The structure of mesons and hadron resonance gas in ***strong*** magnetic fields

($|eB| \gg \Lambda_{QCD}^2$)

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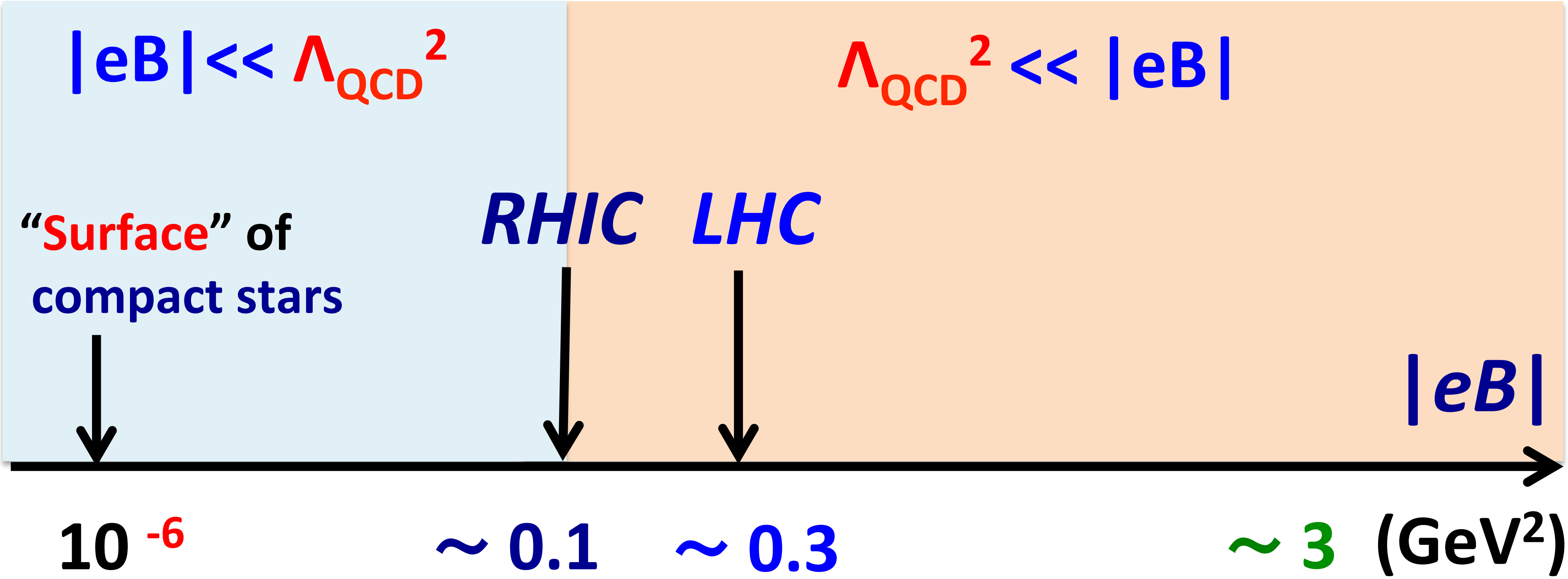
with Nan Su & Koichi Hattori

Refs) PLB720 (2013), PLB726 (2013), NPA951(2016)

Strength of B-fields



$|eB| << \Lambda_{QCD}^2$

$\Lambda_{QCD}^2 << |eB|$

"Surface" of compact stars

RHIC

LHC

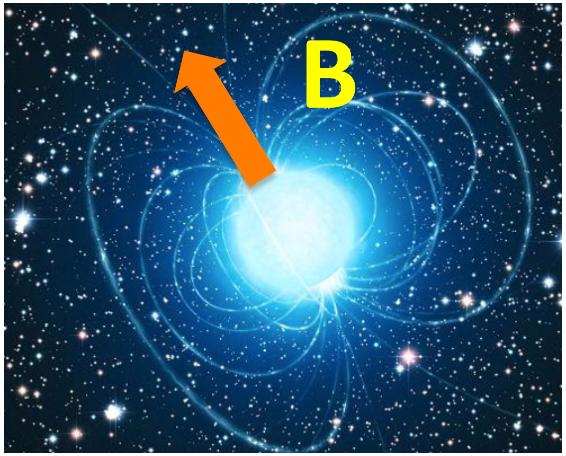
$|eB|$

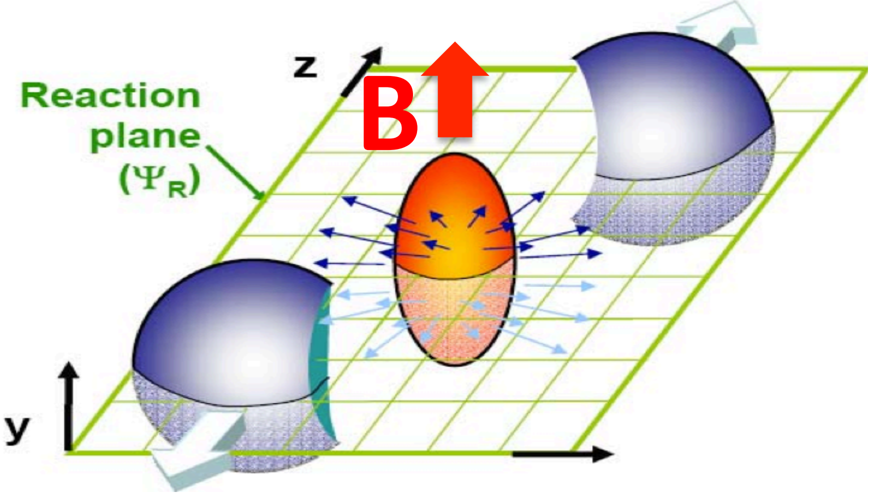
10^{-6}

~ 0.1

~ 0.3

~ 3 (GeV^2)





Strength of B-fields

$|eB| \ll \Lambda_{QCD}^2$

$\Lambda_{QCD}^2 \ll |eB|$

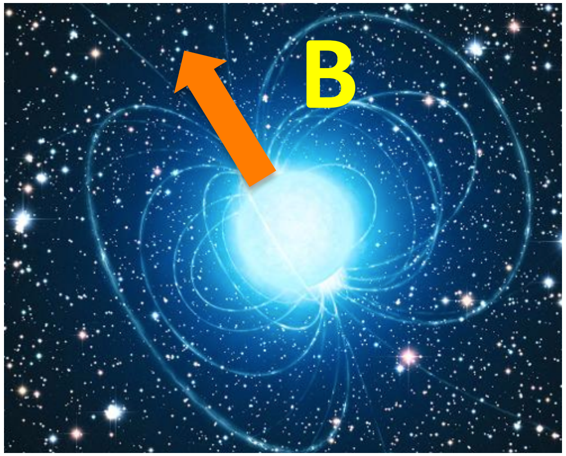
Lattice QCD

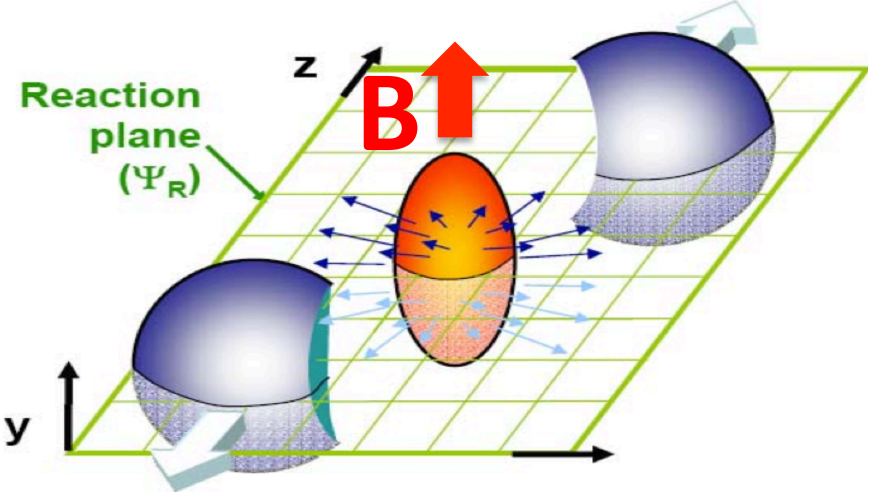
"Surface" of compact stars

RHIC *LHC*

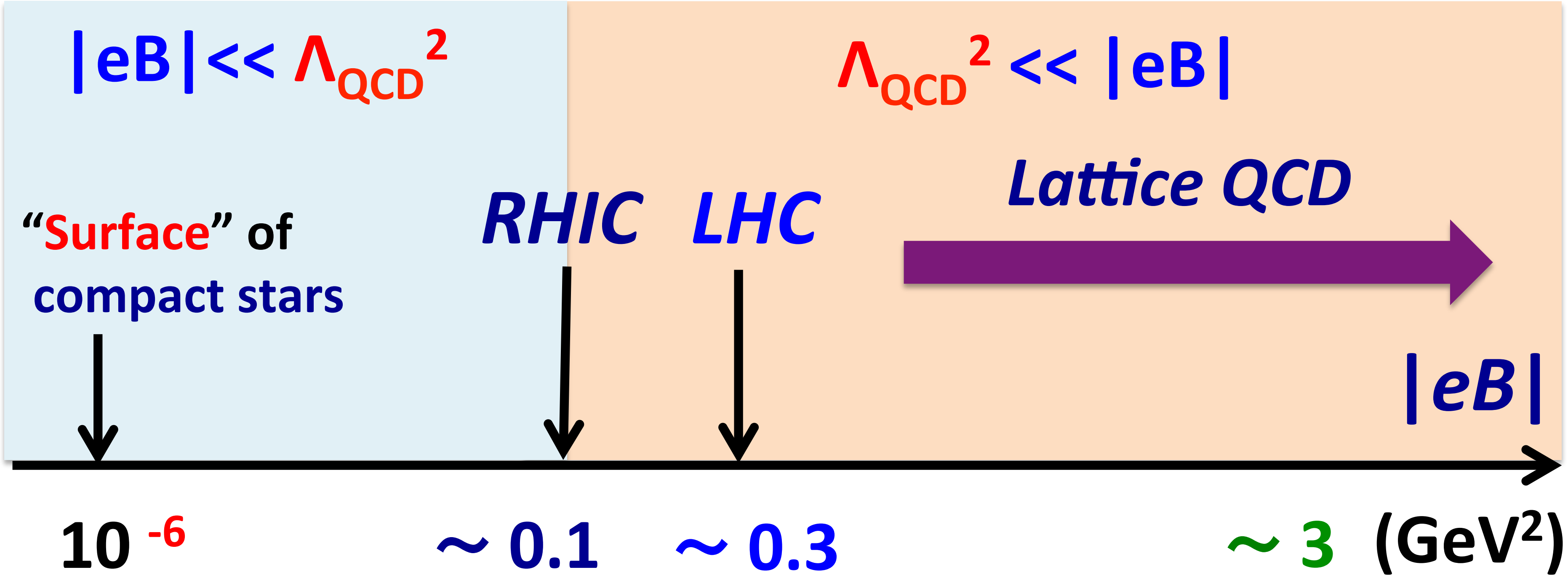
$|eB|$

10^{-6} ~ 0.1 ~ 0.3 ~ 3 (GeV^2)





Strength of B-fields



$|eB| \ll \Lambda_{QCD}^2$

$\Lambda_{QCD}^2 \ll |eB|$

“Surface” of compact stars

RHIC *LHC*

Lattice QCD

$|eB|$

10^{-6} ~ 0.1 ~ 0.3 ~ 3 (GeV^2)

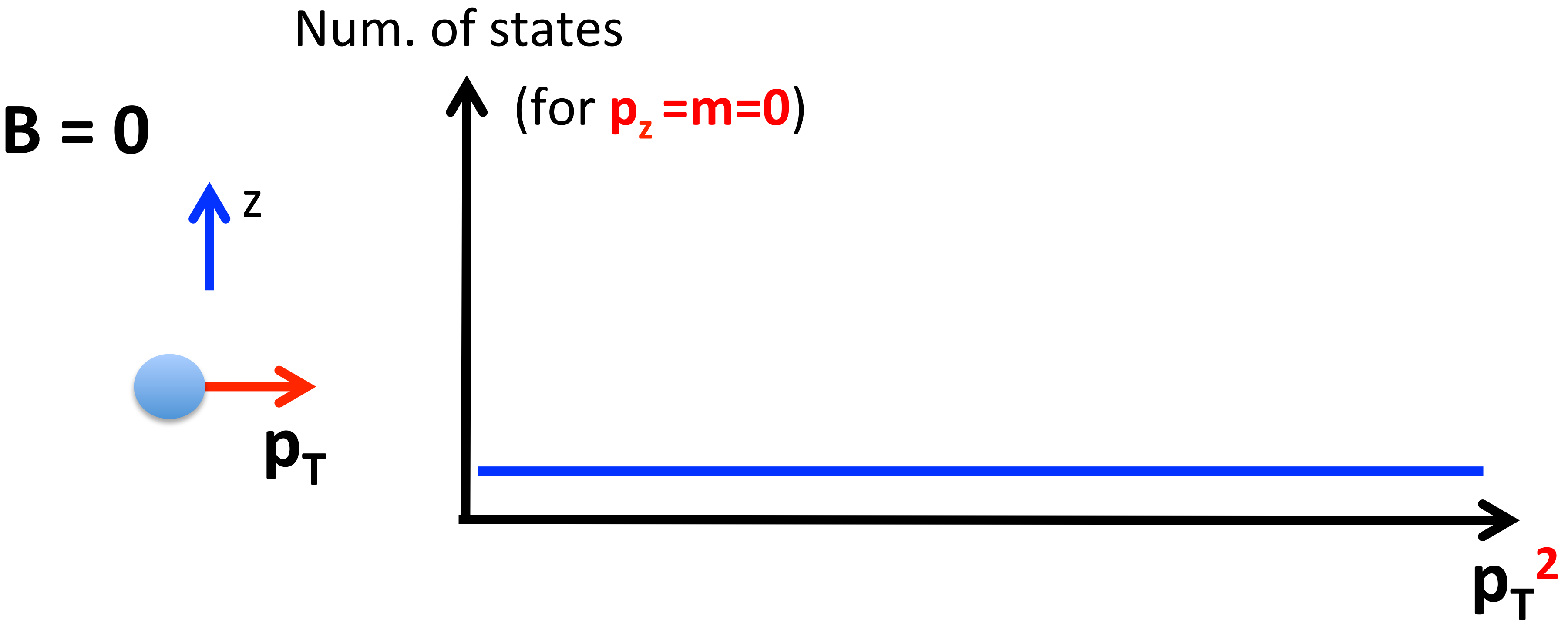
Lattice QCD ~ “Laboratory” on the computer

→ *test* theoretical *ideas* and *concepts* usable for other extreme environments, e.g., *dense QCD*

Contents

Quantum mechanics in mag. fields

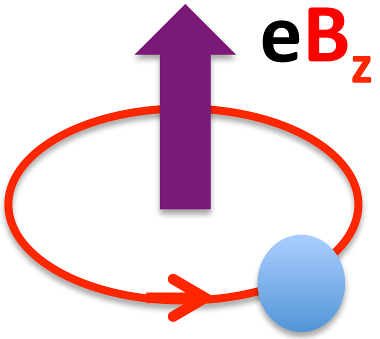
(spinless, free particles)



Quantum mechanics in mag. fields

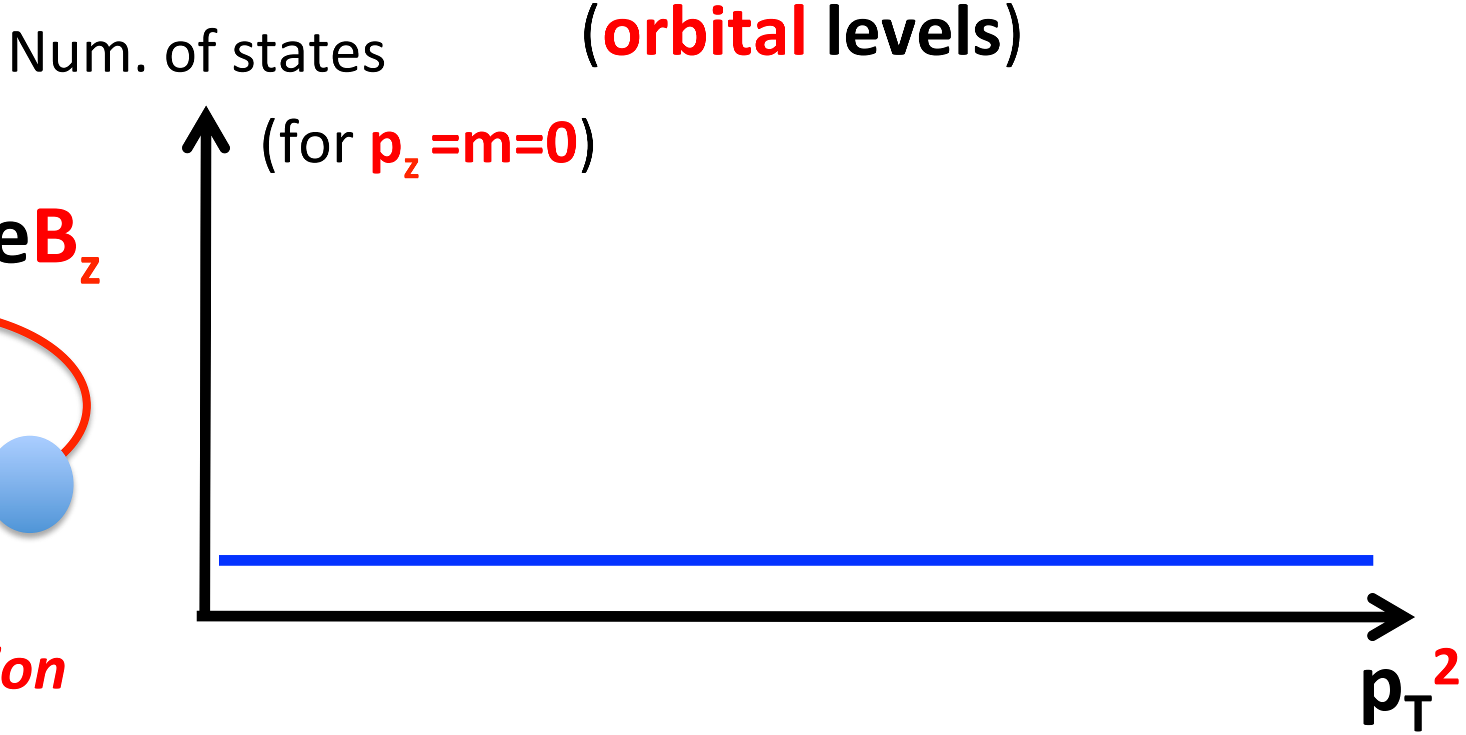
(spinless, free particles)

B ≠ 0



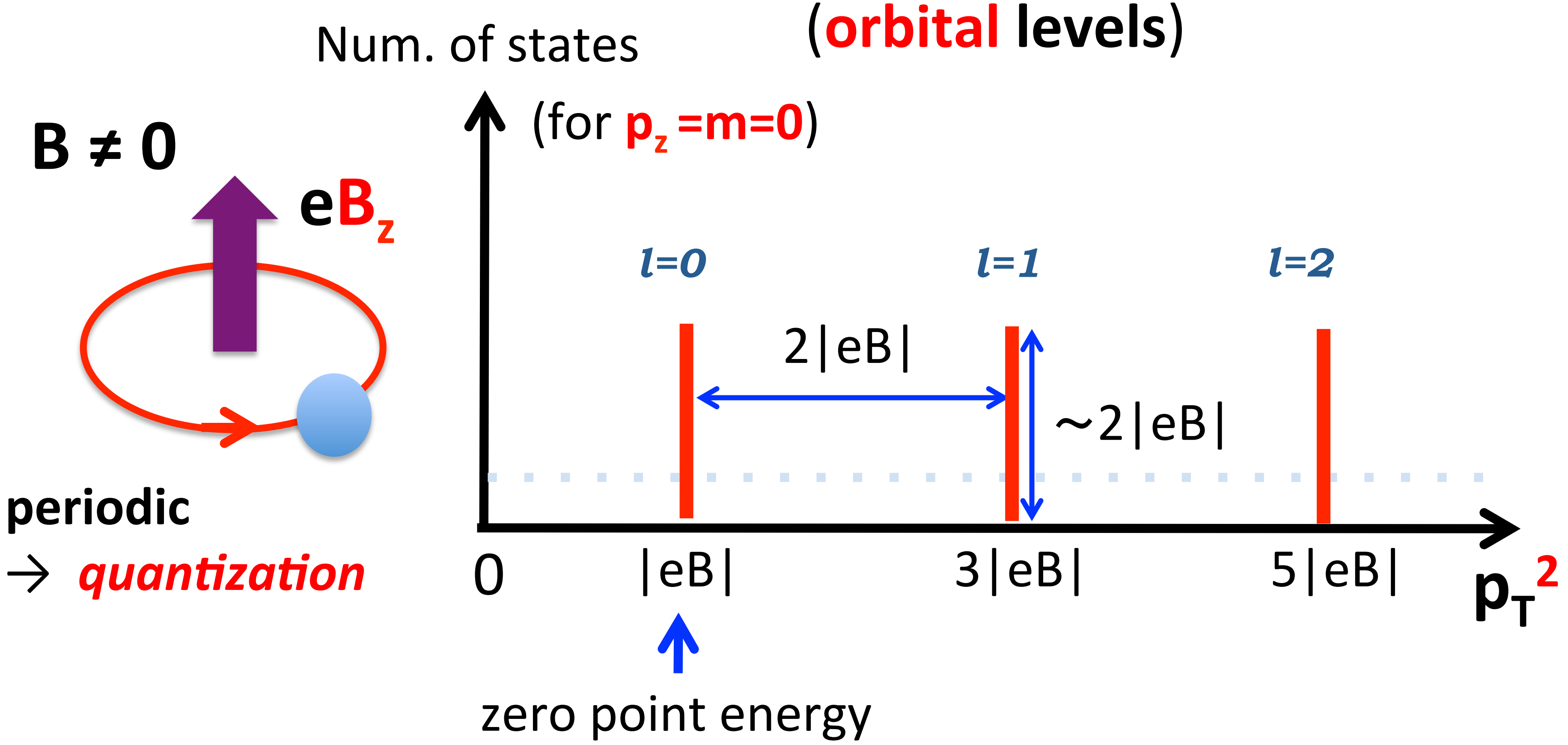
periodic
→ ***quantization***

(**orbital levels**)



Quantum mechanics in mag. fields

(spinless, free particles)



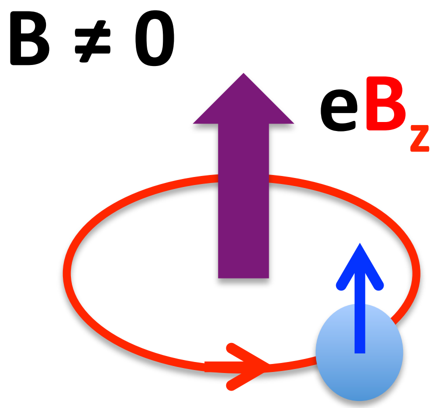
periodic

→ ***quantization***

Quantum mechanics in mag. fields

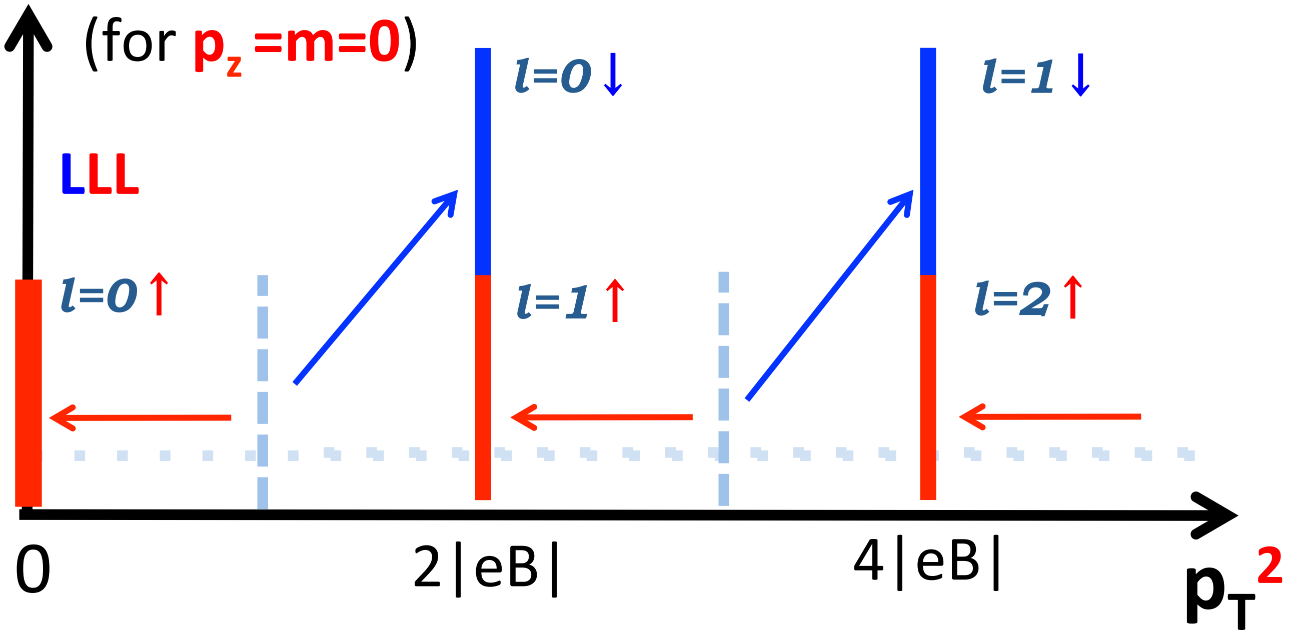
(spin 1/2, free particles)

(orbital + Zeeman splitting)



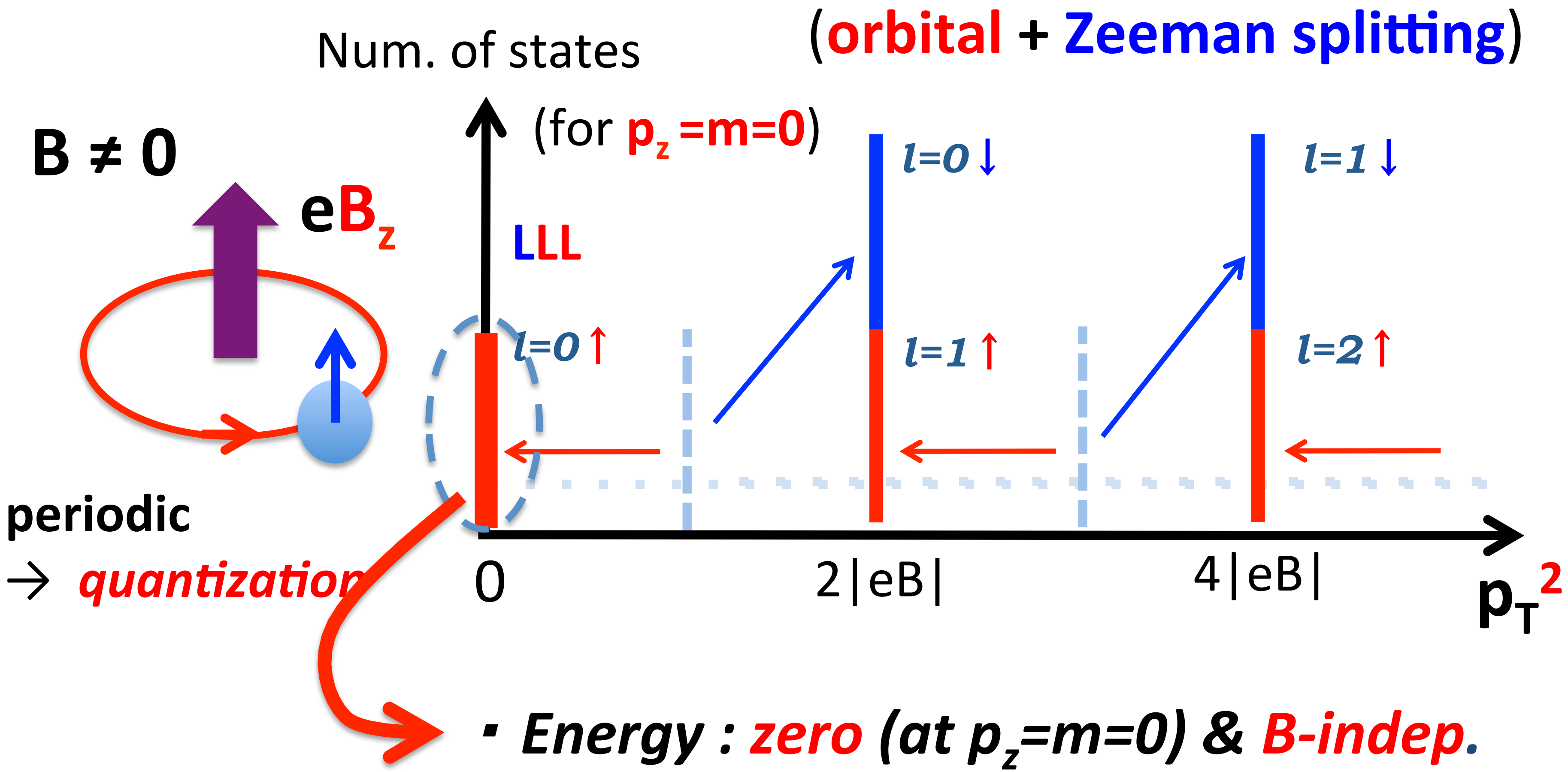
periodic

→ ***quantization***



Quantum mechanics in mag. fields

(spin 1/2, free particles)



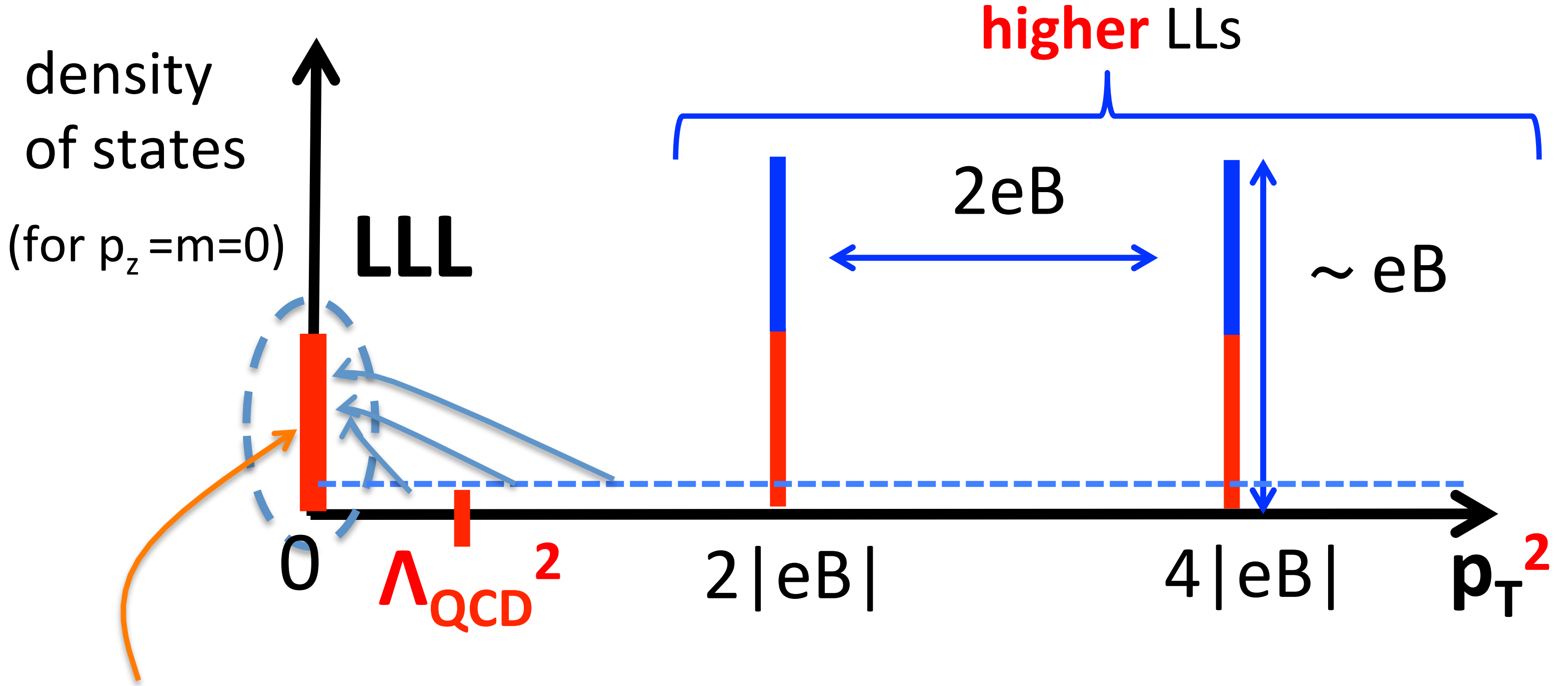
periodic

→ ***quantization***

- ***Energy : zero (at p_z=m=0) & B-indep.***

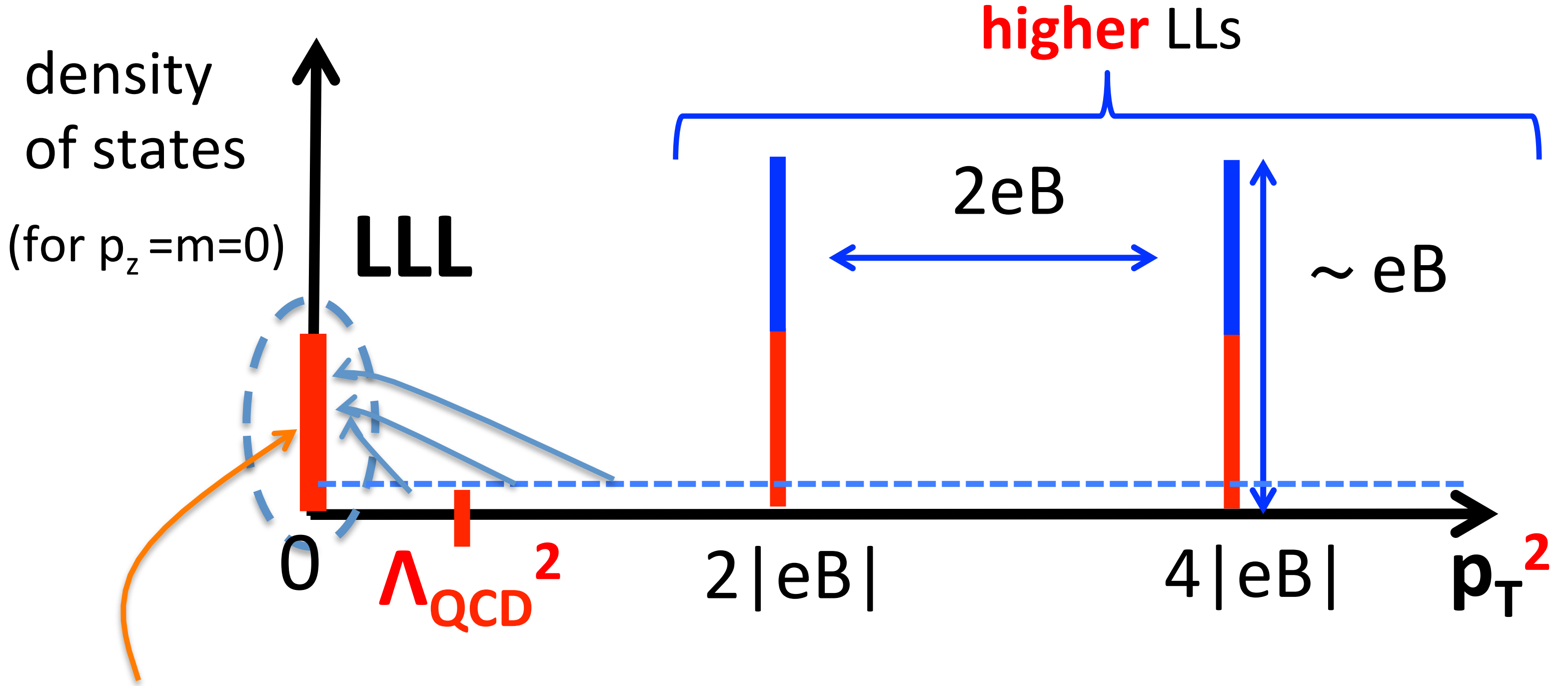
(at tree level)

"*Enhanced*" **IR** phase space for quarks



Larger B → More **quarks can stay** **at low energy.**

“*Enhanced*” IR phase space for quarks

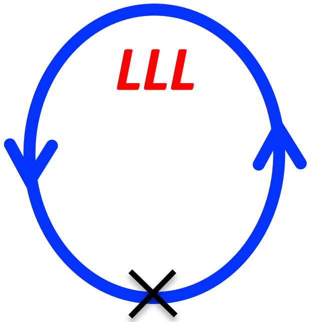


Larger B → More quarks can stay at low energy.

➡ ***Enhanced* quark loop corrections in *IR***

We may change the structure of QCD in the *IR* region

Examples of quark loops



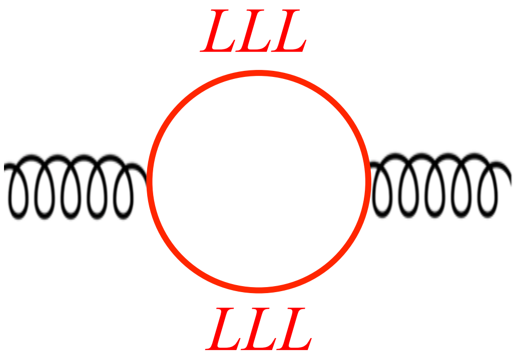
chiral condensate

$$\langle \bar{\psi}(x)\psi(x) \rangle_{4\mathrm{D}} = \underline{\frac{|eB|}{2\pi}}\ \underline{\langle \bar{\psi}(x_L)\psi(x_L) \rangle_{2\mathrm{D}}}$$

degeneracy
(universal)

non-universal

gluon polarization
(perturbative screening)



$$\alpha_s \underline{|eB|} \times \begin{cases} \dfrac{q_{\parallel}^2}{M_q^2(B)} & (q_{\parallel}^2 < M_q^2(B)) \\ 1 & (M_q^2(B) < q_{\parallel}^2) \end{cases}$$

degeneracy
(universal)

(Miranski-Shovkovy 02)

(*Naively*) ***Both effects are enhanced by B***

2, Theoretical Problems:

Lattice *vs* *Models*

$|eB| \gg \Lambda_{QCD}^2$

(For $|eB| \sim \Lambda_{QCD}^2$ → **Mao's talk**)

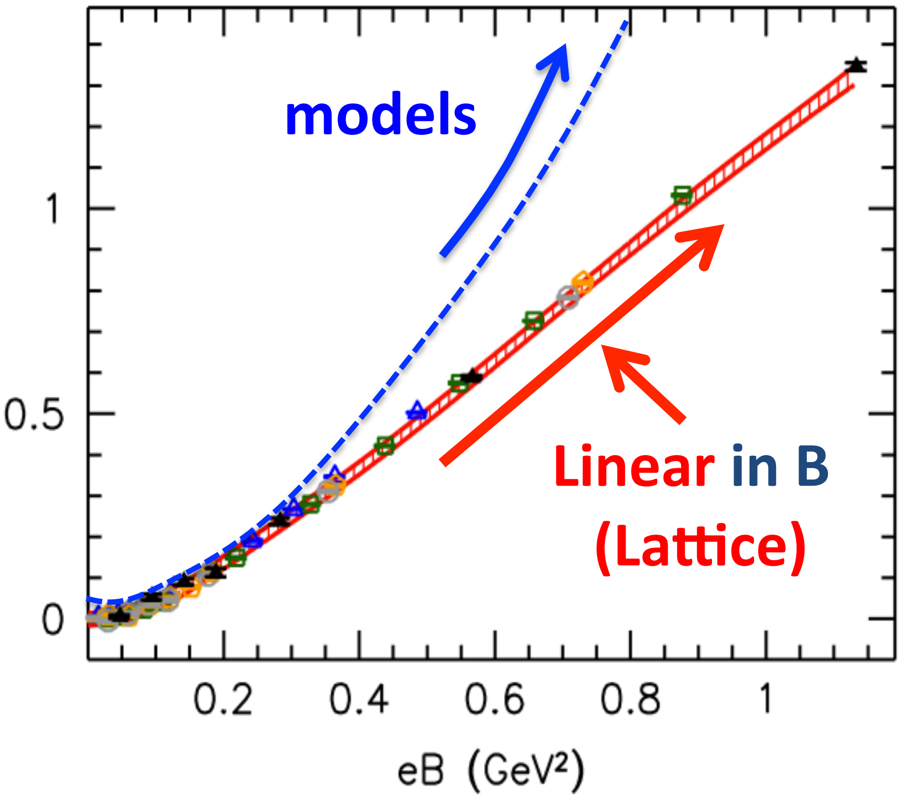
Problems: Lattice vs Models

(Bali et al, 11)

Problem 1)

$$\Delta\langle\bar{\psi}\psi\rangle_B/\langle\bar{\psi}\psi\rangle_{B=0}$$

(T = 0)



Problems: Lattice vs Models

(Bali et al, 11)

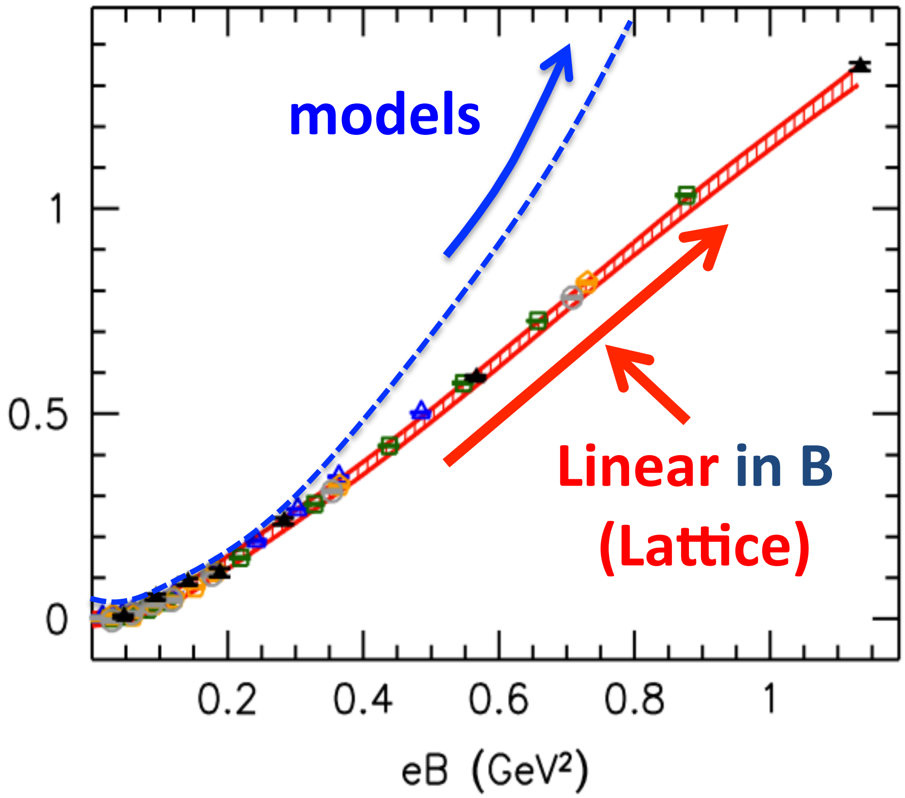
Problem 1)

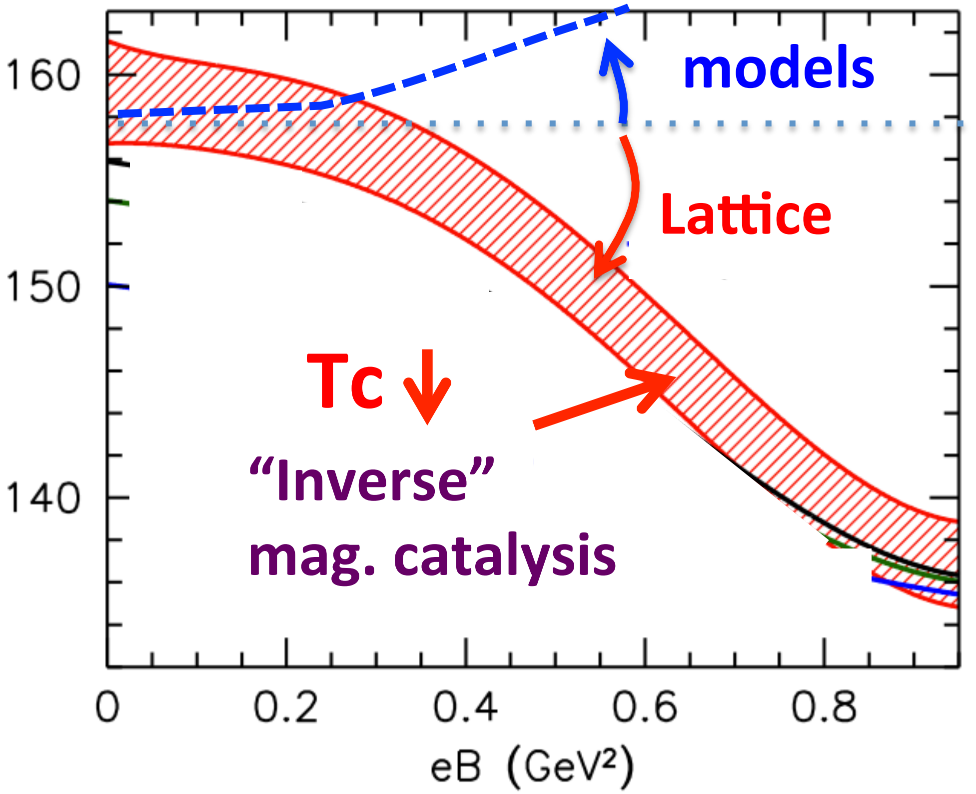
$$\Delta\langle\bar{\psi}\psi\rangle_B/\langle\bar{\psi}\psi\rangle_{B=0}$$

(T = 0)

Problem 2)

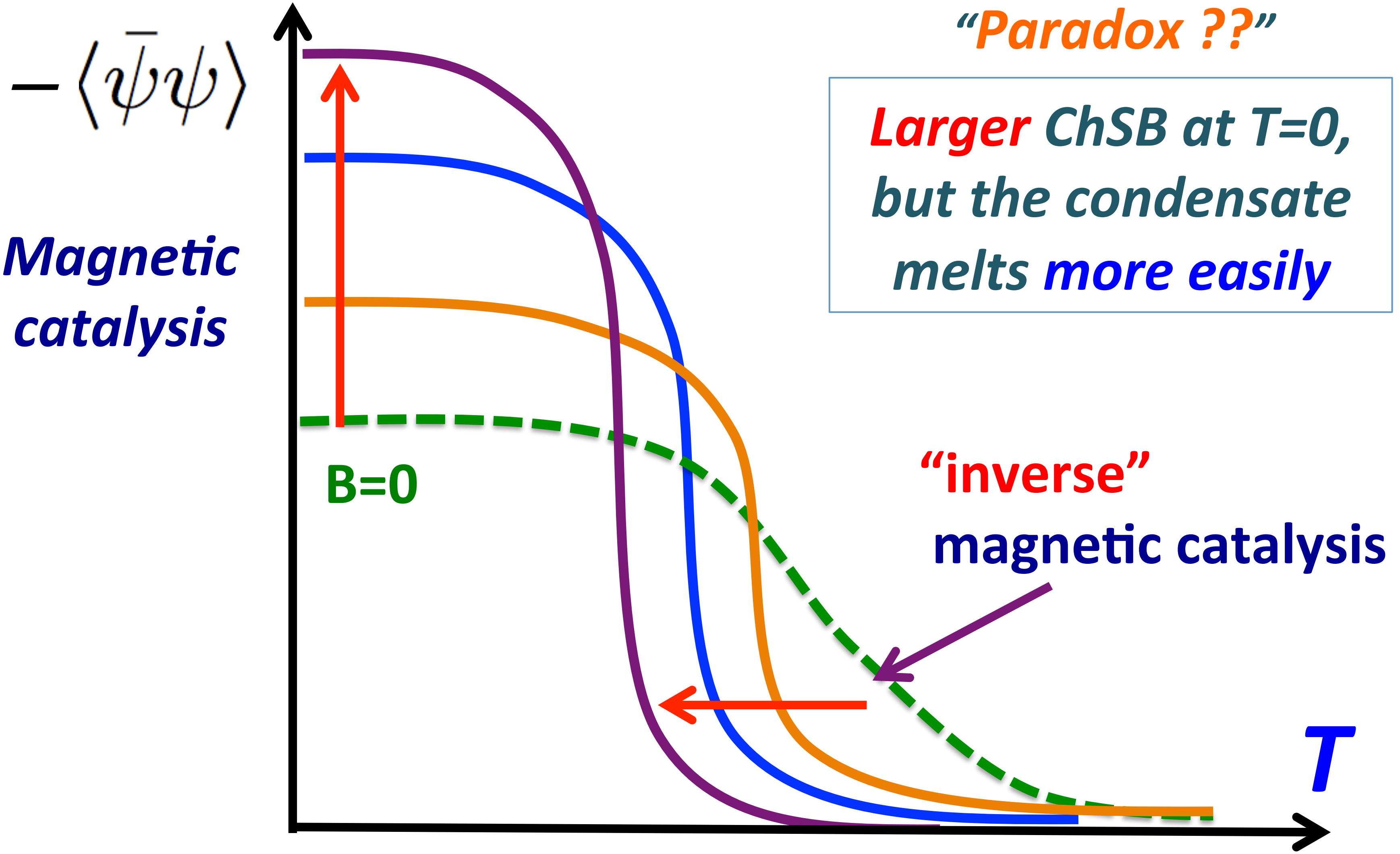
T_{chiral} $(\sim T_{deconf.})$





Problems 1&2: Lattice vs Models

Lattice results look like



Magnetic catalysis

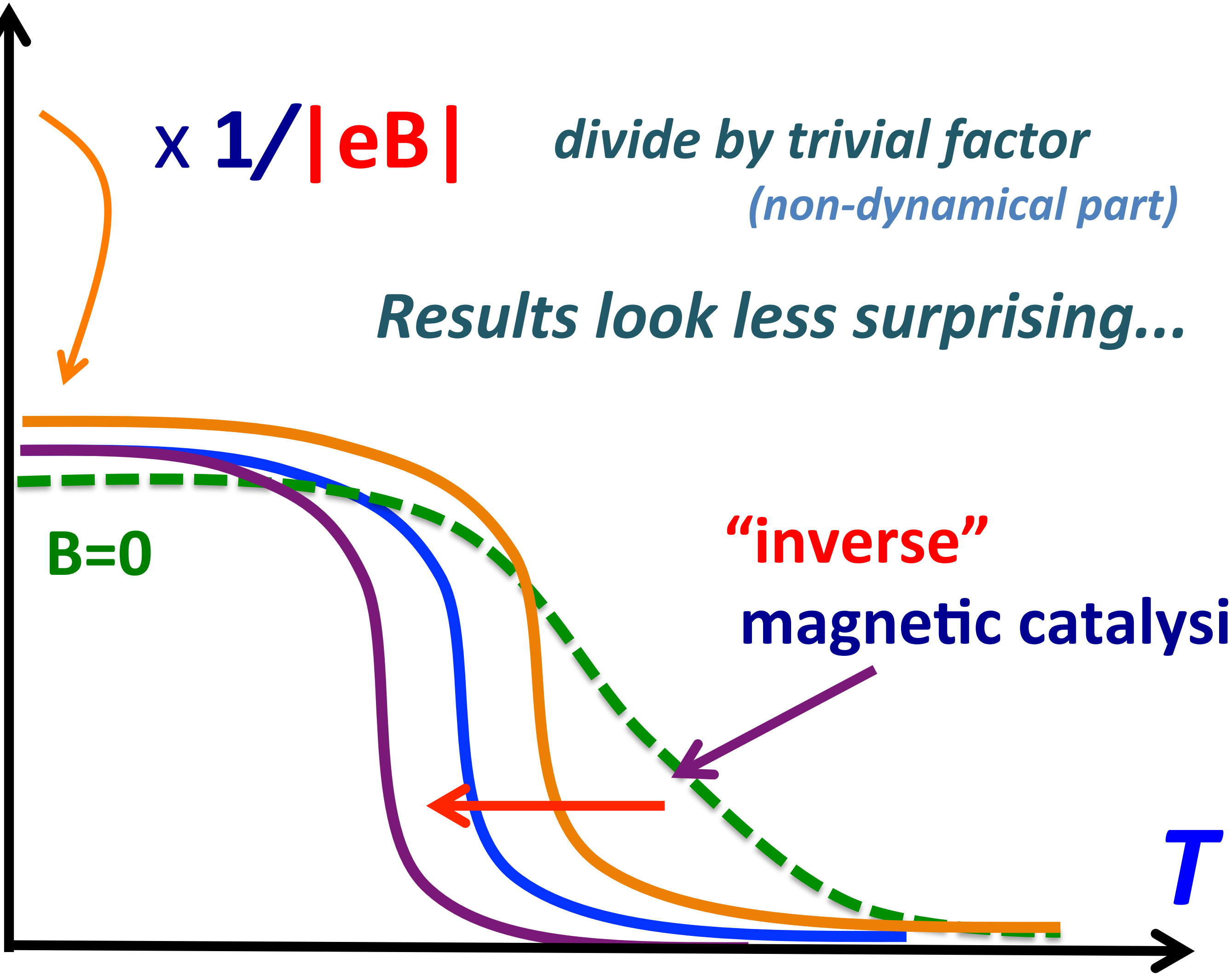
"Paradox ??"

Larger ChSB at T=0, but the condensate melts more easily

"inverse" magnetic catalysis

Problems 1&2: Lattice vs Models

more proper way to look



x 1/|eB|

divide by trivial factor

(non-dynamical part)

non-universal part

$-\langle\bar{\psi}\psi\rangle_{\underline{\text{2D}}}$

Results look less surprising...

"inverse"

magnetic catalysis

Origin of problems (models)

(The NJL-type, ...)

$$M_q \sim |eB|^{1/2}$$ **(?)**

Origin of problems (models)

(The NJL-type, ...)

$$M_q \sim |eB|^{1/2}$$ **(?)**

Problem 1)

$$\begin{tikzcd}
\langle \bar{\psi}\psi \rangle_{2D} \sim |eB|^{1/2} \arrow[d, "\times |eB|"] \\
\langle \bar{\psi}\psi \rangle_{4D} \sim |eB|^{3/2}
\end{tikzcd}$$

$\neq$ **lattice data** $\propto$ **|eB|**

Origin of problems (models)

(The NJL-type, ...)

$$M_q \sim |eB|^{1/2}$$ (?)

Problem 1)

$$\langle \bar{\psi}\psi \rangle_{2D} \sim |eB|^{1/2}$$

$$\Big\downarrow \times |eB|$$

$$\langle \bar{\psi}\psi \rangle_{4D} \sim |eB|^{3/2}$$

≠ **lattice data** ∝ |eB|

Problem 2)

Too massive

to thermally excite :

we need

$$T \sim M_q \sim \underline{|eB|^{1/2}}$$

→ T_c grows as B ↑

≠ **lattice data**

Our Goal

We are going to claim : for QCD

$$M_q \sim \Lambda_{\mathrm{QCD}} \quad \neq \quad |eB|^{1/2}$$ (**models**)

even at **$|eB| >> \Lambda_{QCD}^2$**

Our Goal

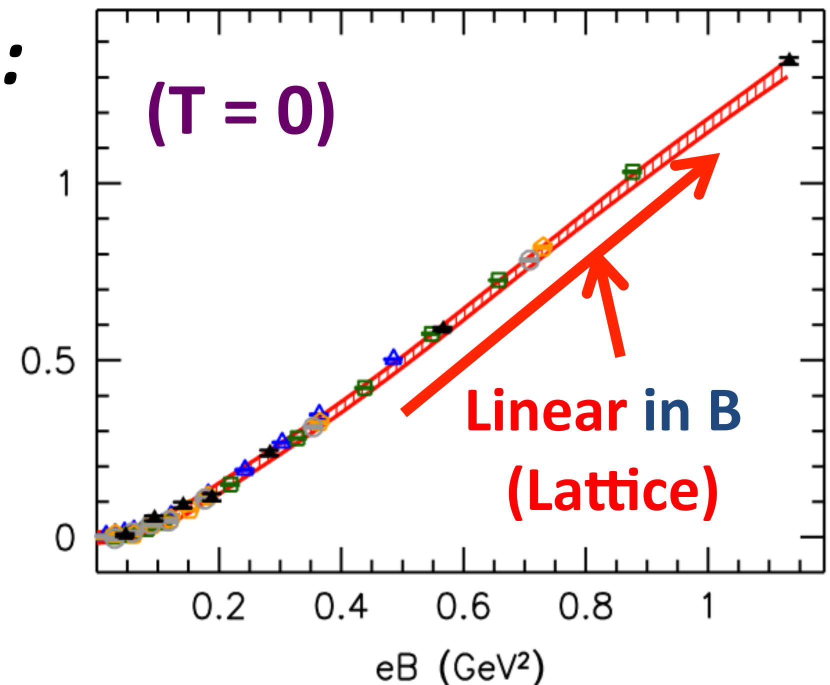
We are going to claim : for QCD

$$M_q \sim \Lambda_{\rm QCD} \quad \neq \quad |eB|^{1/2} \text{ (models)}$$

even at $|eB| \gg \Lambda_{\rm QCD}{}^2$

If so, "problem 1" is solved :

$$\langle \bar{\psi}\psi \rangle_{\rm 4D} = \frac{|eB|}{2\pi} \underbrace{\langle \bar{\psi}\psi \rangle_{\rm 2D}}_{\sim \Lambda_{\rm QCD}}$$



Linear in B (Lattice)

3, The quark mass gap in strong magnetic fields

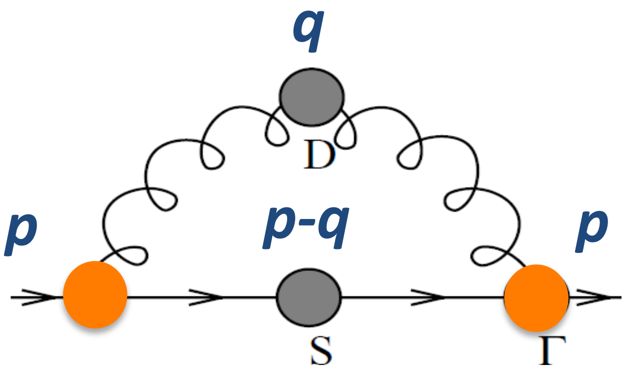
Keep only LLL **(well-justified, see TK-Su 13')**

Structure of the Schwinger-Dyson eq.

(*for LLL*)

1) No explicit B-dep. for the LLL

2) No p_T-dep.
→ " factorization "



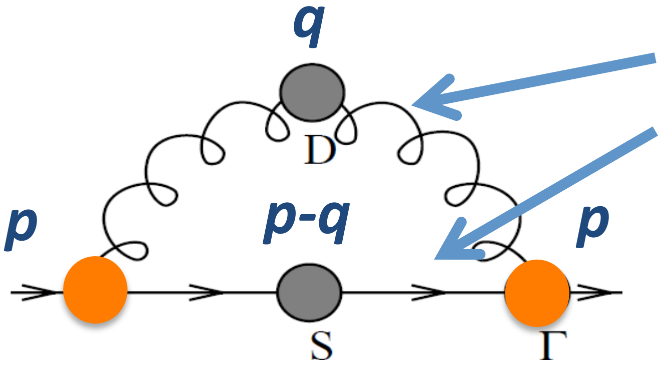
$$M(p_L) \sim \int_{q_L} S^{\rm 2D}_{\rm LLL}(p_L - q_L; M) \boxed{\int_{q_\perp} {\rm e}^{-\frac{q_\perp^2}{2|eB|}} \, D^{\rm 4D}_{\rm NP}(q_L, q_\perp)}$$

Structure of the Schwinger-Dyson eq.

(*for LLL*)

1) **No explicit B-dep. for the LLL**

2) **No p_T-dep.**
→ **" factorization "**



Different bases

$$M(p_L) \sim \int_{q_L} S^{\rm 2D}_{\rm LLL}(p_L - q_L; M) \boxed{\int_{q_\perp} {\rm e}^{-\frac{q_\perp^2}{2|eB|}} \, D^{\rm 4D}_{\rm NP}(q_L, q_\perp)}$$

Structure of the Schwinger-Dyson eq.

(*for LLL*)

1) **No explicit B-dep. for the LLL**

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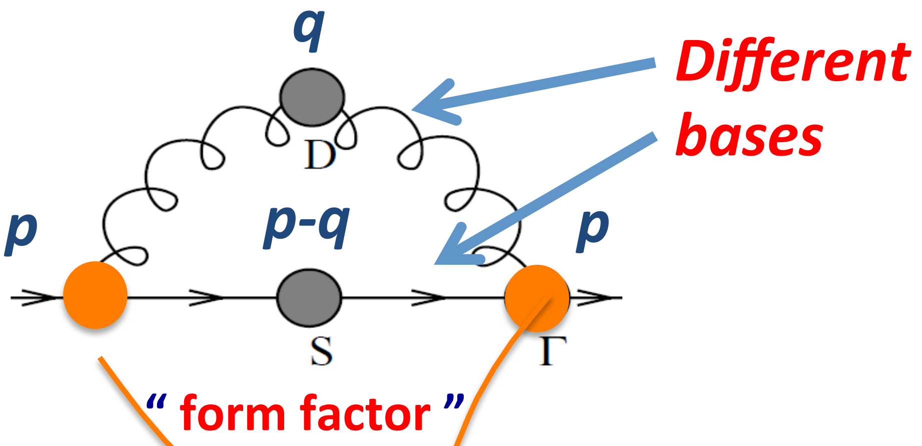


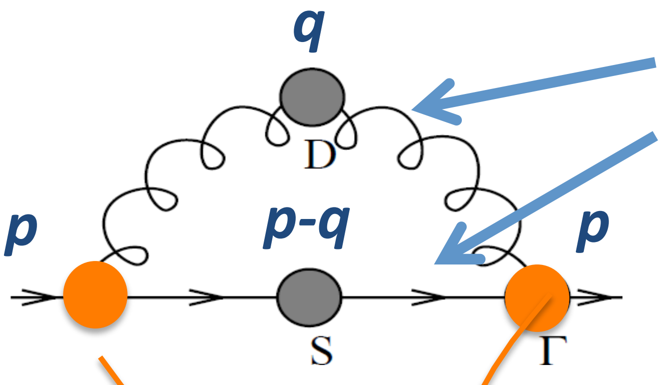
$$M(p_L) \sim \int_{q_L} S^{\mathrm{2D}}_{\mathrm{LLL}}(p_L - q_L; M) \boxed{\int_{q_\perp} \mathrm{e}^{-\frac{q_\perp^2}{2|eB|}} D^{\mathrm{4D}}_{\mathrm{NP}}(q_L, q_\perp)}$$

Structure of the Schwinger-Dyson eq.

(*for* **LLL**)

1) **No explicit B-dep.**
   for the LLL

2) **No p_T-dep.**
   → " **factorization** "



Different bases

" **form factor** "

$$M(p_L) \sim \int_{q_L} S^{\mathrm{2D}}_{\mathrm{LLL}}(p_L - q_L; M) \boxed{\int_{q_\perp} \mathrm{e}^{-\frac{q_\perp^2}{2|eB|}} D^{\mathrm{4D}}_{\mathrm{NP}}(q_L, q_\perp)}$$

Key observation : **All the *B-dep.* will come out from 2D " *smeared* " force !!**

IR vs *UV* interactions

smeared forces

$$\int_{q_\perp} \underline{\mathrm{e}^{-\frac{q_\perp^2}{2|eB|}}}\, D^{\mathrm{4D}}_{\mathrm{NP}}(q_L, q_\perp)$$

← **Origin of all B-dep.**

“ IR ”

“ UV ”

$$= \int_0^{\underline{\sim\Lambda^2_{\mathrm{QCD}}}} \mathrm{d}q_\perp^2\, \mathrm{e}^{-\frac{q_\perp^2}{2|eB|}}\, D^{\mathrm{4D}}_{\mathrm{NP}}(q) + \int_{\underline{\sim\Lambda^2_{\mathrm{QCD}}}}^{\infty} \mathrm{d}q_\perp^2\, \mathrm{e}^{-\frac{q_\perp^2}{2|eB|}}\, D^{\mathrm{4D}}_{\mathrm{NP}}(q)$$

IR vs *UV* interactions

smeared forces

$$\int_{q_\perp} \underline{\mathrm{e}^{-\frac{q_\perp^2}{2|eB|}}}\, D^{\mathrm{4D}}_{\mathrm{NP}}(q_L, q_\perp)$$

← **Origin of all B-dep.**

“ IR ” *“ UV ”*

$$= \int_0^{\underline{\sim\Lambda^2_{\mathrm{QCD}}}} \mathrm{d}q_\perp^2\, \mathrm{e}^{-\frac{q_\perp^2}{2|eB|}}\, D^{\mathrm{4D}}_{\mathrm{NP}}(q) + \int_{\underline{\sim\Lambda^2_{\mathrm{QCD}}}}^{\infty} \mathrm{d}q_\perp^2\, \mathrm{e}^{-\frac{q_\perp^2}{2|eB|}}\, D^{\mathrm{4D}}_{\mathrm{NP}}(q)$$

$$\mathrm{e}^{-\frac{q_\perp^2}{2|eB|}} \sim 1$$

$$\Big\downarrow \qquad\qquad\qquad \Big\downarrow$$

$$\int_0^{\underline{\sim\Lambda^2_{\mathrm{QCD}}}} \mathrm{d}q_\perp^2\, D^{\mathrm{4D}}_{\mathrm{NP}}(q) \qquad\qquad \int_{\underline{\sim\Lambda^2_{\mathrm{QCD}}}}^{\underline{\sim 2|eB|}} \mathrm{d}q_\perp^2\, D^{\mathrm{4D}}_{\mathrm{NP}}(q_L)$$

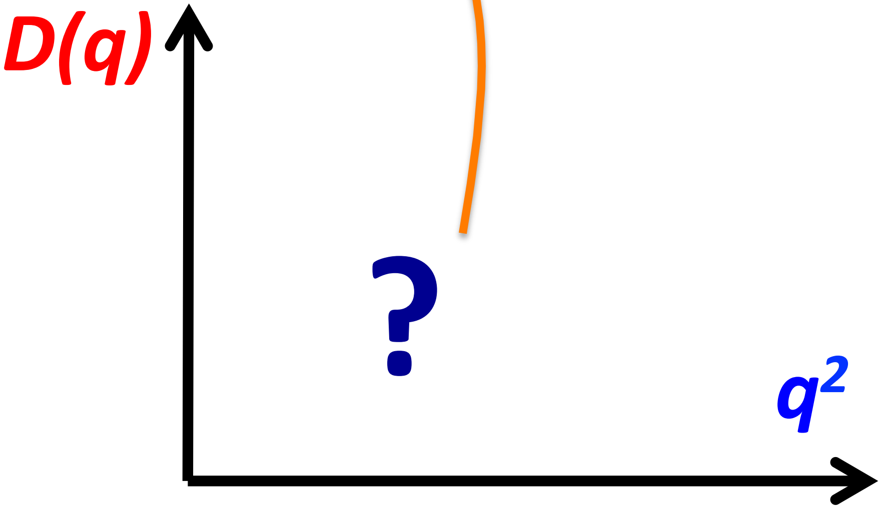
IR → *weak B-dep.*

UV → *strong B-dep.*

Comparison of forces, 1

$$\int_{q_\perp} \underline{\mathrm{e}^{-\frac{q_\perp^2}{2|eB|}}}\, D^{\mathrm{4D}}_{\mathrm{NP}}(q_L, q_\perp)$$

Origin of all B-dep.

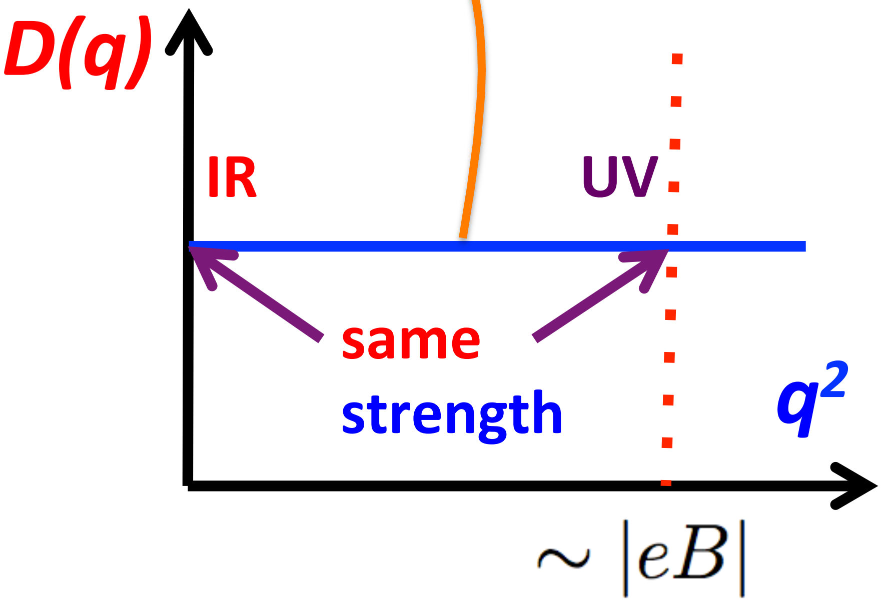


Comparison of forces, 1

$$\int_{q_\perp} \underline{\mathrm{e}^{-\frac{q_\perp^2}{2|eB|}}}\, D^{\mathrm{4D}}_{\mathrm{NP}}(q_L, q_\perp)$$

Origin of all B-dep.

1) Contact int. (NJL, etc.)



Comparison of forces, 1

$$\int_{q_\perp} \underline{\mathrm{e}^{-\frac{q_\perp^2}{2|eB|}}}\, D^{\mathrm{4D}}_{\mathrm{NP}}(q_L, q_\perp)$$

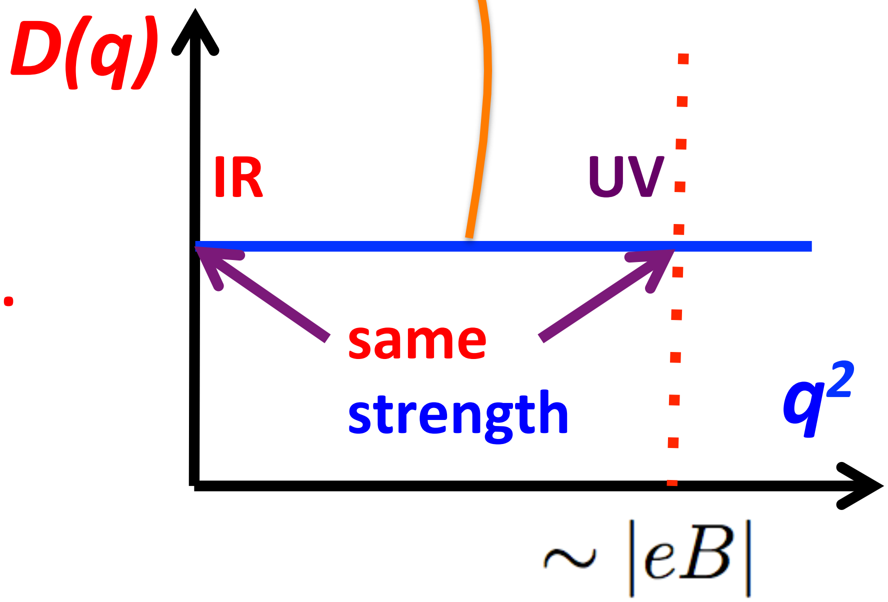
Origin of all B-dep.

---

1) Contact int. (NJL, etc.)

$$\boxed{\sim \underline{|eB|} \times \text{const.}}$$

2D Force is strongly B-dep.



Comparison of forces, 1

$$\int_{q_\perp} \underline{\mathrm{e}^{-\frac{q_\perp^2}{2|eB|}}}\, D^{\mathrm{4D}}_{\mathrm{NP}}(q_L, q_\perp)$$

Origin of all B-dep.

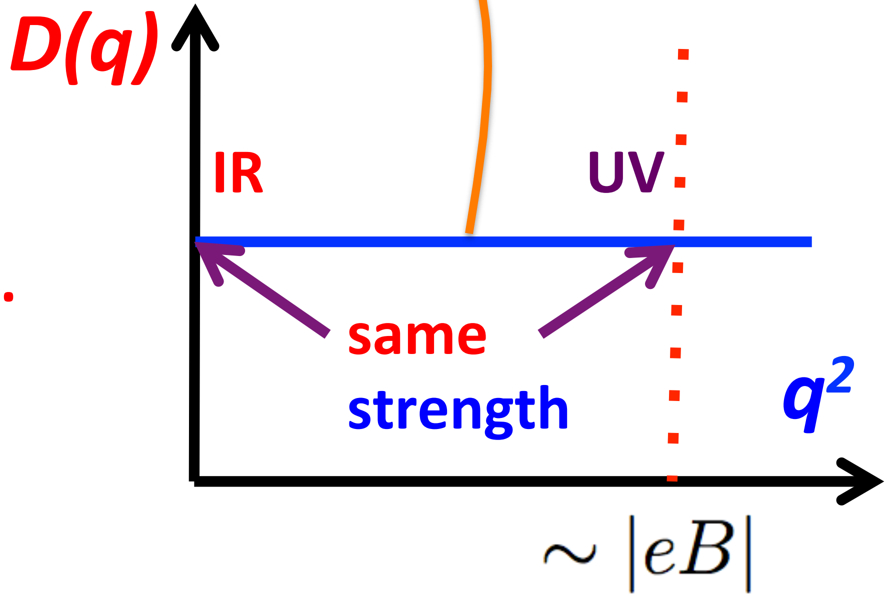
---

1) Contact int. (NJL, etc.)

$$\boxed{\sim \underline{|eB|} \times \mathrm{const.}}$$

2D Force is strongly B-dep.

$$M \sim |eB|^{1/2}$$

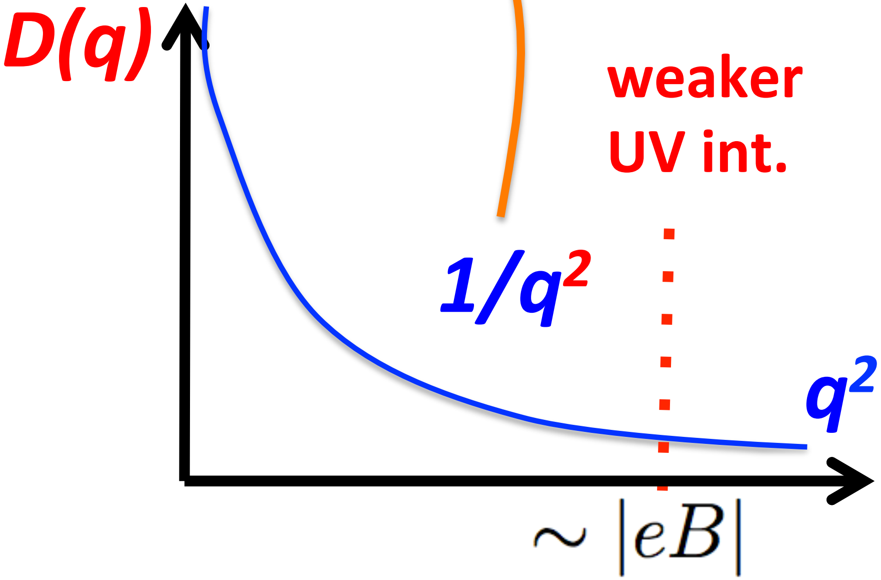


Comparison of forces, 2

$$\int_{q_\perp} \underline{\mathrm{e}^{-\frac{q_\perp^2}{2|eB|}}} \, D^{4\mathrm{D}}_{\mathrm{NP}}(q_L, q_\perp)$$

Origin of all B-dep.

2) QED case (1/q² force)



Comparison of forces, 2

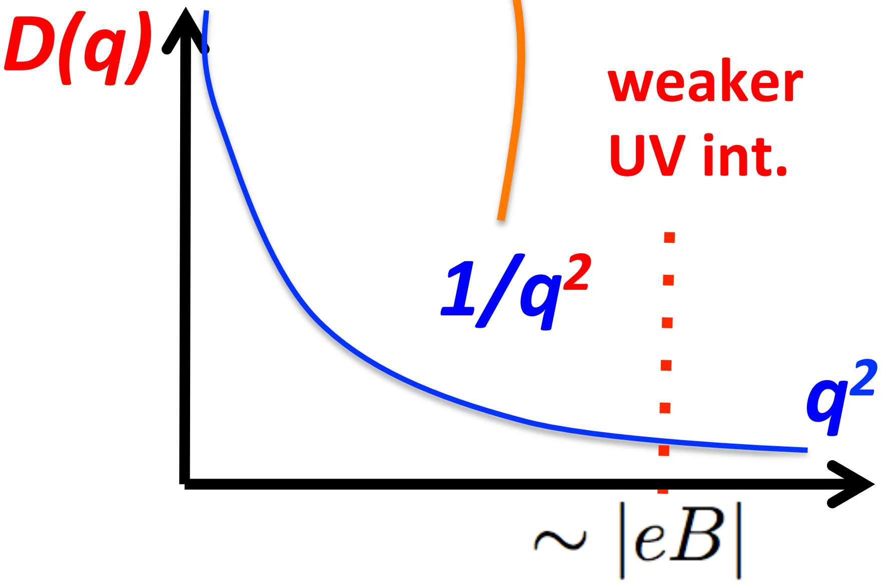
$$\int_{q_\perp} \underline{\mathrm{e}^{-\frac{q_\perp^2}{2|eB|}}}\, D^{\mathrm{4D}}_{\mathrm{NP}}(q_L, q_\perp)$$

Origin of all B-dep.

2) QED case (1/q^2 force)

$$\boxed{\sim \ln \frac{q_L^2}{\underline{|eB|}}}$$

weaker B-dependence



Comparison of forces, 2

$$\int_{q_\perp} \underline{\mathrm{e}^{-\frac{q_\perp^2}{2|eB|}}}\, D^{4\mathrm{D}}_{\mathrm{NP}}(q_L, q_\perp)$$

Origin of all B-dep.

---

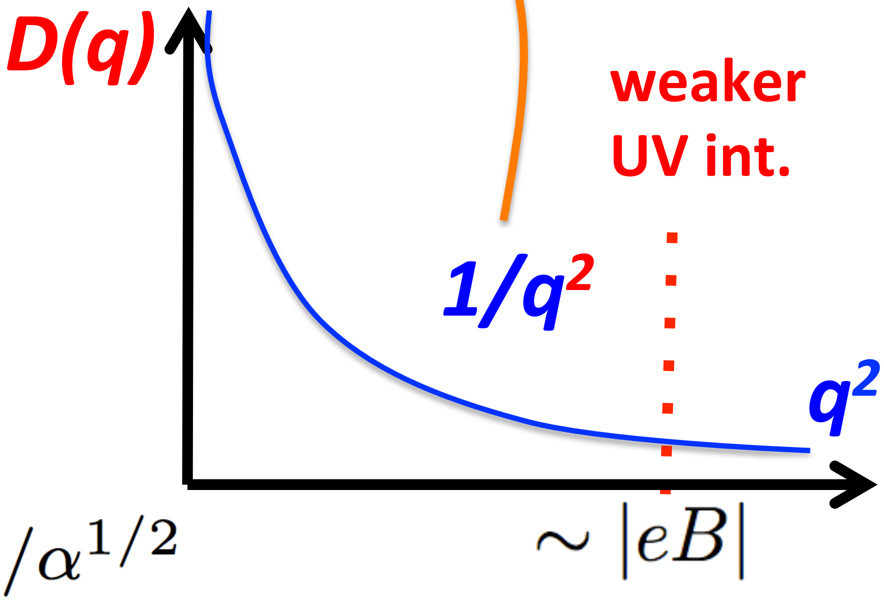
2) **QED** case (1/q^2 force)

$$\boxed{\sim \ln \frac{q_L^2}{\underline{|eB|}}}$$

weaker B-dependence

$$M \sim |eB|^{1/2}\, \underline{\mathrm{e}^{-O(1)/\alpha^{1/2}}}$$

(exponentially small)

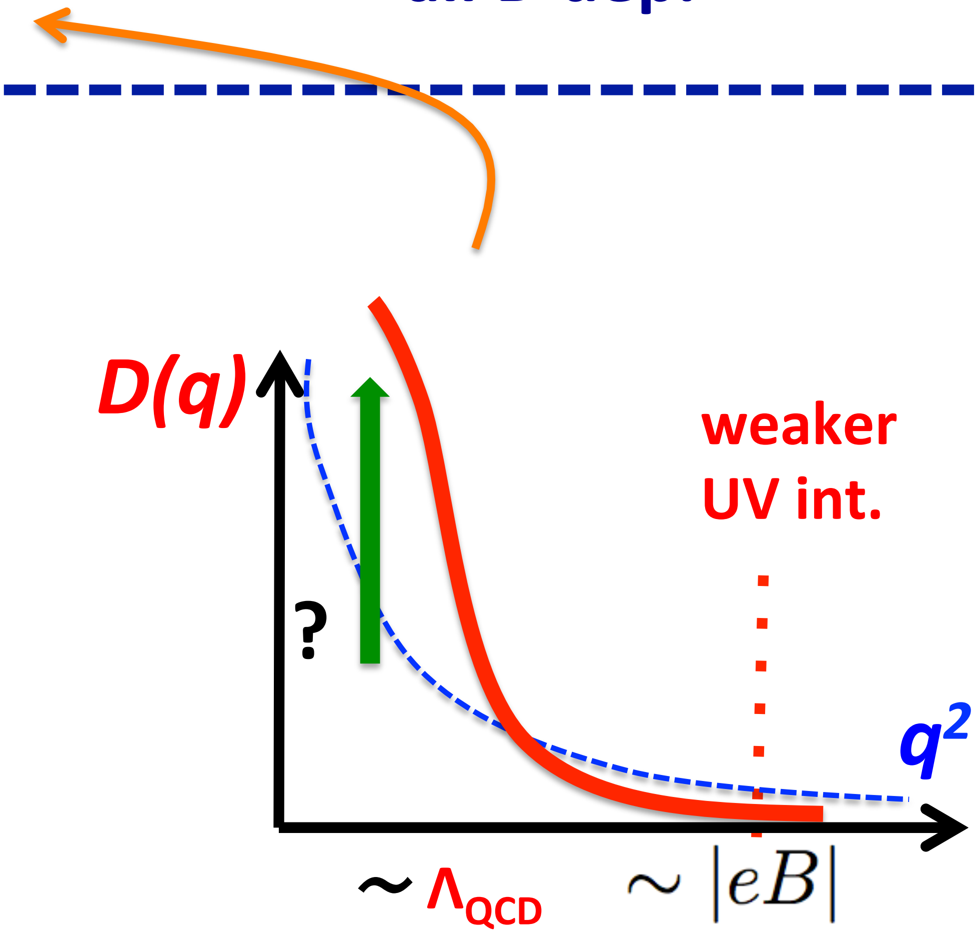


Comparison of forces, 3

$$\int_{q_\perp} \underline{\mathrm{e}^{-\frac{q_\perp^2}{2|eB|}}}\, D^{\mathrm{4D}}_{\mathrm{NP}}(q_L, q_\perp)$$

Origin of all B-dep.

3) QCD case (?)



Comparison of forces, 3

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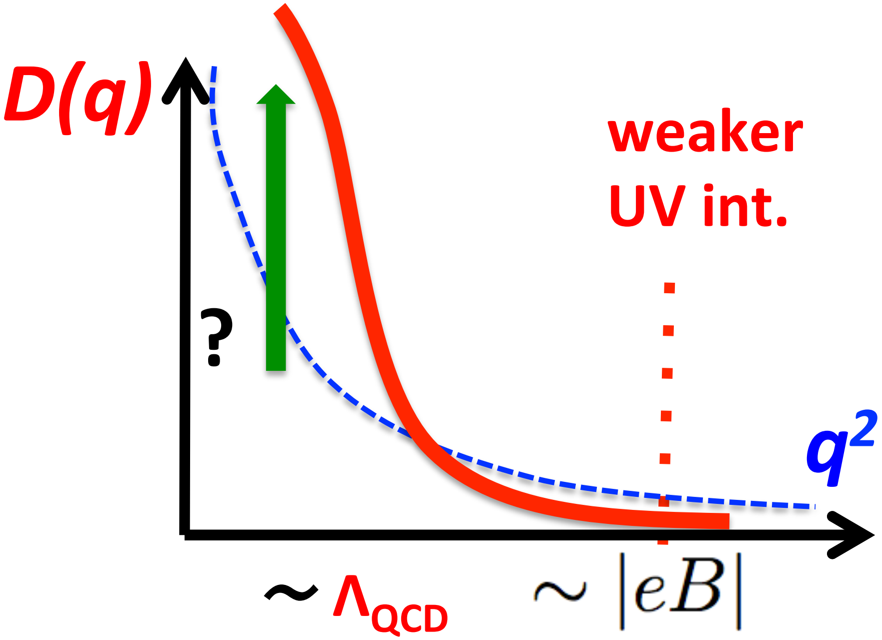
Origin of all B-dep.

3) QCD case (?)

***IR* int. >> *UV* int.**

→ *B-indep. forces*



Comparison of forces, 3

$$\int_{q_\perp} \underline{\mathrm{e}^{-\frac{q_\perp^2}{2|eB|}}}\, D^{\rm 4D}_{\rm NP}(q_L, q_\perp)$$

Origin of all B-dep.

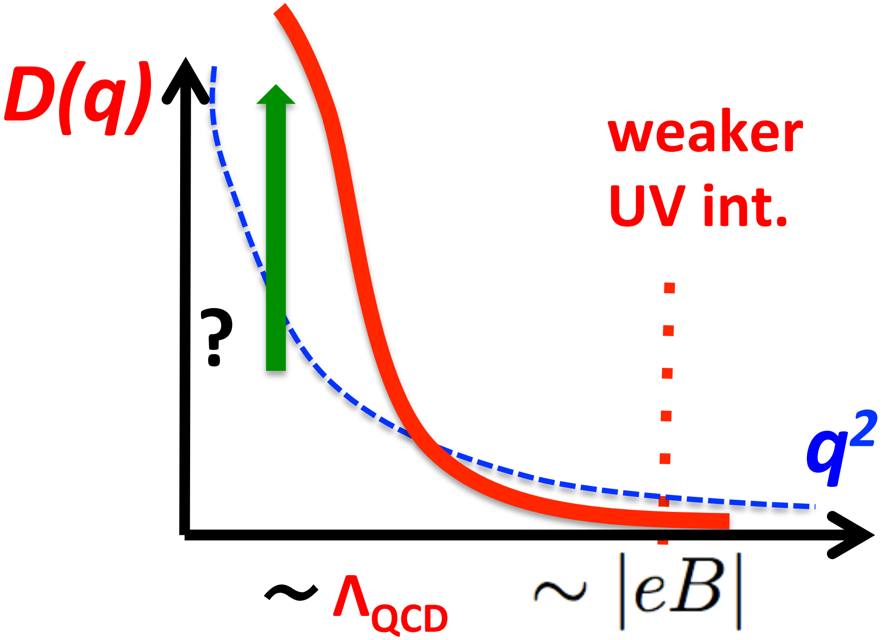
3) QCD case (?)

IR int. >> UV int.

→ B-indep. forces

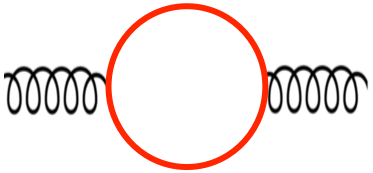
$$M \sim \Lambda_{\rm QCD}$$

"nearly B-indep."

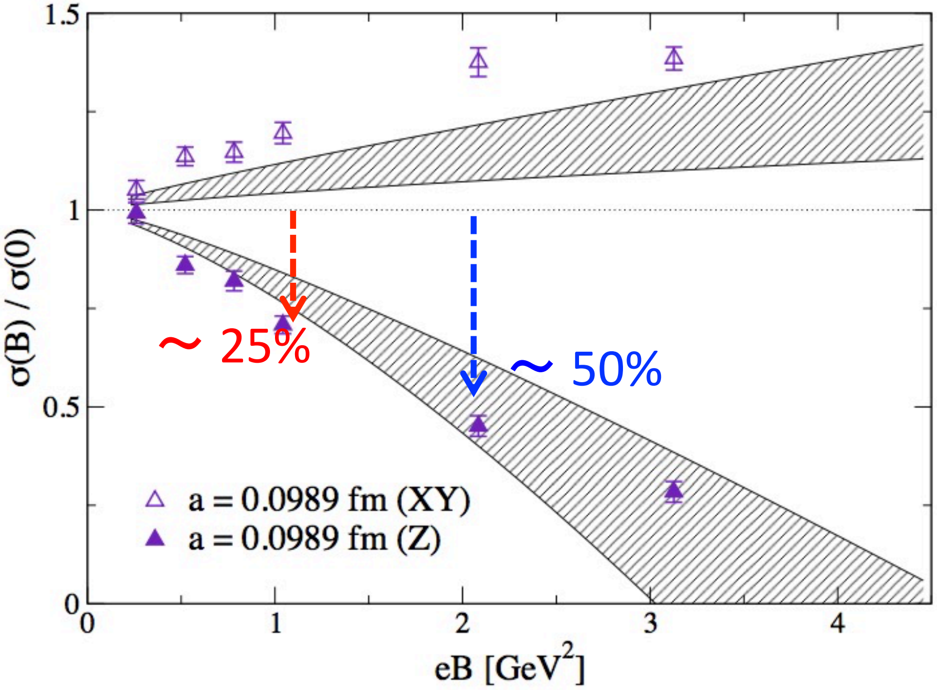


Enhanced screening

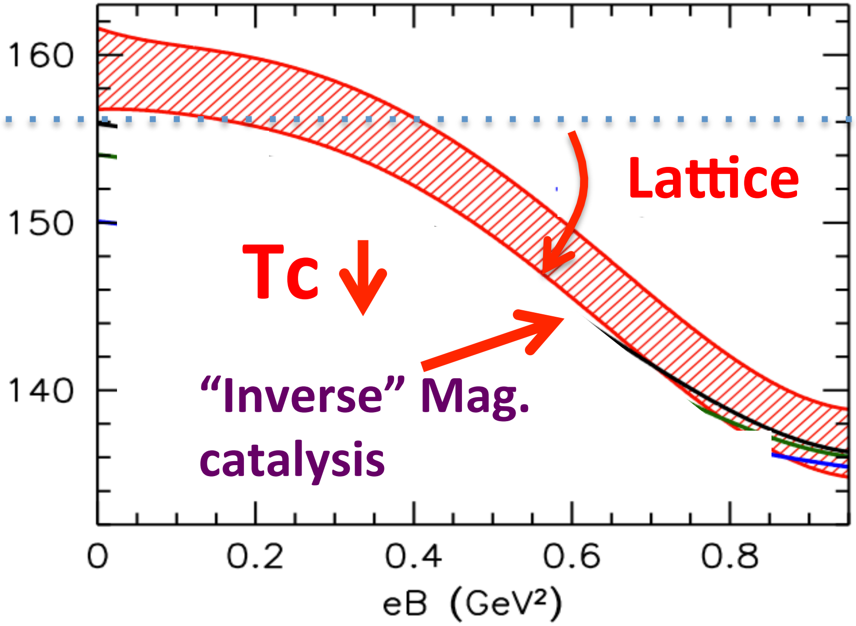
$M \sim$ fixed & $B \rightarrow$ large



string tension $\sigma(B)$ [bonati et al16]



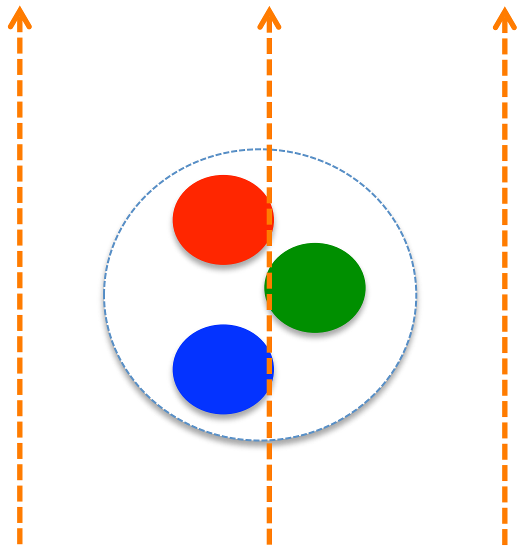
inverse mag. catalysis



this picture seems consistent with the lattice data

4, Mesons and HRG
in strong magnetic fields

Hadrons at ***weak*** *B*

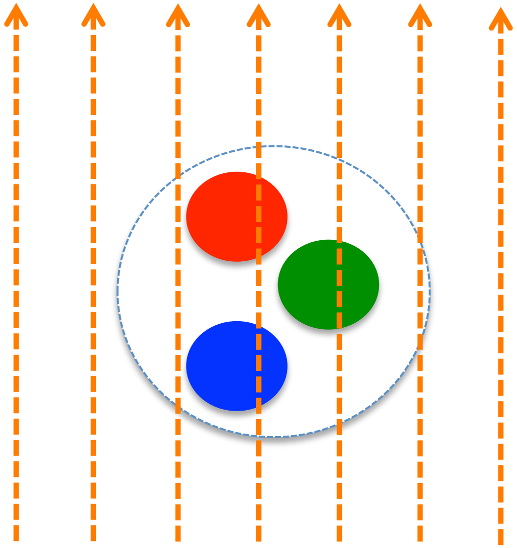


Dilute magnetic flux lines

B observes only
total spin & charge of hadrons

e.g.) **neutral** hadrons decouple from B

Hadrons at strong B



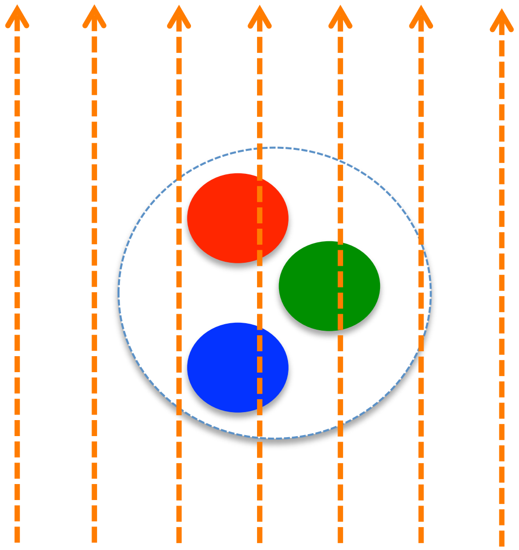
Dense magnetic flux lines

B observes
quarks inside of hadrons

→ structural changes in hadrons

[Fukushima-Hidaka, Simonov, Mao, Taya...]

Hadrons at strong B



Dense magnetic flux lines

B observes
quarks inside of hadrons

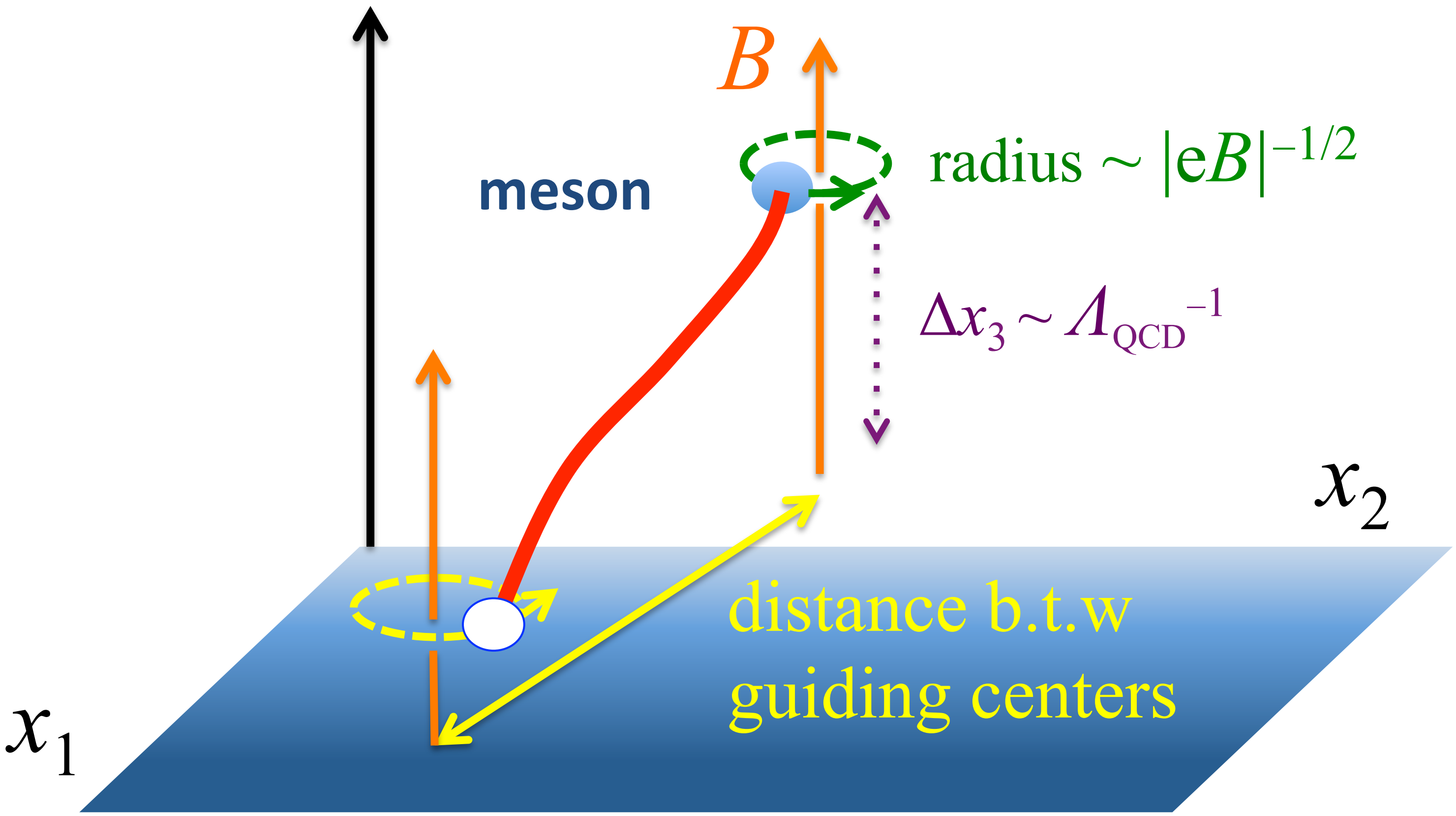
→ structural changes in hadrons

[Fukushima-Hidaka, Simonov, Mao, Taya...]

As an illustration, we study the Bethe-Salpeter eq.

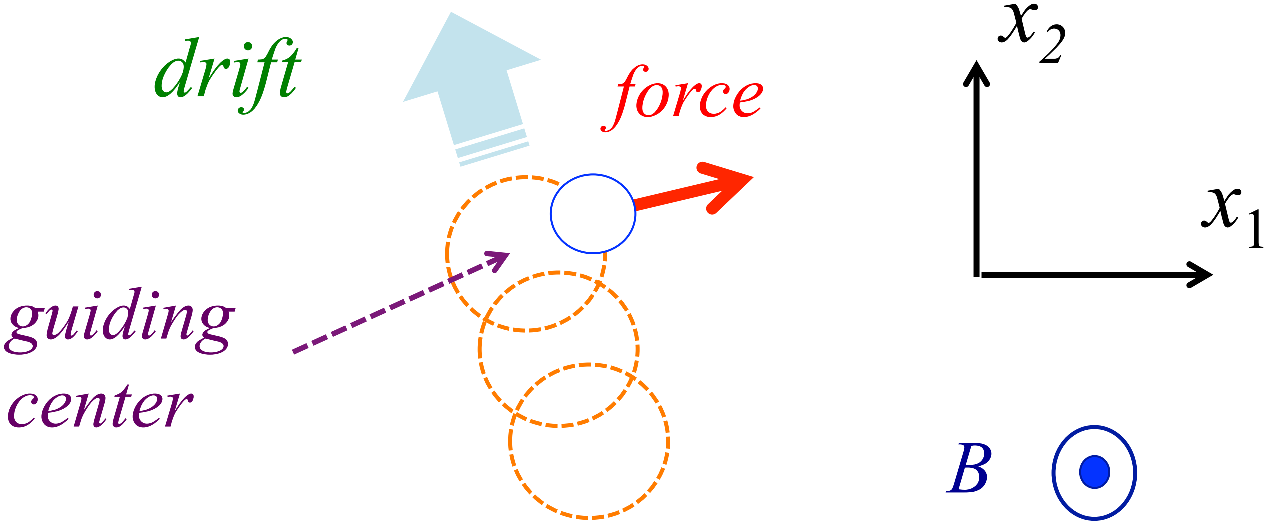
(within the LLL approximation)

Bound state problems



To characterize the bound state,
first we look for the *constants of motion*

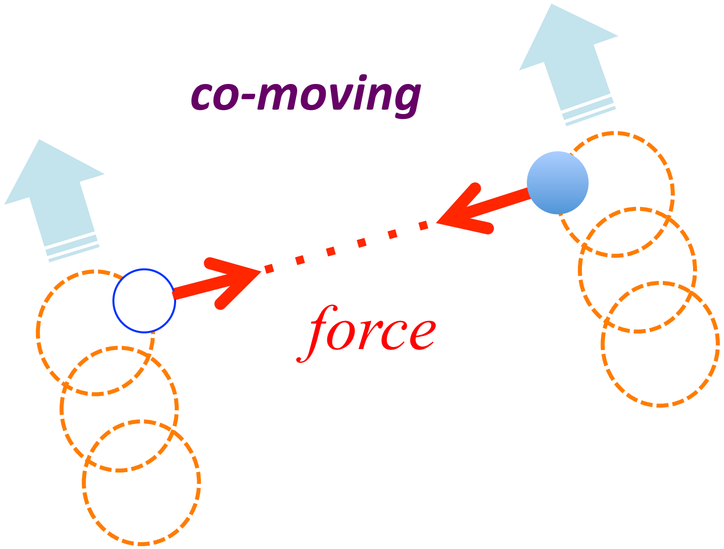
Hall drift



2-*body* problems : *Hall drift*

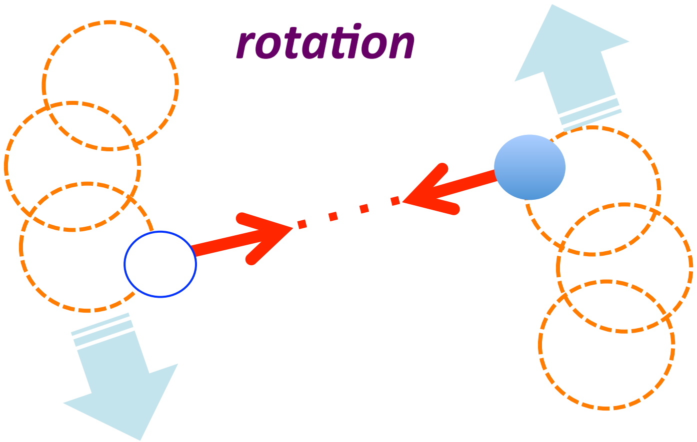
$$Q = - Q'$$

(e.g. neutral mesons)



$$Q = Q'$$

(e.g. di-electrons)



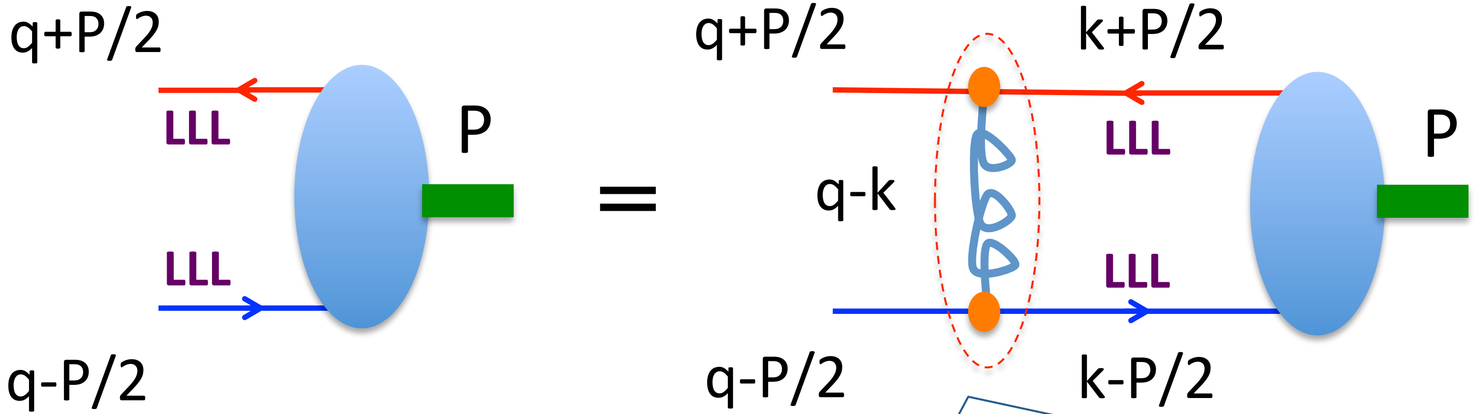
conserved quantities

total momenta (P_x P_y)

angular momentum (L)

(location of center - redundant)

BS-eq. for *neutral* mesons



$$\mathcal{V}^B_{2D}(q_3 - k_3; \underline{\vec{P}_\perp}) = \int_{\vec{k}_\perp} e^{i\Pi(\vec{q}_\perp - \vec{k}_\perp;\vec{P}_\perp)} \underbrace{e^{-\frac{(\vec{q}_\perp - \vec{k}_\perp)^2}{2|eB|}}}_{\text{form factor}} V_{4D}(\vec{q} - \vec{k})$$

“ Schwinger phase “

For neutral mesons : [heavy Q; Strickland et al. (13) & Bonati et al.(15)]

$$\Pi(\vec{q}_\perp - \vec{k}_\perp;\vec{P}_\perp) = \frac{\vec{B}_f \times \vec{P}_\perp}{B_f^2} \cdot (\vec{q} - \vec{k})_\perp$$

$\sim O(P_{perp}/B)$

negiligible for small P

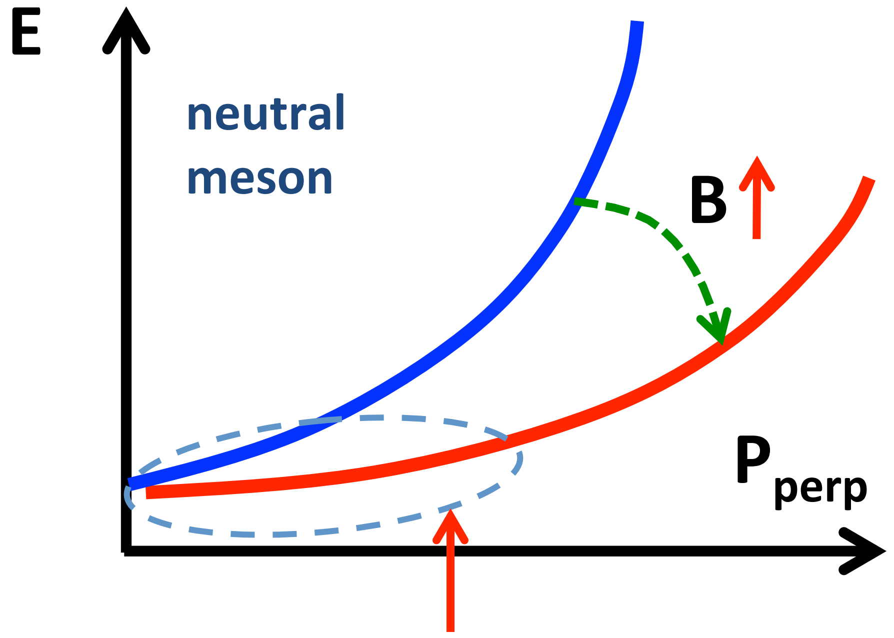
Spectrum : results of *long-range* forces

$$E^{ff'}_{n_3,\vec{P}_\perp}(P_3) \simeq \sqrt{\underline{\left(M^{\text{neutral}}_{n_3}\right)^2} + P_3^2} + c_1 \Lambda^3_{\text{QCD}} \underline{\frac{P_\perp^2}{|B|^2}} + \cdots$$

nearly B-indep.

P_{perp}-correction

[see also Fukushima-Hidaka, ...]



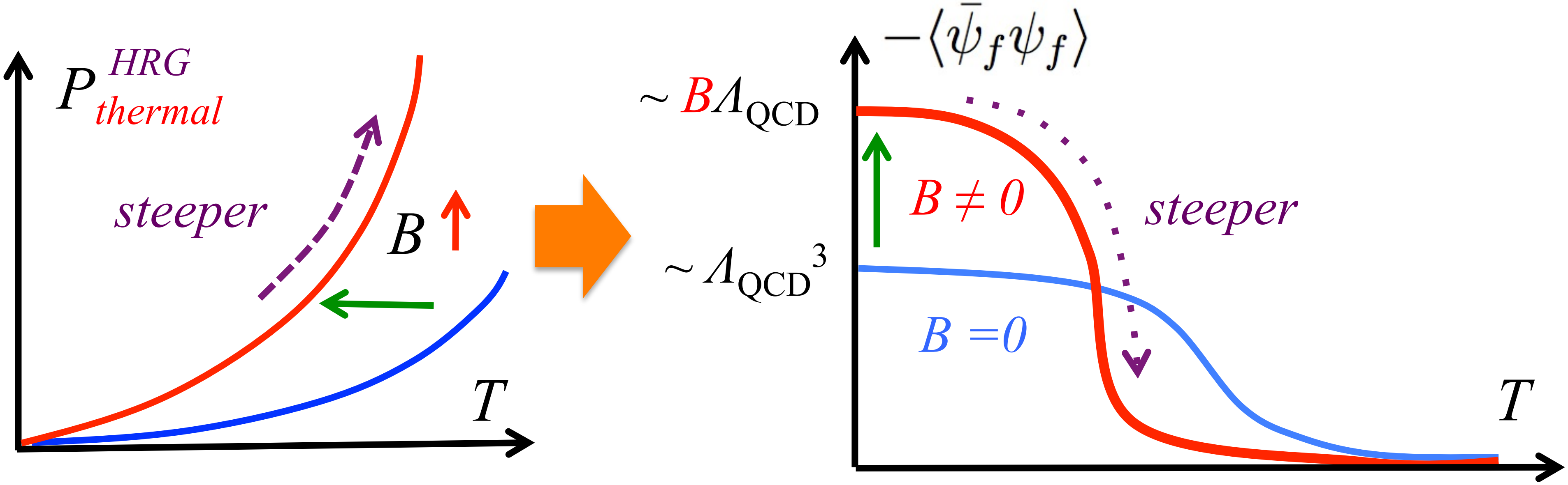
More states at low energy

Percolation & chiral restoration

$$-\langle\bar{\psi}_f\psi_f\rangle_T = \frac{\partial P_{\mathrm{vac}}}{\partial m_f} + \frac{\partial P_{\mathrm{excited}}}{\partial m_f}$$

tend to cancel

$$\simeq -\langle\bar{\psi}_f\psi_f\rangle_{T=0} - \sum_n \sum_P \frac{\partial E_n(P)}{\partial m_f} \frac{1}{\mathrm{e}^{E_n(P)/T} - 1}$$



earlier percolation ~ earlier chiral restoration

(HRG description of *inverse mag. catalysis*)

Summary

- QCD in *strong* magnetic fields

  → A new regime to study *low E* QCD

- The quark mass gap, chiral condensates, meson spectra

  → highly depends on the properties of interactions,
  especially on the *range of interactions*

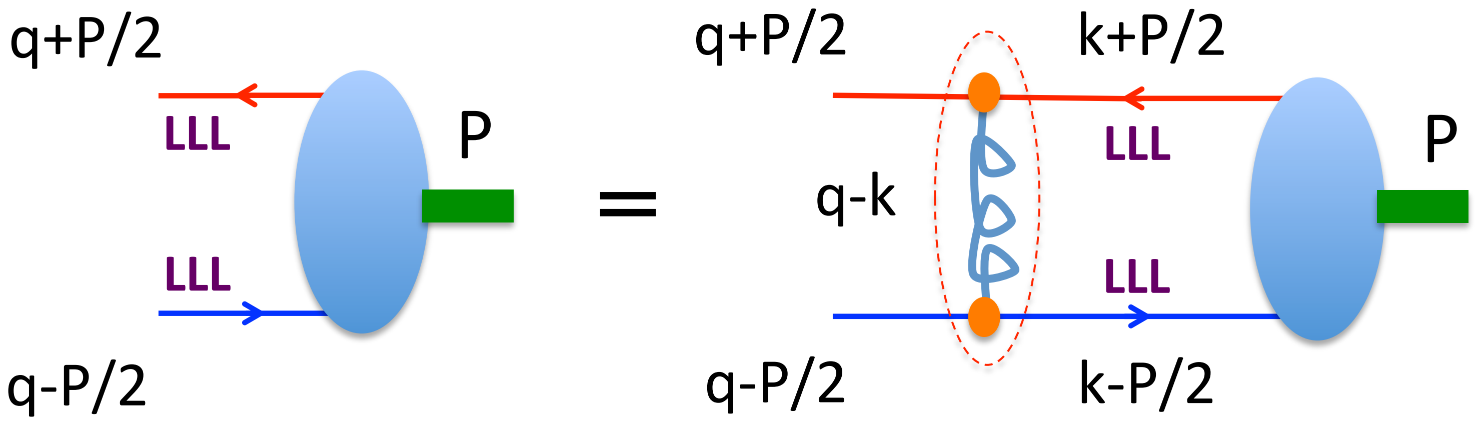
- The *long-range* interactions allow
  stronger fluctuations in quark and meson sectors

  → help to understand the inverse magnetic catalysis

- Outlook : more detailed *quantitative* studies

Bethe-Salpeter equations for LLLs

BS-eqs can be dimensionally reduced from 4D to 2D



2D effective interaction

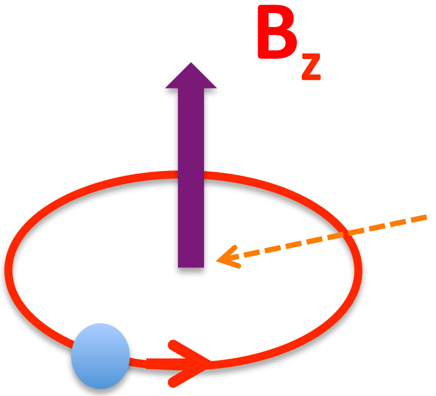
$$D_{\rm 2D}(q_L - k_L) = \int_{\vec{k}_\perp} D_{\rm 4D}(q-k)\, \underbrace{\mathcal{F}_{ff'}(\vec{q}_\perp - \vec{k}_\perp)}_{\text{form factor}}\, \underbrace{{\rm e}^{{\rm i}\left[\Xi^{f}_{q_+,k_+} - \Xi^{f'}_{q_-,k_-}\right]}}_{\text{" Schwinger phase "}}$$

Again, B-dep. arises only from 2D effective interaction

Spatial wavefunctions

Necessary ingredients for bound state problems :

Guiding center coordinate



$$\begin{cases} X = x + \dfrac{\Pi_y}{QB} \\ Y = y - \dfrac{\Pi_x}{QB} \end{cases}$$

$$\Pi_j = p_j - QA_j$$

(conserved in free theory)

X and Y commute with free Hamiltonian, but:

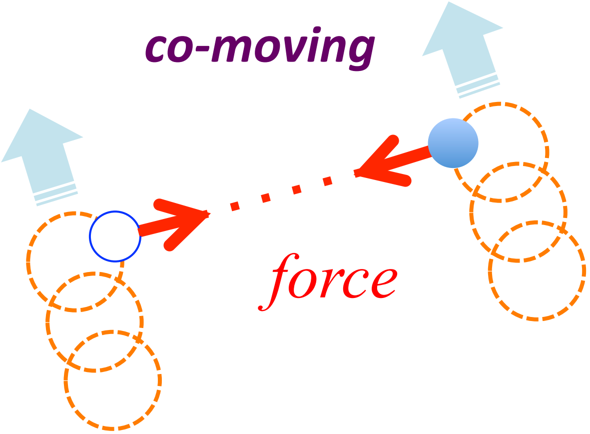
$$[X, Y] = -\frac{\mathrm{i}}{QB}$$

uncertainty relation

→ eigenstates can be labeled only by either X or Y

(e.g. in Landau gauge, $X = p_y/QB$ → p_y is conserved)

Neutral mesons



Neutral states are special :

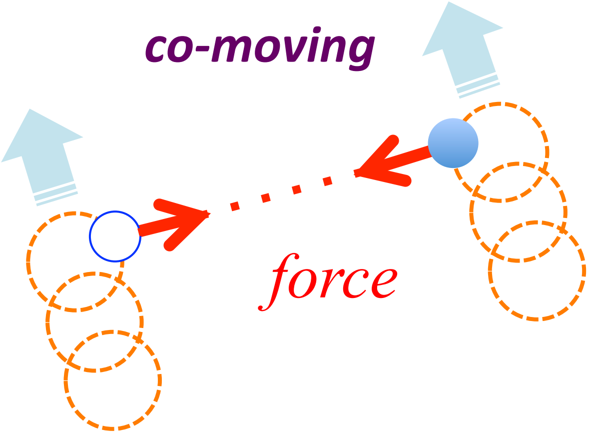
$$[X - X', Y - Y'] = -\frac{\mathrm{i}}{B}\left(\frac{1}{Q} + \frac{1}{Q'}\right)$$ **= 0 !**

→ **one can label eigenstates by 2-continuous parameters :**

X-X' & Y-Y' (conserved)

quark & antiquark co-move with ***fixed guiding center separation***

Neutral mesons



Neutral states are special :

$$[X - X', Y - Y'] = -\frac{i}{B}\left(\frac{1}{Q} + \frac{1}{Q'}\right)$$ **= 0 !**

→ one can label eigenstates by **2**-continuous parameters :

X-X’ & Y-Y’ (conserved)

quark & antiquark co-move with ***fixed guiding center separation***

Specifically, in Landau gauge ;

$$X - X' = \frac{p_y + p'_y}{QB} = \frac{P_y}{QB} \qquad Y - Y' = -\frac{p_x + p'_x}{QB} = -\frac{P_x}{QB}$$

guiding center separation → O(P_{perp} /B)

Structure of 2D interaction

coordinate space

guiding center separation

2D

4D

$$\mathcal{W}^f_{LL'}(r_L;P_\perp)=C_F\,\frac{|B_f|}{2\pi}\int_{\vec{r}_\perp} D_{LL'}(r_L,\vec{r}_\perp)\,\mathrm{e}^{-\frac{|B_f|}{2}\left(\vec{r}_\perp-\underline{\vec{\xi}^{\,f}_P}\right)^2}$$

|X-X'| ~ P/B

Structure of 2D interaction

coordinate space

guiding center separation

2D

4D

$$\mathcal{W}^f_{LL'}(r_L;P_\perp) = C_F\,\frac{|B_f|}{2\pi}\int_{\vec{r}_\perp} D_{LL'}(r_L,\vec{r}_\perp)\,\mathrm{e}^{-\frac{|B_f|}{2}(\vec{r}_\perp\underline{-\vec{\xi}^{\,f}_P})^2}$$

|X-X'| ∼ P/B

Contact int :

$$D_{\text{contact}}(\vec{r}) \sim -\Lambda_{\text{QCD}}^{-2}\,\delta(\vec{r})$$

$$\rightarrow\quad \mathcal{W}_{\text{contact}}(r_3;P_\perp) \simeq -\underline{\frac{|B_f|}{\Lambda_{\text{QCD}}^2}}\,\underline{\mathrm{e}^{-\frac{P_\perp^2}{2|B_f|}}}\,\delta(r_3)$$

very strong at large B

$P_{perp} > B^{1/2}$ decouple

Structure of 2D interaction

coordinate space

guiding center separation

2D

4D

$$\mathcal{W}^f_{LL'}(r_L;P_\perp) = C_F \frac{|B_f|}{2\pi} \int_{\vec r_\perp} D_{LL'}(r_L,\vec r_\perp)\, \mathrm{e}^{-\frac{|B_f|}{2}(\vec r_\perp - \underline{\vec\xi^f_P})^2}$$

|X-X'| ~ P/B

Contact int :

$$D_{\mathrm{contact}}(\vec r) \sim -\Lambda^{-2}_{\mathrm{QCD}}\, \delta(\vec r)$$

$$\rightarrow \quad \mathcal{W}_{\mathrm{contact}}(r_3;P_\perp) \simeq -\frac{|B_f|}{\underline{\Lambda^2_{\mathrm{QCD}}}}\, \underline{\mathrm{e}^{-\frac{P_\perp^2}{2|B_f|}}}\, \delta(r_3)$$

very strong at large B

$P_{perp} > B^{1/2}$ decouple

Linear potential :

$$D^{\mathrm{linear}}(\vec r) = \frac{\sigma}{2\pi C_F}\sqrt{r_3^2 + r_\perp^2}$$

$$\rightarrow \quad \mathcal{W}^f_{00}(r_L;P_\perp) \sim \sigma\sqrt{r_3^2 + \underline{(\vec\xi^f_P)^2}}$$

B-dep. drops off

until $P_{perp} \sim B/\Lambda_{QCD}$

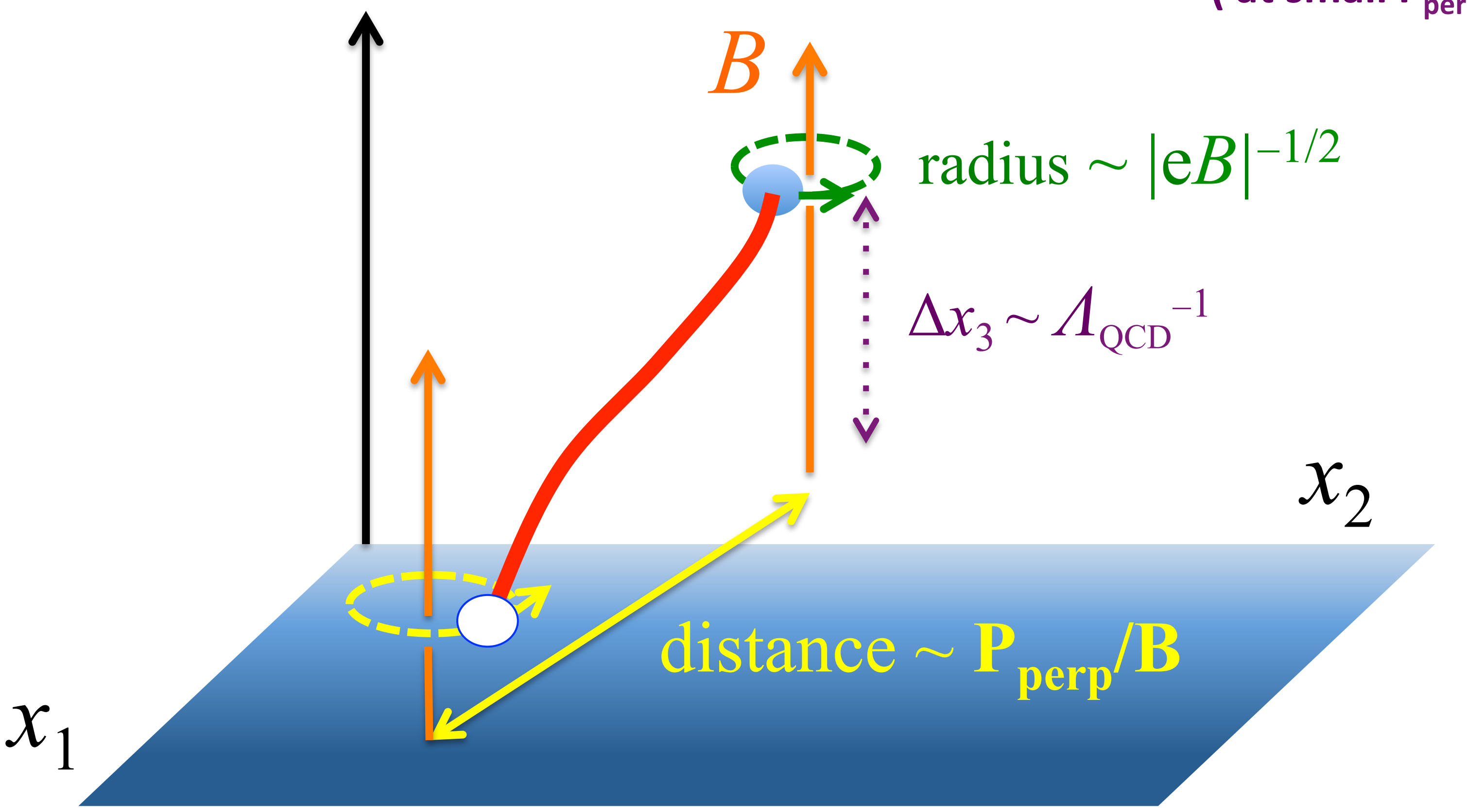
Spectrum : results of *long-range* forces

$$E^{ff'}_{n_3,\vec{P}_\perp}(P_3) \simeq \sqrt{\underline{\left(M^{\rm neutral}_{n_3}\right)^2} + P_3^2} + c_1\Lambda^3_{\rm QCD}\,\underline{\frac{P_\perp^2}{|B|^2}} + \cdots$$

nearly B-indep.

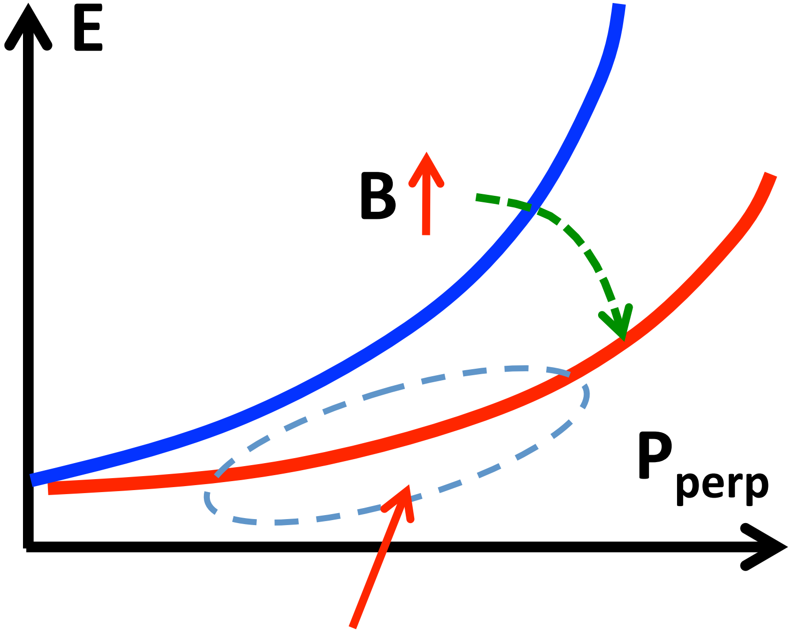
P_{perp}-correction

(at small P_{perp})



Spectrum : results of *long-range* forces

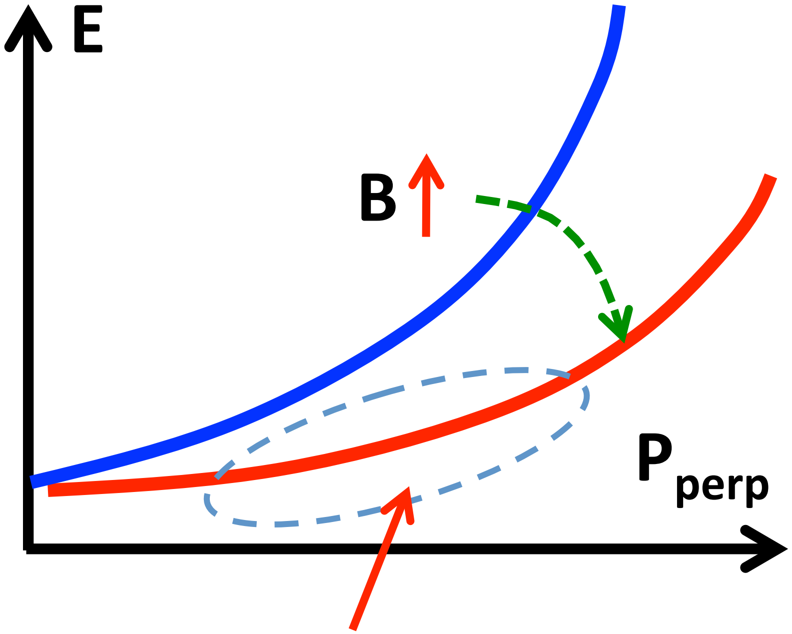
$$E^{ff'}_{n_3,\vec{P}_\perp}(P_3) \simeq \sqrt{\underbrace{\left(M^{\text{neutral}}_{n_3}\right)^2}_{\text{nearly B-indep.}} + P_3^2} + c_1 \Lambda^3_{\text{QCD}} \underbrace{\frac{P_\perp^2}{|B|^2}}_{P_{\text{perp}}\text{-correction}} + \cdots$$



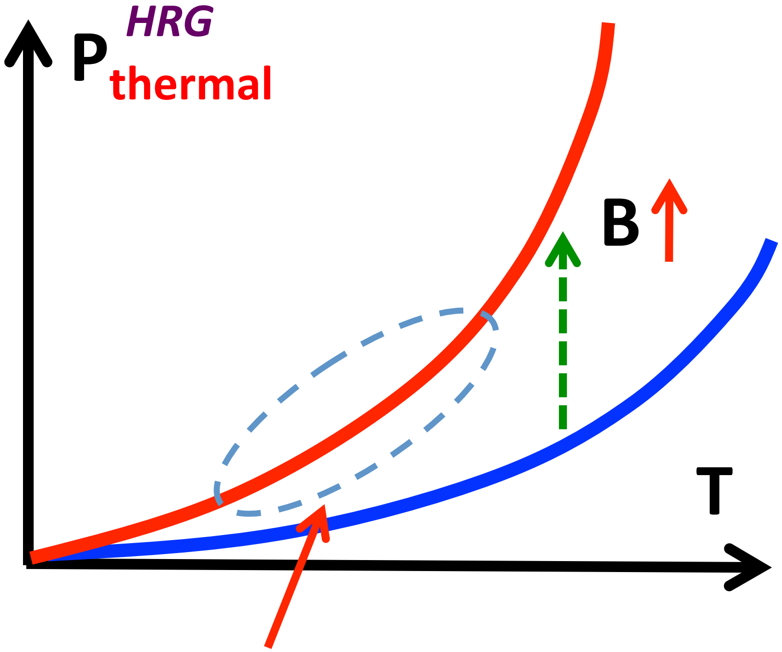
More states at low energy

Spectrum : results of *long-range* forces

$$E^{ff'}_{n_3,\vec{P}_\perp}(P_3) \simeq \sqrt{\underbrace{\left(M^{\mathrm{neutral}}_{n3}\right)^2}_{\text{nearly B-indep.}} + P_3^2} + c_1 \Lambda^3_{\mathrm{QCD}} \underbrace{\frac{P_\perp^2}{|B|^2}}_{P_{\mathrm{perp}}\text{-correction}} + \cdots$$



More states at low energy



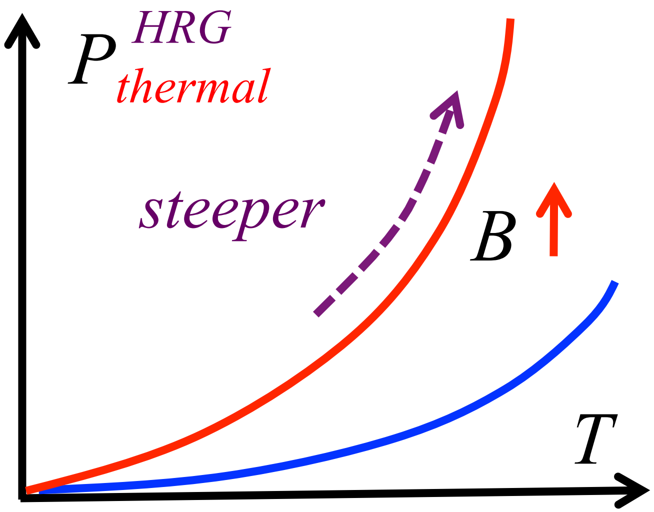
More pressure at given T

Percolation & chiral restoration

tend to cancel

$$-\langle\bar{\psi}_f\psi_f\rangle_T = \frac{\partial\mathcal{P}_{\rm vac}}{\partial m_f} + \frac{\partial\mathcal{P}_{\rm excited}}{\partial m_f}$$

$$\simeq -\langle\bar{\psi}_f\psi_f\rangle_{T=0} - \sum_n\sum_P \frac{\partial E_n(P)}{\partial m_f}\frac{1}{{\rm e}^{E_n(P)/T}-1}$$



Percolation & chiral restoration

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$$-\langle\bar{\psi}_f\psi_f\rangle_T = \frac{\partial P_{\rm vac}}{\partial m_f} + \frac{\partial P_{\rm excited}}{\partial m_f}$$

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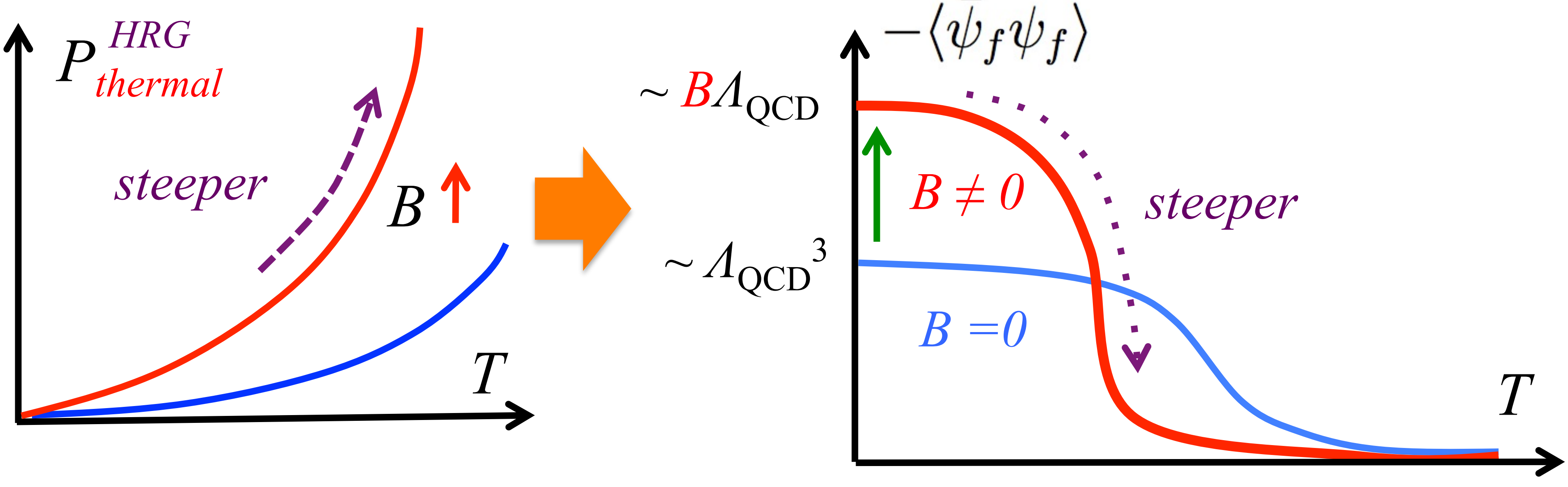

Percolation & chiral restoration

tend to cancel

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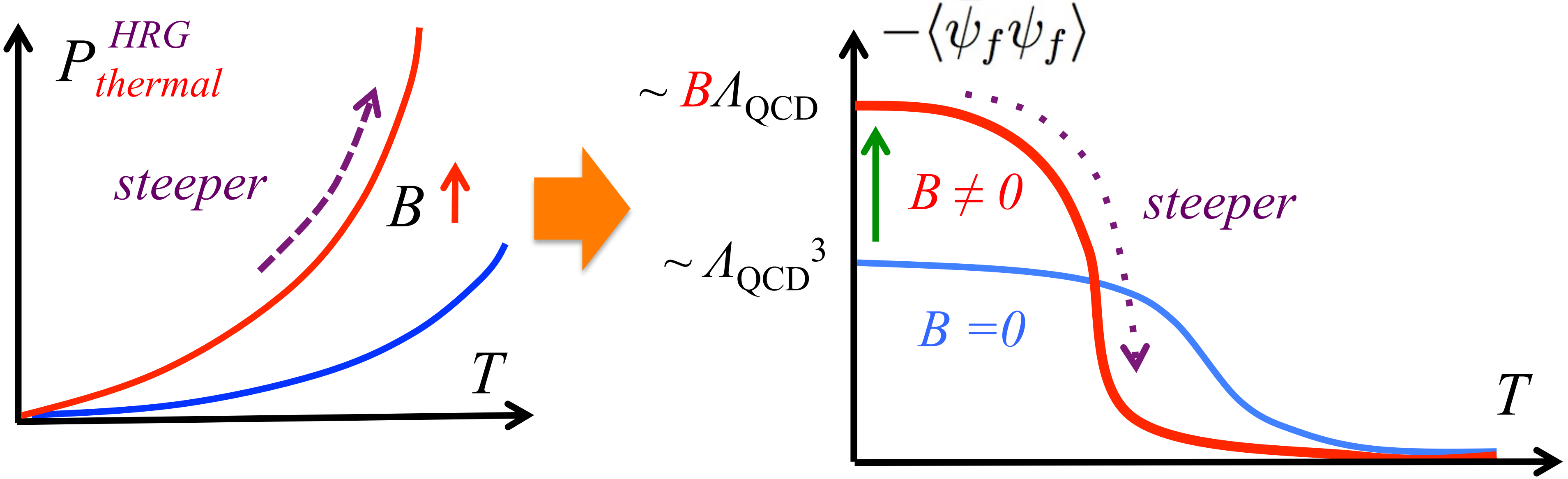


earlier percolation ~ earlier chiral restoration

(HRG description of inverse mag. catalysis)

Summary

- QCD in *strong* magnetic fields

  → A new regime to study *non-pert. aspects* of QCD

- The quark mass gap, chiral condensates, meson spectra

  → highly depends on the properties of interactions,
  especially on the *range of interactions*

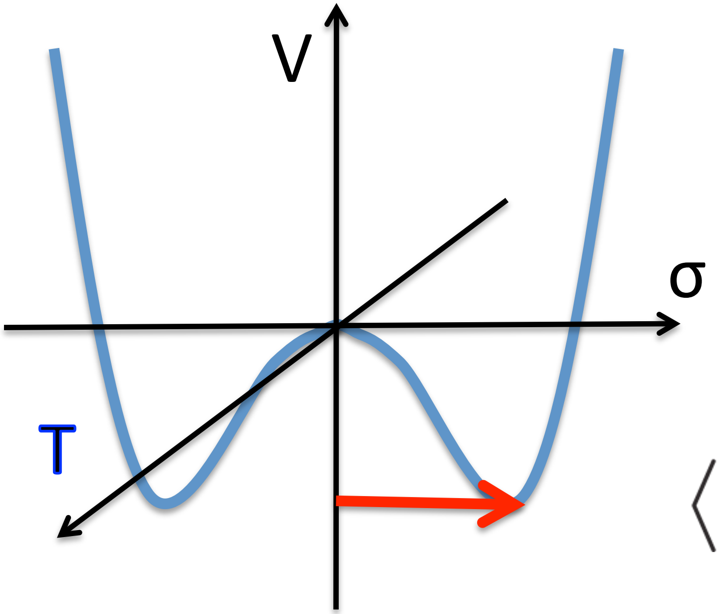
- The *long-range* interactions allow
  stronger fluctuations in quark and meson sectors

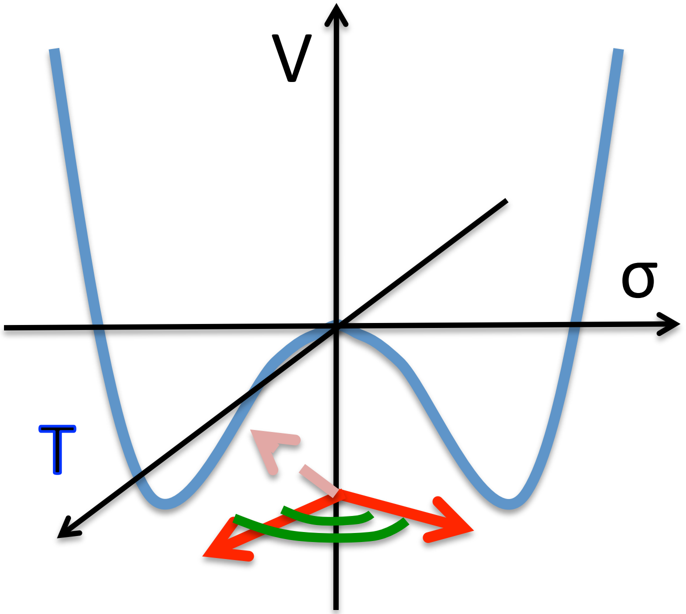
  → help to understand the inverse magnetic catalysis

- **Outlook** : more detailed *quantitative* studies

Backup

Phase fluctuations





$\langle \rho \rangle \neq 0$

$\langle e^{i\theta} \rangle \neq 0$ (SSB)

$\langle e^{i\theta} \rangle = 0$ (No SSB)

IR divergence in (1+1)D phase dynamics

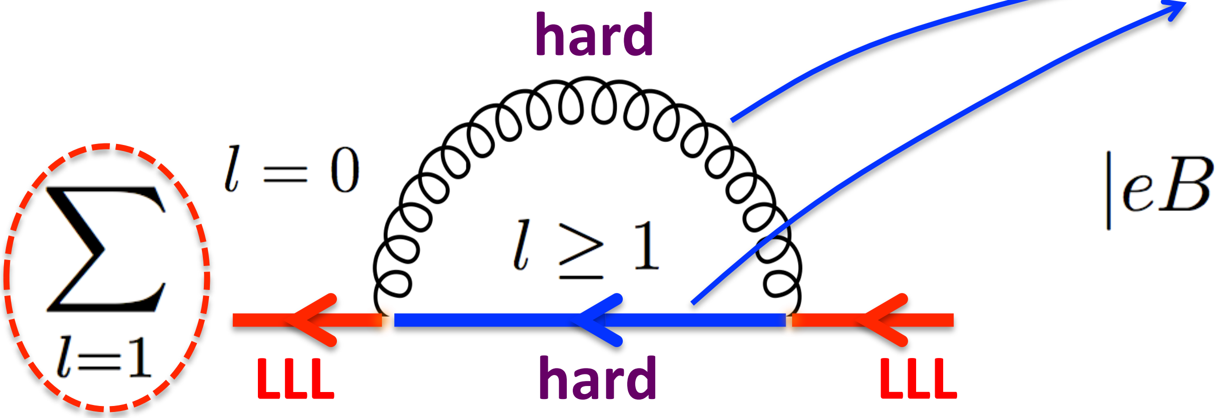
- Phase fluctuations belong to:

Excitations
(physical pion spectra)

ground state properties
(No pion spectra)

LLL mass gap : 3-distinct contributions

1) Coupling with $l \geq 1$



"Perturbative"

under control for

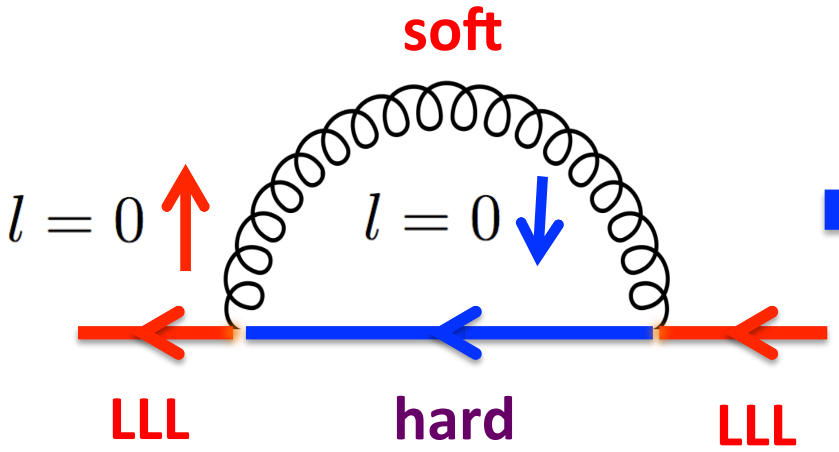
$$|eB| \geq (0.1-0.3)\,\mathrm{GeV}^2$$

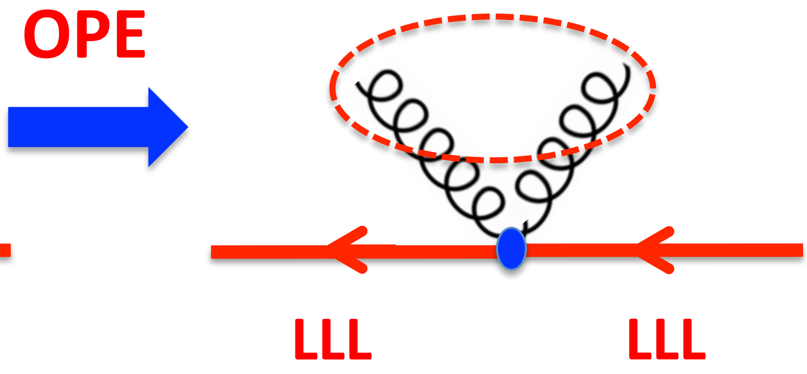
& very small B-dep.

T.K., Nan Su (2013)

2) Coupling with 1st LL but $l=0\downarrow$

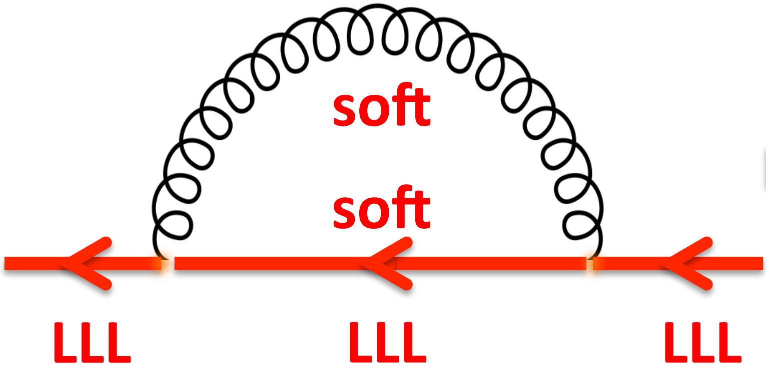
$$\sim \frac{m\langle G^2\rangle}{|eB|^2} \ll \Lambda_{\mathrm{QCD}}$$





LLL mass gap : 3-distinct contributions

3) Couplings within LLLs



→ Everything must be treated "Non-perturbatively"

Natural framework → Schwinger-Dyson eq.

with

Non-perturbative " *force* "

e.g.) full gluon propagator × full vertex for quenched QCD

Example)　a *toy* model study

" Linear rising " potential for color charges

$$D_{\mu\nu} = C_F \times g_{\mu 0} g_{\nu 0} \times \frac{\sigma}{(\vec{p}^2)^2}$$

string tension

- Motivated by Coulomb gauge studies.

(ref: Gribov, Zwanziger)

- The model has " *IR enhancement* ".
- *Confining*, in the sense that

"No $q\bar{q}$ continuum in the meson spectra."

- *Oversimplifications* : *No $1/p^2$ tail, No color mag. int.*, etc.
- *We will solve eqs. within " rainbow ladder "*

Schwinger-Dyson eq. for the LLL

e.g.) scalar part

$$M(p_L) = \int_{q_L} \gamma_0 \, S^{2\mathrm{D}}_{\mathrm{LLL}}(p_L - q_L; M)\, \gamma_0 \otimes \boxed{\int_{q_\perp} \mathrm{e}^{-\frac{q_\perp^2}{2|eB|}} \, D^{4\mathrm{D}}_{00}(q)}$$

for large B

$$\int_0^\infty \mathrm{d}q_\perp^2 \, \frac{\sigma \, \mathrm{e}^{-\frac{q_\perp^2}{2|eB|}}}{(q_\perp^2 + q_z^2)^2} \Longrightarrow \frac{\sigma}{q_z^2} - \cancel{\frac{\sigma}{q_z^2 + \underline{2|eB|}}}$$

(confining in 2D)

The *B-dependence* dropped out, and we get

$$M(p_L) \simeq \int_{q_L} \gamma_0 \, S^{2\mathrm{D}}_{\mathrm{LLL}}(p_L - q_L; M)\, \gamma_0 \times \frac{\sigma}{q_z^2}$$

SD-eq. for *'t Hooft model* (QCD$_2$) in A_z =0 gauge

(the *Bethe-Salpeter* eq. can be also reduced to QCD$_2$)

Bethe-Salpeter eq. for the LLLs

Consider ***meson currents*** for which

both quark & anti-quark can couple to the ***LLL states***.

(Some currents **CAN NOT**, see next slide.)

Dim. reduction can be carried out in the same way :

Both ***total*** *&* ***relative*** *momenta are* ***indep. of trans. momenta.***

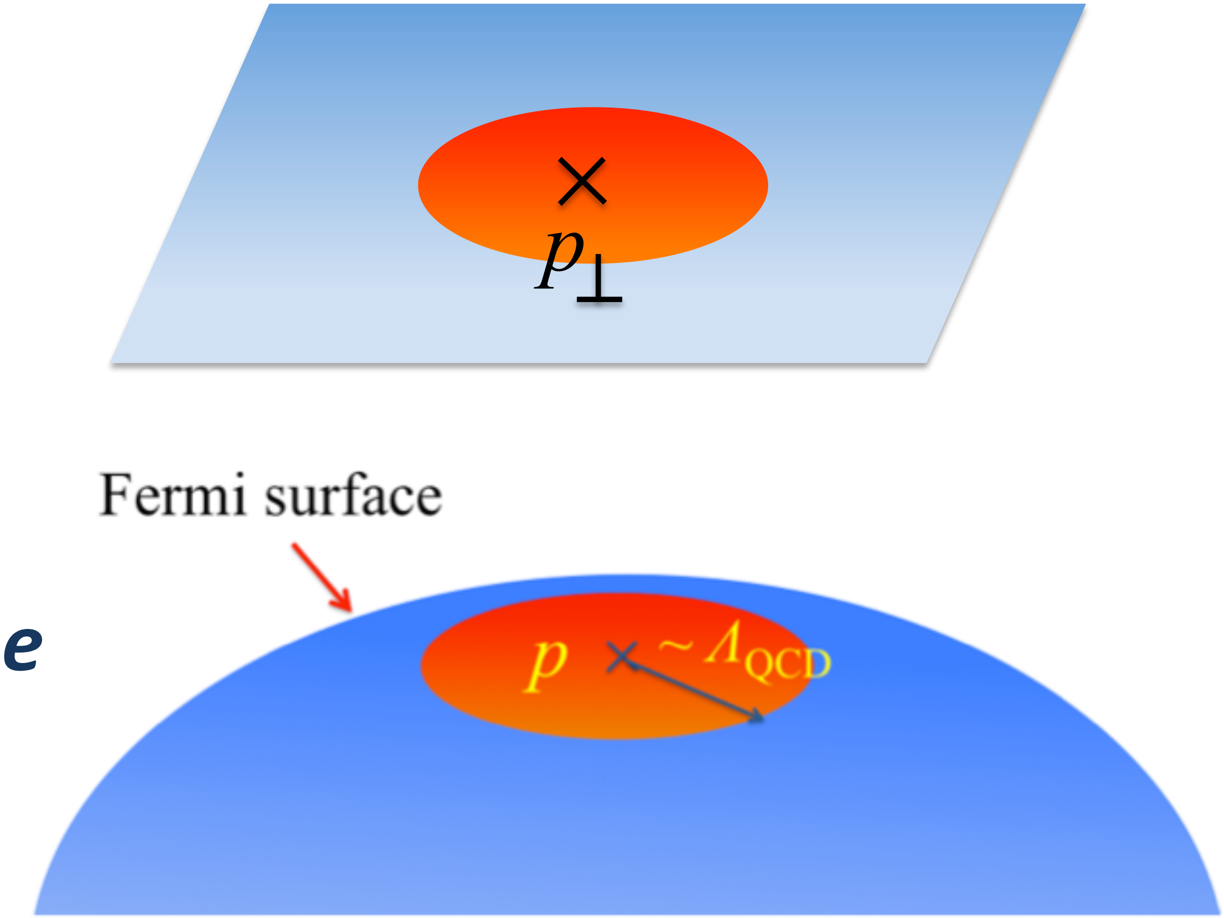
- *Quark & anti-quark* ***align*** *in the z-direction.*

Implications for dense QCD ?

Physics of the LLL

↕

Physics near the Fermi surface



Similar modulo Fermi surface curvature

What's new? : History

1) *ChSB in mag. fields (concept) : 1989 -*

Klevansky-Lemmer (89), Suganuma-Tatsumi (90),

Gusynin-Miransky-Shovkovy (94-), ... **(for NJL, QED,...)**

(*Not specific to QCD*, "*universal* aspects" of fermions at B)

2) *QCD in mag. fields (paradigm shift) : 2007 -*

Kharzeev-McLerran-Warringa (07), Fukushima-Kharzeev-Warringa (08),..

(QCD topology & Its phenomenological applications)

3) *Lattice* studies on *ChSB* & *Deconf.* : *2008 -*

Buividovich et al. (2008) **(quenched)**

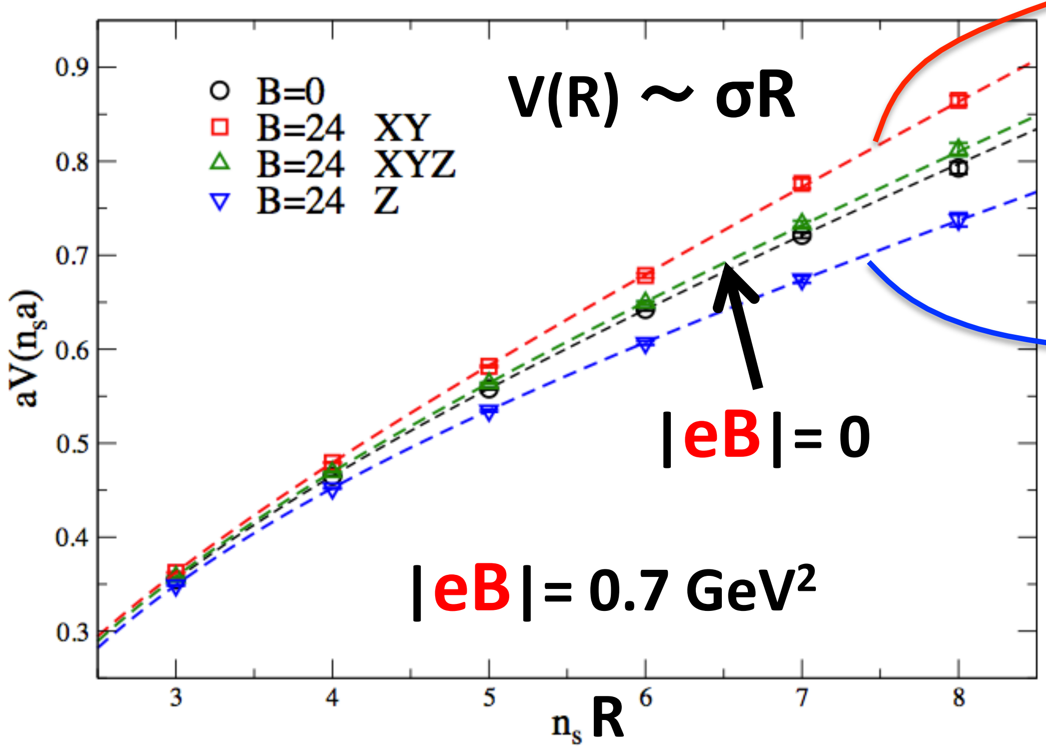
D'Elia-Muckherjee-Sanflippo (2010) **(full, heavy pion)**

Bali et al. (2012) **(full, physical pion)**

Problems 3: Lattice vs Models

Heavy quark potential (T=0)

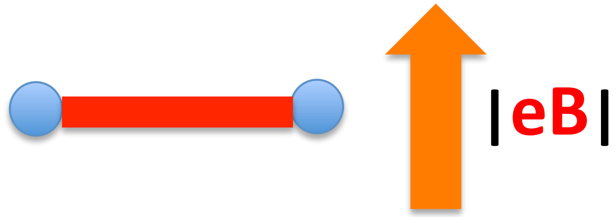
Lattice, (2+1) phys. pion (Bonati et al, 2014)



$V(R) \sim \sigma R$

$|eB| = 0$

$|eB| = 0.7\ \mathrm{GeV}^2$

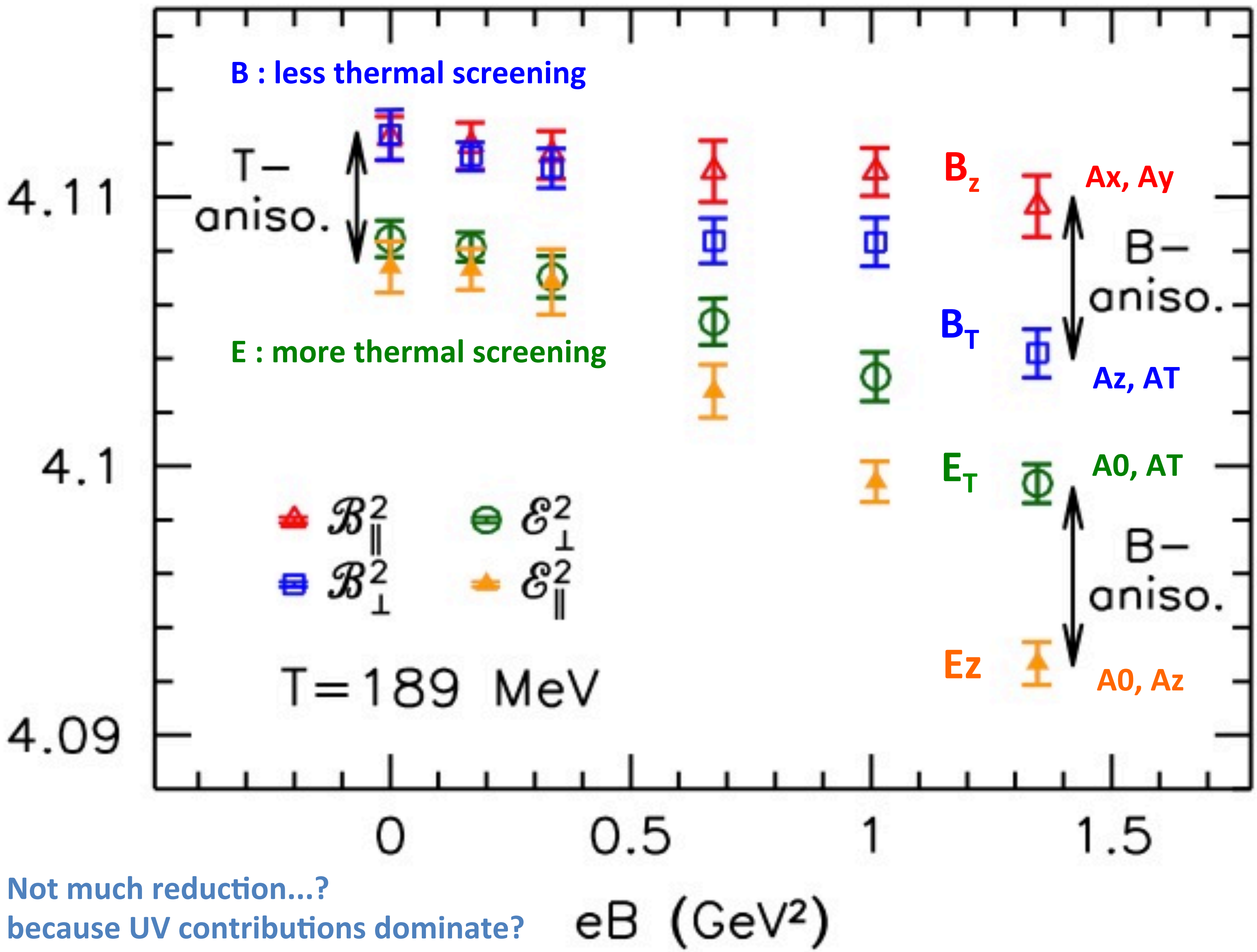


σ → 10 % enhancement

σ → 10 % “ reduction ”

hard to explain *if* $M \sim |eB|^{1/2}$ (models)

(because *back-reaction* is suppressed)



B : less thermal screening

E : more thermal screening

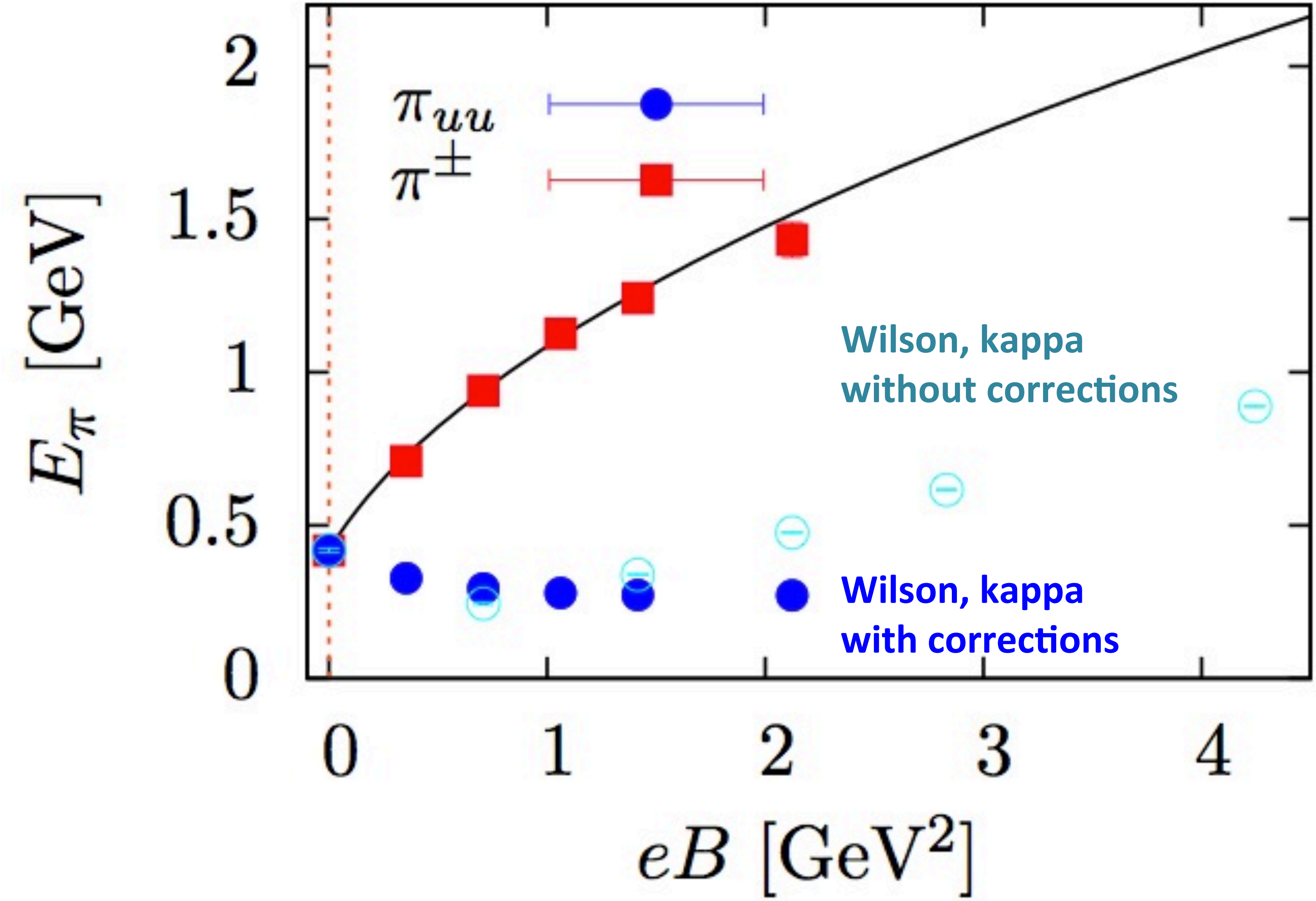
B_z Ax, Ay

B_T Az, AT

E_T A0, AT

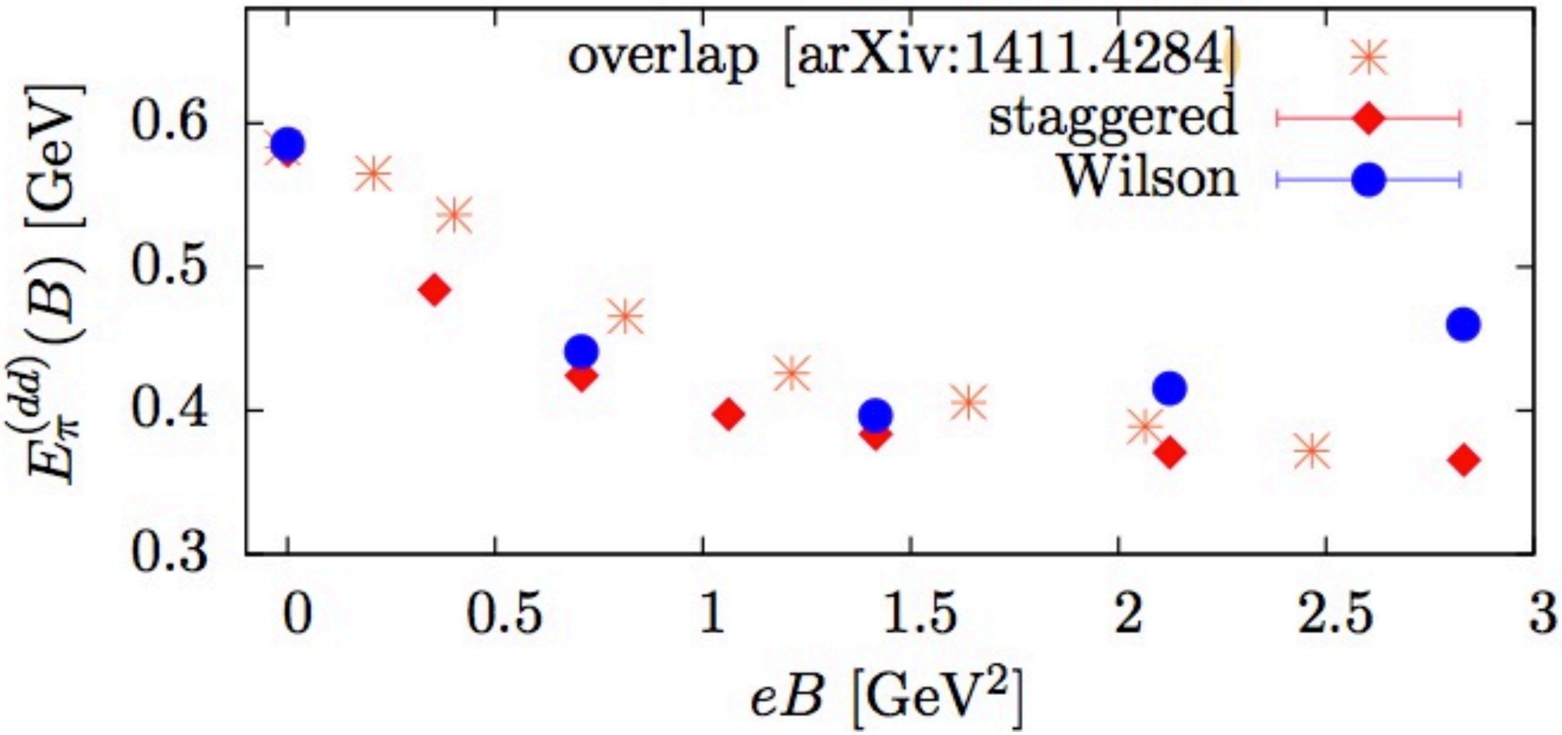
Ez A0, Az

Not much reduction...?
because UV contributions dominate?



Wilson, kappa without corrections

Wilson, kappa with corrections



mpi = 580 MeV at eB=0

Field theory bases : fermion part

" Ritus bases for non-int. fermions at finite B "

1) Choose the gauge for EM fields : e.g.) $A_2^{\mathrm{em}} = Bx_1$

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(σz : spin)

$$\psi_\pm \equiv \mathcal{P}_\pm \psi \qquad \mathcal{P}_\pm = \frac{1 \pm \mathrm{i}\gamma_1\gamma_2\,\mathrm{sgn}(e_f B)}{2}$$

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3) Expand by proper spatial wavefunctions :

$$\psi_\pm(x) = \sum_{l=0} \int \frac{\mathrm{d}^2 p_L \mathrm{d}p_2}{(2\pi)^3}\, \psi^\pm_{l,p_2}(p_L)\, \underline{H_l\Big(x_1 - \frac{p_2}{B}\Big)}\, \mathrm{e}^{-\mathrm{i}p_2 x_2}\, \mathrm{e}^{-\mathrm{i}p_L x_L}$$

$$p_L \equiv (p_0, p_z)$$

Harmonic oscillator w.f. with

$$m\omega = |eB|$$

Field theory bases : fermion part

The action for the **LLL (n=0)**:

$$\chi = \psi_+^{l=0}$$

$$S_{\mathrm{LLL}} = \int_{p_L, p_2} \bar{\chi}_{p_2}(p_L) \, \underline{(-\mathrm{i}\not{p}_L + m)} \, \chi_{p_2}(p_L)$$

(No B-dep. !)

for the **n**-th LLs :

$$\psi_n = \psi_+^{l=n} + \psi_-^{l=n-1}$$

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The propagators :

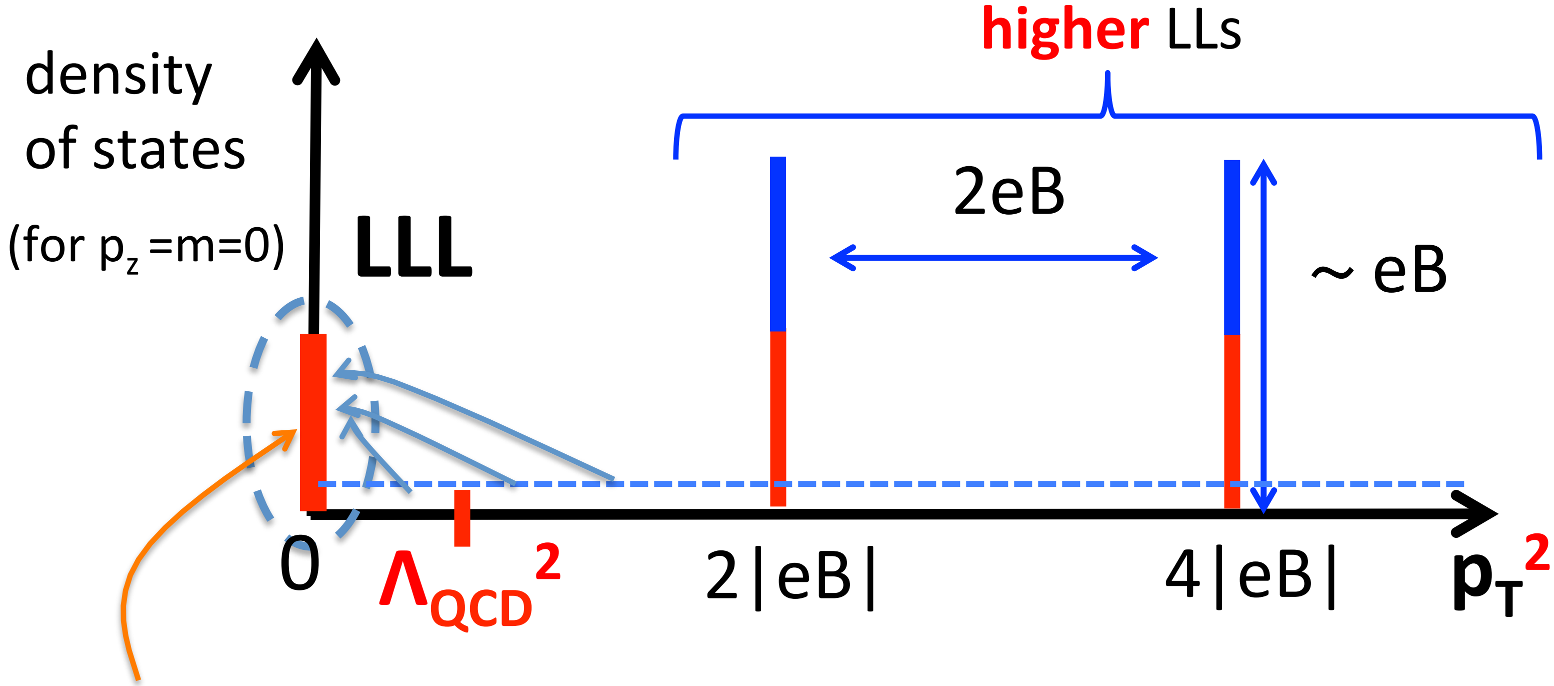
diagonal

$$\langle \psi_{n,p_2}(p_L) \bar{\psi}_{n',p_2'}(p_L') \rangle = \underline{S_n^{2\mathrm{D}}(p_L)} \times \delta_{nn'} \delta(p_2 - p_2') \delta^2(p_L - p_L')$$

(1+1)-dimensional for each index "n"

(*depend only on* p_L)

“*Enhanced*” IR phase space for quarks



Larger B → More quarks can stay at low energy.

- ***Enhanced* ChSB ~ *Magnetic Catalysis***
- ***Screened* gluon dynamics**

Important formula

"Ritus bases"

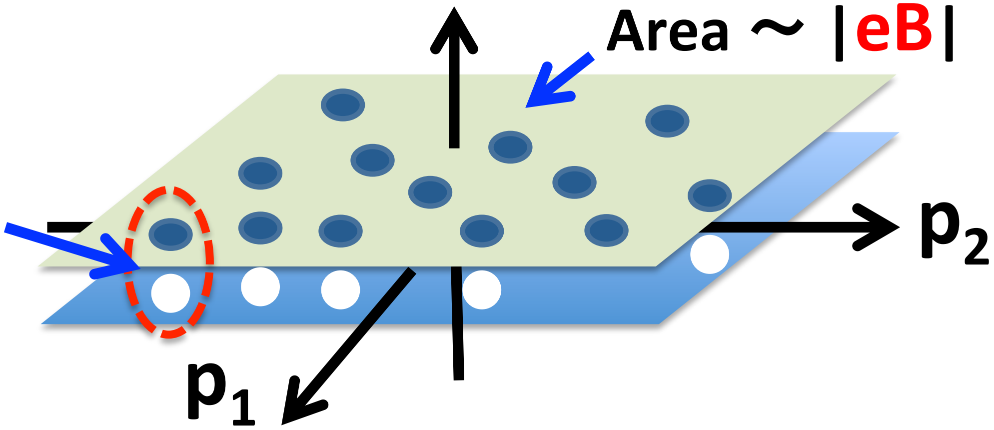
$$\langle \bar{\psi}(x)\psi(x)\rangle_{4\mathrm{D}} = \frac{|eB|}{\underline{2\pi}} \int_{p_L} (-1)\,\mathrm{tr}\left[S^{2\mathrm{D}}_{\mathrm{LLL}}(\underline{p_L}) + \sum_{n=1} S^{2\mathrm{D}}_{n\mathrm{LL}}(\underline{p_L}) \right]$$

$$\boxed{\langle \bar{\psi}(x)\psi(x)\rangle_{4\mathrm{D}} = \frac{|eB|}{\underline{2\pi}} \langle \bar{\psi}(x_L)\psi(x_L)\rangle_{\underline{2\mathrm{D}}}}$$

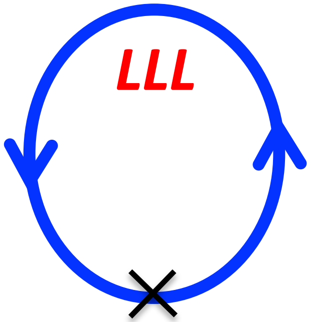
(*degeneracy factor*)

Intuitively,

$\langle \bar{\psi}\psi\rangle_{\underline{2\mathrm{D}}}$



Examples

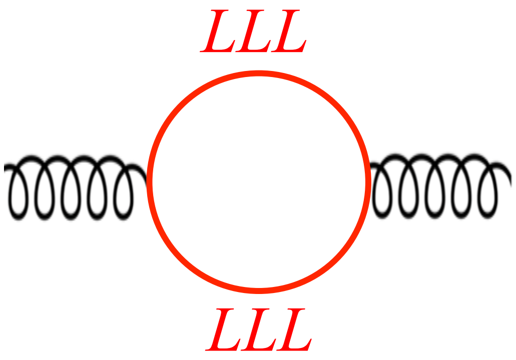


chiral condensate

$$\langle\bar{\psi}(x)\psi(x)\rangle_{4\mathrm{D}} = \underbrace{\frac{|eB|}{2\pi}}_{\substack{\text{degeneracy}\\ \text{(universal)}}} \underbrace{\langle\bar{\psi}(x_L)\psi(x_L)\rangle_{2\mathrm{D}}}_{\substack{\text{dynamical}\\ \text{contents}}}$$

gluon polarization

(perturbative screening)



$$\alpha_s|eB|$$

$$\underbrace{|eB|}_{\substack{\text{degeneracy}\\ \text{(universal)}}} \times \left\{ \begin{array}{ll} \dfrac{1}{M_q^2(B)} & (q_\parallel^2 < M_q^2(B)) \\ \dfrac{1}{q_\parallel^2} & (M_q^2(B) < q_\parallel^2) \end{array} \right.$$

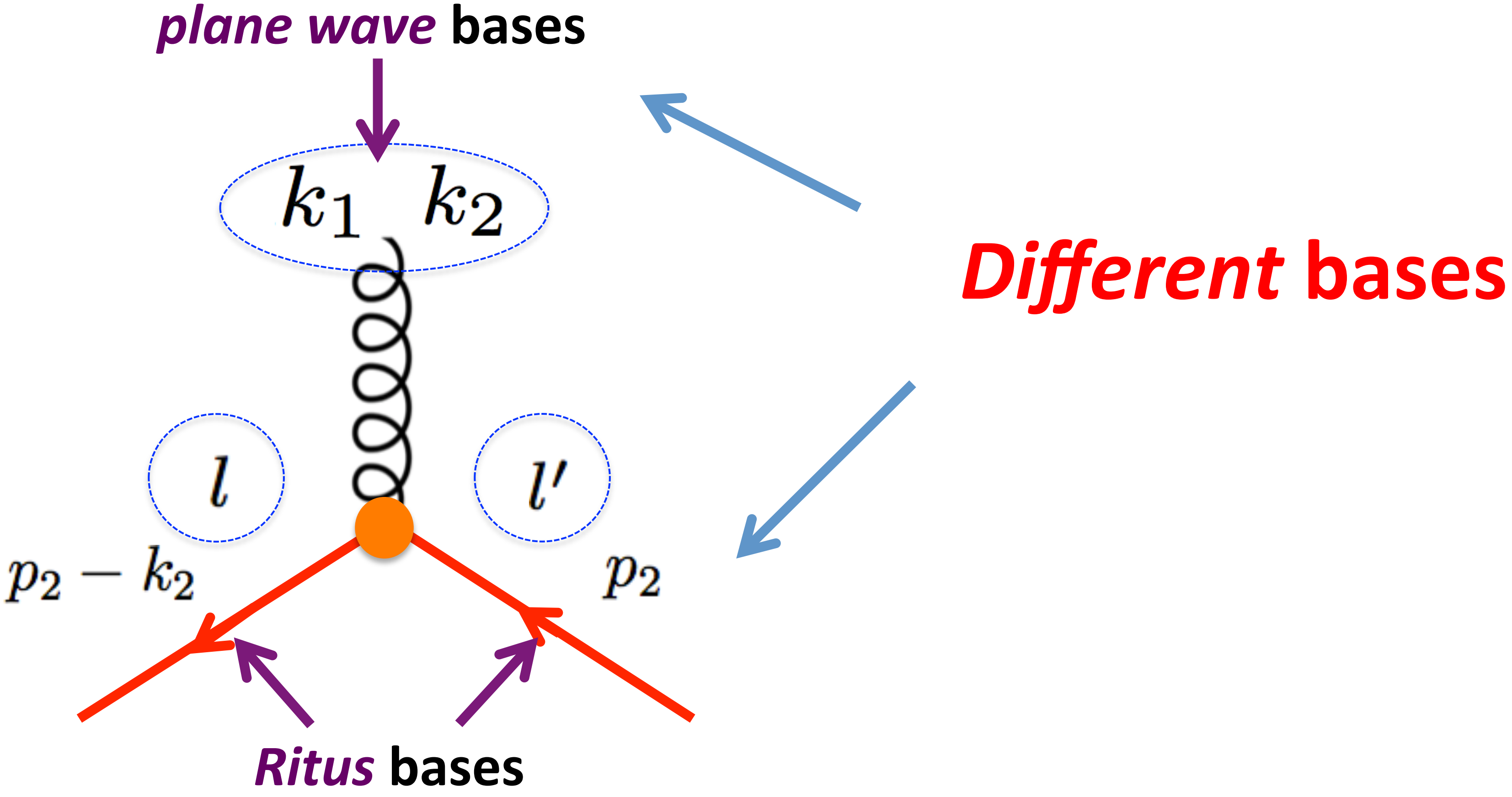
(Miranski-Shovkovy 02)

(*naively*) **Both quantities are enhanced by B**

Couplings b.t.w. different LLs

$$S_{\text{int}} = \int_x \bar{\psi}(x)\gamma_\mu t_a \psi(x)\, A^a_\mu(x)$$

4D Gluons couple to different LLs.



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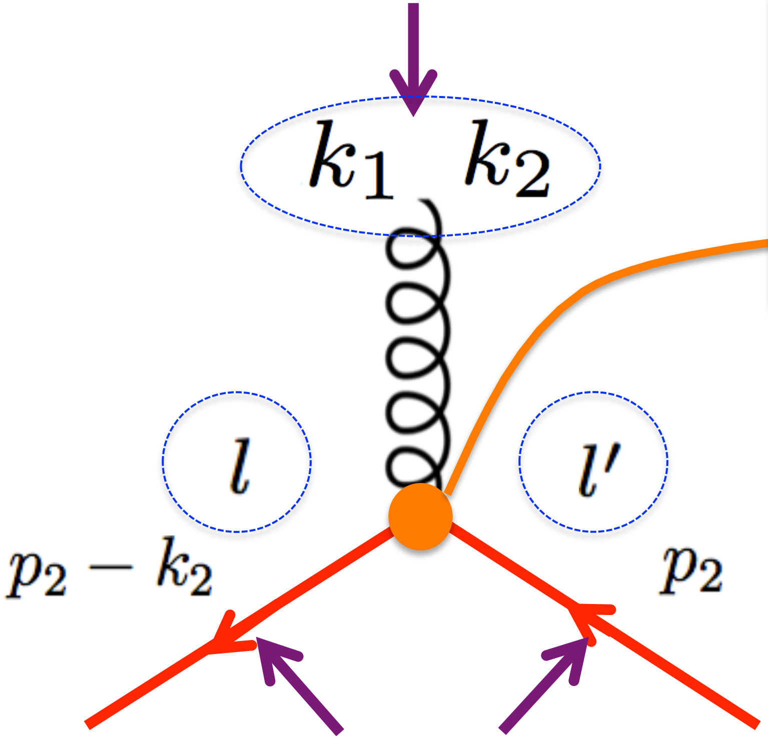
plane wave bases



Ritus bases

“ form factor ”

$$\Delta l = |l - l'|$$

$$I_{l,l'}(\vec{k}_\perp) \propto \left(\frac{k_\perp^2}{2|eB|}\right)^{\frac{\Delta l}{2}} \mathrm{e}^{-k_\perp^2/4|eB|}$$

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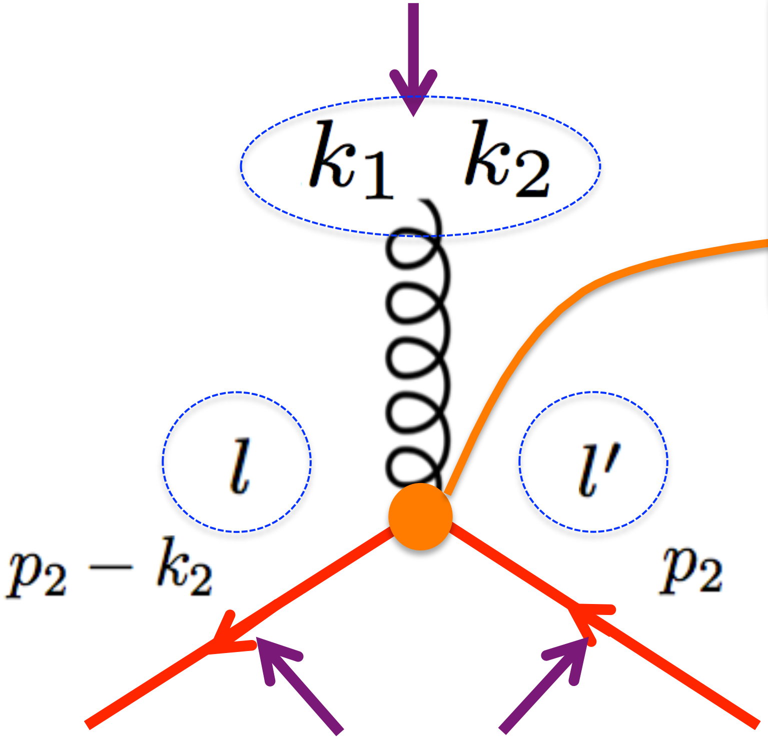

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For *Δl ≠ 0* processes :

small overlap with soft gluons

(Only *Δl = 0* process are dangerous)

LLL decoupling from hLLs

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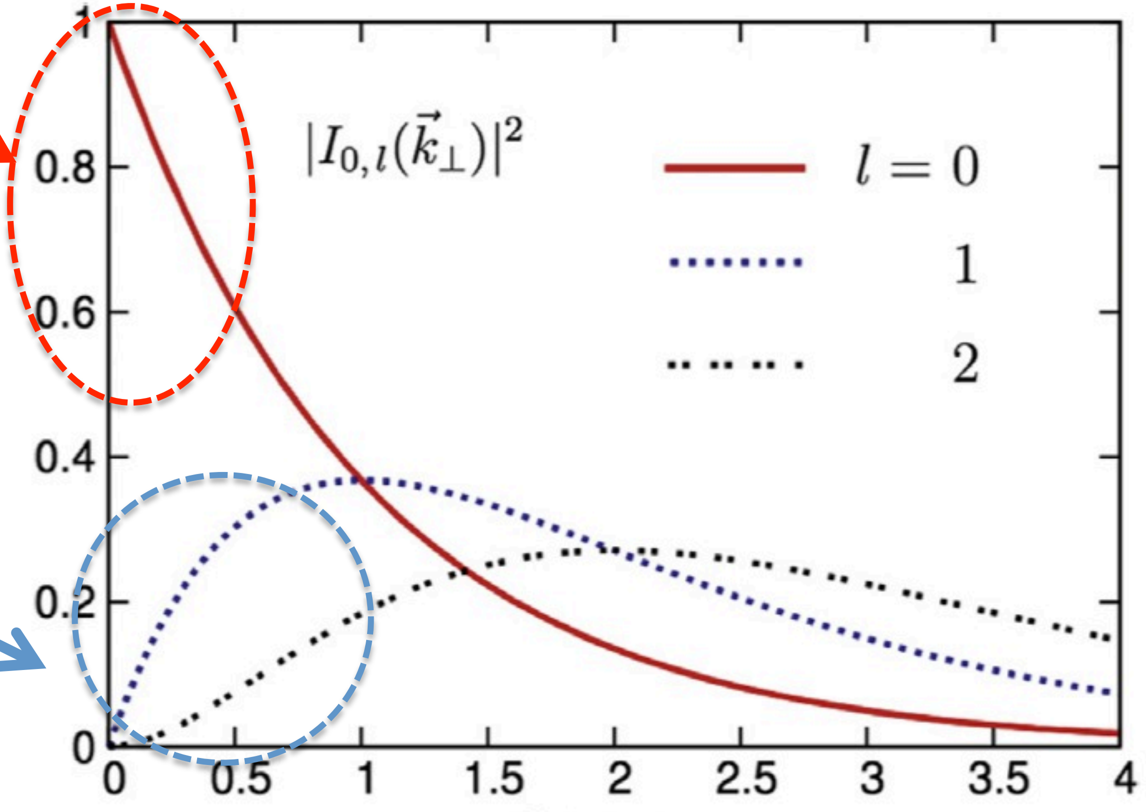
$\Delta l = 0$ (LLL-LLL)

IR gluons couple

$\Delta l \neq 0$ (LLL-hLL)

IR gluons *decouple*



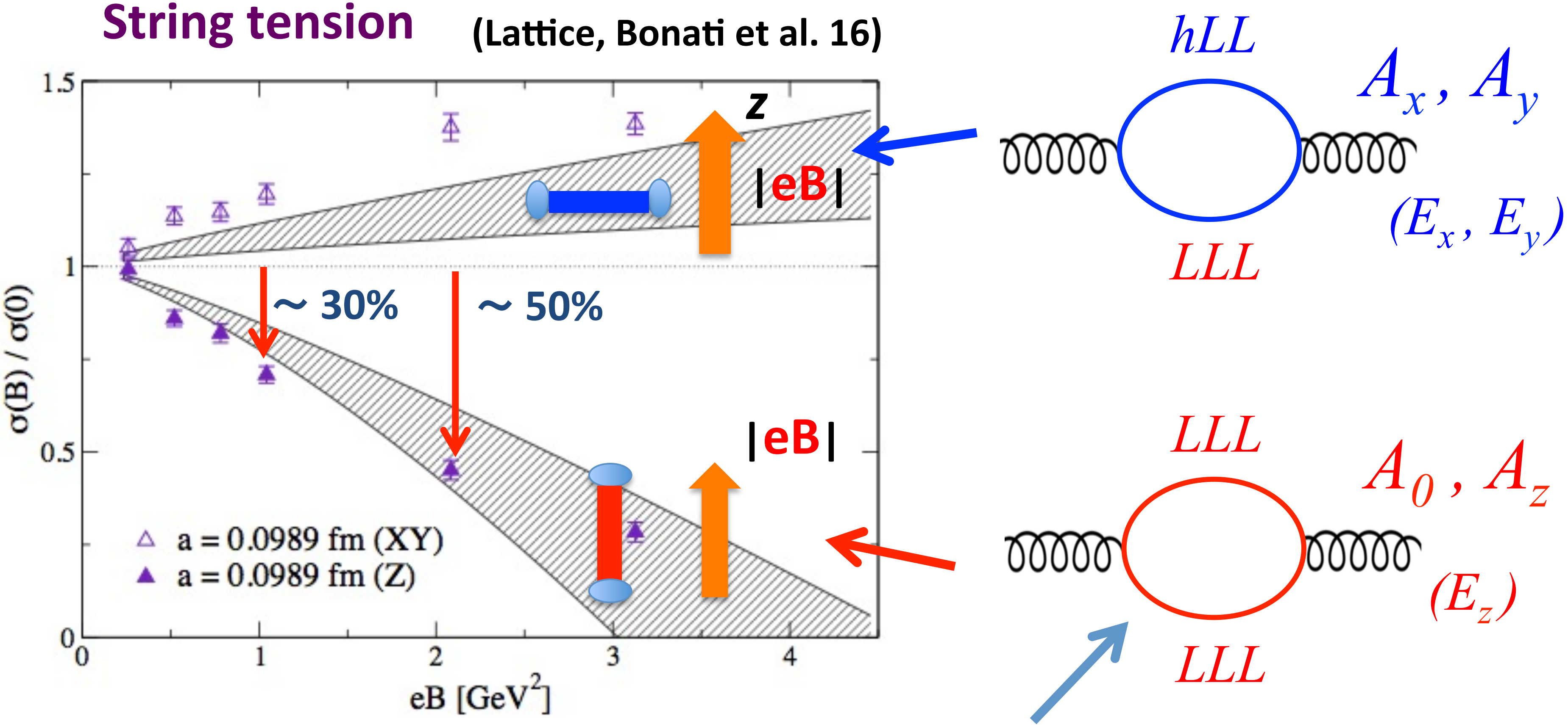
IR

UV

→ In QCD, the LLL tends to decouple from hLLs at large B.

Impact on gluon sectors

String tension (Lattice, Bonati et al. 16)



LLL is light & phase space ∝ |eB|

→ *more screening* of A_0, A_z (& E_z)

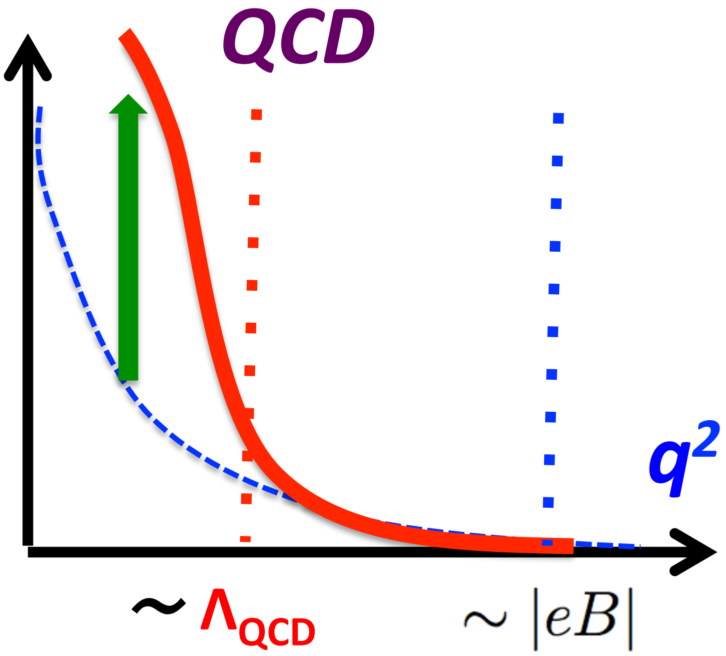
Comparison of forces, 3

Suppose: QCD force has stronger "*IR enhancement*"

$$\int_{q_\perp} \mathrm{e}^{-\frac{q_\perp^2}{2|eB|}} D^{4\mathrm{D}}(q_L, q_\perp)$$

For small $q_{\mathrm{perp}} \sim \Lambda_{\mathrm{QCD}}$:

we can set : $\underline{\mathrm{e}^{-\frac{q_\perp^2}{2|eB|}} \sim 1}$



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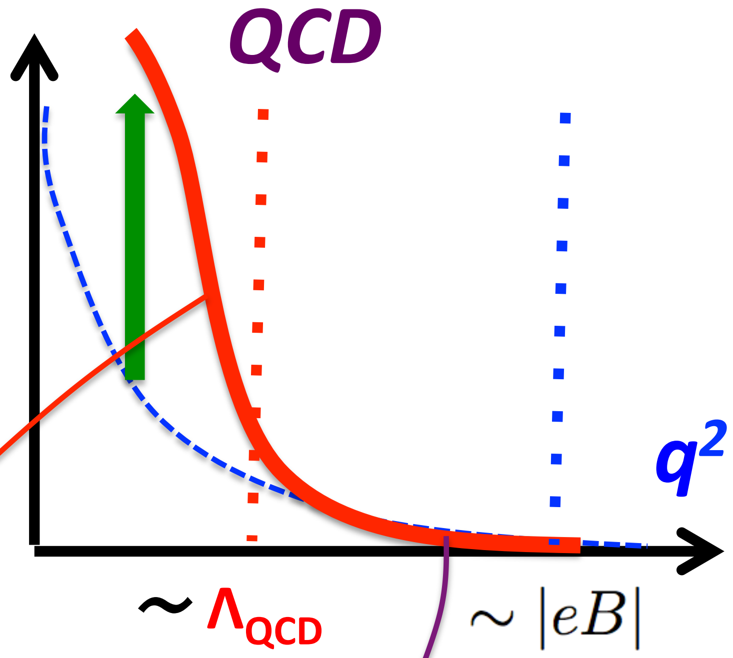

$$\sim \int_0^{\underline{\sim\Lambda_{\mathrm{QCD}}^2}} \mathrm{d}q_\perp^2 D^{4\mathrm{D}}(q_L, q_\perp)$$

\+ *small B-dep. corrections*

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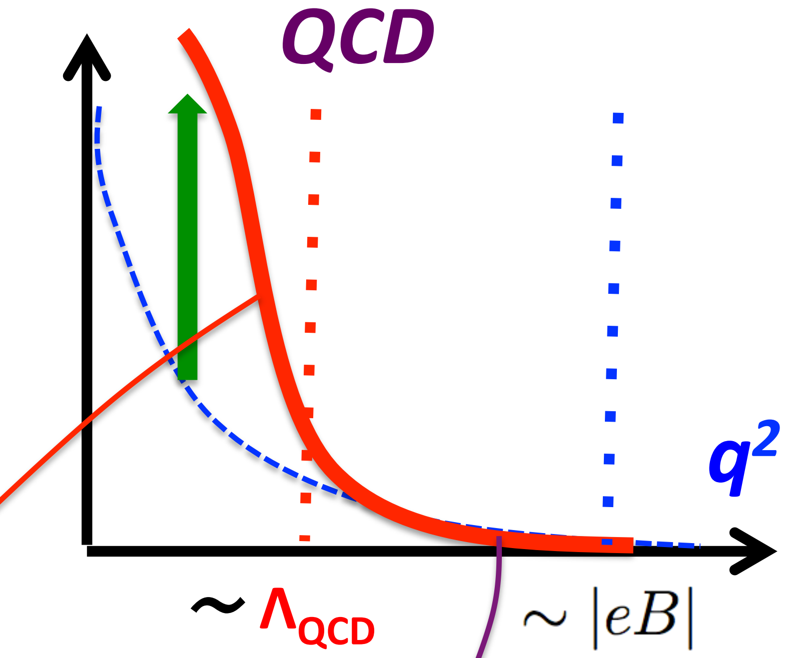


For small $q_{\rm perp}$ $\sim \Lambda_{\rm QCD}$:

we can set : $\underline{\mathrm{e}^{-\frac{q_\perp^2}{2|eB|}} \sim 1}$

$$\sim \int_0^{\underline{\sim\Lambda_{\rm QCD}^2}} \mathrm{d}q_\perp^2 \, D^{4\mathrm{D}}(q_L, q_\perp)$$

+ *small B-dep. corrections*

The dominant part ➡ $M \sim \Lambda_{\rm QCD}$ *"nearly B-indep."*

Our scenario

$$M_q \sim \Lambda_{\mathrm{QCD}}$$

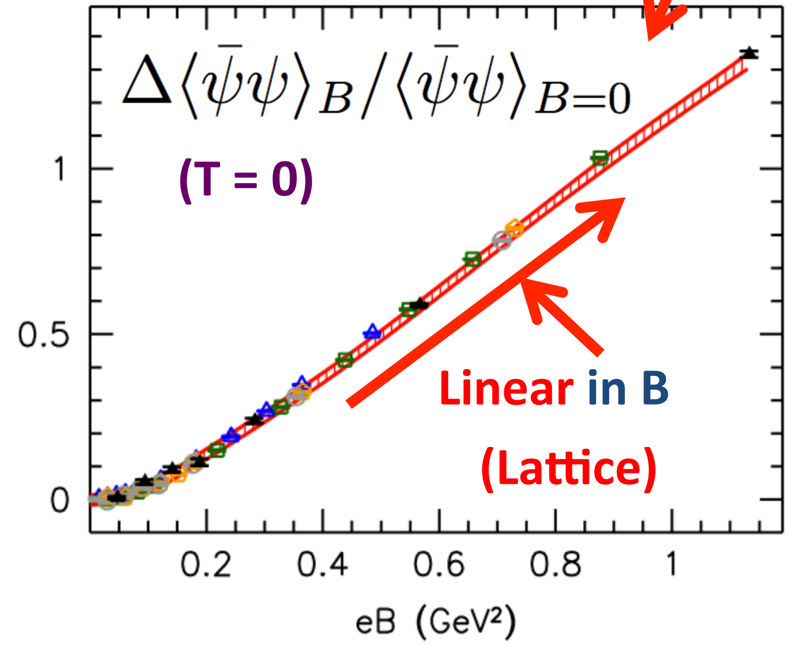
(instead of |eB|$^{1/2}$)

$$\langle\bar{\psi}\psi\rangle_{4\mathrm{D}} = \frac{|eB|}{2\pi}\langle\bar{\psi}\psi\rangle_{2\mathrm{D}}$$

phase space increases as

~ |eB| Λ_{QCD}

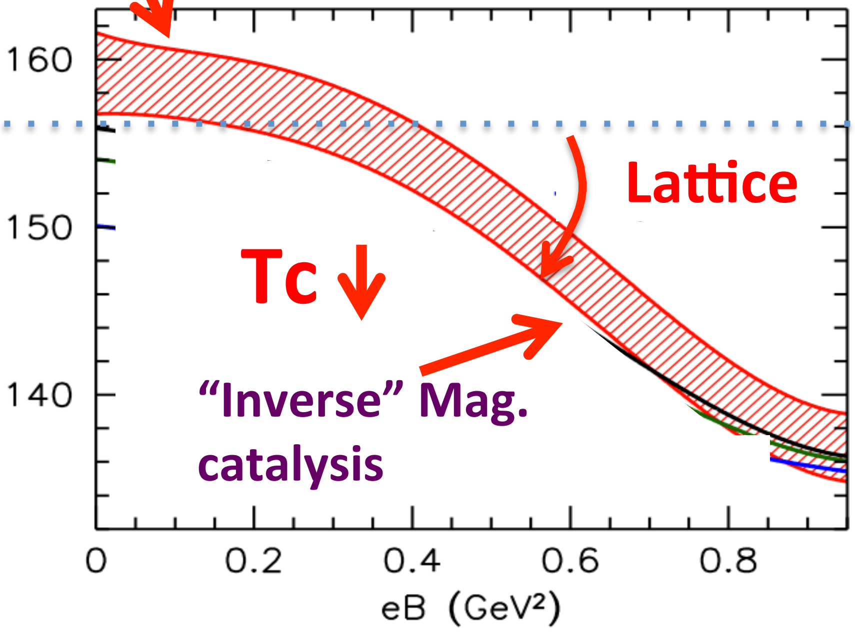
"Enhanced" thermal fluct.



(T = 0)

Linear in B

(Lattice)



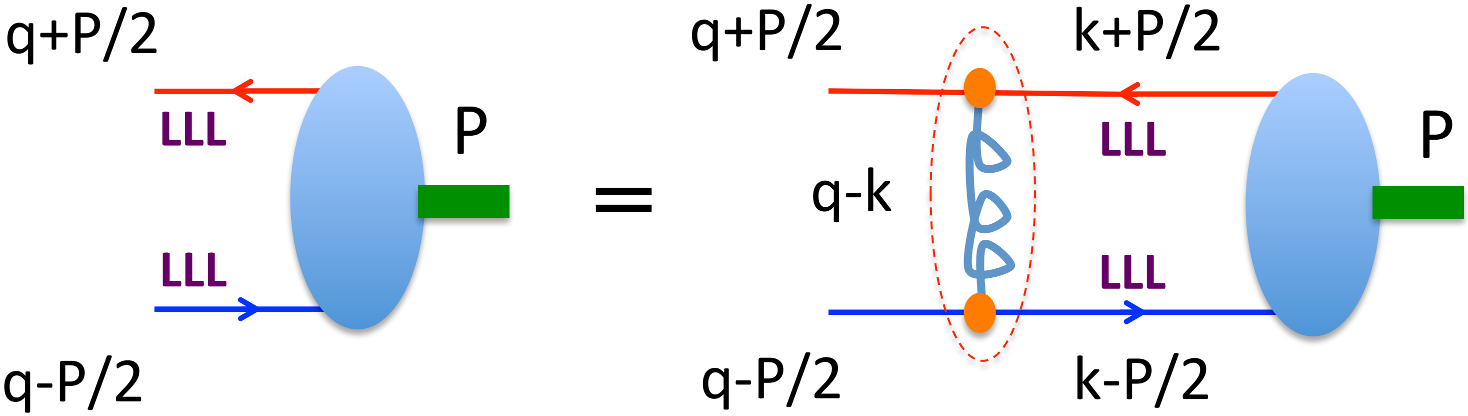
Lattice

Tc ↓

"Inverse" Mag. catalysis

Bethe-Salpeter equations for LLLs

BS-eqs can be dimensionally reduced from 4D to 2D



2D effective interaction

$$\mathcal{V}^{B}_{2\mathrm{D}}(q_3 - k_3; \underline{\vec{P}_\perp}) = \int_{\vec{k}_\perp} \underbrace{\mathrm{e}^{\mathrm{i}\Pi(\vec{q}_\perp - \vec{k}_\perp; \vec{P}_\perp)}}_{\text{“ Schwinger phase “}} \underbrace{\mathrm{e}^{-\frac{(\vec{q}_\perp - \vec{k}_\perp)^2}{2|eB|}}}_{\text{form factor}} V_{4\mathrm{D}}(\vec{q} - \vec{k})$$

Again, B-dep. arises only from 2D effective interaction

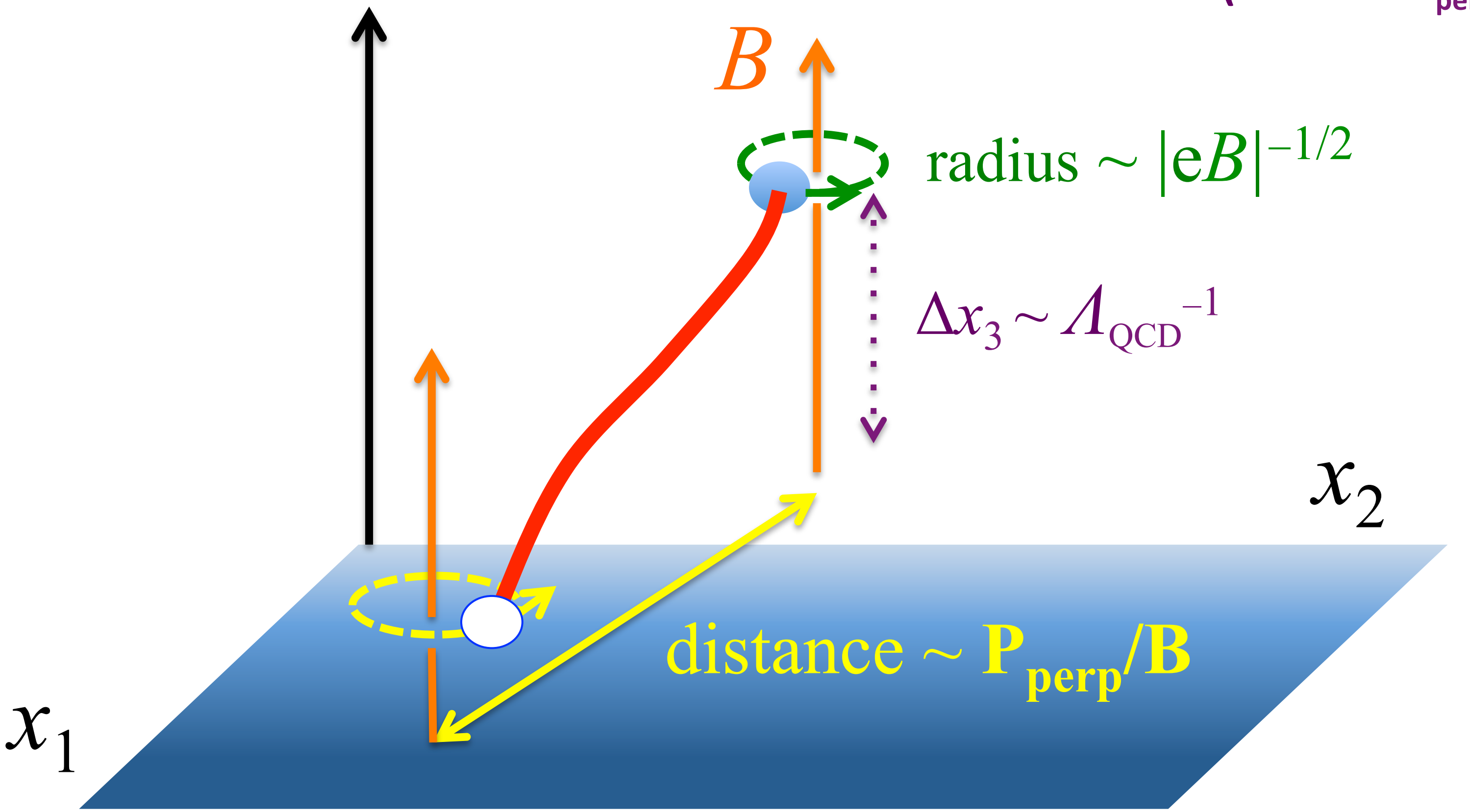
Spectrum : results of *long-range* forces

$$E^{ff'}_{n_3,\vec{P}_\perp}(P_3) \simeq \sqrt{\underline{\left(M^{\rm neutral}_{n_3}\right)^2} + P_3^2} + c_1\Lambda^3_{\rm QCD}\,\underline{\frac{P_\perp^2}{|B|^2}} + \cdots$$

nearly B-indep.

$P_{\rm perp}$-correction

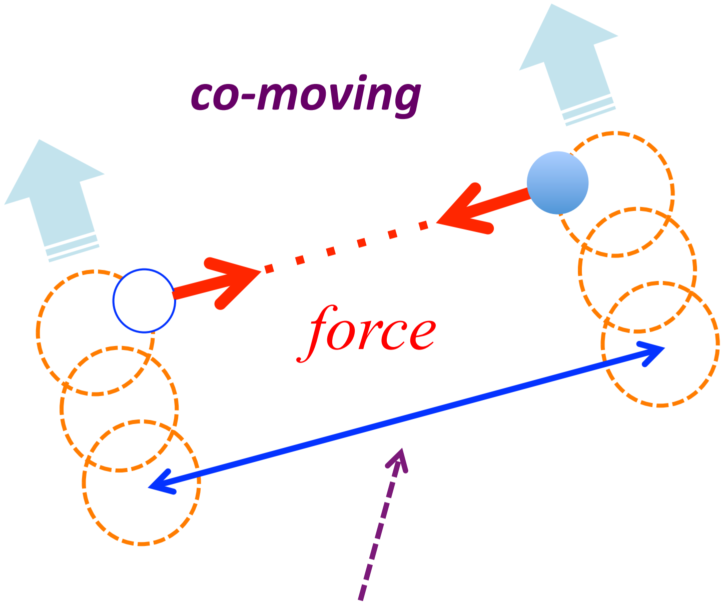
(at small $P_{\rm perp}$)



2-body problems : Hall drift

$Q = - Q'$

(e.g. neutral mesons)



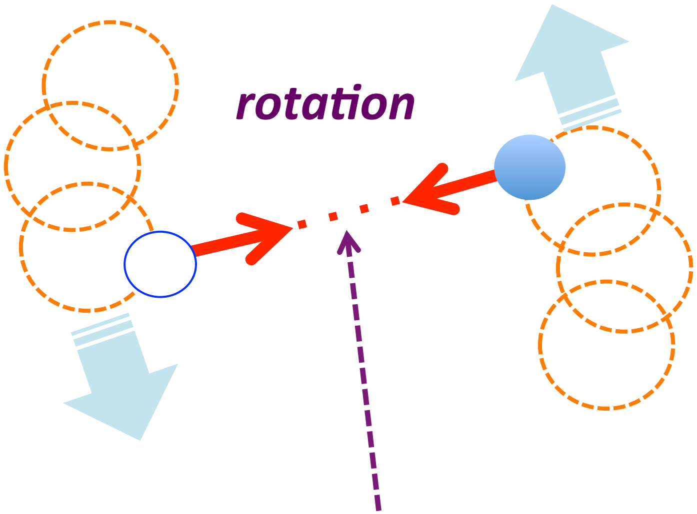
co-moving

force

distance conserved

$Q = Q'$

(e.g. di-electrons)



rotation

center conserved

Classification of states

$u_{\uparrow}$ $\bar{u}_{\uparrow}$ $d_{\downarrow}$ $\bar{d}_{\downarrow}$ **can couple to LLL**

The possible states (saturated by LLL only):

π_0 $\rho_0(s_z=0)$ $\rho_+(s_z=+1)$ $\rho_-(s_z=-1)$, ... etc.

G.S. can be saturated by LLL only → light

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$u_\downarrow \quad \bar{u}_\downarrow \quad d_\uparrow \quad \bar{d}_\uparrow$ **must** couple to **hLLs**

$\pi_\pm$ $\rho_0(s_z\neq0)$ $\rho_+(s_z\neq+1)$ $\rho_-(s_z\neq-1)$, ..., etc.

contain hLLs → mass grows as $|eB|^{1/2}$

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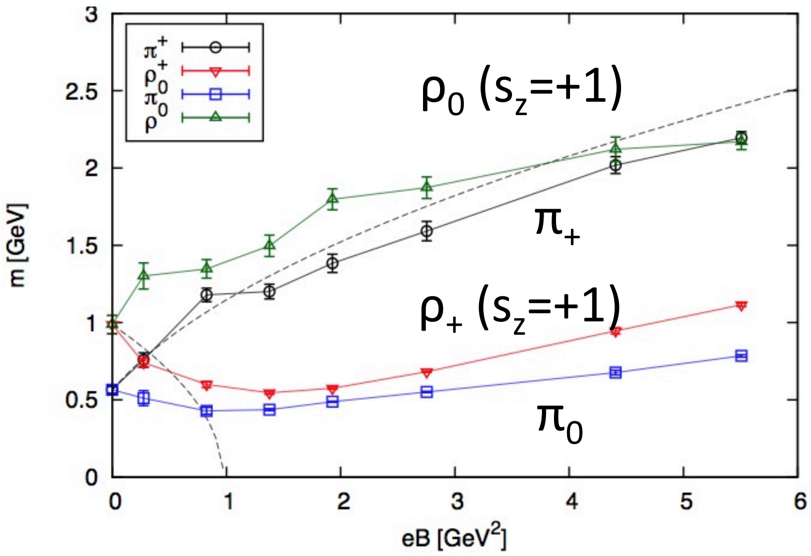
Lattice results (*quenched* only)



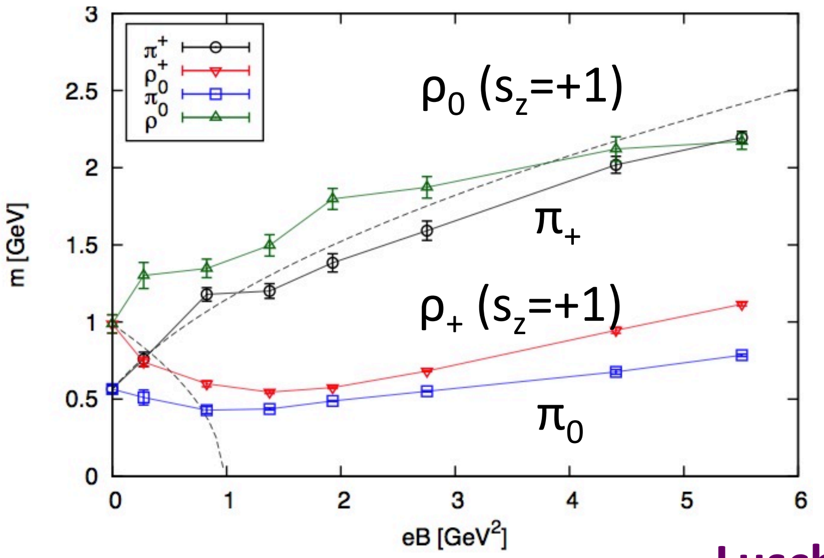
Hidaka-Yamamoto 12

Wilson fermion

problems at large B?

(see Bali et al. 1510.03899)

Lattice results (*quenched* only)



Hidaka-Yamamoto 12

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Luschevskaya et al. 15 *overlap fermion*