Towards multi-jet processes at NNLO

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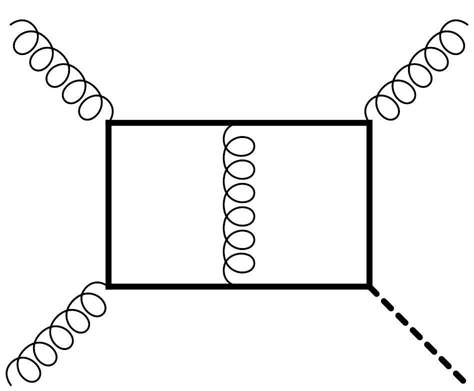
Introduction

- in recent years, many processes from the *Les Houches wishlist* were computed
- this includes many 2 to 2 processes at NNLO, such as VV' production and double H production
- multi-jet processes at NNLO are a new challenge
- in this talk, I will focus on progress and techniques for the virtual contributions



- for techniques relevant for real and real-virtual, see Daniel de Florian's talk

‘Ideal’ and ‘real’ scattering amplitudes

Is there some simpler version of QCD that allows to understand key properties of scattering amplitudes?

How can we obtain numerical results for cross sections at the LHC

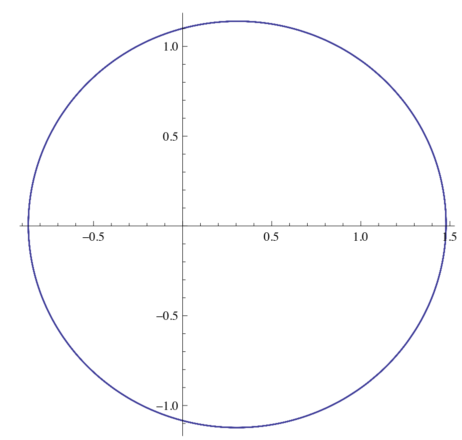
This talk: tools for ‘real’ QCD coming from ‘ideal’ amplitudes

Idealized ‘toy’ theories: from Kepler to QFT

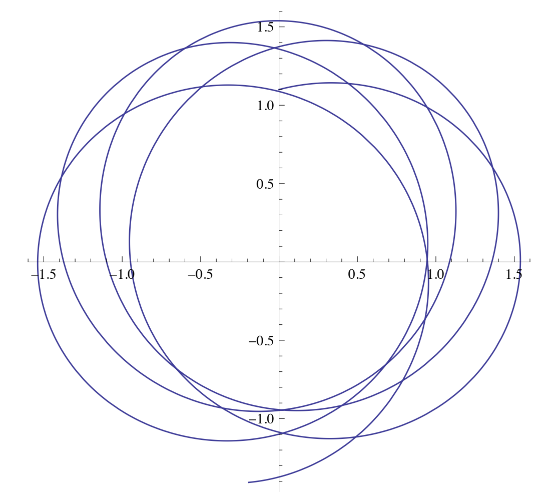
<u>Classical mechanics: Kepler problem</u>

- Laplace-Runge-Lenz (LRL) vector is conserved
- consequence: orbits do not precess

$V = 1/r$



$V = 1/r^{0.9}$



<u>Quantum mechanics: Hydrogen atom</u>

- LRL-operator commutes with hamiltonian
- gives elegant algebraic way to find spectrum

<u>Is there a quantum (gauge) field theory with this symmetry?</u>

Maximally supersymmetric Yang-Mills theory

Particle content similar to QCD:

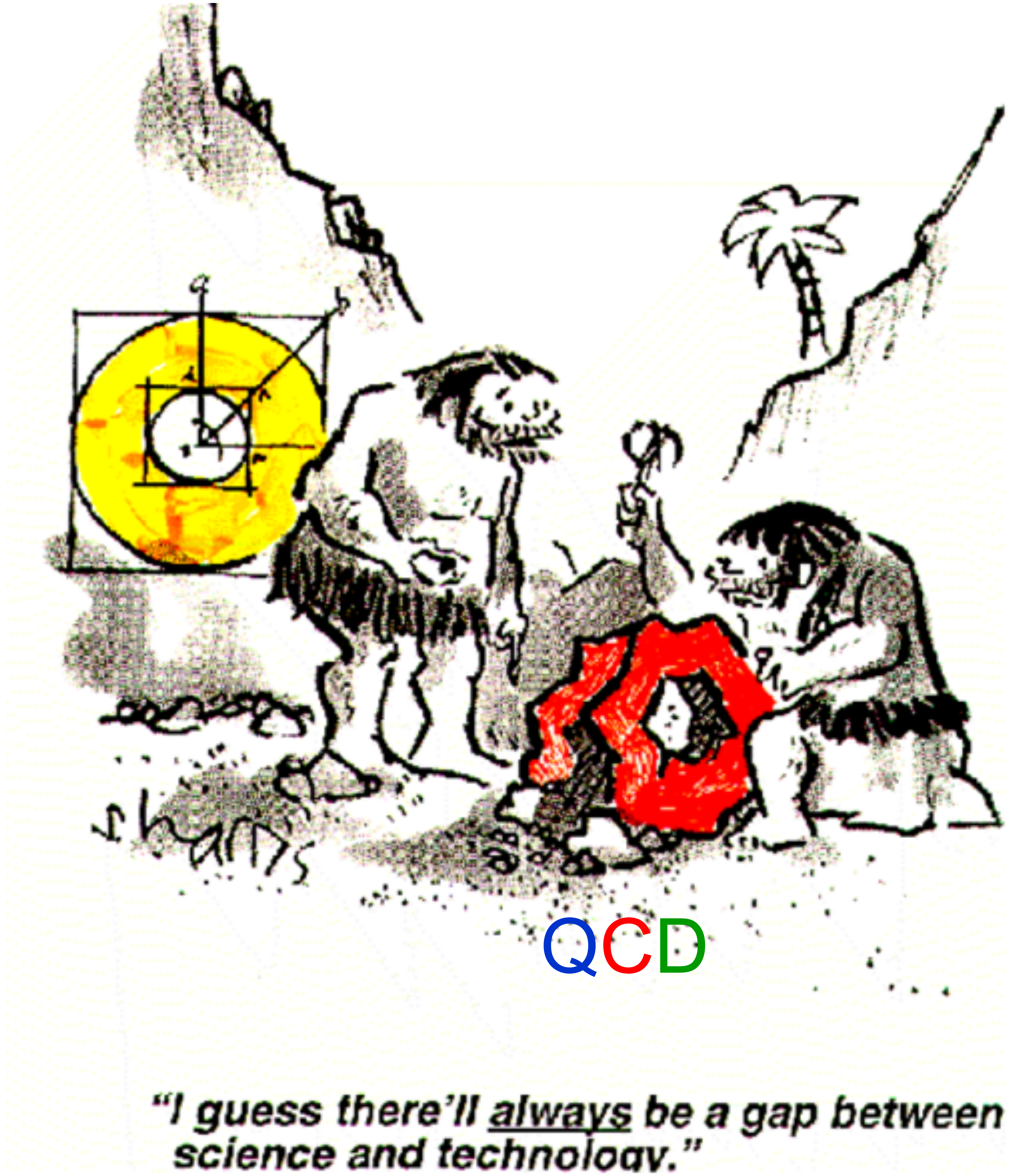
QCD	N=4 supersymmetric Yang-Mills theory
• SU(3) Yang-Mills theory (gluons) • fermions in fundamental representation	• SU(Nc) Yang-Mills theory • nf fermions, adjoint repr. • ns scalars

Bonus features:

- supersymmetry; only one coupling, zero beta function
- planar theory has Yangian symmetry - origin is LRL-symmetry!

From ‘science’ to ‘technology’

N=4 SYM



QCD

(slide from Lance Dixon’s talk at EPS HEP11 Grenoble)

On-shell techniques

- original idea: perturbative unitarity of S matrix
- on-shell recursions for tree amplitudes

[Britto, Cachazo, Feng, Witten, PRL 94 (2005)]

- construction of one-loop amplitudes

[Bern, Dixon, Dunbar, Kosower, Nucl. Phys. B425 (1994)]

[Anastasiou, Britto, Feng, Kunszt, Mastrolia, Phys. Lett. B645 (2007)]

[Ossola, Papadopoulos, Pittau, Nucl. Phys. B763 (2007)]

- today: automated computations of one-loop amplitudes

NLO revolution

On-shell techniques

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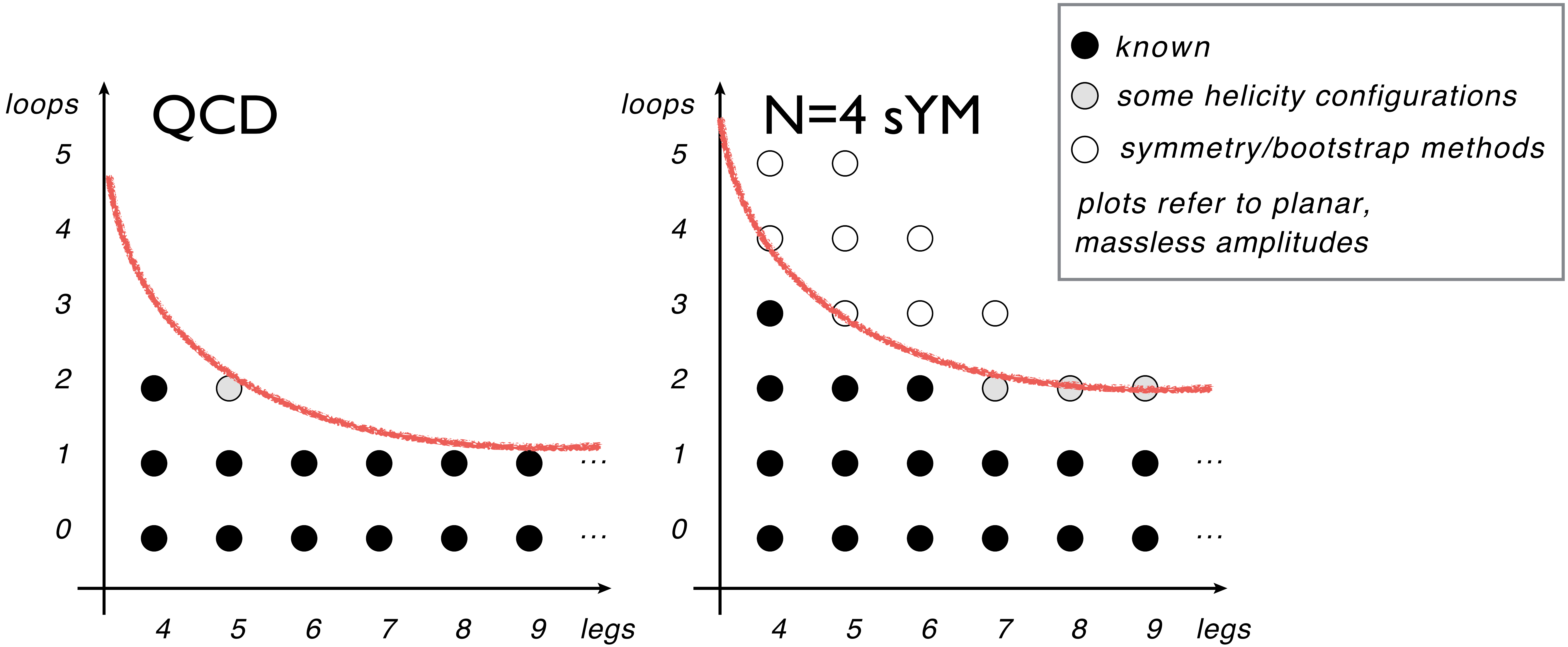
NLO revolution

This talk: towards a NNLO revolution

State of the art two-loop amplitudes



- frontier of knowledge pushed forward continuously
- N=4 sYM a good prediction what we can hope to achieve next in QCD

State of the art two-loop amplitudes

- integrand in suitable form

  on-shell techniques at two-loops e.g. [Badger, Mogull, Peraro, 2016]

  new ideas about appropriate integrand basis

  [Arkani-Hamed, Bourjaily, Cachazo, Trnka, 2010][JMH, 2013]

- reduction to integral basis

  new ideas based on algebraic geometry

  [Georgoudis, Larsen, Zhang, 2016]

  finite field methods for systems of linear equations

  e.g. [von Manteuffel, Schabinger, 2014]

- evaluation of Feynman integrals

  progress in differential equations method [JMH, 2013]

  improved understanding of special functions

  [Goncharov, Spradlin, Vergu, Volovich 2010]

Analytic progress for loop integrals example: integrals with massless internal lines

- all two-loop integrals for vector boson production pp to V V'

[JMH, Melnikov, Smirnov] JHEP 1405 (2014) 090

[Caola, JMH, Melnikov, Smirnov] JHEP 1409 (2014) 043

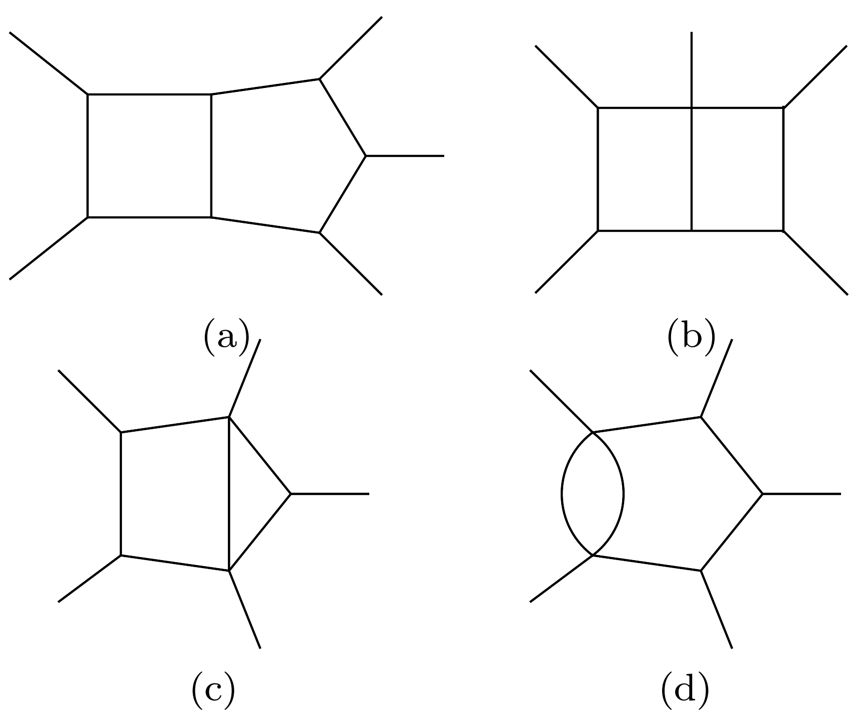
for pp to VV: [Gehrmann, von Manteuffel, Tancredi, Weihs] JHEP 1406 (2014) 032

[Gehrmann, Tancredi, Weihs] JHEP 1308 (2013) 070

- planar two-loop 5-point integrals

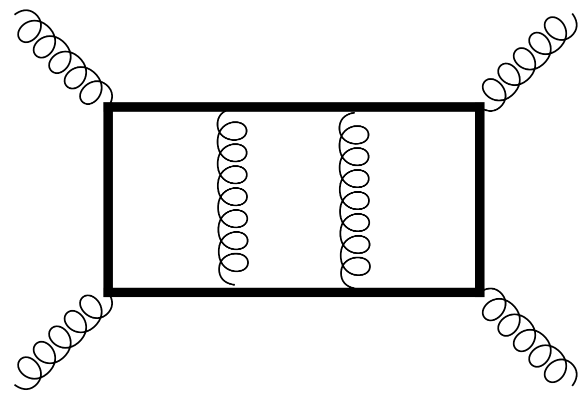
[Gehrmann, JMH, Lo Presti, PRL 116 (2016)]

[Papadopoulos, Tommasini, Wever, JHEP 1604 (2016) 078]



Examples: integrals with massive internal particles

- scattering amplitudes & cross sections in massive toy model in N=4 sYM



[JMH, S. Caron-Huot, JHEP 1406 (2014) 114]

s, t, m^2

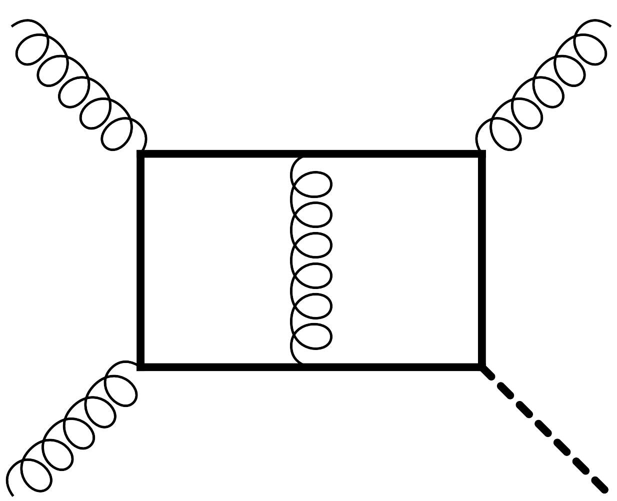
3 loops and 3 scales!

- NLO QCD corrections to H to Z gamma

[Bonciani, Del Duca, Frellesvig, JMH, Moriello, Smirnov, JHEP 1508 (2015) 108]
[Gehrmann, Guns, Kara, JHEP 1509 (2015) 038]

- mixed QCD-EW [Bonciani, di Vita, Mastrolia, Schubert, JHEP 1609 (2016) 091]

- planar integrals for Higgs to 3 partons



involves elliptic polylogarithms

[Bonciani, Del Duca, Frellesvig, JMH, Moriello, Smirnov, JHEP 1612 (2016) 096]

Key tools for analytic progress

- tools for special functions

    buzzwords: ‘symbol’, ‘alphabet’ of iterated integrals

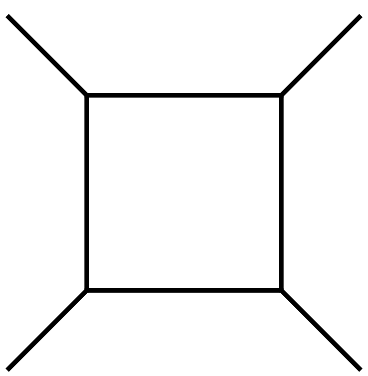
- <u>leading singularities</u> of loop integrands

- differential equation technique

Analyzing loop integrands: maximal cuts, leading singularities

- maximal cuts / leading singularities

$$D_1 = k^2 \quad D_2 = (k+p_1)^2 \quad D_3 = (k+p_1+p_2)^2 \quad D_4 = (k+p_1+p_2+p_3)^2$$



$$= \int d^4k \delta(D_1)\delta(D_2)\delta(D_3)\delta(D_4) \sim \frac{1}{st}$$

residues of integrand at poles: leading singularities

- observation: integrals with constant leading singularities have very nice properties

Simple example leading singularities

'Loop' Integrand I, x, y integration variables

$$I(x,y) = \frac{1+x+by}{xy(1+x+y)} = \frac{1}{xy} + \frac{b-1}{x(1+x+y)}$$

Step 1: take residues in first variable (e.g. y)

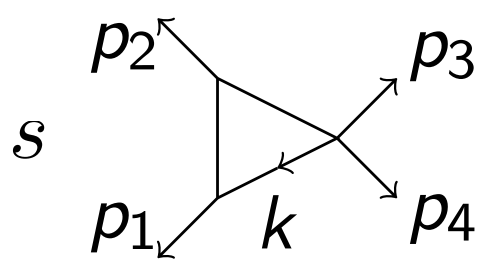
$$\oint_{y=0} I(x,y) = \frac{1}{x} \qquad \oint_{y=-1-x} I(x,y) = \frac{b-1}{x}$$

Step 2: take residues in second variable

$$\oint_{x=0} \oint_{y=0} I(x,y) = 1 \qquad \oint_{x=0} \oint_{y=-1-x} I(x,y) = b-1$$

Leading singularities: $\{1, b-1\}$

One-loop triangle integral



$$= \frac{s\, d^4k}{k^2(k-p_1)^2(k-p_1-p_2)^2} \qquad s = (p_1+p_2)^2$$

$$p_1 = \lambda_1\tilde{\lambda}_1, \quad p_2 = \lambda_2\tilde{\lambda}_2, \quad \lambda_1\tilde{\lambda}_2, \quad \lambda_2\tilde{\lambda}_1$$

$$k = \alpha_1\lambda_1\tilde{\lambda}_1 + \alpha_2\lambda_2\tilde{\lambda}_2 + \frac{\langle 23\rangle}{\langle 13\rangle}\alpha_3\lambda_1\tilde{\lambda}_2 + \frac{\langle 13\rangle}{\langle 23\rangle}\alpha_4\lambda_2\tilde{\lambda}_1$$

$$d\mathcal{I}_3 = \frac{d\alpha_1 \wedge d\alpha_2 \wedge d\alpha_3 \wedge d\alpha_4}{(\alpha_1\alpha_2-\alpha_3\alpha_4)(\alpha_1\alpha_2-\alpha_2-\alpha_3\alpha_4)(1-\alpha_1-\alpha_2+\alpha_1\alpha_2-\alpha_3\alpha_4)}$$

With this normalization, all leading singularities are ± 1

Leading singularities as guiding principle for an integral basis

- conjecture: integrals with constant leading singularities give rise to 'pure' functions

[Arkani-Hamed et al, 2012]

[Arkani-Hamed, Bourjaily, Cachzao, Trnka, 2010]

- pure functions are (rational linear combinations of) polylogarithmic functions of uniform weight

e.g. $\mathrm{Li}_3(1-x/y)+\frac{1}{2}\log^3(x)+\pi^2\log(y)$

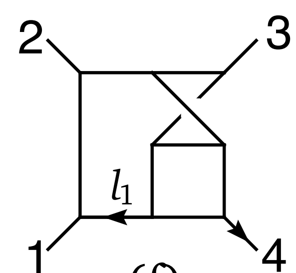
- pure functions satisfy simple differential equations
- although first understood in N=4 sYM, this also applies for integrals needed for QCD

From N=4 sYM to QCD

Original examples involved massless, planar, dual conformal, finite integrals

Today the conjecture is being used more generally:

- non - planar



(f)

- masses (more complicated factorizations)
- generic power counting (e.g. triangles)
- generalization to dimensional regularization

e.g. $${}_2\mathrm{F}_1(1+2\epsilon,\epsilon,1+\epsilon,1-x)$$
$$= 1-\epsilon\log x+\epsilon^2\log^2 x+\mathrm{Li}_2(1-x)+\mathcal{O}(\epsilon^2)$$

New strategy for computing Feynman integrals

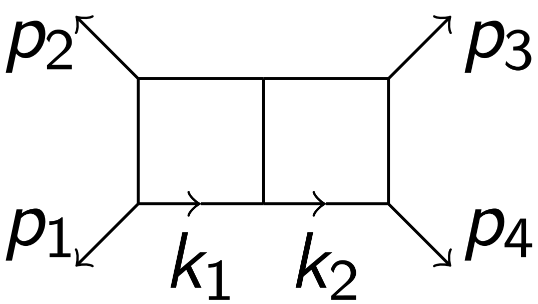
- choose basis of integrals with constant leading singularities
- write down differential equations (algorithmic)
- equations take canonical form
- read off special functions

Leading singularity algorithm

- start with numerator ansatz (power counting constraints)
- parametrize loop momenta; integrand is rational multi-variable function
- take residues consecutively
- normalize leading singularities

Let us look at an example…

Planar Double Box

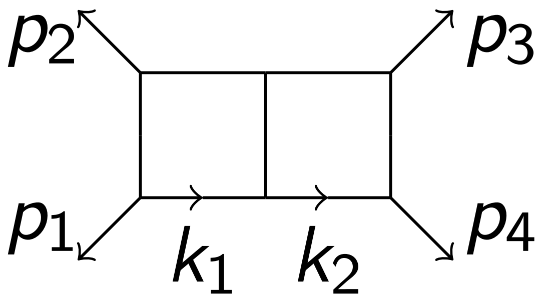


$$p_{12} = p_1 + p_2$$
$$p_{123} = p_1 + p_2 + p_3$$

$$d\mathcal{I}^{\mathrm{P}} = \frac{d^D k_1 d^D k_2 N(k_1, k_2)}{k_1^2 (k_1 + p_1)^2 (k_1 + p_{12})^2 k_2^2 (k_2 + p_{12})^2 (k_2 + p_{123})^2 (k_1 - k_2)^2}$$

$$\begin{array}{llll} k_1^2, & (k_1 + p_1)^2, & (k_1 + p_{12})^2, & (k_1 + p_{123})^2 \\ k_2^2, & (k_2 + p_1)^2, & (k_2 + p_{12})^2, & (k_2 + p_{123})^2 \\ (k_1 - k_2)^2 & & & \end{array}$$

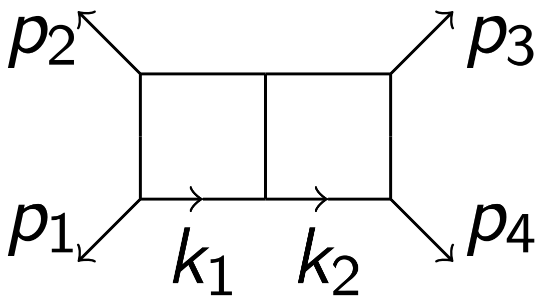
Planar Double Box



$$p_{12} = p_1 + p_2$$
$$p_{123} = p_1 + p_2 + p_3$$

$$\begin{aligned}
N(k_1, k_2) = {} & n_1 + k_1^2 n_2 + (k_1 + p_1)^2 n_3 + (k_1 + p_{12})^2 n_4 \\
& + (k_1 + p_{123})^2 n_5 + k_2^2 n_6 + (k_2 + p_1)^2 n_7 \\
& + (k_2 + p_{12})^2 n_8 + (k_2 + p_{123})^2 n_9 + (k_1 - k_2)^2 n_{10} \\
& + k_1^2 k_2^2 n_{11} + k_1^2 (k_2 + p_1)^2 n_{12} + k_1^2 (k_2 + p_{12})^2 n_{13} \\
& + k_1^2 (k_2 + p_{123})^2 n_{14} + (k_1 + p_1)^2 k_2^2 n_{15} \\
& + (k_1 + p_1)^2 (k_2 + p_1)^2 n_{16} + (k_1 + p_1)^2 (k_2 + p_{12})^2 n_{17} \\
& + (k_1 + p_1)^2 (k_2 + p_{123})^2 n_{18} + (k_1 + p_{12})^2 k_2^2 n_{19} \\
& + (k_1 + p_{12})^2 (k_2 + p_1)^2 n_{20} + (k_1 + p_{12})^2 (k_2 + p_{12})^2 n_{21} \\
& + (k_1 + p_{12})^2 (k_2 + p_{123})^2 n_{22} + (k_1 + p_{123})^2 k_2^2 n_{23} \\
& + (k_1 + p_{123})^2 (k_2 + p_1)^2 n_{24} + (k_1 + p_{123})^2 (k_2 + p_{12})^2 n_{25} \\
& + (k_1 + p_{123})^2 (k_2 + p_{123})^2 n_{26}
\end{aligned}$$

Planar Double Box



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& +(k_1 + p_{123})^2 n_5 + k_2^2 n_6 + (k_2 + p_1)^2 n_7 \\
& +(k_2 + p_{12})^2 n_8 + (k_2 + p_{123})^2 n_9 + (k_1 - k_2)^2 n_{10} \\
& +k_1^2 k_2^2 n_{11} + k_1^2 (k_2 + p_1)^2 n_{12} + k_1^2 (k_2 + p_{12})^2 n_{13} \\
& +k_1^2 (k_2 + p_{123})^2 n_{14} + (k_1 + p_1)^2 k_2^2 n_{15} \\
& +(k_1 + p_1)^2 (k_2 + p_1)^2 n_{16} + (k_1 + p_1)^2 (k_2 + p_{12})^2 n_{17} \\
& +(k_1 + p_1)^2 (k_2 + p_{123})^2 n_{18} + (k_1 + p_{12})^2 k_2^2 n_{19} \\
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& +(k_1 + p_{123})^2 (k_2 + p_1)^2 n_{24} + (k_1 + p_{123})^2 (k_2 + p_{12})^2 n_{25} \\
& +(k_1 + p_{123})^2 (k_2 + p_{123})^2 n_{26}
\end{aligned}$$

$$\boxed{\begin{aligned} n_{16} &= 0 \\ n_{26} &= 0 \\ n_{18} &= -n_{24} \end{aligned}}$$

Leading Singularities of Planar Double Box

$$-\frac{n_5 t+n_9 t-n_1}{s^2 t}, \quad \frac{n_5 t-n_1}{s^2 t}, \quad \frac{n_5-n_{18} s}{s^2}, \quad \frac{n_7 t-n_1}{s^2 t},$$

$$-\frac{n_8 s-n_1}{s^2 t}, \quad -\frac{n_3-n_{17} s}{s^2}, \quad -\frac{n_{18} t^2+n_5 t+n_7 t+n_{10} t-n_1}{s^2 t},$$

$$\frac{n_1}{s^2 t}, -\frac{n_{15} s-n_3}{s^2}, \frac{-n_3 t-n_7 t+n_1}{s^2 t}, \frac{-n_{21} s^2+n_4 s+n_8 s-n_1}{s^2 t},$$

$$\frac{-n_{22} st+n_4 s+n_9 t-n_1}{s^2 t}, \quad -\frac{n_4}{st}, \quad -\frac{n_{20} t-n_4}{st},$$

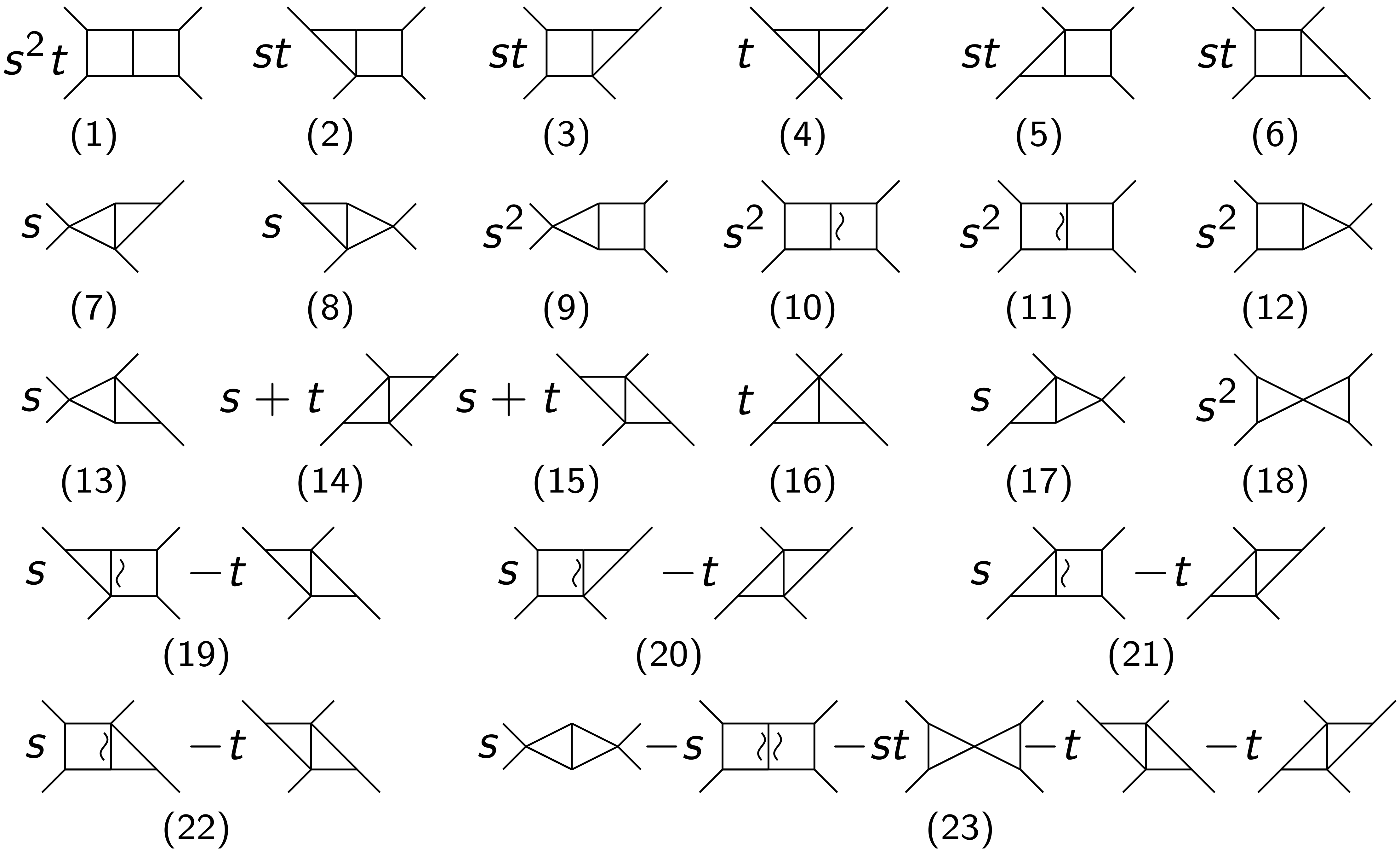
$$\frac{n_{19} s^2 t+n_{20} s^2 t+n_{18} st^2-n_{23} st^2-n_4 s(s+t)+n_5 t(s+t)}{s^2 t(s+t)},$$

$$\frac{n_2 s-n_1}{s^2 t}, \quad -\frac{n_{12} s^2(-t)-n_{13} s^2 t-n_{18} st^2+n_{25} st^2+n_2 s(s+t)-n_5 t(s+t)}{s^2 t(s+t)},$$

$$\frac{n_{25} st-n_8 s-n_5 t+n_1}{s^2 t}, \quad \frac{-n_{12} st+n_2 s+n_7 t-n_1}{s^2 t},$$

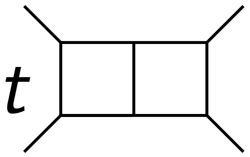
$$-\frac{-n_{14} st+n_2 s+n_9 t-n_1}{s^2 t}, \quad \frac{-n_{11} s^2+n_2 s+n_6 s-n_1}{s^2 t}, \quad \frac{n_{23} s-n_5}{s^2}, -\frac{n_6 s-n_1}{s^2 t}$$

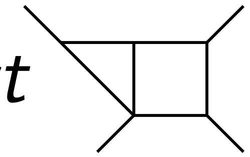
List of Integrands



Uniform Transcendental Weight

Integral basis:

$s^2 t$ 
(1)

st 
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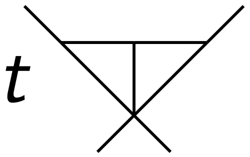
t 
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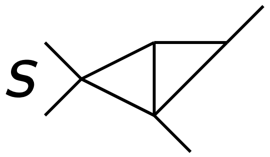
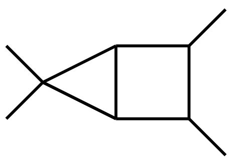
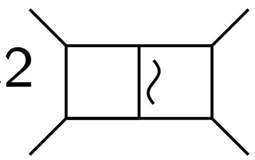
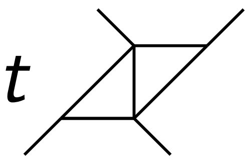
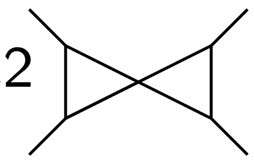
s 
(7)

s^2 
(9)

s^2 
(10)

$s + t$ 
(14)

s^2 
(18)

Differentiate

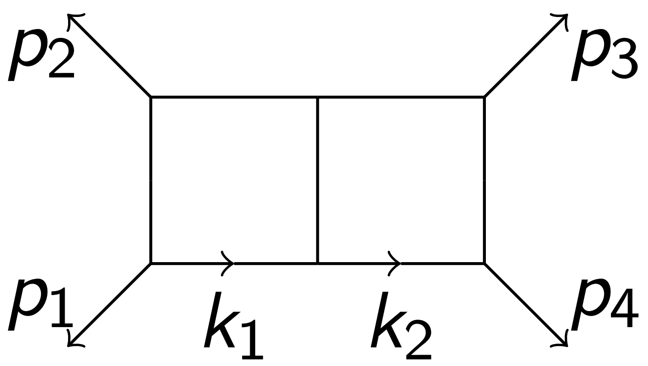
$$\frac{\partial}{\partial x}\vec{f} = \epsilon \left(\frac{a}{x} + \frac{b}{1+x} \right) \vec{f}, \quad x = \frac{t}{s}, \quad D = 4 - 2\epsilon$$

where a and b are constant matrices, to prove uniform transcendental weight property.

Uniform Transcendental Weight

$$f_3 = f_5 = f_6 = f_2, \quad f_8 = f_{13} = f_7, \quad f_{11} = f_{10},$$
$$f_{12} = f_9, \quad f_{15} = f_{14}, f_{16} = f_4, \quad f_{17} = f_7,$$
$$f_{19} = f_{20} = f_{21} = f_{22} = -\frac{1}{3}f_2 + f_4 + f_7,$$
$$f_{23} = -\frac{1}{2}f_1 - 5f_4 - 3f_7 + \frac{1}{2}f_9 + 3f_{14} + f_{18} - \frac{3}{2}f_{10} - \frac{4}{3}f_2$$

Planar Double Box Summary



- Ansatz with 26 integrands
- Integrand basis with 23 integrands
- All integrals have uniform transcendental weight
- Integral basis with 8 integrals (complete)

Conclusions and outlook

- N=4 sYM - inspired methods are useful in QCD
- Feynman integrals are no longer the bottleneck of NNLO calculations
- 2 to 3 processes are within reach
- longer-term: progress for special functions and amplitudes for 2 to n processes

we are seeing the beginning of a NNLO revolution!