Inverse Magnetic Catalysis for chiral and deconfinement phase transitions

refs：

(1) Phys. Lett. B 758, 195-199 (2016)
(2) Phys. Rev. D 94, 036007 (2016)
(3) PNJL results, under preparation

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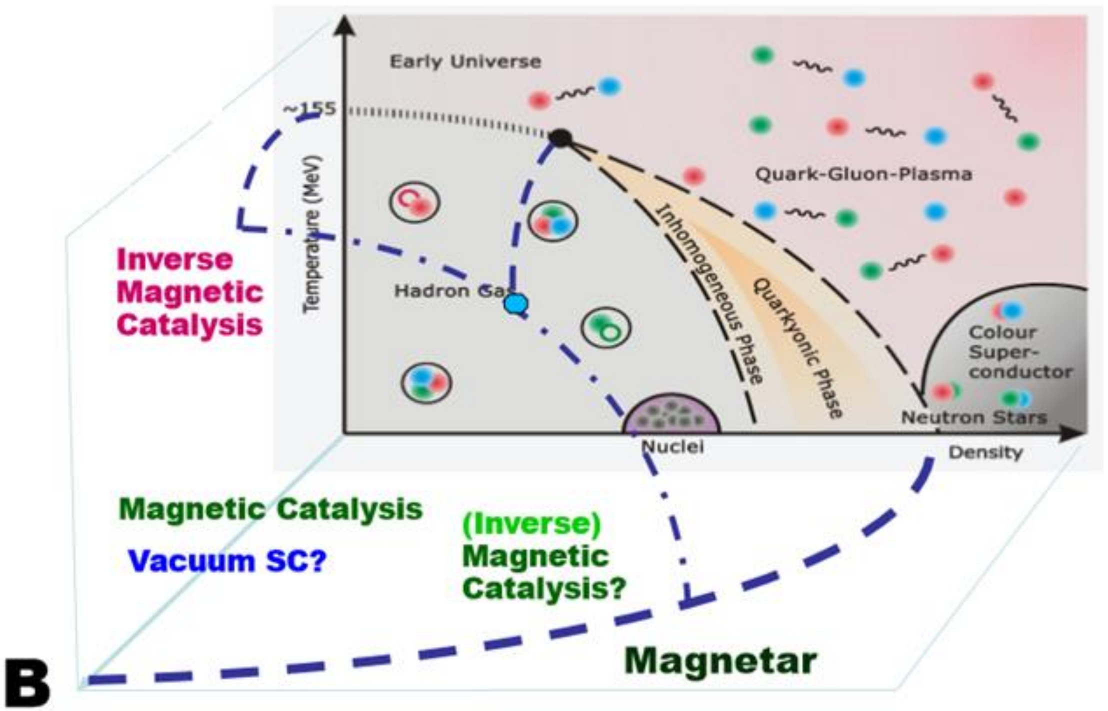
QCD Phase Diagram



Compact stars：$10^{10\sim15}$ Gauss

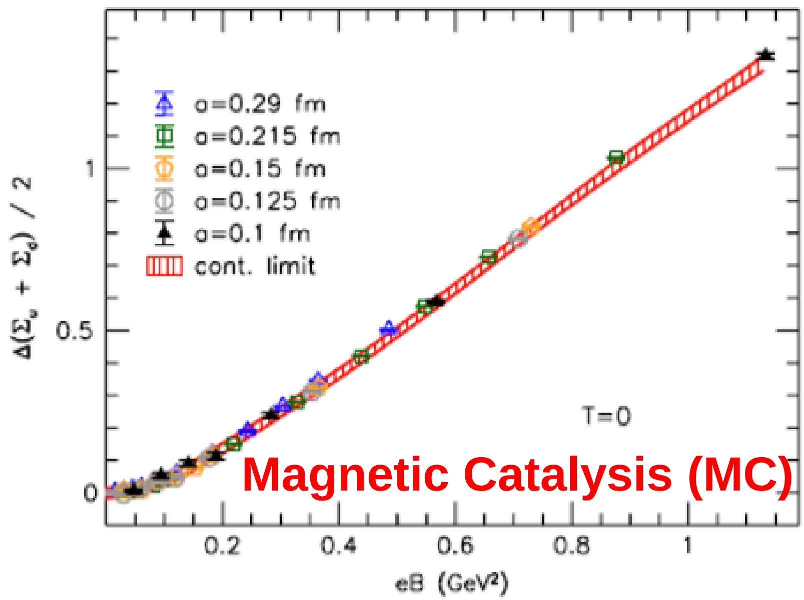
HICs：$10^{18\sim19}$ G

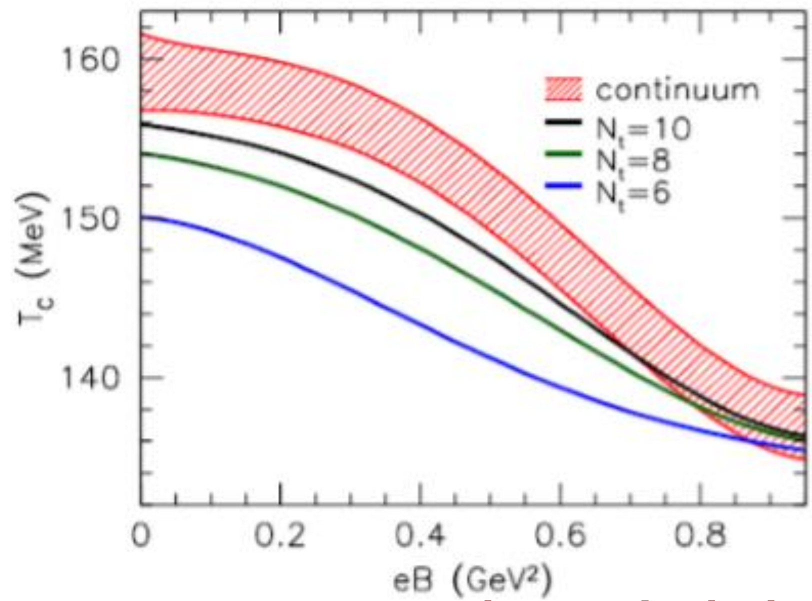
Early Universe：10^{24} Gauss

1MeV^2 =1.7*10^{14}Gauss

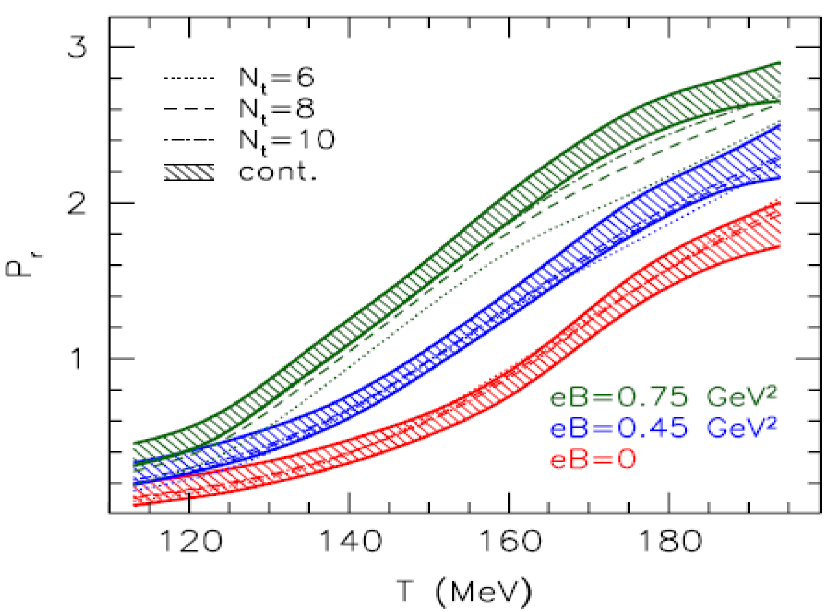
- ➢**High T, μ limit：perturbative QCD;**
- ➢**Phase transition (non-perturbative):**
  LQCD；Dyson Eq； FRG method
  effective models (NJL ，σ model)

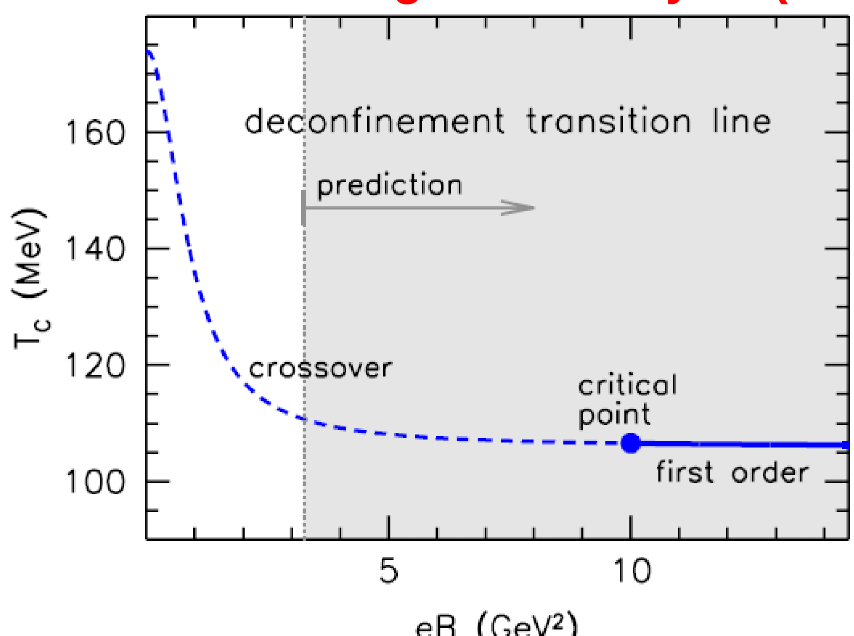
Lattice QCD: chiral+deconfinement





Inverse Magnetic Catalysis (IMC)





Magnetic Catalysis (MC)

Dimension Reduction (Nucl. Phys. B 462, 249(1996); 563, 361 (1999))

IMC is still an open question.

(1) magnetic inhibition: Fukushima et al.**, PRL 110, 031601(2013)**

contribution from neutral pion

(2) mass gap in large N_c limit:

Toru et al. , PLB 720, 192 (2013)

(3) chirality imbalance:

Huang Mei et al. **, PRD 88, 054009 (2013)**

(4) contribution from sea quark (gluon screening effect):

Bruckmann et al. , **JHEP 04, 112(2013)**

(5) weakening of strong coupling:

Pinto et al. **, PRC 90, 025203(2014)**

refs: Phys. Rep. 576,1 (2015);Rev. Mod. Phys. 88, 025001 (2016).

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Theoretical Framework

NJL model

SU(2) Nambu--Jona-Lasinio model

$$\mathcal{L} = \bar{\psi}\,(i\gamma^{\mu}\partial_{\mu} - m_0\,)\,\psi + G\Big[\,(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\tau_i\psi)^2\,\Big]$$

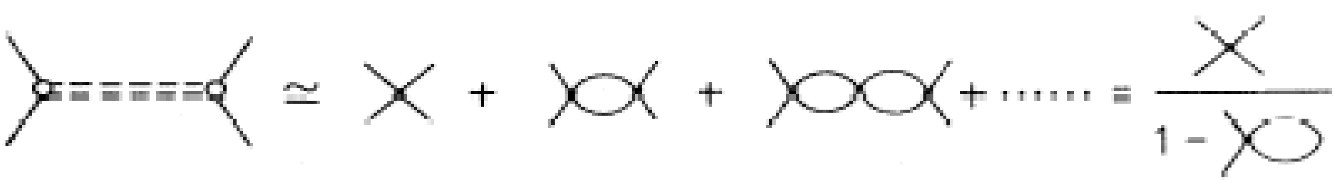
ψ: quark

chiral symmetry $SU(2)_L \otimes SU(2)_R \xrightarrow{\text{B}} U(1)_L \otimes U(1)_R$

NJL模型受BCS理论的启发,被广泛用来研究手征对称性(手征凝聚)(2008, Nobel Prize)。

(1)Quarks：basic degree of freedom

(2)Mesons：RPA resummation (quantum fluctuation)



(3)Q-M system：Feed-down from mesons to quarks

(1) Mean field

SU(2) NJL model

$$\mathcal{L} = \bar{\psi}\left(i\gamma_\nu D^\nu - m_0\right)\psi + \frac{G}{2}\left[\left(\bar{\psi}\psi\right)^2 + \left(\bar{\psi}i\gamma_5\vec{\tau}\psi\right)^2\right]$$

$$D^\nu = \partial^\nu + iQA^\nu \quad Q = diag(Q_u, Q_d) = diag(\tfrac{2}{3}e, -\tfrac{1}{3}e)$$

Magnetic field：$\mathbf{B} = (0, 0, B) = \nabla \times \mathbf{A}$

ψ is quarks; chiral limit m_0=0; coupling constant G ;

order parameter： chiral condensate $\langle\bar{\psi}\psi\rangle$

effective quark mass $m = m_0 - G\langle\bar{\psi}\psi\rangle$

thermodynamic potential :

$$\Omega_{mf} = \frac{m^2}{2G} + \Omega_q$$

$$\Omega_q = -\frac{T}{V}\mathrm{Tr}\,\mathrm{Ln}\,S^{-1}$$

（S为夸克传播子）

gap equation: $\partial\Omega_{mf}/\partial m = 0 \Longrightarrow m\left(\frac{1}{2G} + \frac{\partial\Omega_q}{\partial m^2}\right) = 0$

quark propagator under external magnetic field
---- Ritus propagator

$$S_f(x,y) = i\sum_{n=0}^{\infty}\int \frac{d\tilde{p}}{(2\pi)^3} e^{-i\tilde{p}\cdot(x-y)} P_n(x_1,p_2) D_f(\bar{p}) P_n(y_1,p_2),$$

$$P_n(z,q) = \frac{1}{2}\left[g_n^{s_f}(z,q) + I_n g_{n-1}^{s_f}(z,q)\right] + \frac{is_f}{2}\left[g_n^{s_f}(z,q) - I_n g_{n-1}^{s_f}(z,q)\right]\gamma^1\gamma^2,$$

$$D_f^{-1}(\bar{p}) = \gamma\cdot\bar{p} - m, \quad \bar{p} = (p_0, 0, -s_f\sqrt{2|Q_f B|n}, p_3)$$

$$\tilde{p} = (p_0, 0, p_2, p_3) \qquad I_n = 1 - \delta_{n0}$$

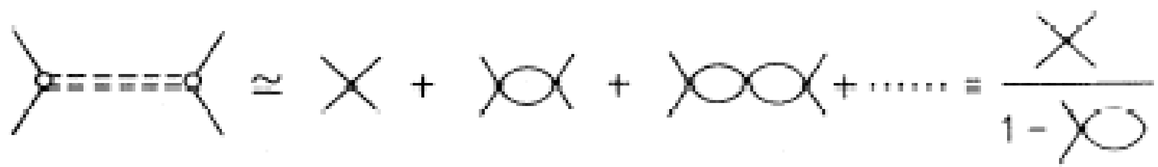
$$g_n^{s_f}(z,q) = \phi_n(z - s_f q/|Q_f B|)$$

$$\phi_n(z) = \left(2^n n! \sqrt{\pi}|Q_f B|^{-1/2}\right)^{-1/2} \times e^{-z^2|Q_f B|/2} H_n\left(z/|Q_f B|^{-1/2}\right)$$

(2) Mesons (quantum fluctuations)

meson propagators @ Random Phase Approximation

$$D_M(k) = \frac{G}{1 - G\Pi_M(k)}$$



polarization functions

$$\Pi_M(k) = -i \int d^4(x-x')e^{ik\cdot(x-x')}\mathrm{Tr}\left[\Gamma_M S(x,x')\Gamma_M^* S(x',x)\right]$$

$$\Gamma_M = \begin{cases} 1 & M=\sigma \\ i\tau_+\gamma_5 & M=\pi_+ \\ i\tau_-\gamma_5 & M=\pi_- \\ i\tau_3\gamma_5 & M=\pi_0\,, \end{cases} \qquad \Gamma_M^* = \begin{cases} 1 & M=\sigma \\ i\tau_-\gamma_5 & M=\pi_+ \\ i\tau_+\gamma_5 & M=\pi_- \\ i\tau_3\gamma_5 & M=\pi_0 \end{cases}$$

meson mass：

$$1 - G\Pi_M(k_0^2 = m_M^2, \mathbf{k}^2 = 0) = 0$$

← **pole equation**

quark-meson coupling:

$$\left(g^{\mu}_{q\bar{q}M}\right)^2 = \left[g^{\mu\mu}\frac{d\Pi_M(k)}{dk_\mu^2}\bigg|_{k^2=(m_M^2,0)}\right]^{-1}$$

$$g^1_{q\bar{q}M} = g^2_{q\bar{q}M} \neq g^0_{q\bar{q}M} = g^3_{q\bar{q}M}$$

(3) beyond mean field

quark-meson system: $$\Omega = \frac{m^2}{2G} + \Omega_q + \sum_M \Omega_M$$ ⟷ **mesons**

meson thermo-dynamic potential:

$$\Omega_M = \int \frac{d^3\mathbf{k}}{(2\pi)^3}\left[\frac{E_M}{2} + T\ln\left(1 - e^{-E_M/T}\right)\right]$$

meson energy:

$$E_M = \sqrt{m_M^2 + k_3^2 + v_\perp^2(k_1^2 + k_2^2)}$$

transverse velocity:

$$v_\perp^2 = \left(g^0_{q\bar{q}M}\right)^2 / \left(g^1_{q\bar{q}M}\right)^2$$

new gap equation: **"running" coupling constant G'(B,T)**

$$m\left(\frac{1}{2G} + \frac{\partial\Omega_q}{\partial m^2} + \sum_M \frac{\partial\Omega_M}{\partial m^2}\right) = 0$$

$$\Omega_M = \sum_n \frac{1}{n!}\frac{\partial^n\Omega_M}{\partial(m^2)^n}\Big|_{m^2_{mf}}\left(m^2 - m^2_{mf}\right)^n \qquad n = 0, 1.$$

$$m\left(\frac{1}{2G'} + \frac{\partial\Omega_q}{\partial m^2}\right) = 0 \qquad \frac{1}{2G'} = \frac{1}{2G} + \sum_M \frac{\partial\Omega_M}{\partial m^2}\Big|_{m^2_{mf}}$$

Theoretical framework

1. Quark level：solving gap equation with mean field ,m_{mf}

$$m\left(\frac{1}{2G}+\frac{\partial\Omega_q}{\partial m^2}\right)=0$$

2. Mesons（quantum fluctuations）

$$1-G\Pi_M(k_0^2=m_M^2,\mathbf{k}^2=0)=0$$

$$v_\perp^2=\left(g^0_{q\bar{q}M}\right)^2/\left(g^1_{q\bar{q}M}\right)^2$$

3. Feed-down from mesons to quarks：

---- beyond mean field gap eq,m_{bmf}

$$m\left(\frac{1}{2G'}+\frac{\partial\Omega_q}{\partial m^2}\right)=0$$

$$\frac{1}{2G'}=\frac{1}{2G}+\sum_M\frac{\partial\Omega_M}{\partial m^2}\Big|_{m^2_{mf}}$$

Mesons

However,
(1)G is constant;
(3)Running G’(B,T)

From eq(1),
B ↑,m ↑;
G ↑,m ↑;

NJL results

model parameters

$m_0 = 0, \quad \Lambda = 1127 \text{ MeV} \quad N = 3, \; G = 9.94 \text{ GeV}^{-2}$

Pauli-Villars regularization

$$m \;\rightarrow\; m_i = \sqrt{m^2 + a_i \Lambda^2}$$

$$E_f(m) \;=\; \sqrt{m^2 + p_z^2 + 2p|Q_f B|}$$

$$\rightarrow\; E_{f_i}(m_i) = \sqrt{m_i^2 + p_z^2 + 2p|Q_f B|}$$

$$\sum_p \int \frac{dp_z}{2\pi} f[E_f(m)] \;\rightarrow\; \sum_p \int \frac{dp_z}{2\pi} \sum_{i=0}^{N} c_i f[E_{f_i}(m_i)]$$

$$a_0 = 0, \; c_0 = 1, \; \sum_{i=0}^{N} c_i m_i^{2L} = 0 \text{ for } L = 0, 1, \cdots N-1$$

π_0 mass and transverse velocity

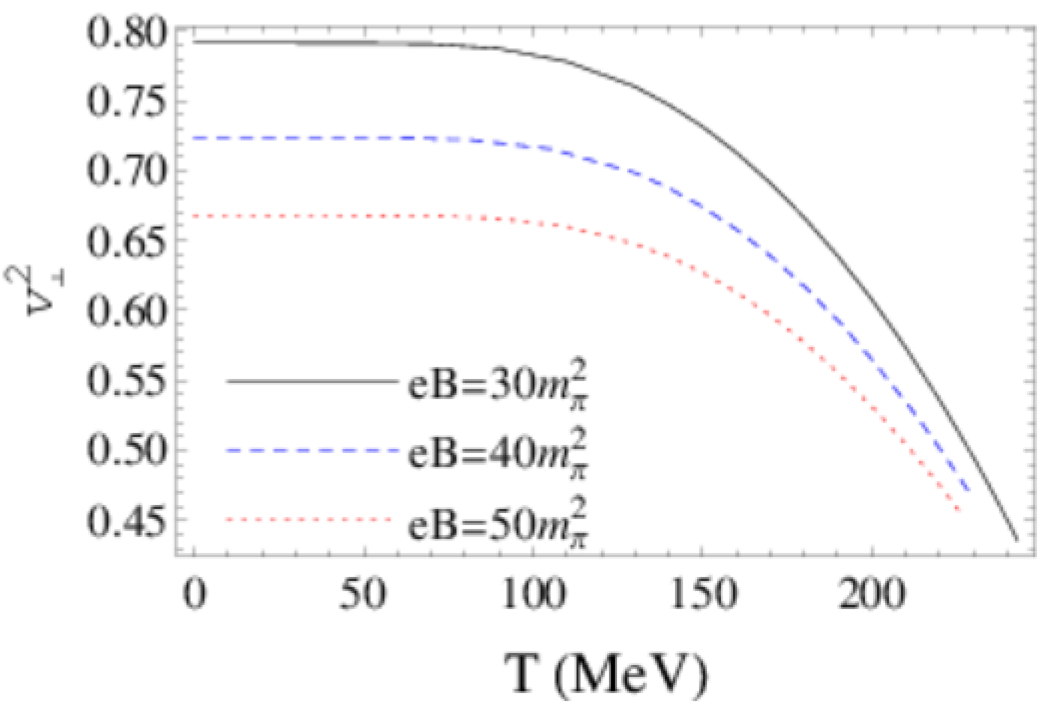
meson mass:

$$1 - G\Pi_M(k_0^2 = m_M^2, \mathbf{k}^2 = 0) = 0 \longrightarrow m_{\pi_0} = 0 \rightleftarrows$$ **Goldstone mode**

$$m\left(\frac{1}{2G} + \frac{\partial \Omega_q}{\partial m^2}\right) = 0$$

longitudinal (z) : $v_{||} = 1$

transverse (x,y): $v_{\perp} < 1$



dimension reduction

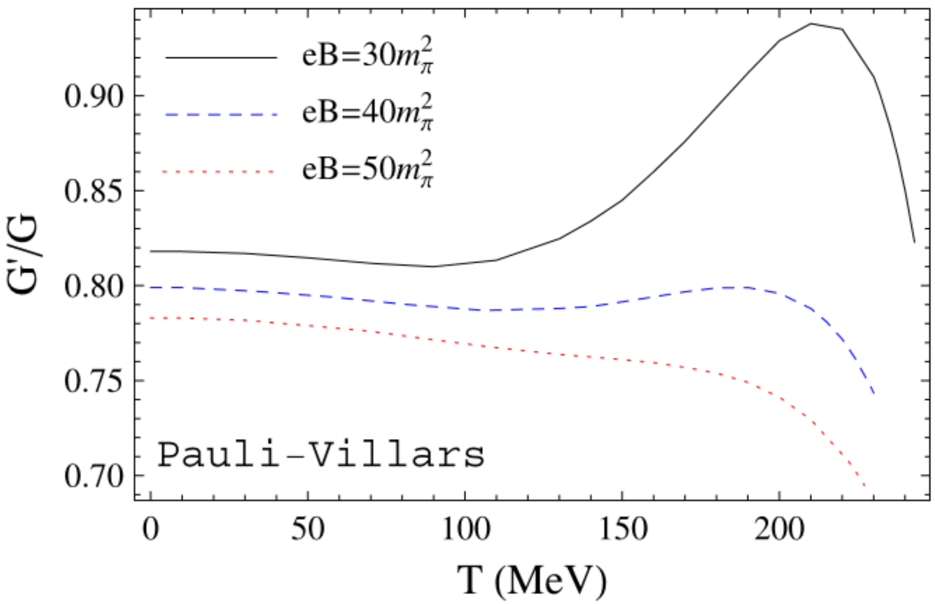
$$E_M = \sqrt{m_M^2 + k_3^2 + v_\perp^2(k_1^2 + k_2^2)}$$

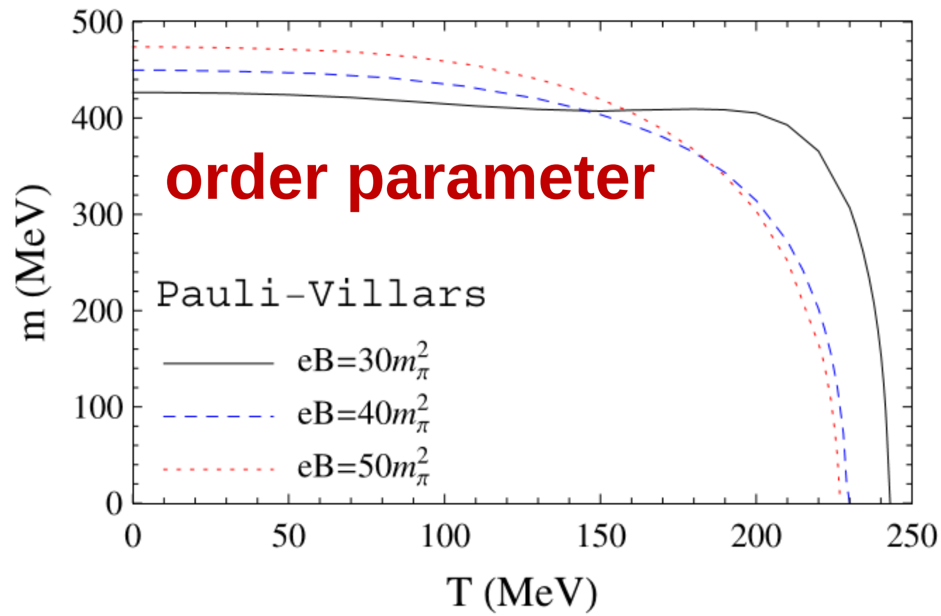
Mermin-Wagner-Coleman theorem

possible IMC

effective quark mass beyond mean field

gap eq BMF: $$m\left(\frac{1}{2G'}+\frac{\partial\Omega_q}{\partial m^2}\right)=0 \qquad \frac{1}{2G'}=\frac{1}{2G}+\sum_M \frac{\partial\Omega_M}{\partial m^2}\Big|_{m^2_{mf}}$$



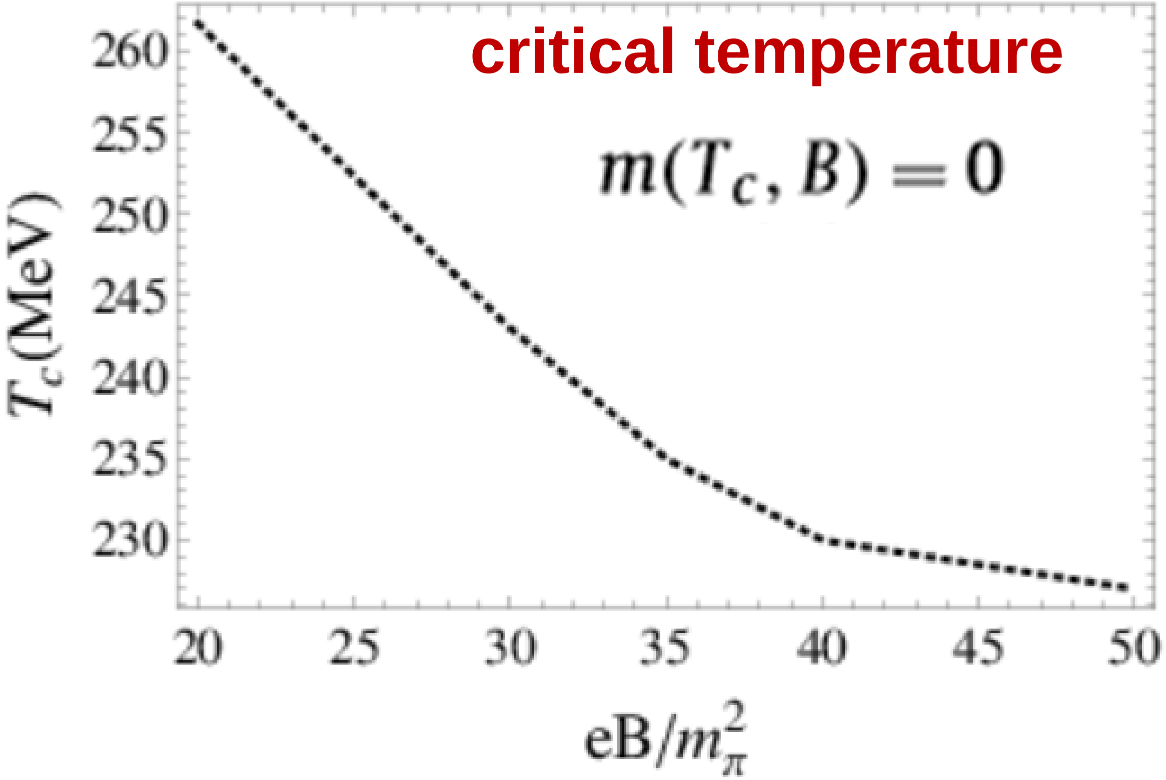


(1)G'/G<1,
meson weakens coupling。
(2)high T, G' changes fast。

G'↓,m↓; B ↑,m ↑;

low T, m ↑ with B, (MC);
high T, m ↓ with B, (IMC);

chiral restoration phase transition

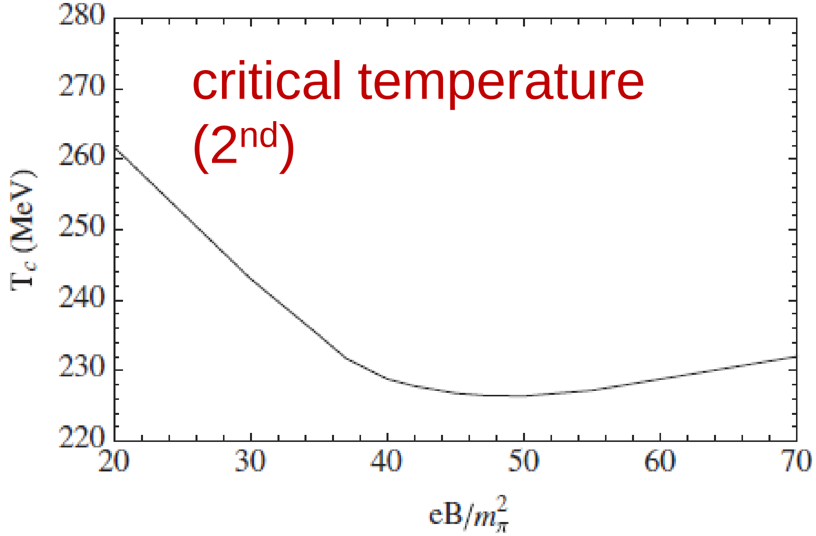


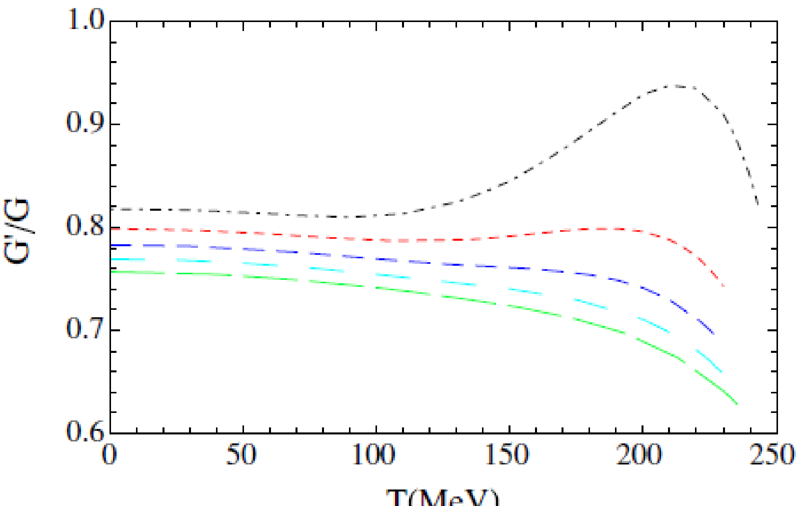
Inverse Magnetic Catalysis:

Tc decreases with B；consistent with LQCD

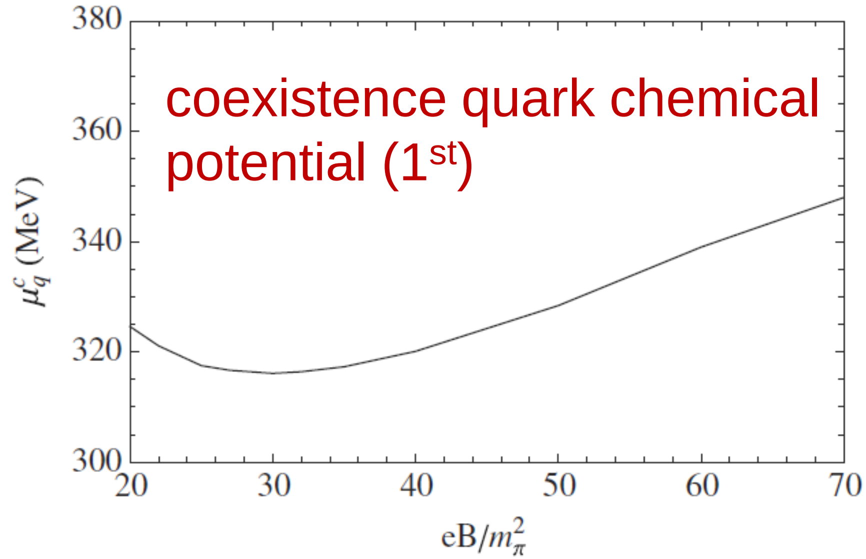
From IMC to MC

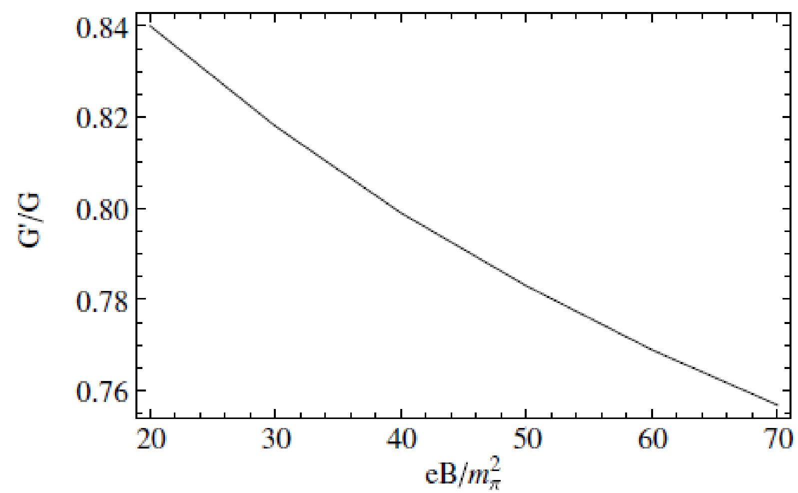
μ_q=0



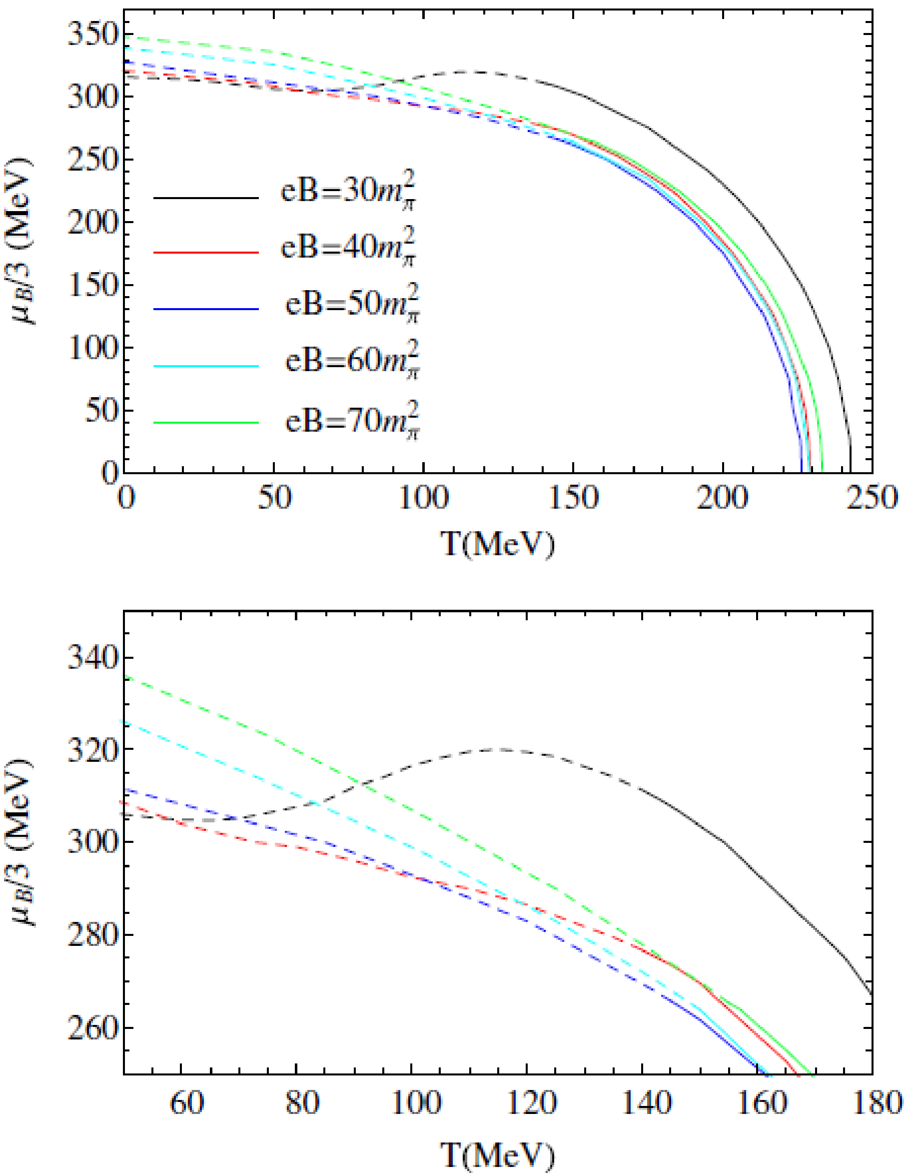


T=0





Phase diagram beyond mean field



IMC effect in
chiral and deconfinement phase transition

Polyakov-extended NJL model

$$\mathcal{L} = \bar{\psi}\left(i\gamma_\mu D^\mu - m_0\right)\psi + \frac{G}{2}\left[\left(\bar{\psi}\psi\right)^2 + \left(\bar{\psi}i\gamma_5\vec{\tau}\psi\right)^2\right] \underline{-\,\mathcal{U}(\Phi,\bar{\Phi})}$$

$$D^\mu = \partial^\mu - iQA^\mu_{EM} \underline{-\,iA^\mu}$$

magnetic field $\mathbf{B} = \nabla \times \mathbf{A}_{EM} = (0, 0, B)$

temporal gluon field $A^\mu = \delta^\mu_0 A^0$ with $A^0 = g\mathcal{A}^0_a \lambda_a/2 = -iA_4$

Polyakov potential: $$\frac{\mathcal{U}}{T^4} = -\frac{b_2(T)}{2}\bar{\Phi}\Phi - \frac{b_3}{6}(\bar{\Phi}^3 + \Phi^3) + \frac{b_4}{4}(\bar{\Phi}\Phi)^2$$

Polyakov loop $\Phi = \left(\mathrm{Tr}_c L\right)/N_c$

$$L(\mathbf{x}) = \mathcal{P}\exp\left[i\int_0^\beta d\tau A_4(\mathbf{x},\tau)\right] = \exp\left[i\beta A_4\right] \text{ with } \beta = 1/T$$

$$m\left(\frac{1}{2G}+\frac{\partial\Omega_q}{\partial m^2}\right)=0,$$
$$\frac{\partial\mathcal{U}}{\partial\Phi}+\frac{\partial\Omega_q}{\partial\Phi}=0.$$

thermodynamic potential

$$\Omega=\mathcal{U}(\Phi)+\frac{m^2}{2G}+\Omega_q+\sum_M\Omega_M$$

$$\Omega_q = -\sum_f\sum_n\alpha_n\int\frac{dp_z}{2\pi}\frac{|Q_fB|}{2\pi}\left[3E_f+2T\ln\left(1+3\Phi e^{-\beta E_f}+3\Phi e^{-2\beta E_f}+e^{-3\beta E_f}\right)\right]$$

$$\text{quark energies } E_f = \sqrt{p_z^2+2n|Q_fB|+m^2}.$$

$$\Omega_M=\int\frac{d^3\mathbf{k}}{(2\pi)^3}\left[\frac{E_M}{2}+T\ln\left(1-e^{-E_M/T}\right)\right]$$

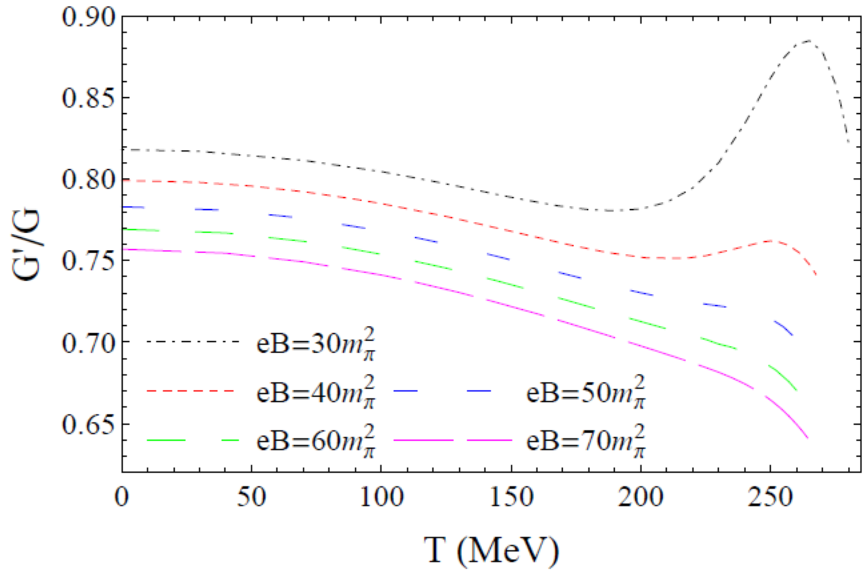
$$\text{meson energy } E_M = \sqrt{m_M^2+k_3^2+v_\perp^2(k_1^2+k_2^2)}$$

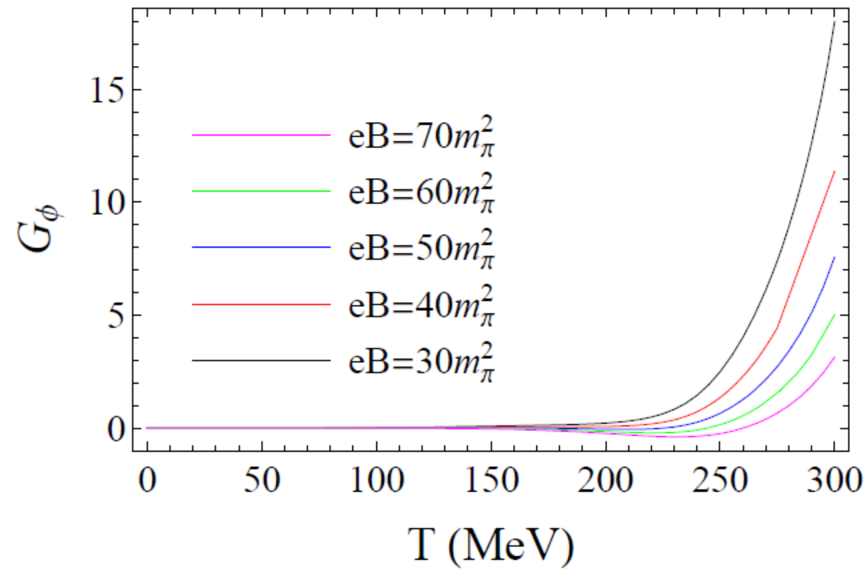
gap eqs beyond mean field

$$m\left(\frac{1}{2G'}+\frac{\partial\Omega_q}{\partial m^2}\right)=0,\qquad \frac{1}{2G'}=\frac{1}{2G}+\sum_M\frac{\partial\Omega_M}{\partial m^2}\Big|_{mf},$$

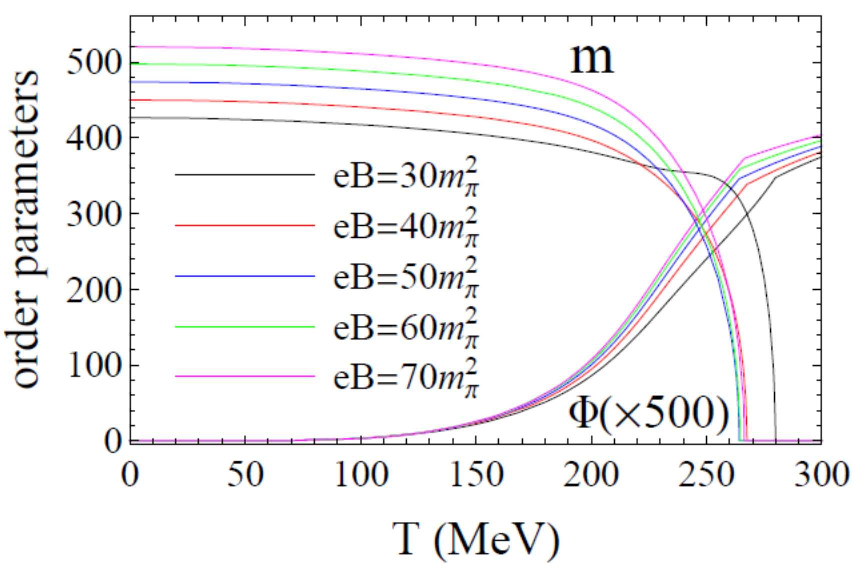
$$\frac{\partial\mathcal{U}}{\partial\Phi}+\frac{\partial\Omega_q}{\partial\Phi}=G_\phi\qquad G_\phi=-\sum_M\frac{\partial\Omega_M}{\partial\Phi}\Big|_{mf},$$

$$m\left(\frac{1}{2G'}+\frac{\partial\Omega_q}{\partial m^2}\right)=0,\ \frac{\partial\mathcal{U}}{\partial\Phi}+\frac{\partial\Omega_q}{\partial\Phi}=G_\phi$$

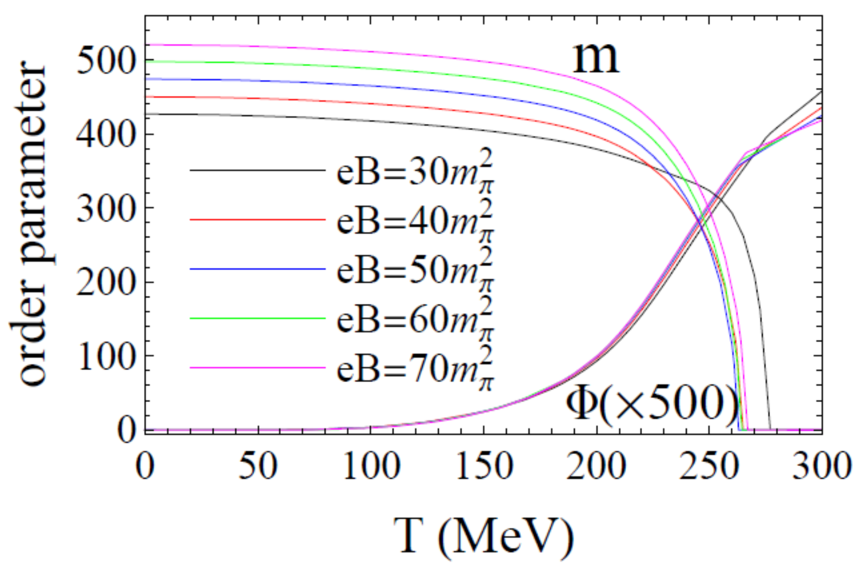




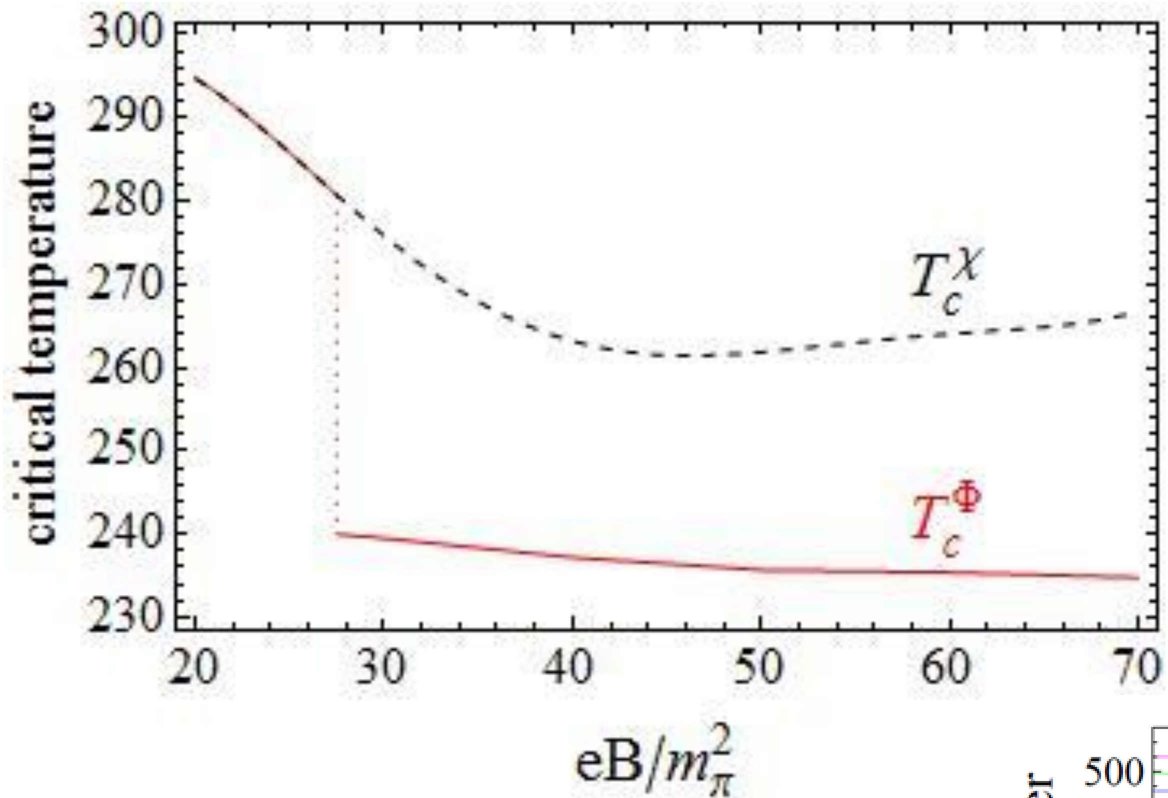
without G_ϕ term



with G_ϕ term

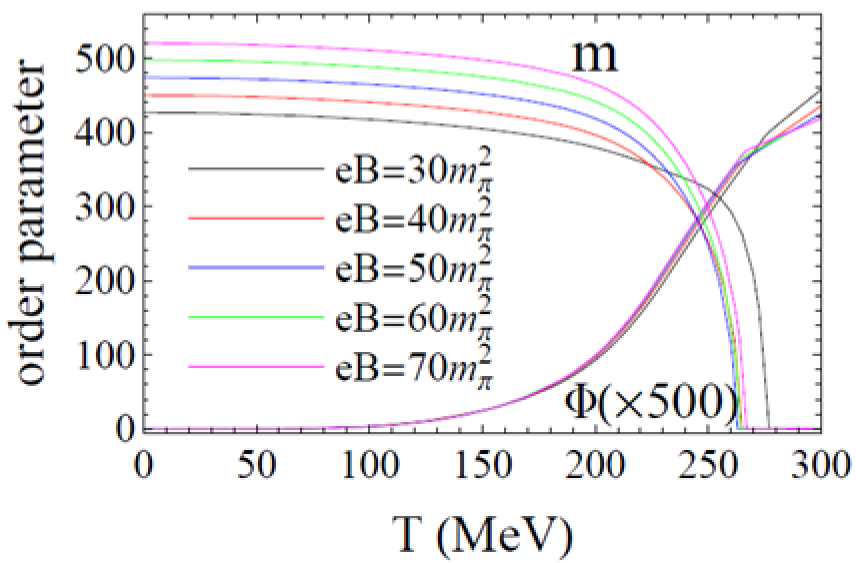


chiral and deconfinement phase transition



$$m(T_c^\chi, B) = 0,$$

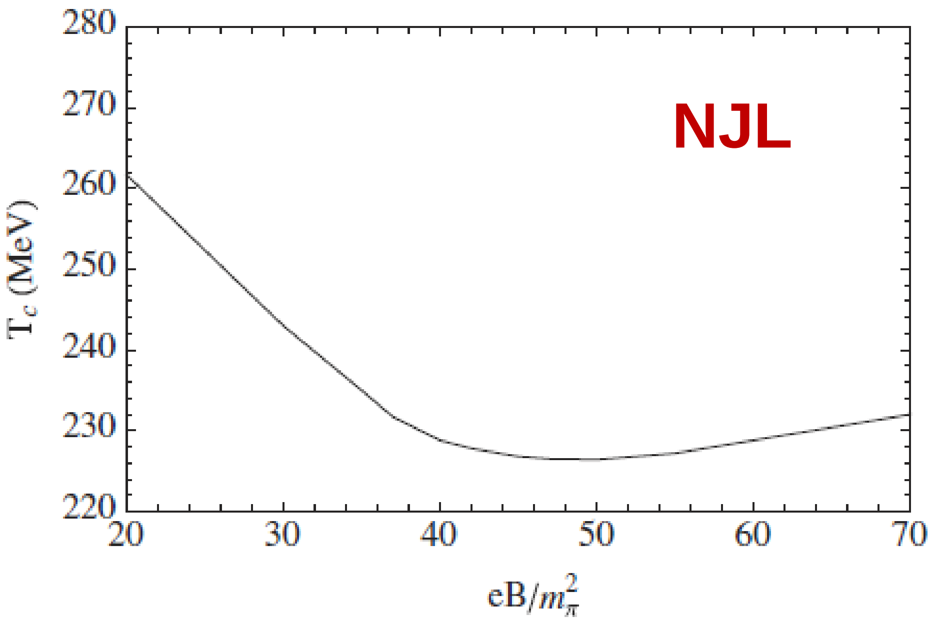
maximum value of $\frac{d\Phi}{dT}$

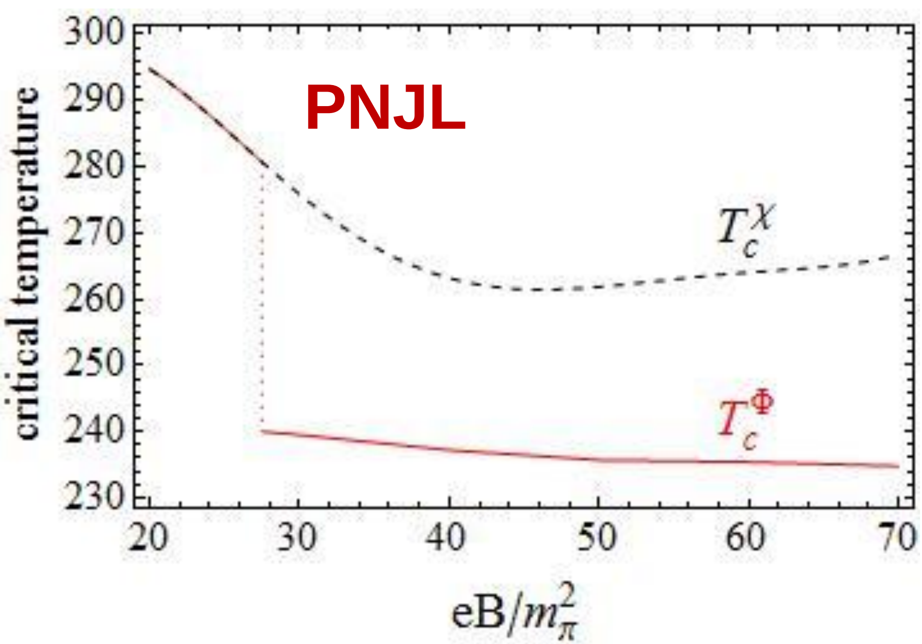


Summary

Inverse magnetic catalysis for chiral and deconfinement phase transition in (P)NJL model

quantum fluctuations ⟶ "running" coupling G'(B,T)





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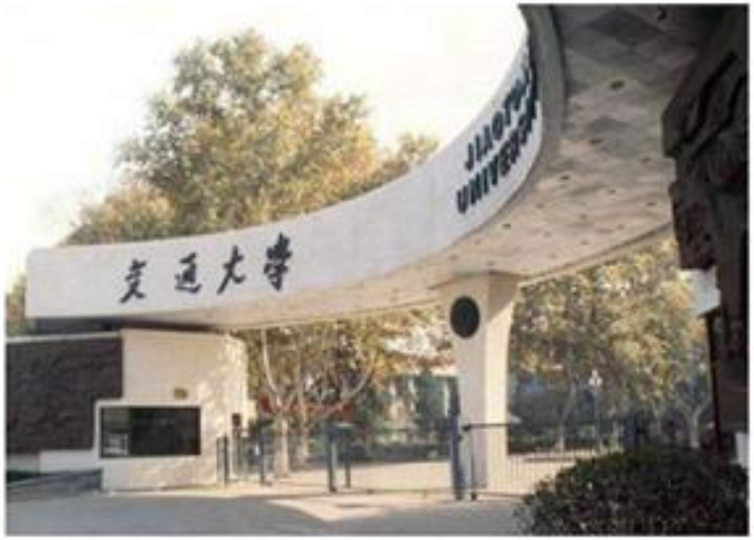

2017.7.21-23 西安(Xi’an)

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端午节快乐！

大鹏一日同风起，扶摇直上九万里。
假令风歇时下来，犹能簸却沧溟水。
时人见我恒殊调，闻余大言皆冷笑。
宣父犹能畏后生，丈夫未可轻年少。

谢谢大家！

summary (1)

NJL模型中，超出平均场地研究磁场对手征相变的影响，
磁催化 or 磁反催化？

量子涨落(介子效应) ⟶ 跑动耦合G'(B,T)
⟶ 实现低温磁催化和高温磁反催化(与格点一致)