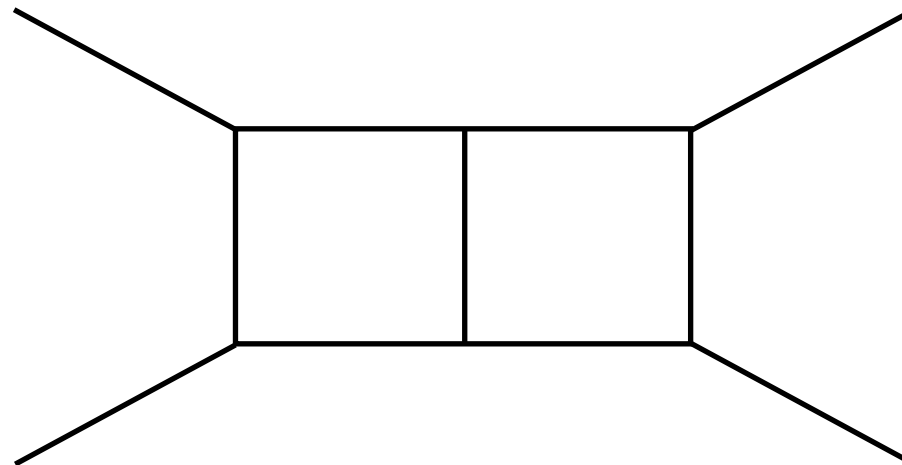
# NLO

## Loop induced processes



# NLO

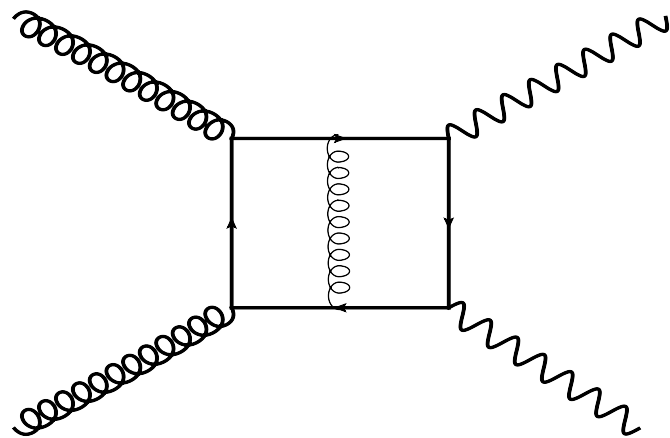
## Loop induced processes



NLO = 2 loops for them...

## Loop induced Processes : start at one loop at LO

- ▶ Enhanced by gluon luminosity
- ▶ Corrections for gg channel usually large (color, logs)



$$gg \rightarrow VV$$

H background

F. Caola, et al (2015-2016)

J. Campbell, K. Ellis, M. Czakon, S. Kirchner (2015)

$$gg \rightarrow (H) \rightarrow VV$$

signal-background interference

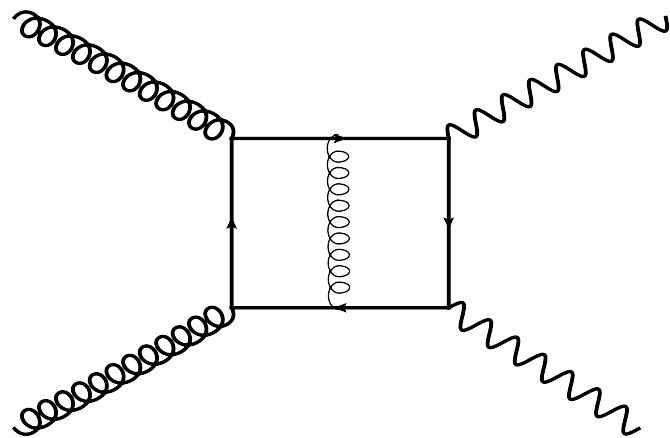
Higgs width

- ▶ Available only for massless partons @NLO (+1/m<sub>T</sub> expansion)
- ▶ But mass effects not-negligible (helicity flip in interference)



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$$gg \rightarrow H + \text{jet}$$

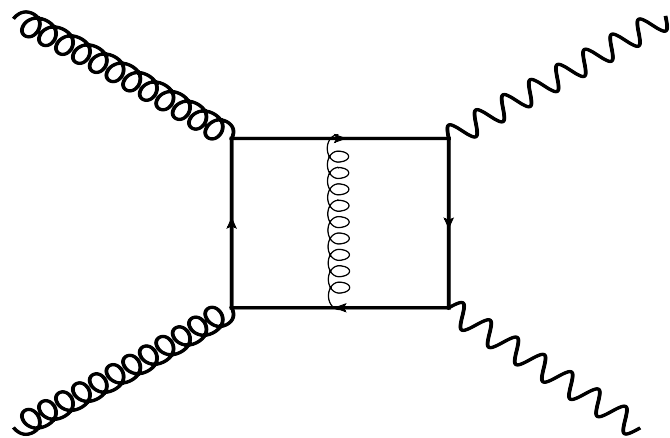
- ▶ usually computed within EFT (large top mass limit)
- ▶ sensitive to top mass at large  $p_T$
- ▶ sensitive to top-bottom interference at low  $p_T$

40-50% correction  
low mass approx.

J. Lindert, K. Melnikov, L. Tancredi, C. Wever (2017)

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H.Frellesvig QCD@LHC17

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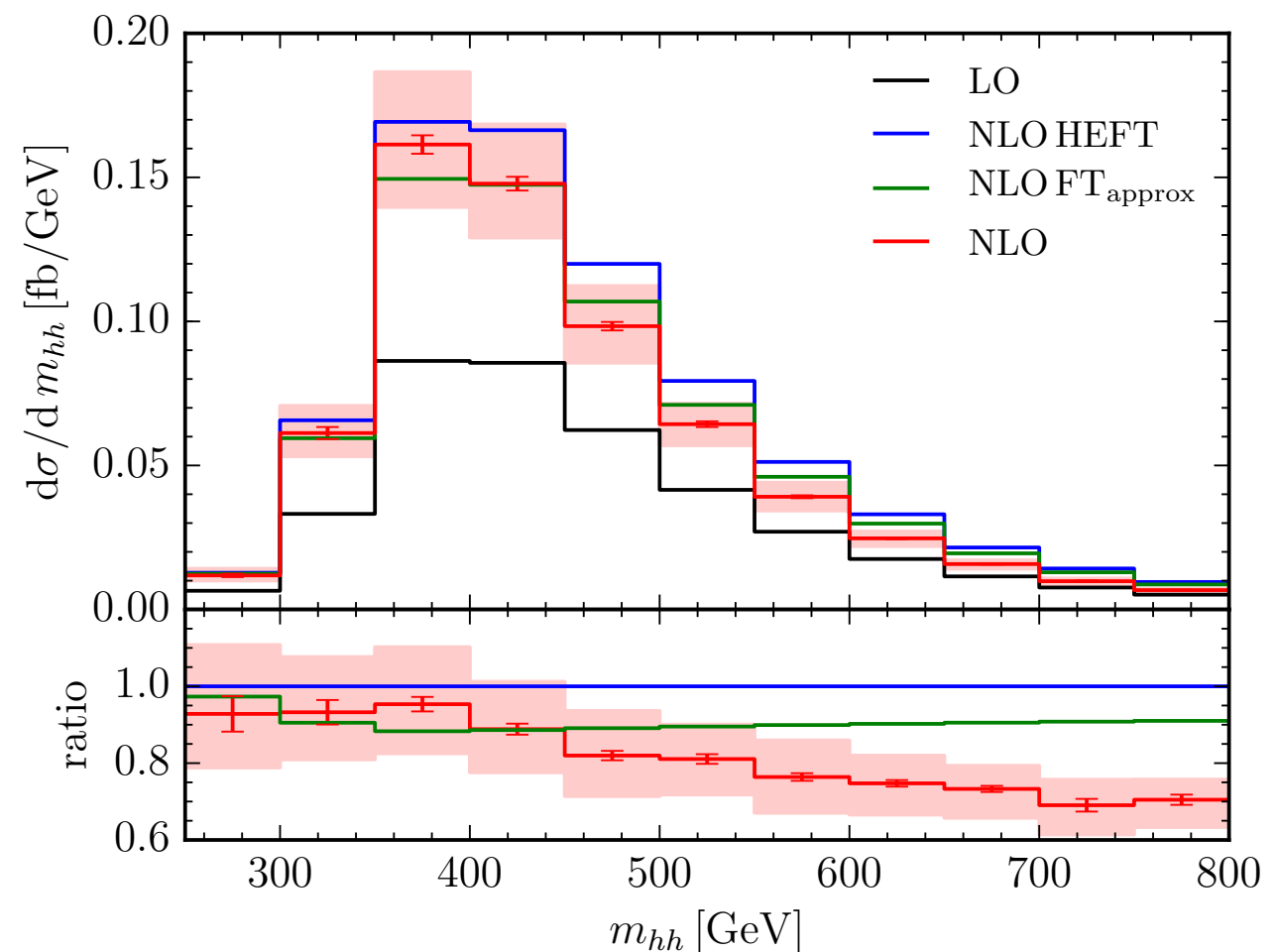
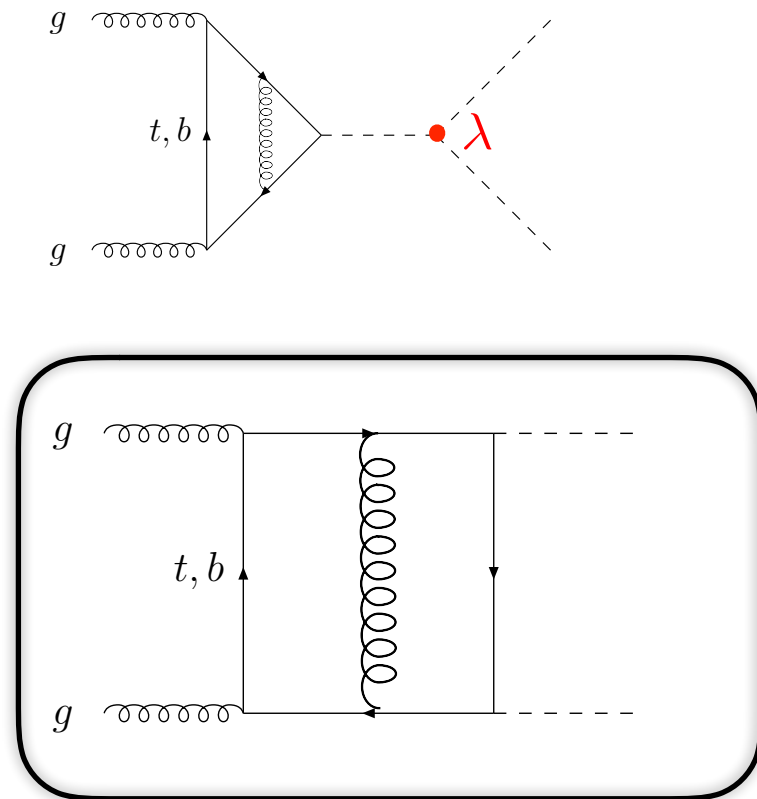
C.Weber QCD@LHC17

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# HH production in gg fusion

- ▶ Full NLO calculation Borowka, Greiner, Heinrich, Jones, Kerner, Schlenk, Schubert, Zirke (2016)
- ▶ 2 loop amplitudes computed numerically with SecDec

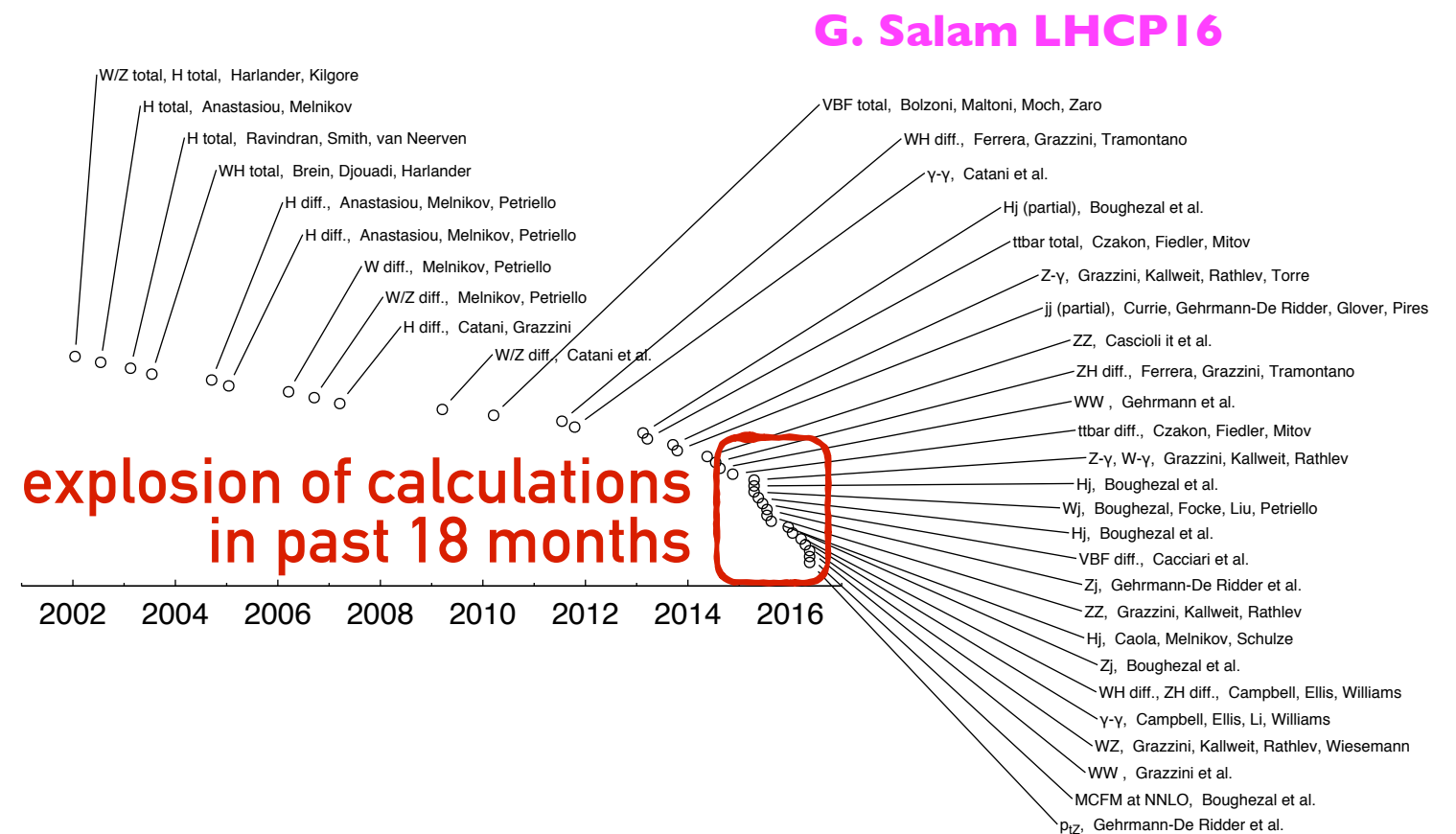


- ▶ -14% wrt EFT / bigger for large invariant masses
- ▶ NNLO available in EFT (learn about approx.) deF, Mazzitelli (2014), deF et al (2016)
- ▶ Technique applicable for other observables?
  - 2-loop reduction/integrals out of analytic reach

# NNLO

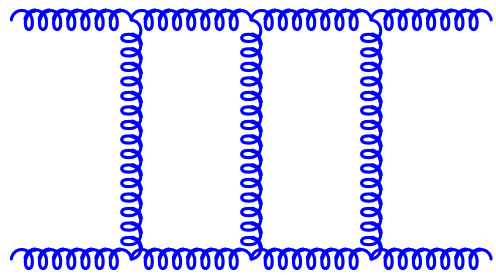
# NNLO

## The NNLO revolution



# Degree of complexity at NNLO

## ► 2 loop

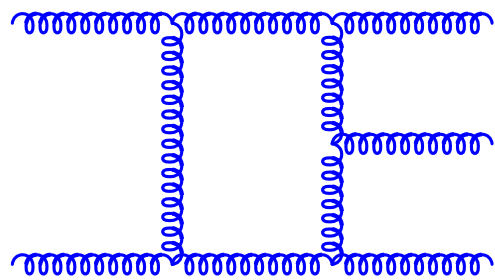


loop integrals  $\longrightarrow$  explicit infrared poles  $\frac{1}{\epsilon^4}$

$2 \rightarrow 2$  **available** (even for VV production)

- **Bottleneck** for larger multiplicities?

## ► 1 loop + single emission



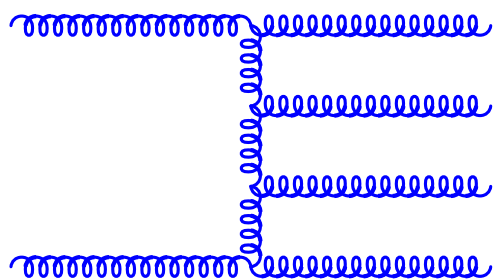
“NLO complexity” : loop  $\longrightarrow \frac{1}{\epsilon^2}$

singular emission  $\longrightarrow \frac{1}{\epsilon^2}$

## ► Double real emission

Tree level Trivial to compute Amplitudes  
a Hell of infrared singularities

- **Bottleneck** for larger multiplicities?



after integration over  
unresolved partons  $\longrightarrow \frac{1}{\epsilon^4}$  poles

Handling singularities

$$\int |M_R|^2 dPS$$

**Subtraction Method** : need **local** subtraction counter-term

$$\int_0^1 (|M_R|^2 - \mathcal{S}) dPS + \int \mathcal{S} dPS + \int |M_V|^2 dPS'$$

Finite

Computed “analytically”  
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- ▶ Subtraction can be fully local (better convergence, but not all)
- ▶ At NNLO many more singular configurations
- ▶ Integration of subtraction term quite complicated (can be numerical)

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different  
approaches

Sector decomposition [Anastasiou, Melnikov, Petriello; Binoth, Heinrich](#)

Antennae subtraction [Gehrmann, Gehrmann-de Ridder, Glover](#)

Sector-Improved residue subtraction [Czakon, Boughezal, Melnikov, Petriello](#)

CoLorFul subtraction [Del Duca, Somogyi, Trocsanyi](#)

Projection-to-Born [Cacciari, Dreyer, Karlberg, Salam, Zanderighi](#)

**Phase space slicing** : split phase space according to singular configurations

$$\int_{\delta}^1 |M_R|^2 dPS + \int_0^{\delta} |M_R|^2 dPS + \int |M_V|^2 dPS'$$

Regularized by cut-off  
(numerically involved)

Can be obtained from  
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- ▶ Simpler to implement (resummation)
- ▶ Count with faster computers for “smaller” correction
- ▶ Can use precise NLO calculations as basis (X+jet)
- ▶ Use local subtraction for NLO-like singularities

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- $q_T$  and Jettiness to characterize “pure” NNLO configurations

$q_T$ -subtraction      Catani, Grazzini; Catani, Cieri, deF, Ferrera, Grazzini

N-jettiness subtraction      Boughezal, Focke, Liu, Petriello; Gaunt, Stahlhofen, Tackmann, Walsh

- So far only “simpler” configurations : one/zero colored particle in f.s.

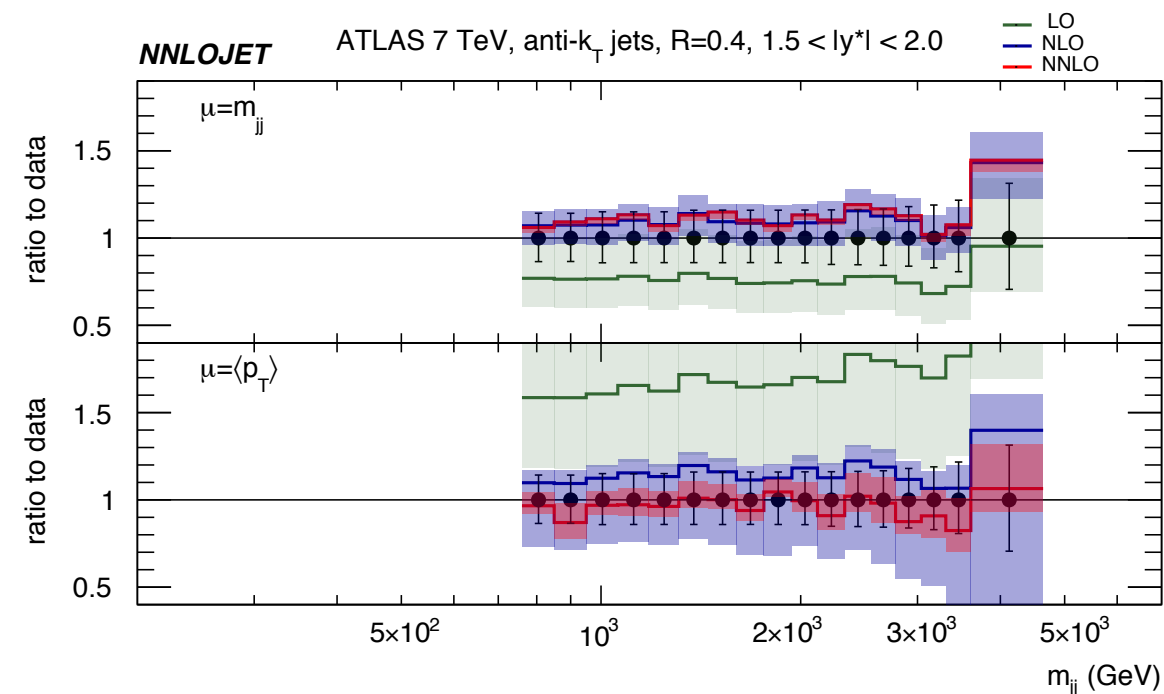
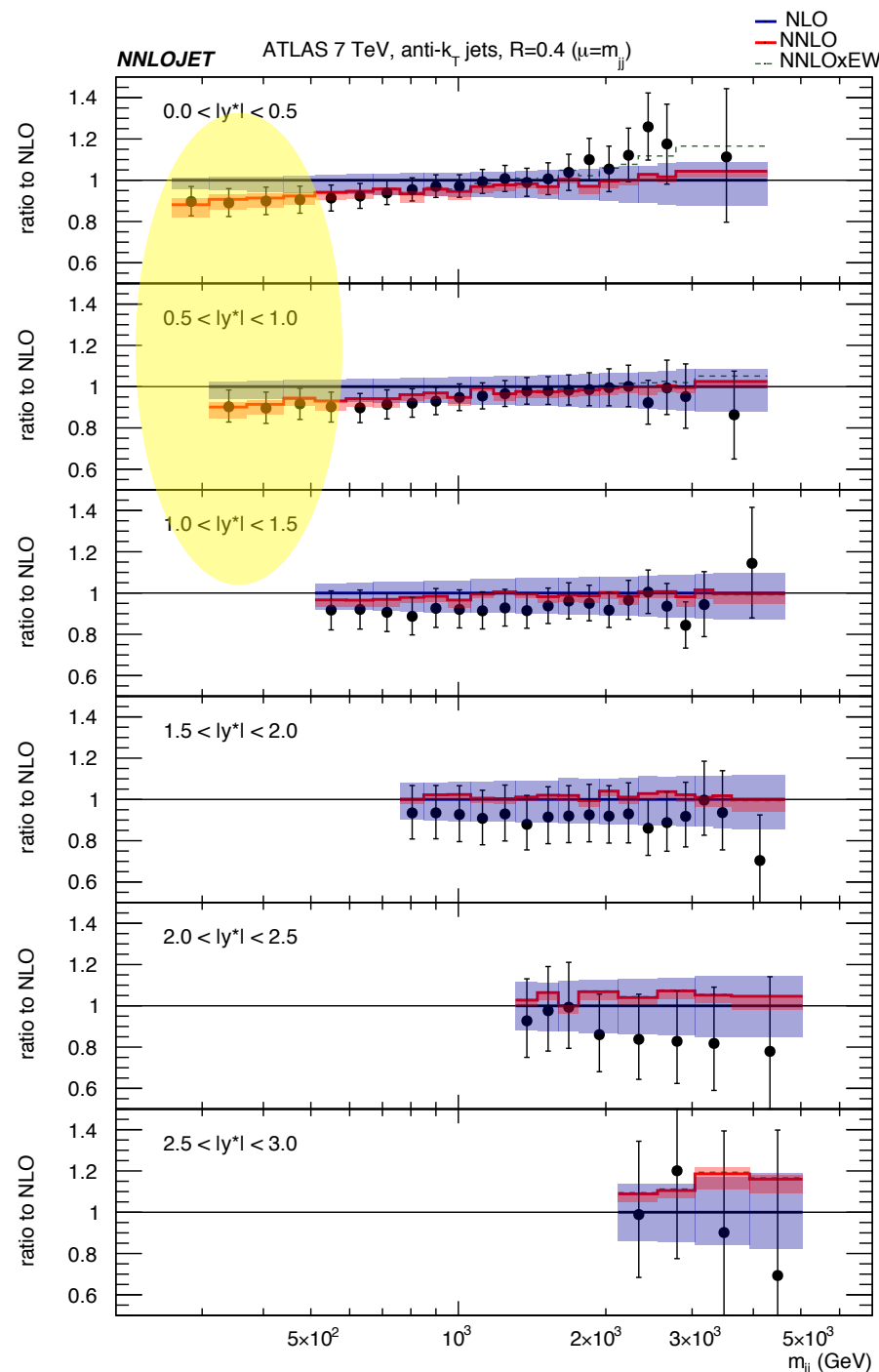
# $pp \rightarrow 2 \text{ jets}$

## ► Leading color using antenna subtraction : NNLOJET (1 and 2 jets)

J.Currie, A. Gehrmann-De Ridder, T. Gehrmann, E.W.N. Glover, A.Huss, J.Pires (2017) J.Currie, E.W.N. Glover, J.Pires (2016)

- Moderate NNLO corrections (<10%)
- Improve description of data for low  $M_{jj}/y^*$
- Invariant mass natural scale (better convergence)
- Cures pathological NLO behavior for  $\langle p_T \rangle$

$$\mu = m_{jj} \qquad \mu = \frac{1}{2}(p_{T1} + p_{T2})$$



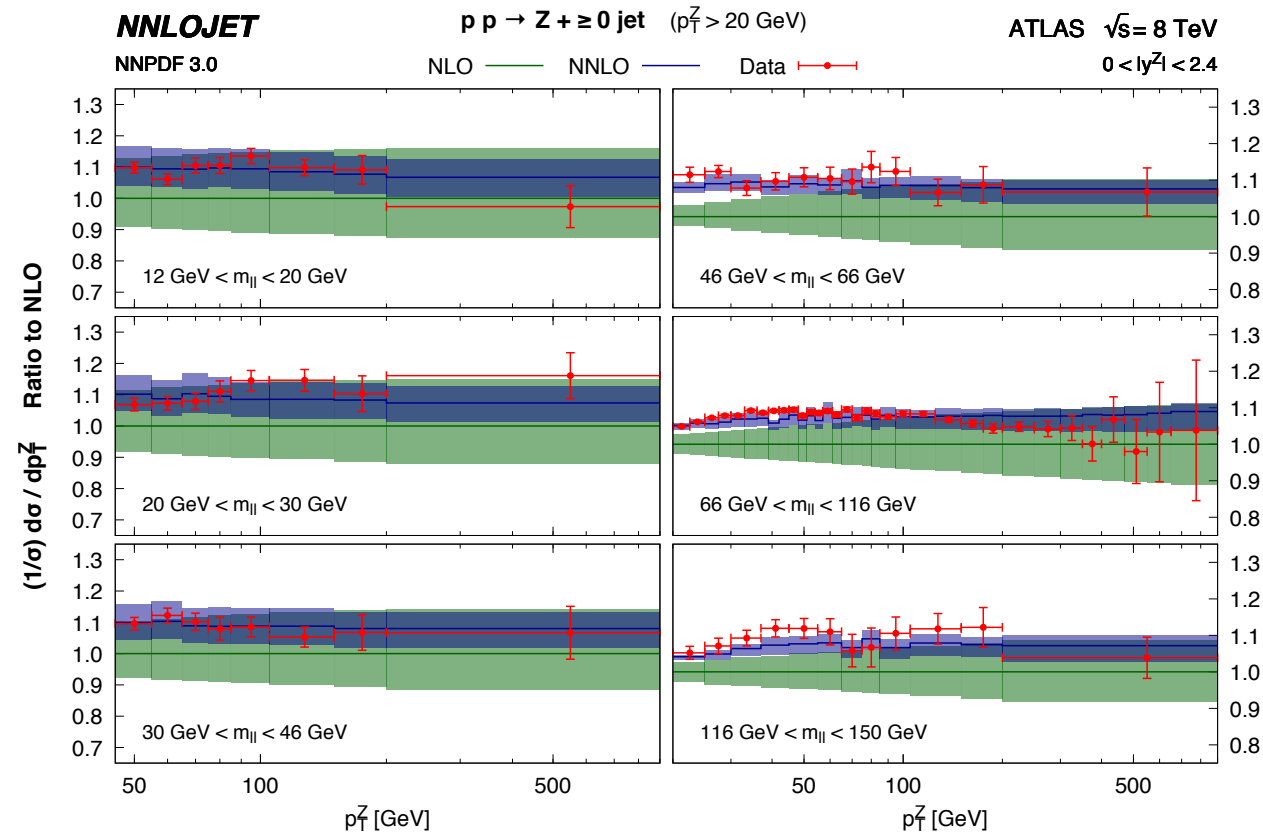
- NNLO scale dep. smaller than EXP errors
- NLO underestimates uncertainty

# $pp \rightarrow Z + \text{jets}$

- ▶ Experimental Uncertainties at the 1% level or below
- ▶ Phenomenological interest : PDF's, luminosity normalization, (W mass)

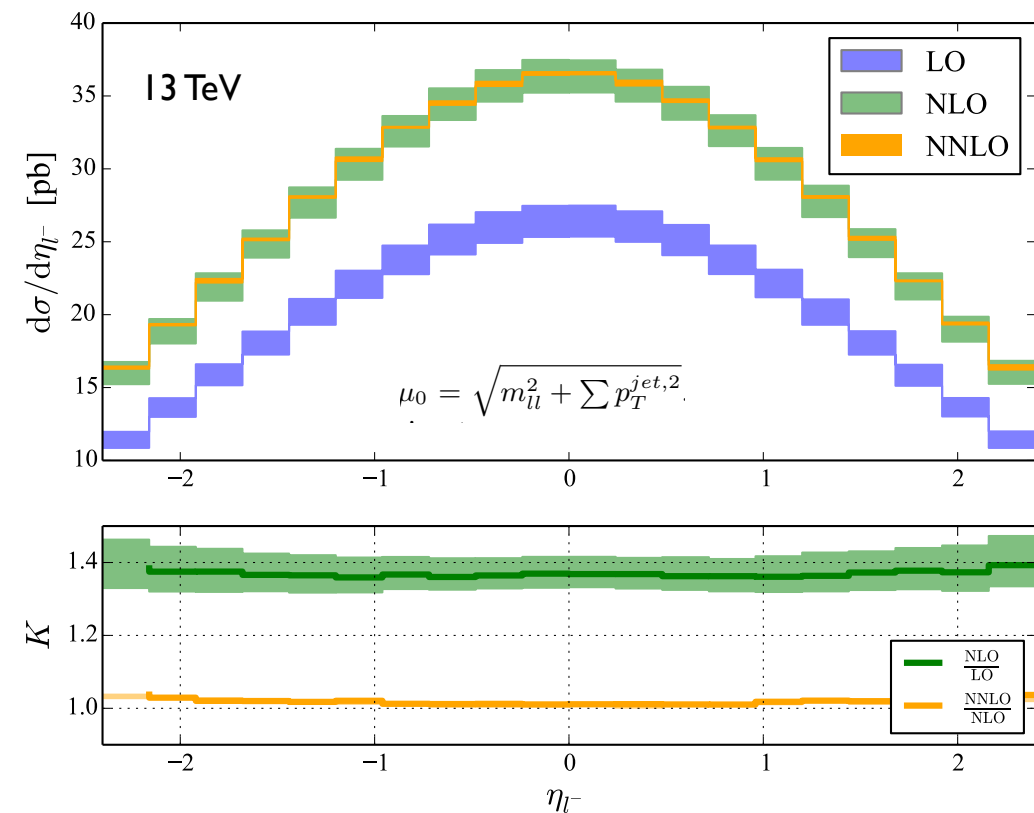
## Antennae subtraction

A.Gehrmann-De Ridder, T. Gehrmann,  
E.W.N. Glover, A.Huss, T.A.Morgan (2016)



## N-Jettiness

R. Boughezal, J. Campbell, K. Ellis, C. Focke,  
W. Giele, X. Liu, F. Petriello (2016)



- ▶ significant reduction in scale dependence
- ▶ substantial improvement in agreement with data
- ▶ W+ jet available R. Boughezal, X. Liu, F. Petriello (2016)



# $pp \rightarrow H + \text{jets}$

## ► Higgs moving from inclusive to fiducial/exclusive distributions

### Antennae subtraction

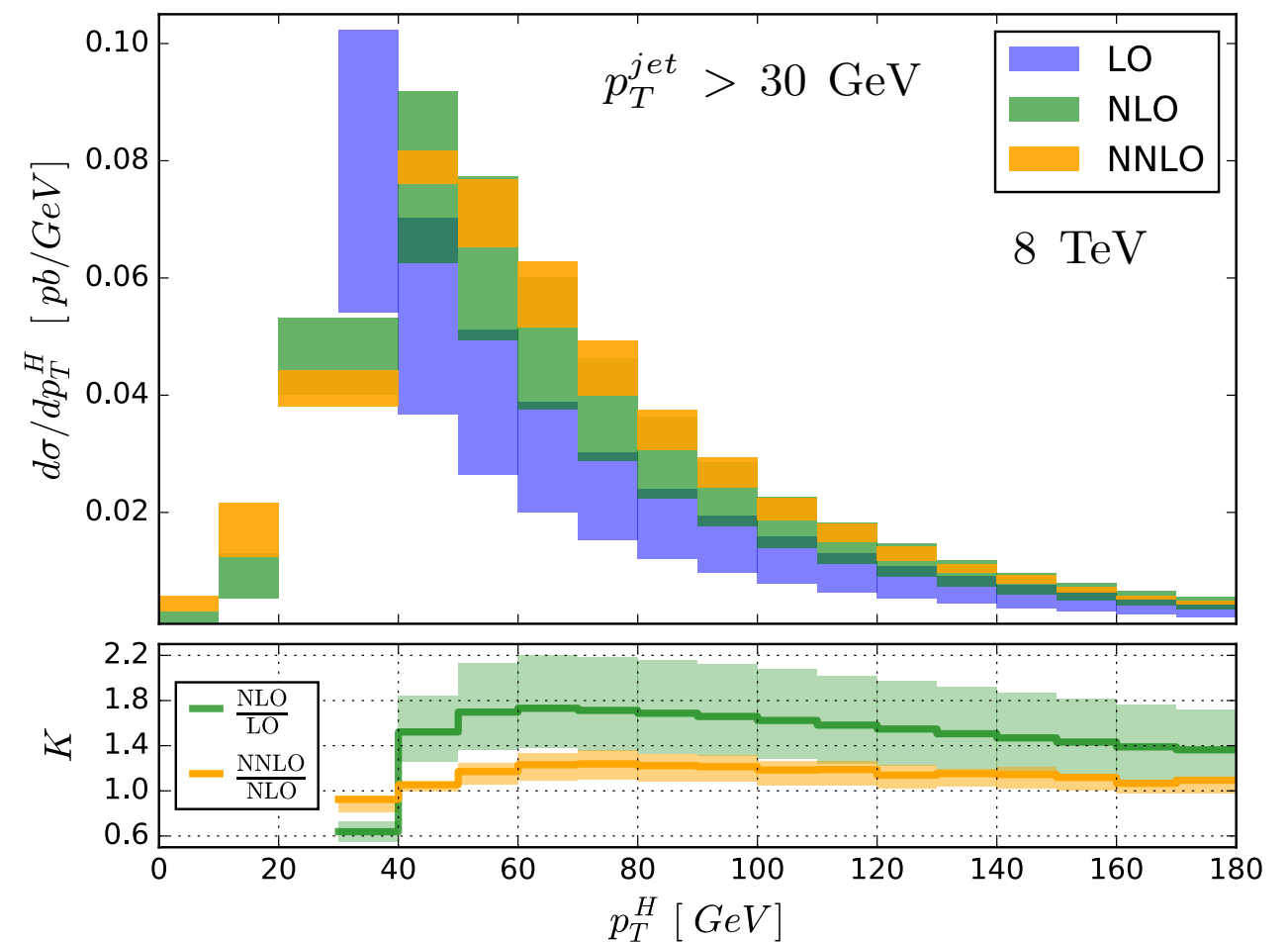
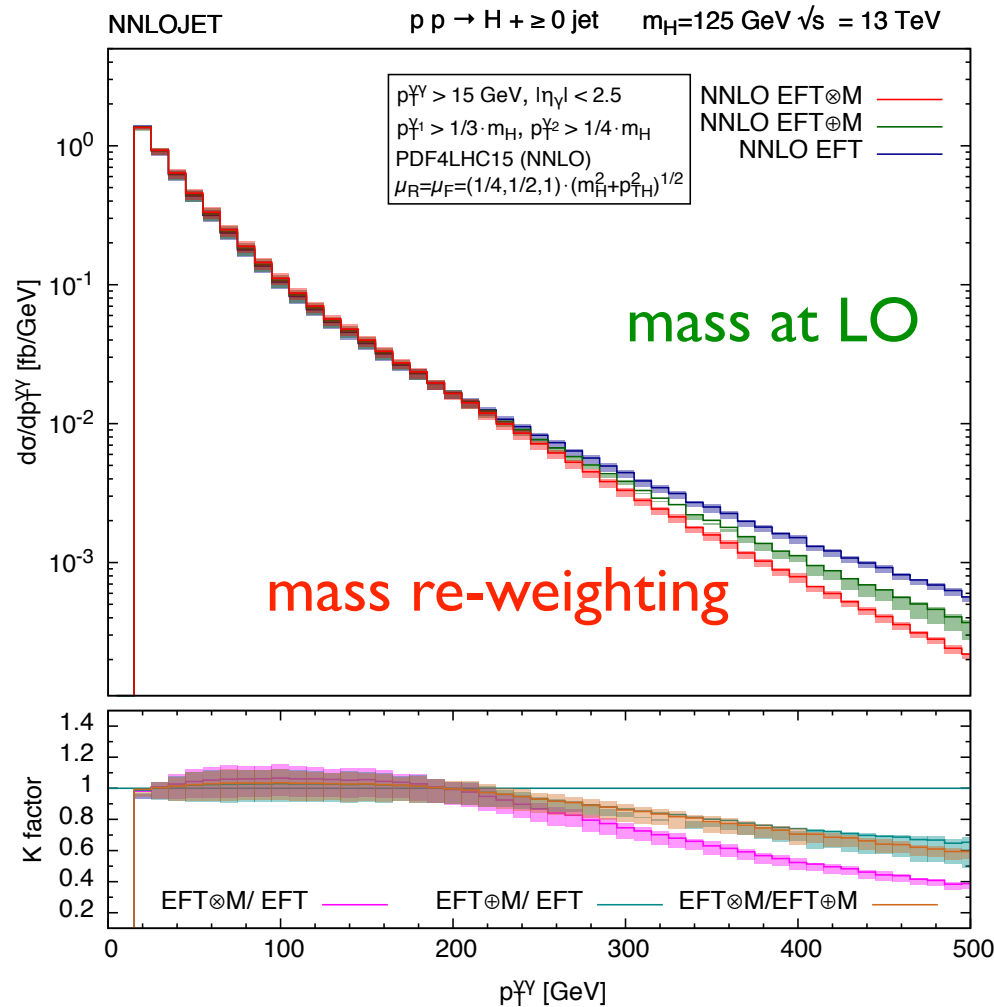
X. Chen, J. Cruz-Martinez, T. Gehrmann,  
E.W.N. Glover, M. Jaquier (2016)

### N-Jettiness

R. Boughezal, C. Focke, W. Giele,  
X. Liu, F. Petriello (2015)

### Sector dec.

R. Boughezal, F. Caola, K. Melnikov,  
F. Petriello, M. Schulze (2015)



► Within approx. of EFT : missing **HQ** effects

H.Frellesvig QCD@LHC17

► Need full mass dependence at NLO (massive two loop)

# Towards automation @ NNLO

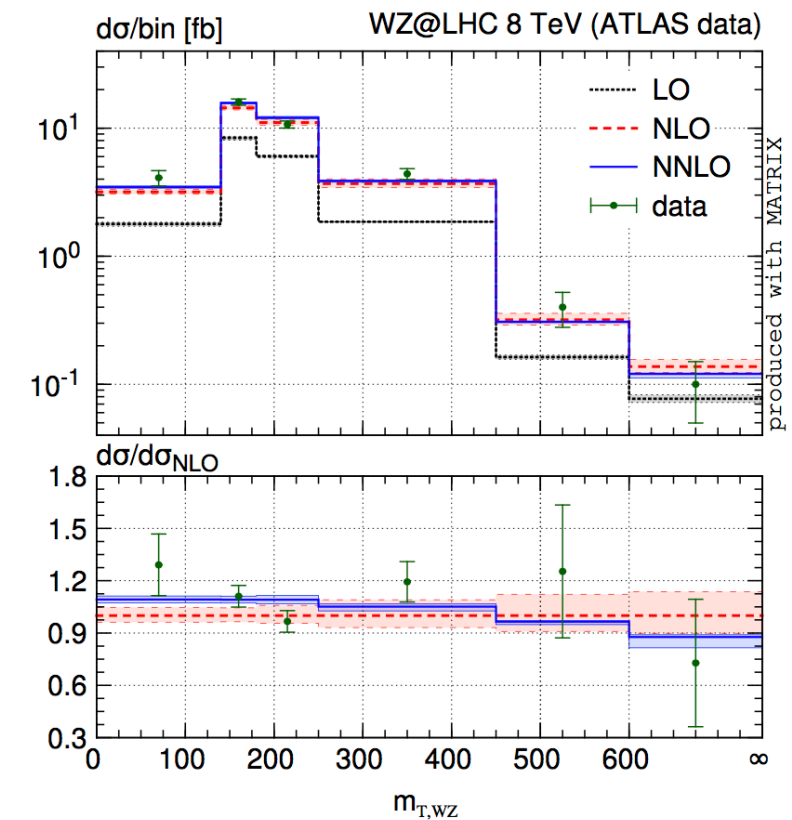
## Matrix @ NNLO

M. Grazzini, S. Kallweit, D. Rathlev, M. Wiesemann (2016)

- $pp \rightarrow Z/\gamma^* (\rightarrow l^+l^-)$  ✓
- $pp \rightarrow W (\rightarrow lv)$  (✓)
- $pp \rightarrow H$  ✓
- $pp \rightarrow \gamma\gamma$  ✓
- $pp \rightarrow W\gamma \rightarrow lv\gamma$  ✓
- $pp \rightarrow Z\gamma \rightarrow l^+l^-\gamma$  ✓
- $pp \rightarrow ZZ (\rightarrow 4l)$  ✓
- $pp \rightarrow WW \rightarrow (lvl'v')$  ✓
- $pp \rightarrow ZZ/WW \rightarrow ll\nu\nu$  ✓
- $pp \rightarrow WZ \rightarrow lvll$  ✓
- $pp \rightarrow HH$  (✓)

► NNLO parton level generator with several processes in unique framework (di-boson)

- qt subtraction
- Open-Loops :  $X+1$  parton
- Will include  $qT$  resummation
- So far, colored singlet final state
- Public version soon



# Towards automation @ NNLO

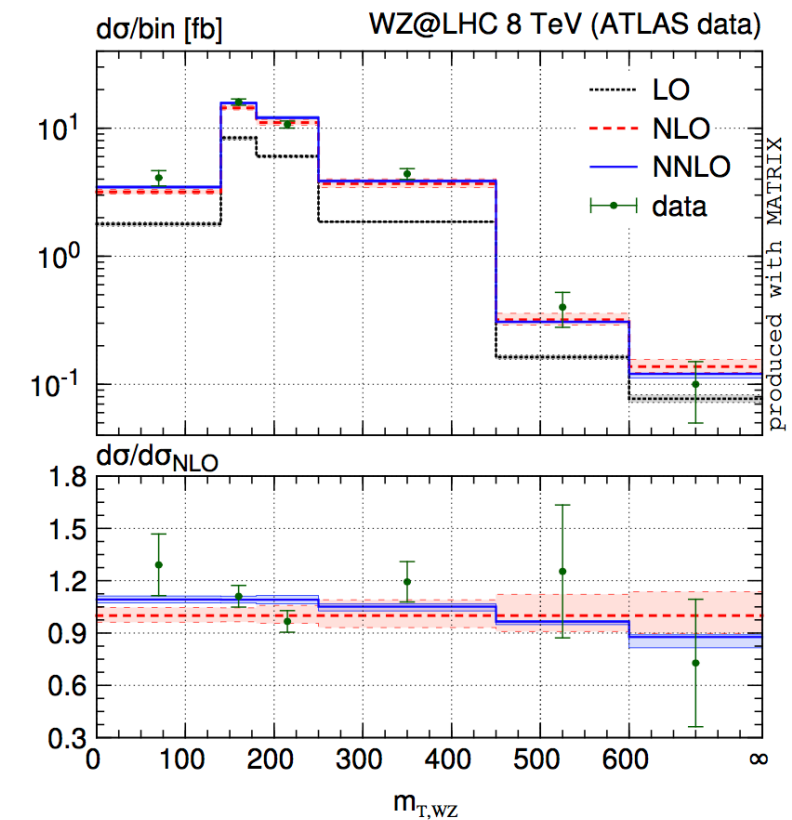
## Matrix @ NNLO

M. Grazzini, S. Kallweit, D. Rathlev, M. Wiesemann (2016)

- $pp \rightarrow Z/\gamma^* (\rightarrow l^+l^-)$  ✓
- $pp \rightarrow W (\rightarrow lv)$  (✓)
- $pp \rightarrow H$  ✓
- $pp \rightarrow \gamma\gamma$  ✓
- $pp \rightarrow W\gamma \rightarrow lv\gamma$  ✓
- $pp \rightarrow Z\gamma \rightarrow l^+l^-\gamma$  ✓
- $pp \rightarrow ZZ (\rightarrow 4l)$  ✓
- $pp \rightarrow WW \rightarrow (lvll'v')$  ✓
- $pp \rightarrow ZZ/WW \rightarrow ll\nu\nu$  ✓
- $pp \rightarrow WZ \rightarrow lvll$  ✓
- $pp \rightarrow HH$  (✓)

► NNLO parton level generator with several processes in unique framework (di-boson)

- qt subtraction
- Open-Loops :  $X+1$  parton
- Will include qT resummation
- So far, colored singlet final state
- Public version soon



## MCFM@ NNLO

R. Boughezal, J. Campbell, K. Ellis, C. Focke, W. Giele, X. Liu, F. Petriello (2016)  
J. Campbell, T. Neumann, C. Williams (2017)

- N-Jettiness
- Less processes available yet :  $V+1$  jet done

$W^+$   
 $W^-$   
 $Z$   
 $H$   
 $\gamma\gamma$   
 $Z\gamma$   
 $W^+H$   
 $W^-H$   
 $ZH$

# Towards automation @ NNLO

- Sector-decomposition + FKS : **Stripper**

R. Poncelet QCD@LHC17

# Towards automation @ NNLO

- ▶ Sector-decomposition + FKS : **Stripper**

R. Poncelet QCD@LHC17

- ▶ 3 jet production in  $e^+e^-$  and event shapes: CoLoRFulNNLO

Z. Tulipánt QCD@LHC17  
Z. Ször

Del Duca, Duhr, Kardos, Somogyi, Ször, Trócsányi, Tulipánt (2016)

- ▶ Fully differential results for  $t\bar{t}$ bar Czakon, Heymes, Mitov(2015-2016)

A. Mitov QCD@LHC17

- ▶ t-channel. Single-top + top-decay (NW) Berger, Gao, Yuan, Zhu (2016)

Slicing (N-jettiness) + subtraction (P2B)

- ▶ VBF at NNLO : projection to Born method

Cacciari, Dreyer, Karlberg, Salam, Zanderighi (2015)

- ▶  $H \rightarrow b\bar{b}$  @ NNLO Del Duca, Duhr, Somogyi, Ször, Tramontano, Trócsányi (2015)

- ▶ + many more computations in just a few years

# $N^3\text{LO}$

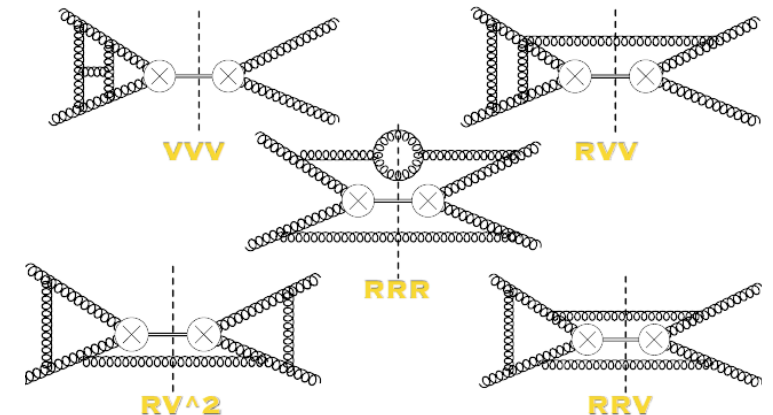
## The new Frontier

# Higgs at $N^3LO$

C. Anastasiou, C. Duhr, F. Dulat, F. Herzog, B. Mistlberger (2015)

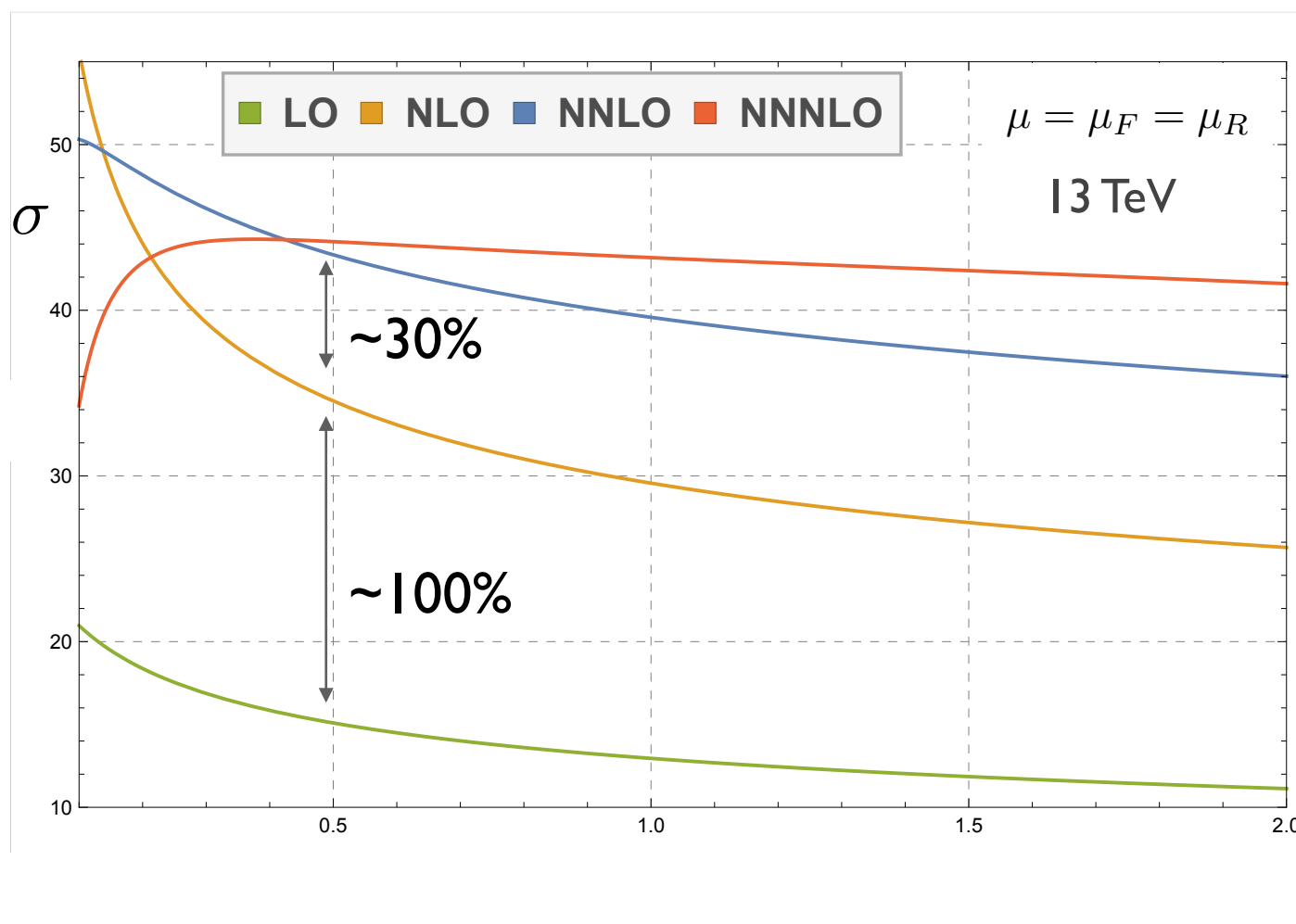
- Very relevant observable called for higher orders (slow convergence)
- Impressive calculation : new techniques
  - ▶ Threshold expansion (very high order)
  - ▶ Within (excellent) heavy top approximation
  - ▶ Could be used for DY
  - ▶ Differential distributions

S. Lionetti QCD@LHC17



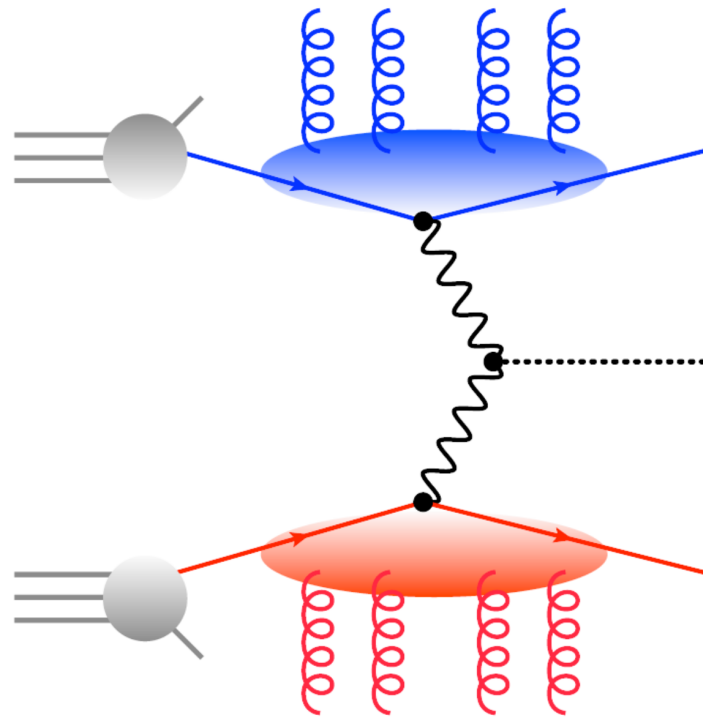
68273802 loop and phase space integrals

C. Anastasiou, C. Duhr, F. Dulat, E. Furlan, T. Gehrmann, F. Herzog, A. Lazopoulos, B. Mistlberger (2016)

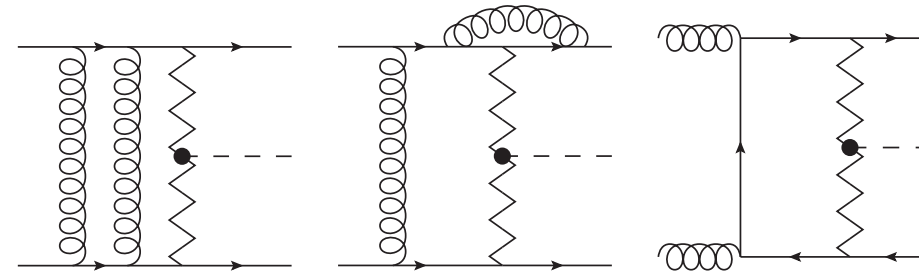


- ▶ Inclusive over parton radiation
- ▶ Observe stabilization of expansion
- ▶ Small correction (2% at  $M_H/2$ )
- ▶ Scale variation at  $N^3LO$  ~2%

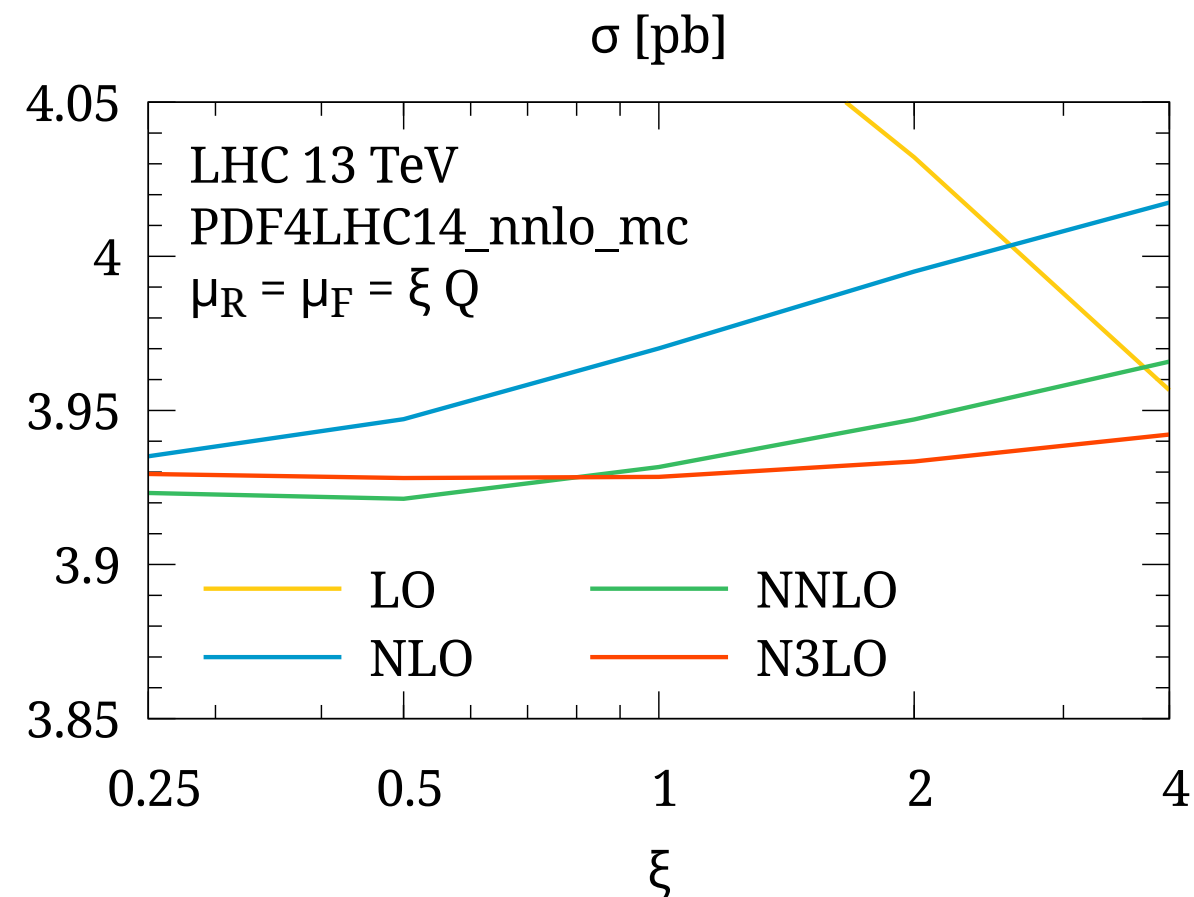




► DISxDIS like approach  $\sim 1\%$  accurate picture



neglect exchange between lower and upper legs



- Inclusive on parton radiation
- small corrections  $\sim 1-2\%$
- within NNLO band
- sizable reduction in scale dep.

► Exclusive at NNLO

M. Cacciari, F. Dreyer, A. Karlberg, G. Salam, G. Zanderighi (2015)

NNLO differential larger (5-10%) than for inclusive (1%) and beyond NLO band



# N<sup>3</sup>LO Splitting functions

S. Moch, B. Ruijl, T. Ueda, J. Vermaseren, A. Vogt (2017)

- ▶ Non-Singlet 4 loop splitting function
- ▶ N=20 Mellin moments (large N<sub>c</sub>)
- ▶ Enough to provide a reconstruction in terms of Harmonic sums
- ▶ N=16 beyond large N<sub>c</sub>
- ▶ Precise for  $x \gtrsim 10^{-4}$

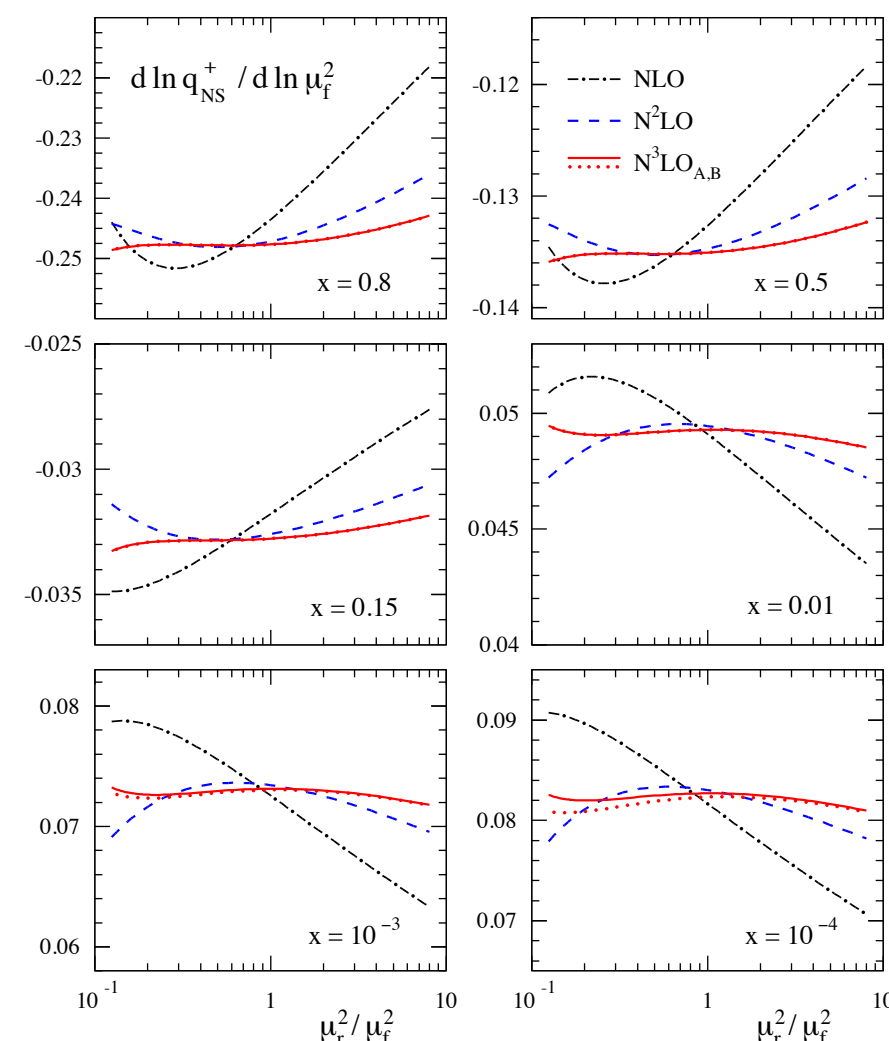
$$xq_{\text{ns}}^{\pm, \nu}(x, \mu_0^2) = x^{0.5}(1-x)^3$$

$$\alpha_s(\mu_0^2) = 0.2$$

- Visible improvement of scale stability

Singlet and Gluon splitting functions feasible

QED corrections G. Sborlini QCD@LHC17



# TH Uncertainties

$$\sigma = 48.58 \text{ pb}^{+2.22 \text{ pb } (+4.56\%)}_{-3.27 \text{ pb } (-6.72\%)} (\text{theory}) \pm 1.56 \text{ pb } (3.20\%) (\text{PDF}+\alpha_s)$$

what is the meaning of that?

- Usually obtained by performing scale variations

$$\log \frac{Q}{\mu} \quad \log \frac{\mu_F}{\mu_R} \quad \log \frac{Q}{\mu_{F,R}} \quad \text{keep logs small}$$

$$\mu_{F,R} = \left( r, \frac{1}{r} \right) Q$$

- Lack of probabilistic framework : how to combine with other?
- Several examples showing that “ $r=2$ ” might be short to account for true uncertainties

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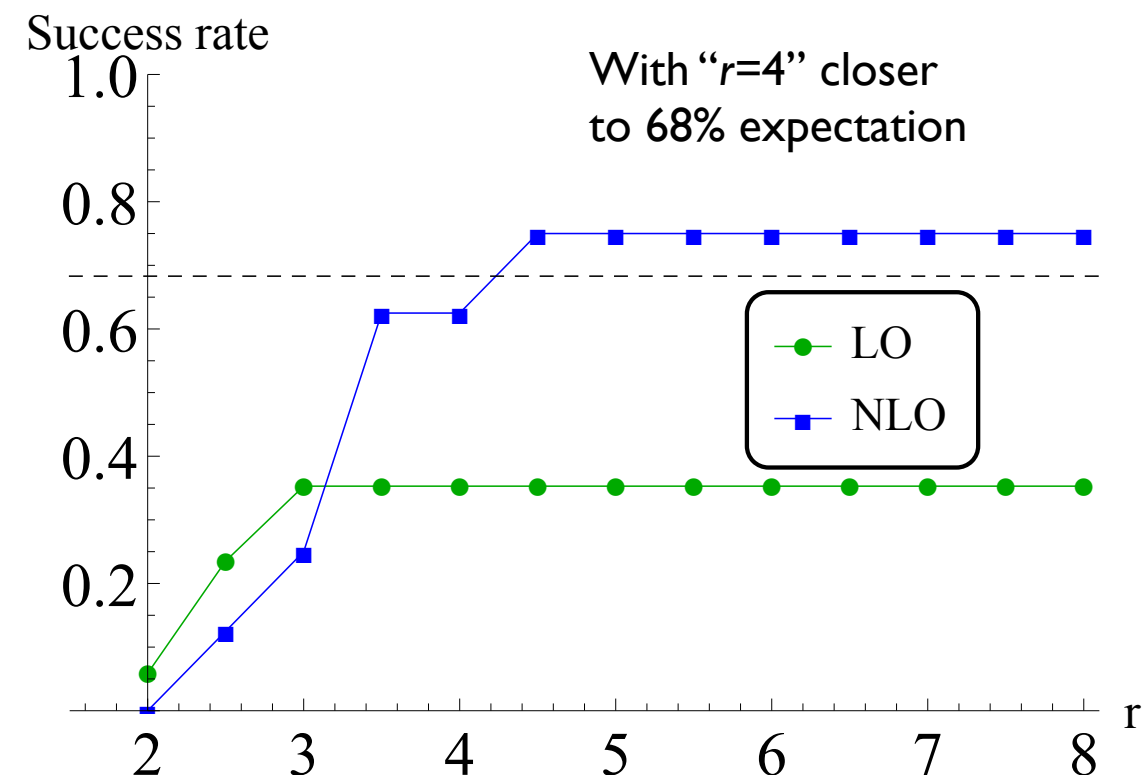
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- Several examples showing that “ $r=2$ ” might be short to account for true uncertainties

- Fraction of hadronic observables ( $\sim 15$ ) whose h.o. correction is contained in the scale variation interval

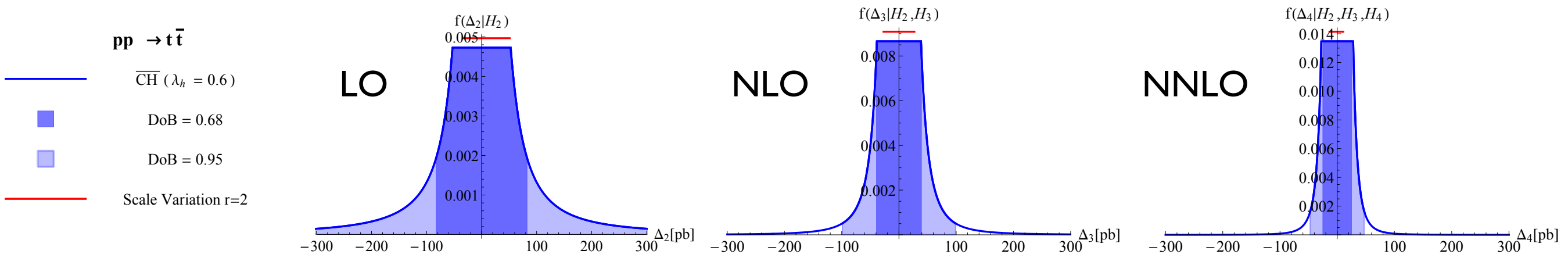
E. Bagnaschi, M. Cacciari, A. Guffanti, L. Jenniches (2014)

- But *rescaling* depends on order: might be better from NNLO



- Bayesian approach: Introduce conditional density  
compute credibility interval with degree of belief (68%, 95%)

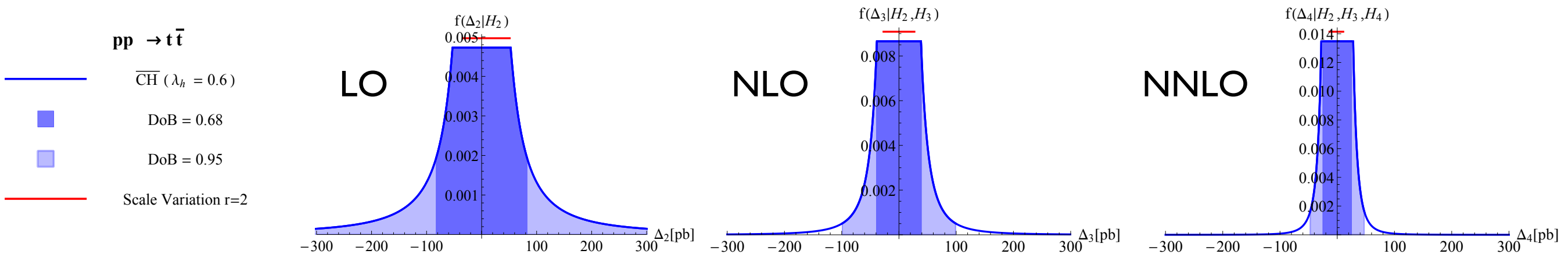
M. Cacciari, N. Houdeau (2011); E. Bagnaschi, M. Cacciari, A. Guffanti, L. Jenniches (2014)



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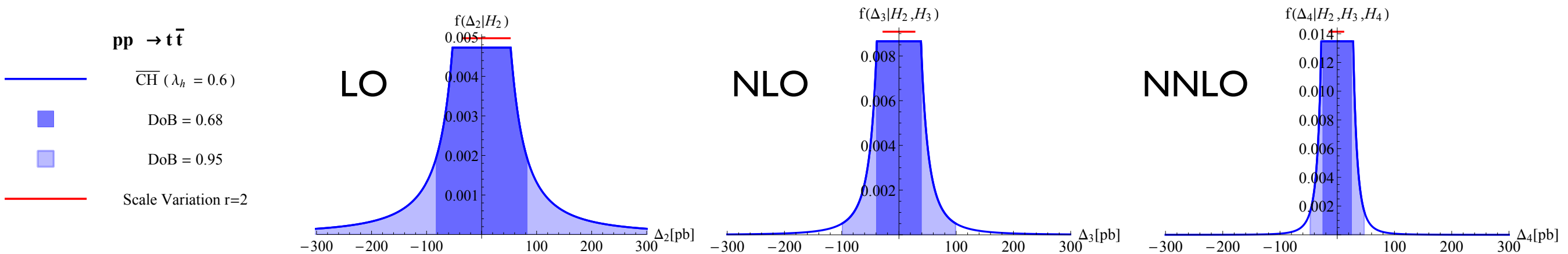


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- Evaluate “higher order” terms from resummation framework

DdeF, J. Mazzitelli, S. Moch, A. Vogt (2014)  
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Too much effort to reach  $N^n\text{LO}$  to avoid the search for a more rigorous handling of TH uncertainties in perturbative calculations



# Conclusions

- ▶ Amazing progress in fixed order calculations during the last (>) decade

Automation of NLO

Several NNLO processes  $2 \rightarrow 2$  ✓

**Driven by LHC**

Even N<sup>3</sup>LO for simpler kinematics and first set of splitting functions

- ▶ But... **Reaching new bottlenecks**
- ▶ Large multiplicity at NLO still needs *manual*-work
- ▶ Loop induced processes (massive) yet hard to tackle
- ▶ NNLO very difficult for more than 2 particles in final state
  - Virtual amplitudes (massive)
  - Real radiation not trivial (numerical infrared treatment)

**Will need significant development**

- ▶ Need a more rigorous treatment of TH uncertainties



Thanks to Costas Papadopoulos and Marco Zaro for discussions

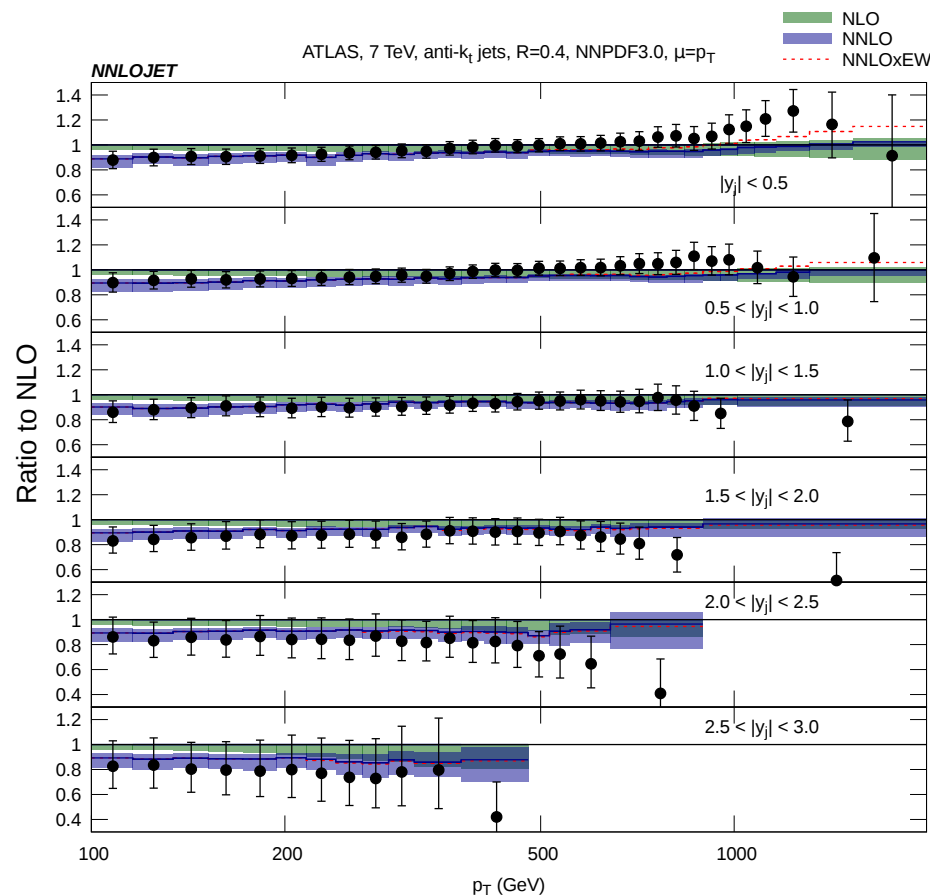
# Backup slides

# Single-jet production

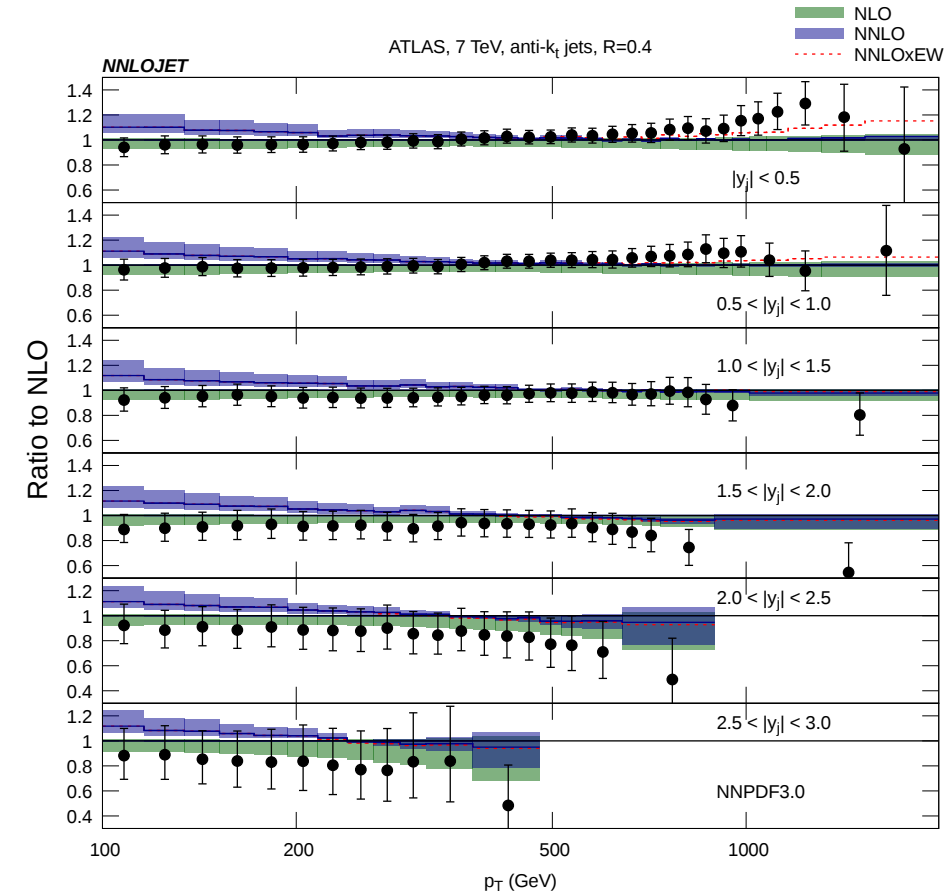
## ► Leading color using antenna subtraction : NNLOJET

J.Currie, E.W.N. Glover, J.Pires (2016)

J.Currie, A. Gehrmann-De Ridder, T. Gehrmann, E.W.N. Glover, A.Huss, J.Pires (2017)



$$\mu = p_T$$



$$\mu = p_{T_1}$$

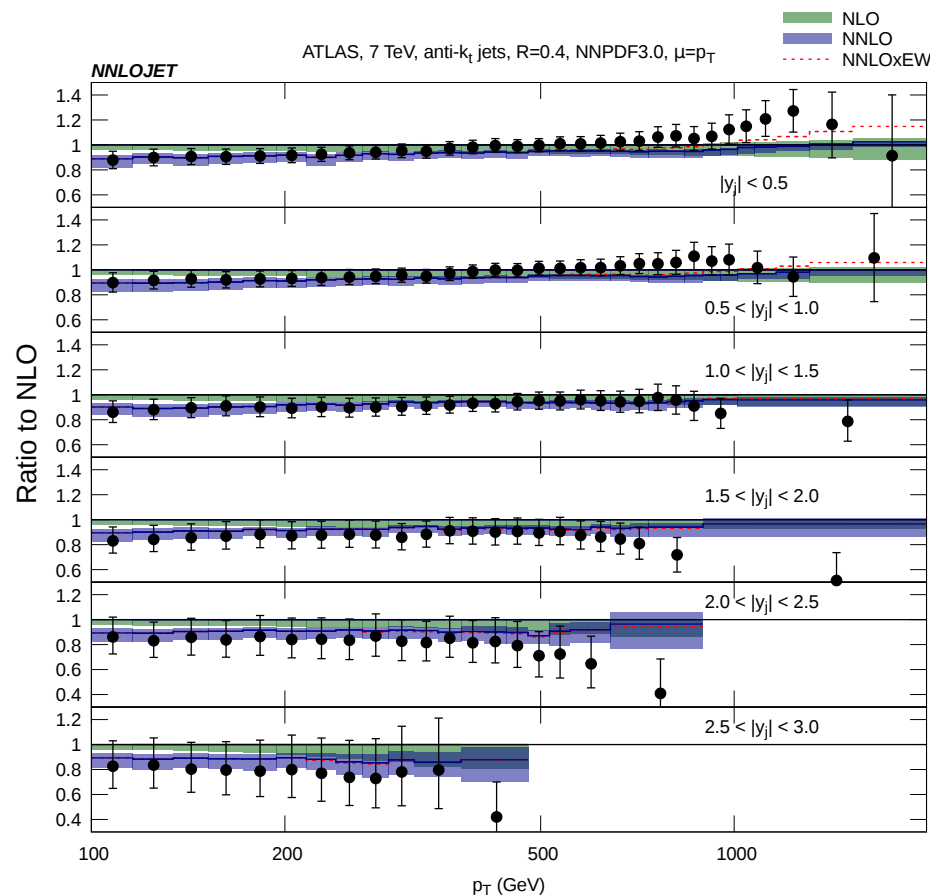
- Moderate NNLO corrections
- Two different central scales: leading jet vs individual jet

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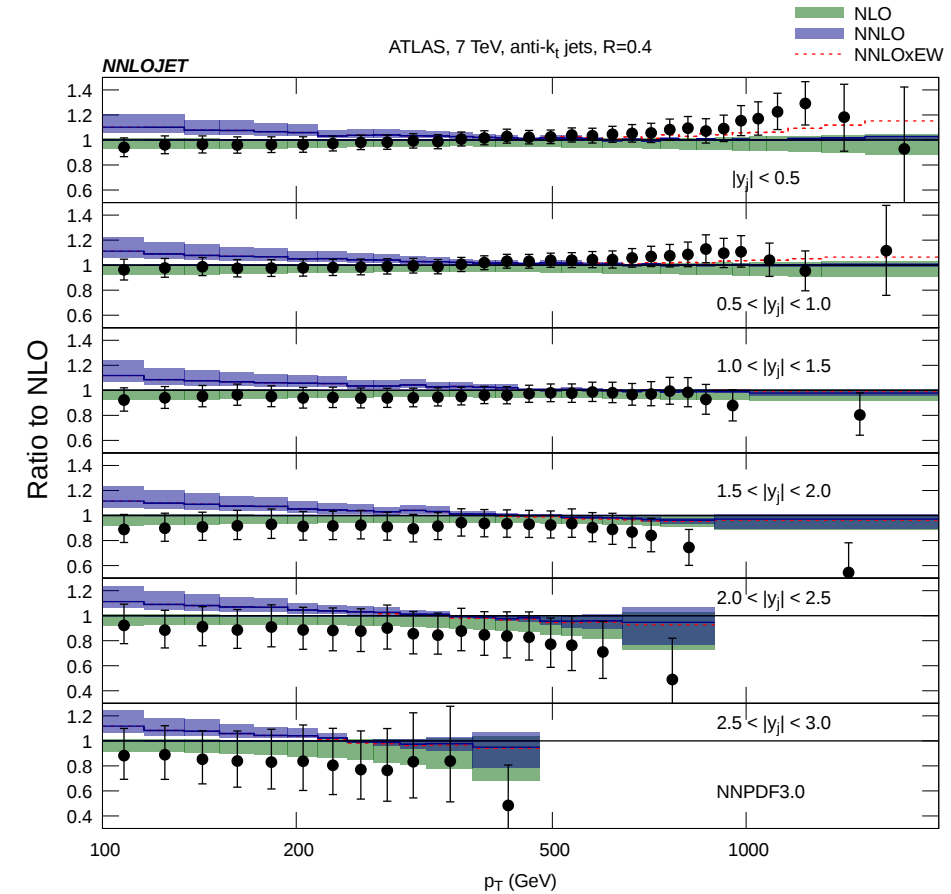
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$$\mu = p_T$$



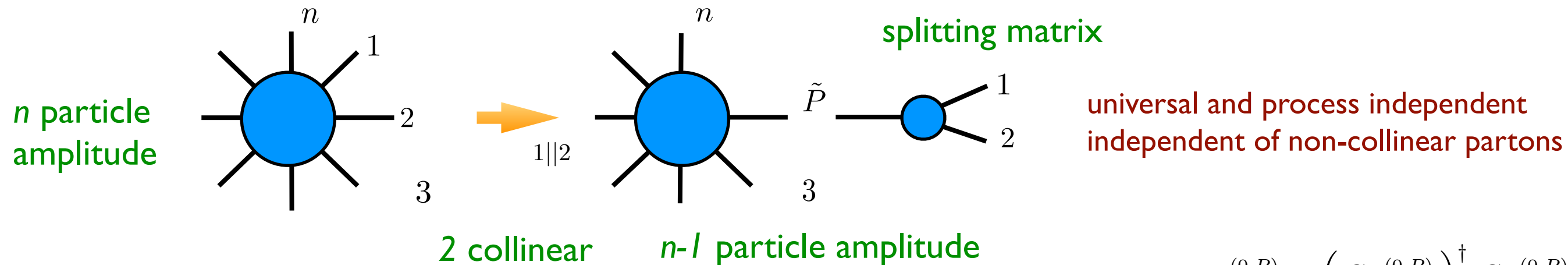
$$\mu = p_{T_1}$$

- Moderate NNLO corrections
- Two different central scales: leading jet vs individual jet
  - ▶ Equivalent at large transverse momentum
  - ▶ Differences outside scale band at low momentum
  - ▶  $p_T$  provides better description (with larger corrections and scale dep.)
  - ▶ Requires further studies to LHC data (scale, shape, cone, pdfs)

# Infrared Structure of QCD

- ▶ H.O. computations possible: understanding infrared structure of amplitudes
- ▶ Key for cancellation of singularities : **factorization of amplitudes**

## Strict factorization in collinear limit



$$|\mathcal{M}^{(0)}(p_1, p_2, \dots, p_n)\rangle \simeq \mathbf{Sp}^{(0)}(p_1, p_2; \tilde{P}) |\mathcal{M}^{(0)}(\tilde{P}, \dots, p_n)\rangle$$

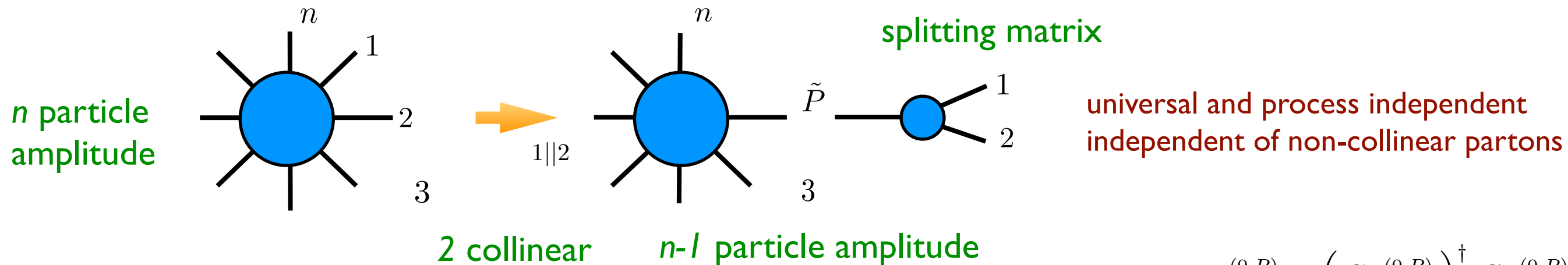
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**AP kernel**

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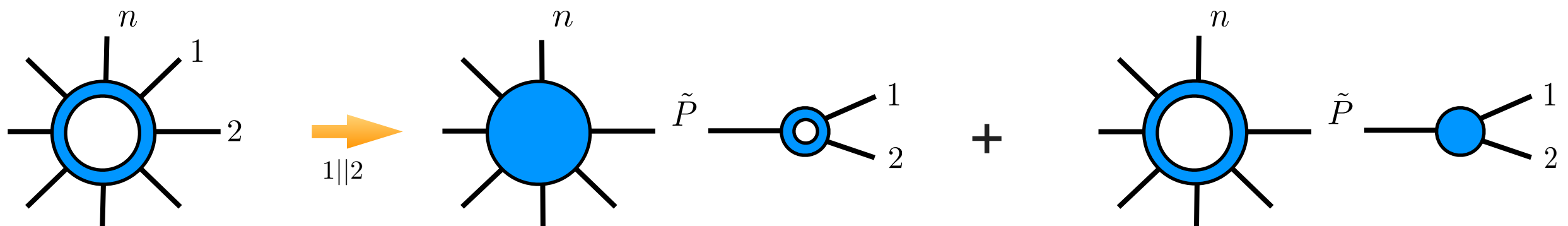
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AP kernel

- ▶ Similar approach for virtual amplitudes

$$|\mathcal{M}^{(1)}(p_1, p_2, \dots, p_n)\rangle \simeq \mathbf{Sp}^{(1)}(p_1, p_2; \tilde{P}) |\mathcal{M}^{(0)}(\tilde{P}, \dots, p_n)\rangle + \mathbf{Sp}^{(0)}(p_1, p_2; \tilde{P}) |\mathcal{M}^{(1)}(\tilde{P}, \dots, p_n)\rangle$$





► Factorization fails in space-like region: hadronic colliders Catani, deF., Rodrigo (2012)

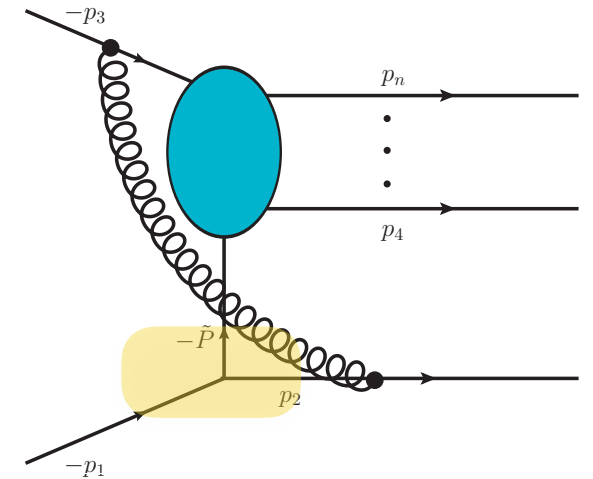
$Sp^{(1)}(p_1, p_2; \tilde{P}; p_3, \dots, p_n)$  depends on non-collinear partons

➡ Violation of strict factorization for **one loop** amplitudes

fact. breaking  
divergent part

$$\Delta_{mC}^{(1)}(\varepsilon) = \frac{\alpha_S(\mu^2)}{2\pi} \frac{i\pi}{\varepsilon} \sum_{\substack{i \in C \\ j \in NC}} \mathbf{T}_i \cdot \mathbf{T}_j \Theta(-z_i) \text{sign}(s_{ij})$$

- Cancels in TL (and DIS) due to color Coherence
- Absorptive (Imaginary): cancels in cross section





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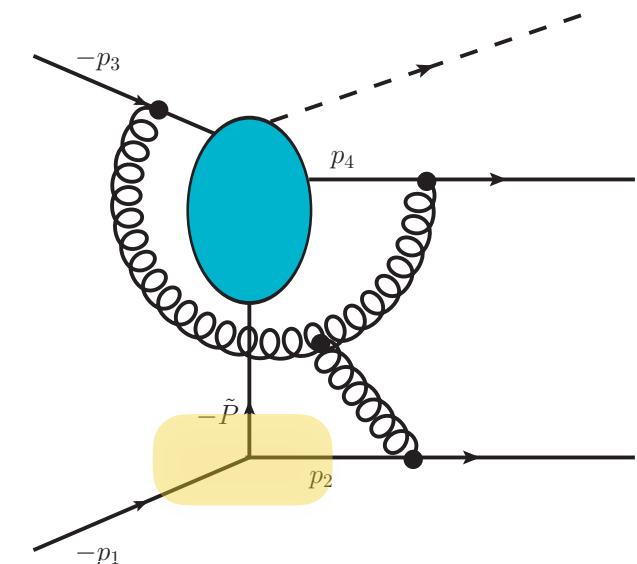
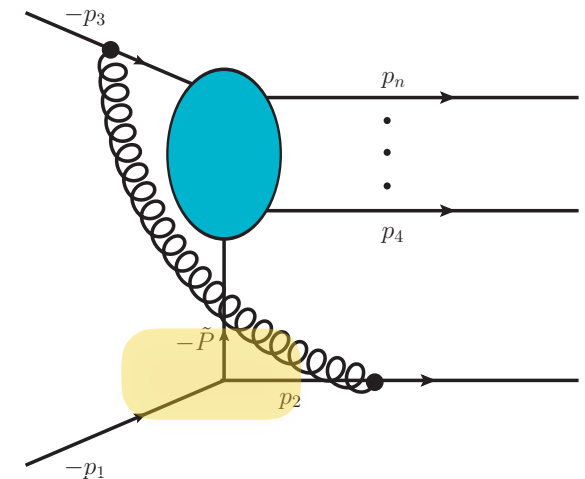
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- real contribution: only cancels in *pure* QCD
- non-vanishing with EW interference (or CP/width)
- Induces 3 loop fact. breaking term

Forshaw, Seymour, Siódmok (2012)  
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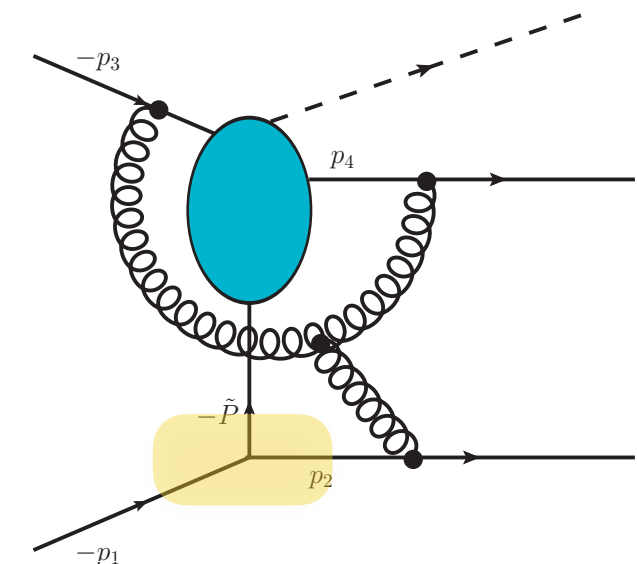
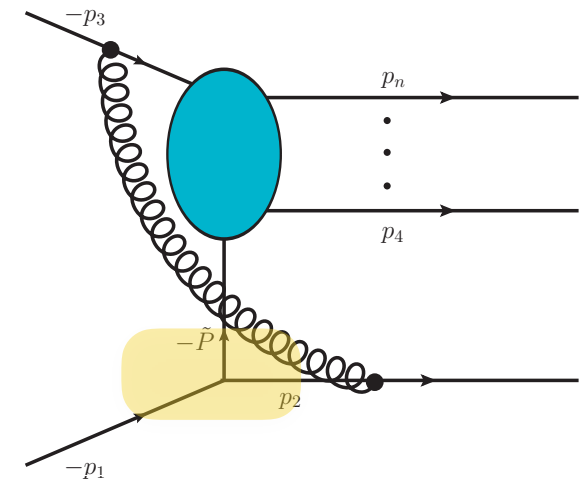
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► Produces super-leading logarithms in ‘gaps–between–jets’ cross sections

