Finite-size effect on rotating and/or magnetized fermionic matter

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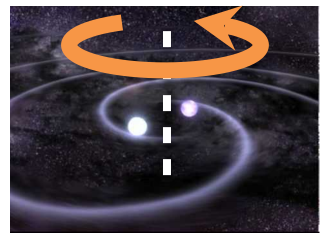
HL. Chen, K. Fukushima, XG. Huang, KM, PRD 93, 104052 (2016)
S. Ebihara, K. Fukushima, KM, PLB 764, 94 (2017)
HL. Chen, K. Fukushima, XG. Huang, KM, arXiv:XXXX.XXXX (2017)

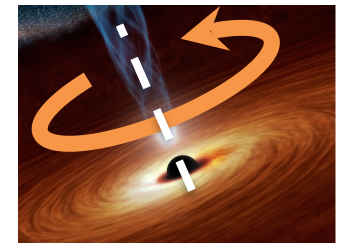
Rotating Relativistic systems

binary star merger

NASA/Tod Strohmayer

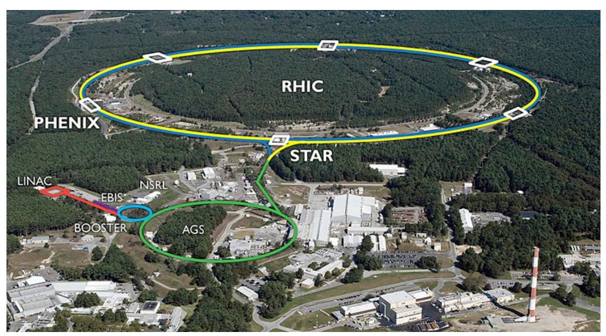


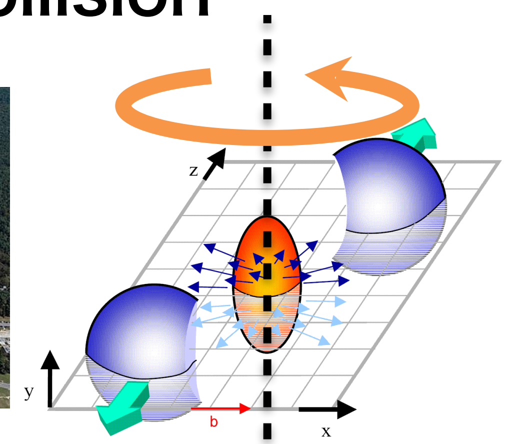
black holes



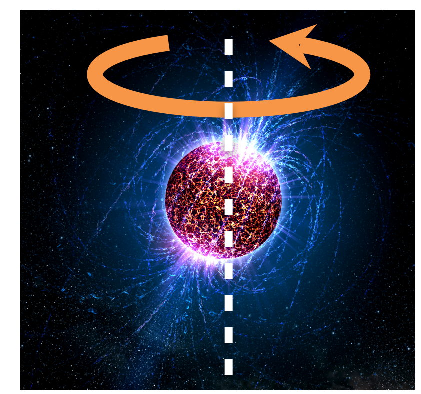
NASA/JPL-Caltech

heavy-ion collision





neutron stars



rotating (and magnetized) QCD systems

Heavy-ion collisions $eB \sim 10^{18}\,\mathrm{G}$ Skokov et al. (2009)

$\Omega \sim 10^{-1}\,\mathrm{MeV}$ (local rotation) Jiang, Lin, Liao (2016) Deng, Huang (2016)

Magnetar $eB \sim 10^{15}\,\mathrm{G}$ (surface) $eB \sim 10^{18-20}\,\mathrm{G}$ (interior)

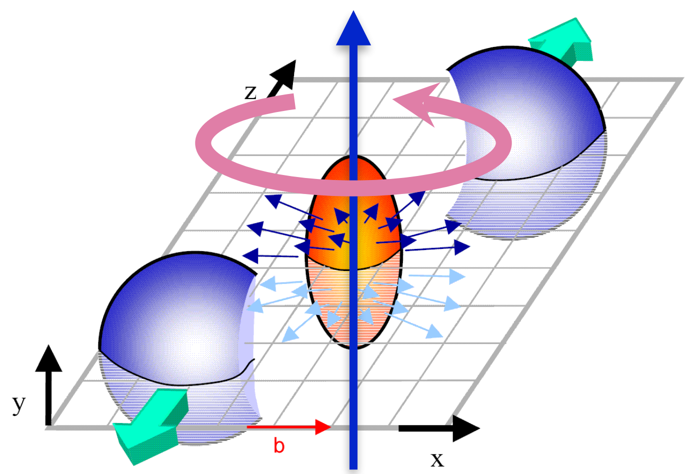
Duncan, Thompson (1992) Lai, Shapiro (1991) Ferrer et al. (2010)

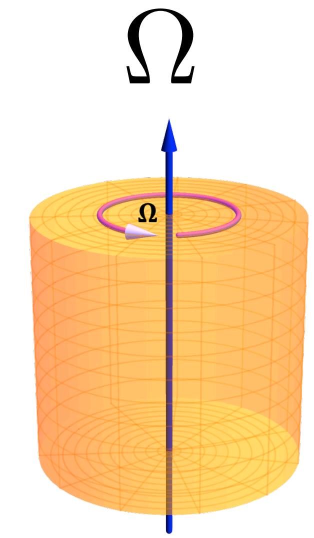
$R\Omega \sim 10^{-1}$ Marshall et al. (2004)

Contents

Part I Finite-size system with Ω

Rigidly Rotating System



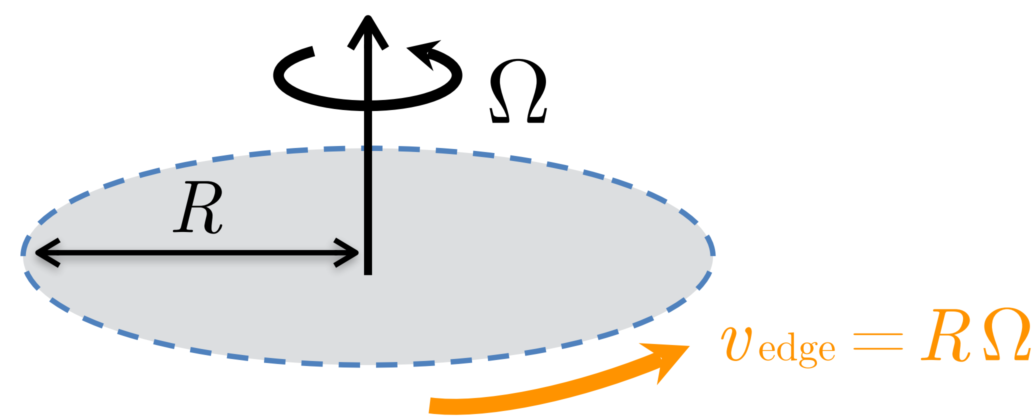


Chernodub, Gongyo (2016)

causality constraint

$$v_{\mathrm{edge}} = \Omega R \leq 1$$

$$\longrightarrow \quad R \leq 1/\Omega < \infty$$



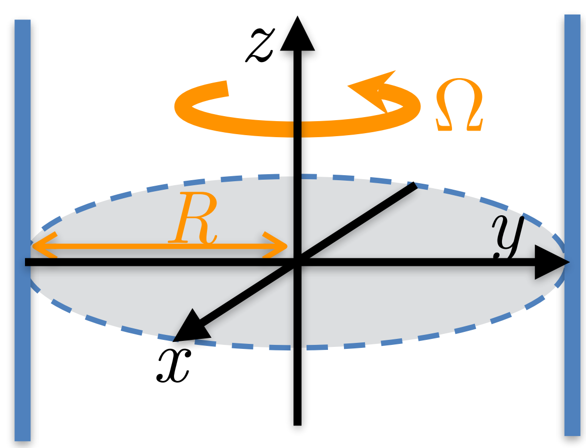
Rotating systems must be finite-size

coordinate transformation

$$x^{\mu}_{\mathrm{lab}} \longrightarrow x^{\mu}_{\mathrm{rot}}$$

Rotating Fermions

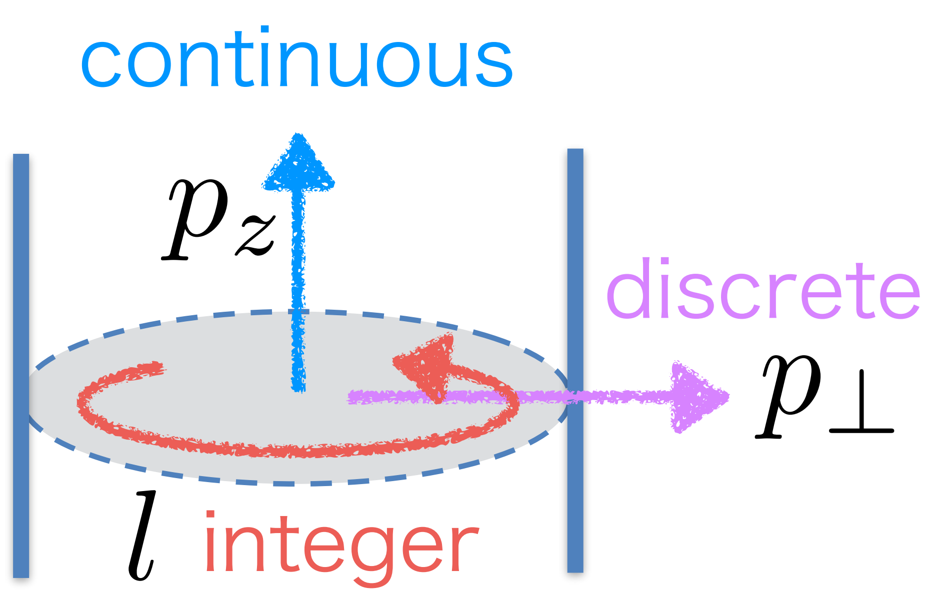
$$[i\gamma^\mu(\partial_\mu + \Gamma_\mu) - m]\psi = 0$$



$$g_{\mu\nu} = \begin{pmatrix} 1 - r^2\Omega^2 & y\Omega & -x\Omega & 0 \\ y\Omega & -1 & 0 & 0 \\ -x\Omega & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$\left[i\gamma^0 \left\{\partial_0 + \Omega\left(-x\partial_2 + y\partial_1 - \frac{i}{2}\sigma^{12}\right)\right\} + i\gamma^1\partial_1 + i\gamma^2\partial_2 + i\gamma^3\partial_3 - m\right]\psi = 0$$

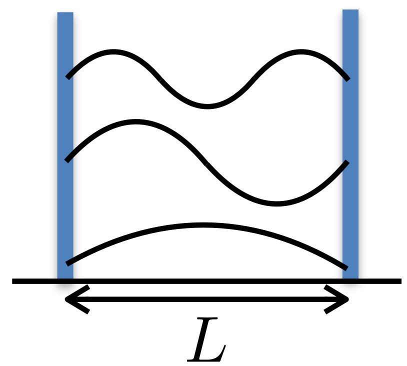
rotational energy



finite-size effect

Momentum Discretization

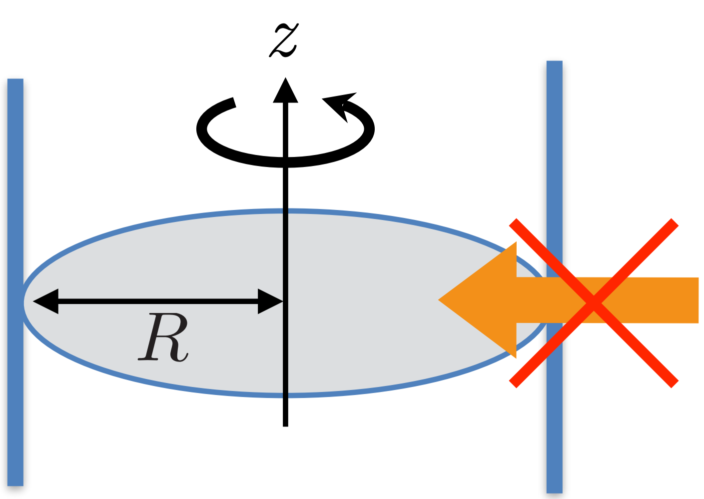
Ex.) Dirichlet b.c.



$$\sin(px)|_{x=L} = 0$$

$$\longrightarrow \quad p = \frac{n\pi}{L} \geq \frac{\pi}{L}$$

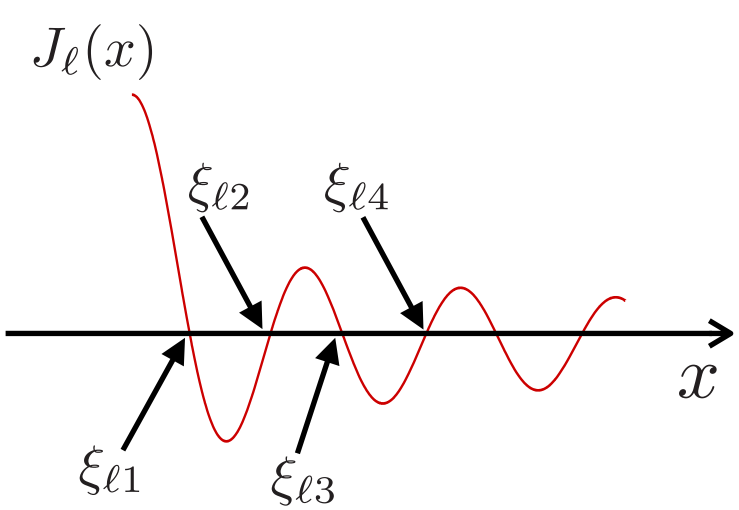
NO incoming current



$$\hat{e}_r \bar{\psi} \gamma^r \psi \Big|_{r=R} = 0$$

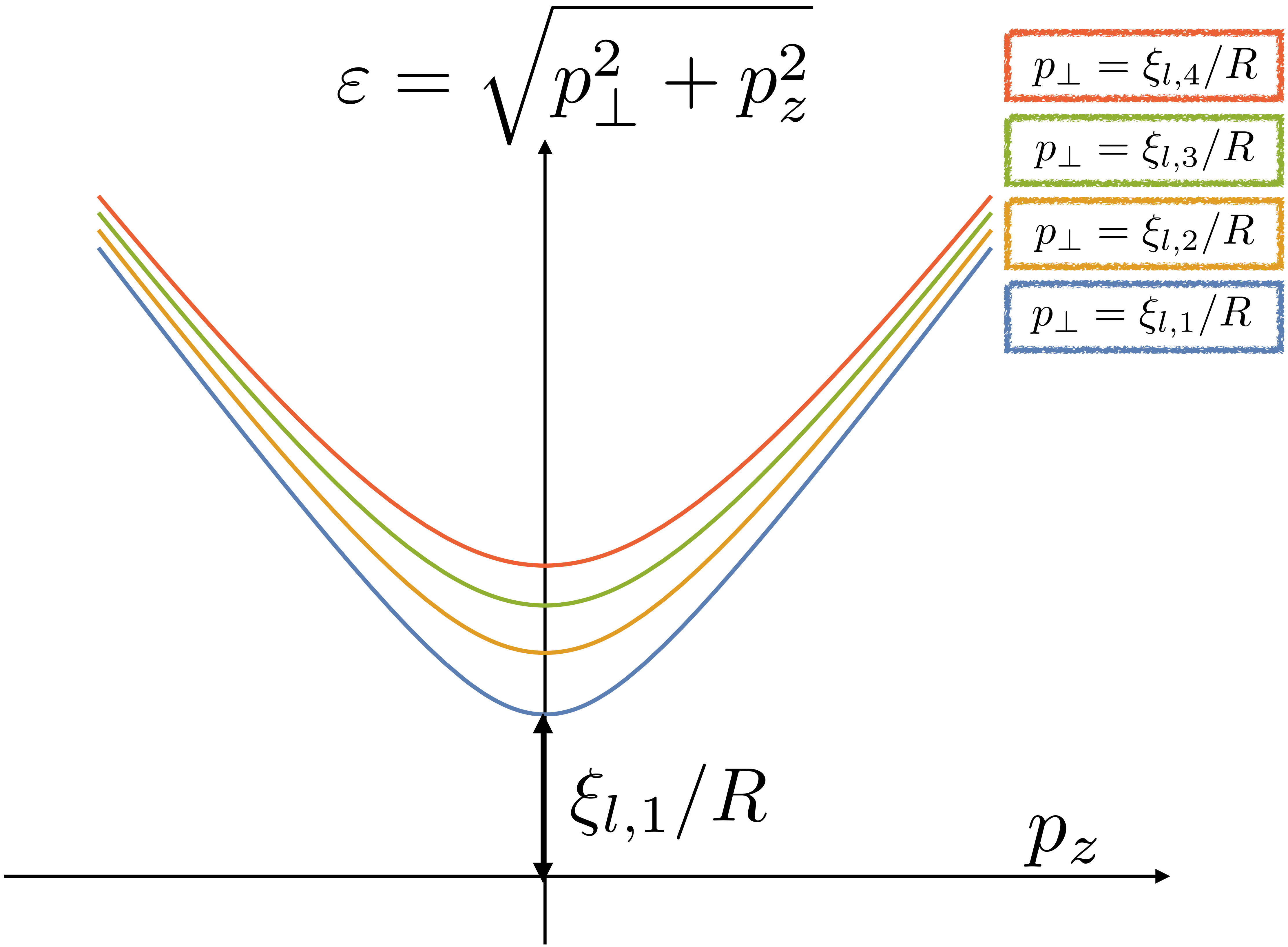
$$\longrightarrow \quad p_\perp \geq \frac{\xi_{l,1}}{R} \simeq \frac{2.4}{R}$$

IR gapped mode



$$x = \xi_{l,k} \ : \ \text{the } k\text{th root of } J_l(x)$$

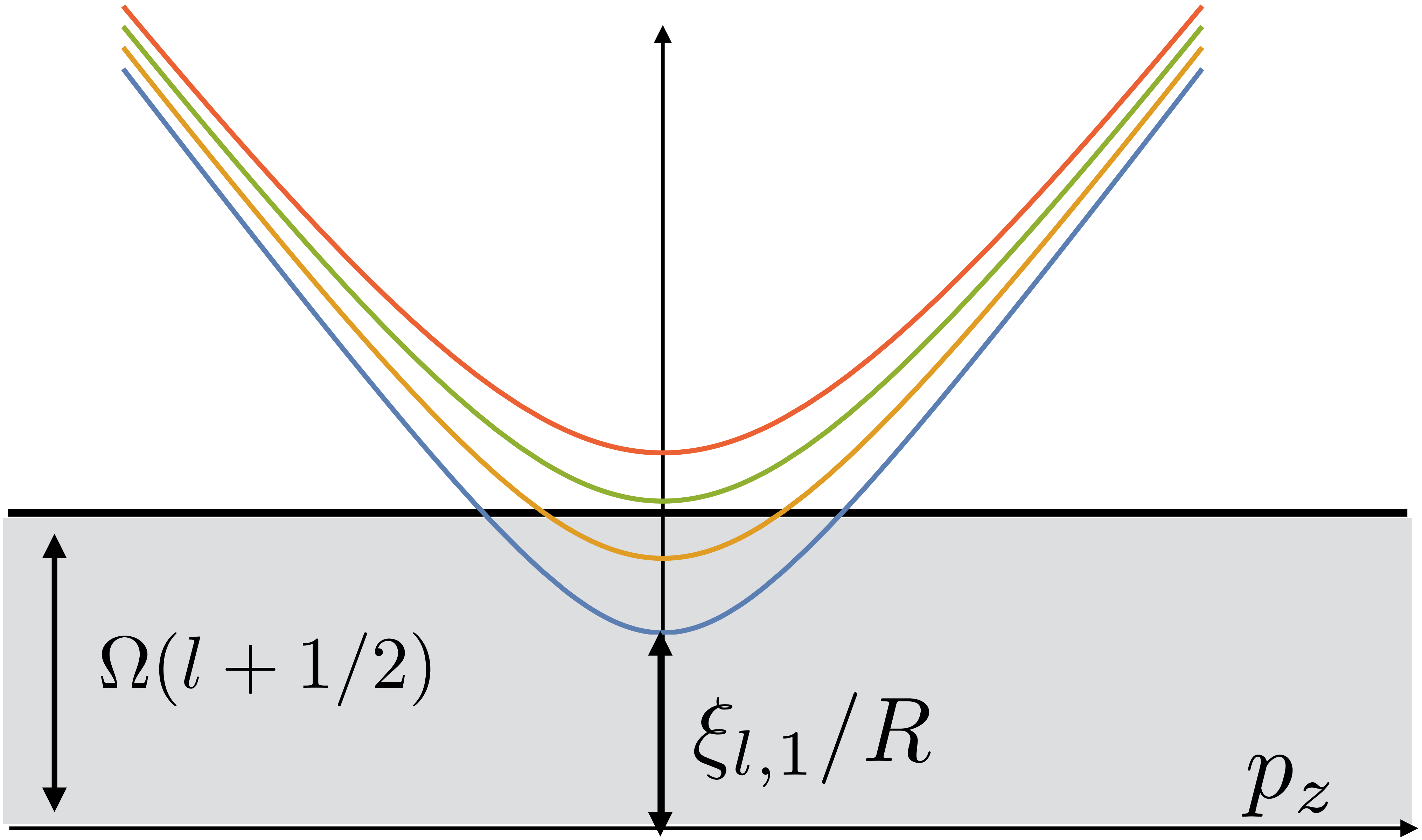
Rotational Effect at $T = 0$



Rotational Effect at $T = 0$

$$f(\varepsilon, j) = \frac{1}{e^{\beta(\varepsilon - \Omega j)} + 1} \longrightarrow f(\varepsilon, j) = \theta(\Omega j - \varepsilon)$$

Ωj = effective chemical potential

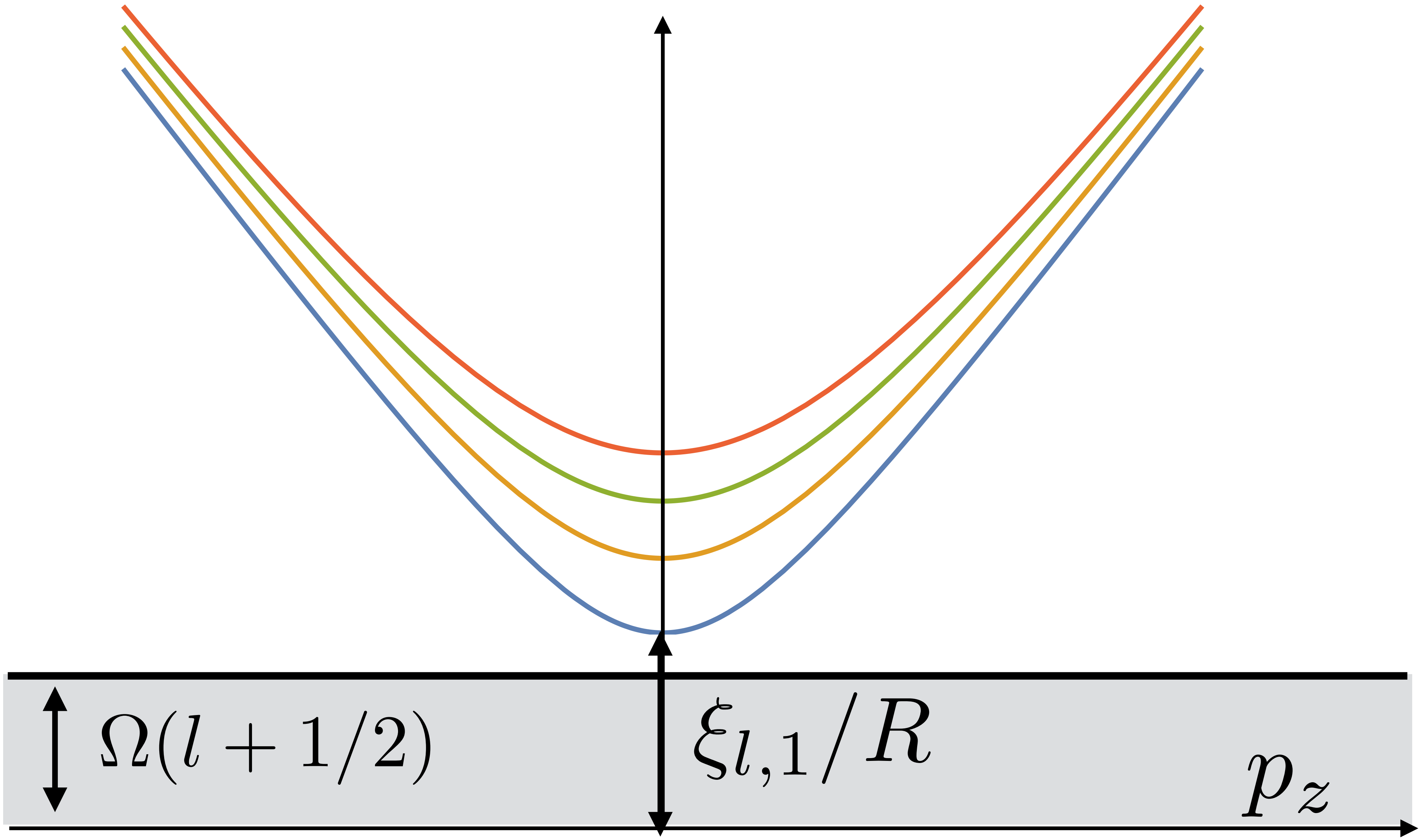


visible

Rotational Effect at $T = 0$

$$f(\varepsilon, j) = \frac{1}{e^{\beta(\varepsilon - \Omega j)} + 1} \longrightarrow f(\varepsilon, j) = \theta(\Omega j - \varepsilon)$$

Ωj = effective chemical potential



invisible

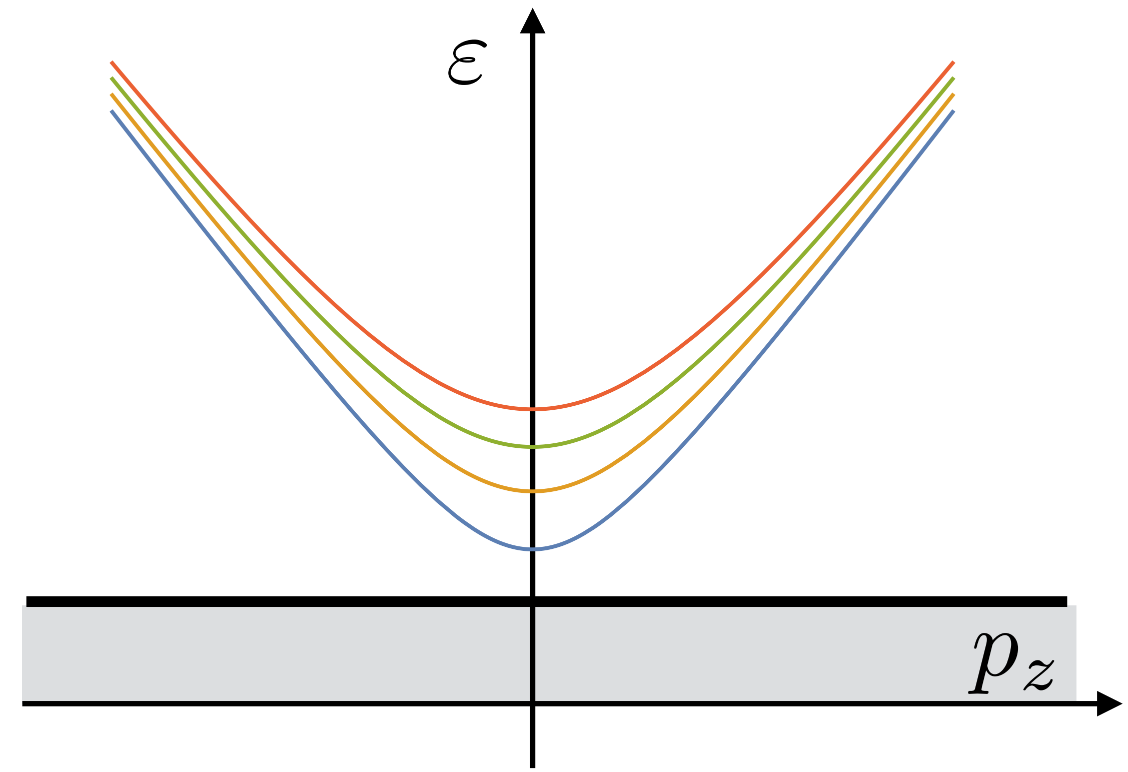
cf. Silver Blaze

Rotational Effect at $T = 0$

causality $\Omega \le 1/R$

$$\boxed{\xi_{l,1}/R > \Omega(l + 1/2)}$$

for arbitrary l



finite-size effect → NO rotational effect at $T = 0$

Ebihara, Fukushima, KM (2016)

Note : visible at finite temperature

CVE $j_5 = \dfrac{T^2}{12}\Omega$

Vilenkin (1979)

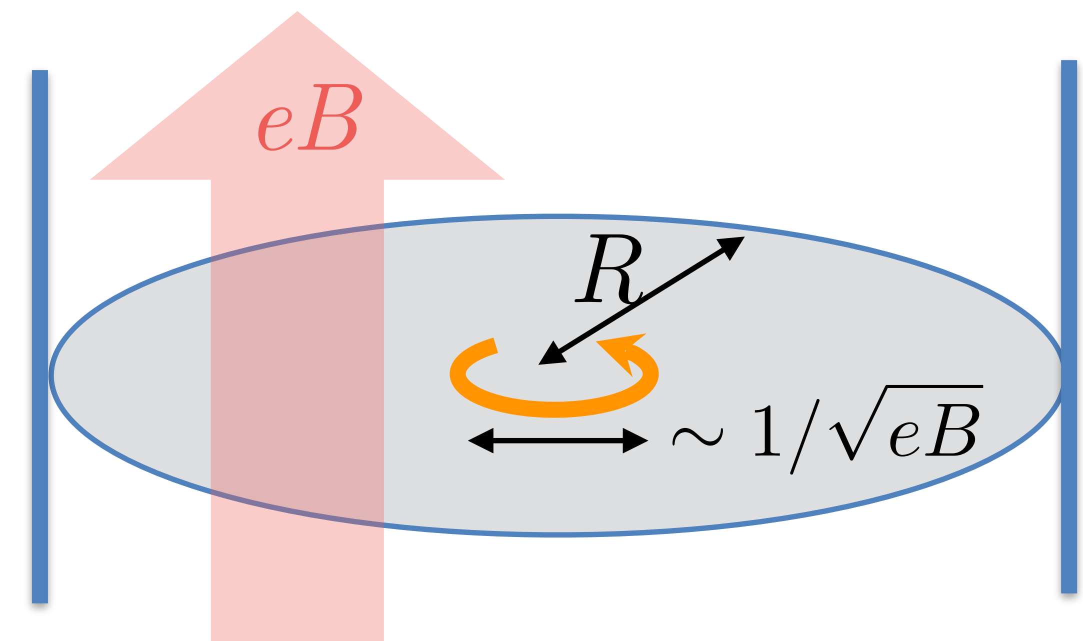
Part II Finite-size system with eB

Cyclotron Motion

(1) $1/\sqrt{eB} \ll R$

$\longrightarrow \quad p_\perp = \sqrt{2neB}$

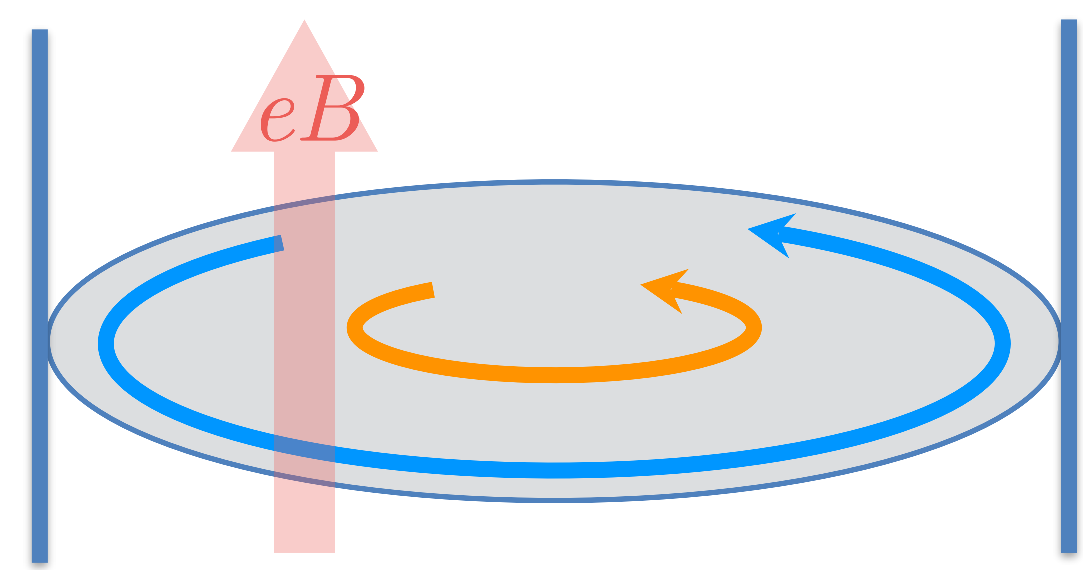
independent of l



(2) $1/\sqrt{eB} \lesssim R$

small l $\longrightarrow$ still $p_\perp \simeq \sqrt{2neB}$

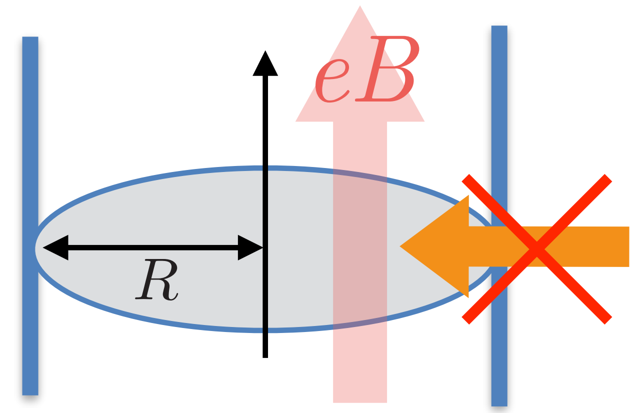
large l $\longrightarrow$ modified?



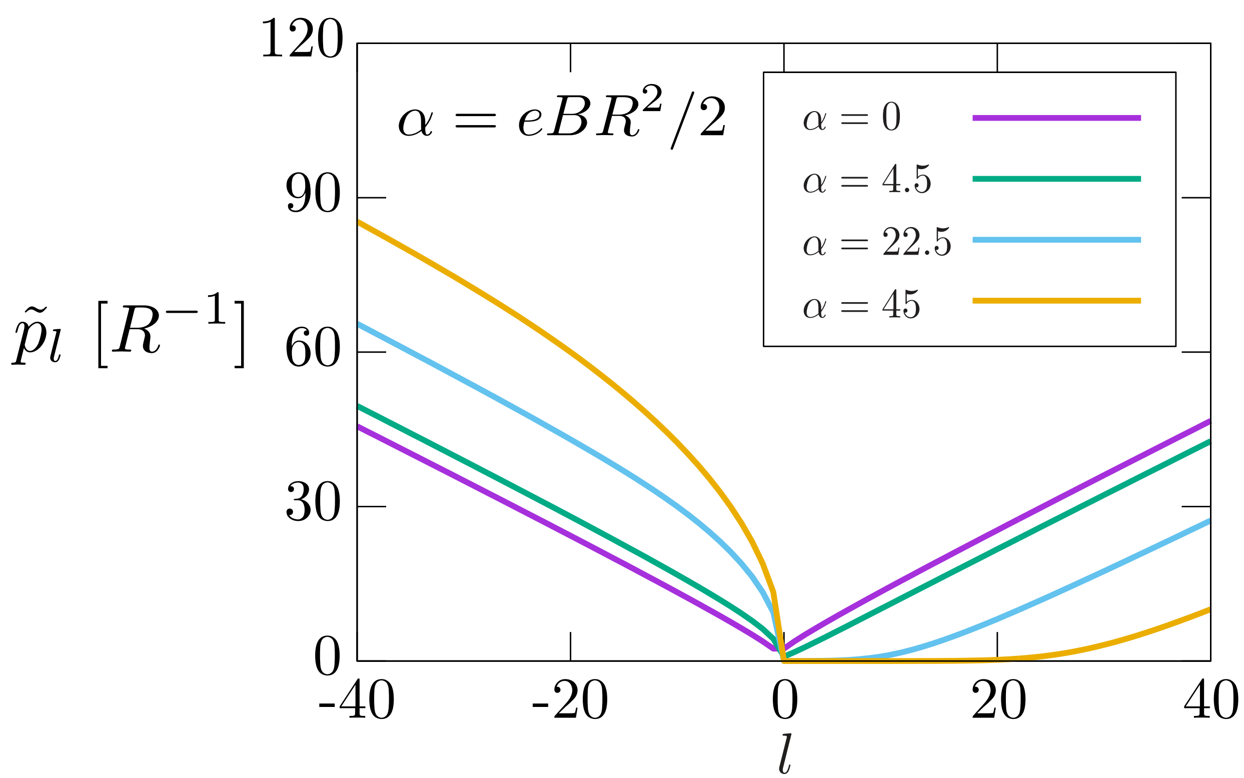
"Incomplete Landau quantization"

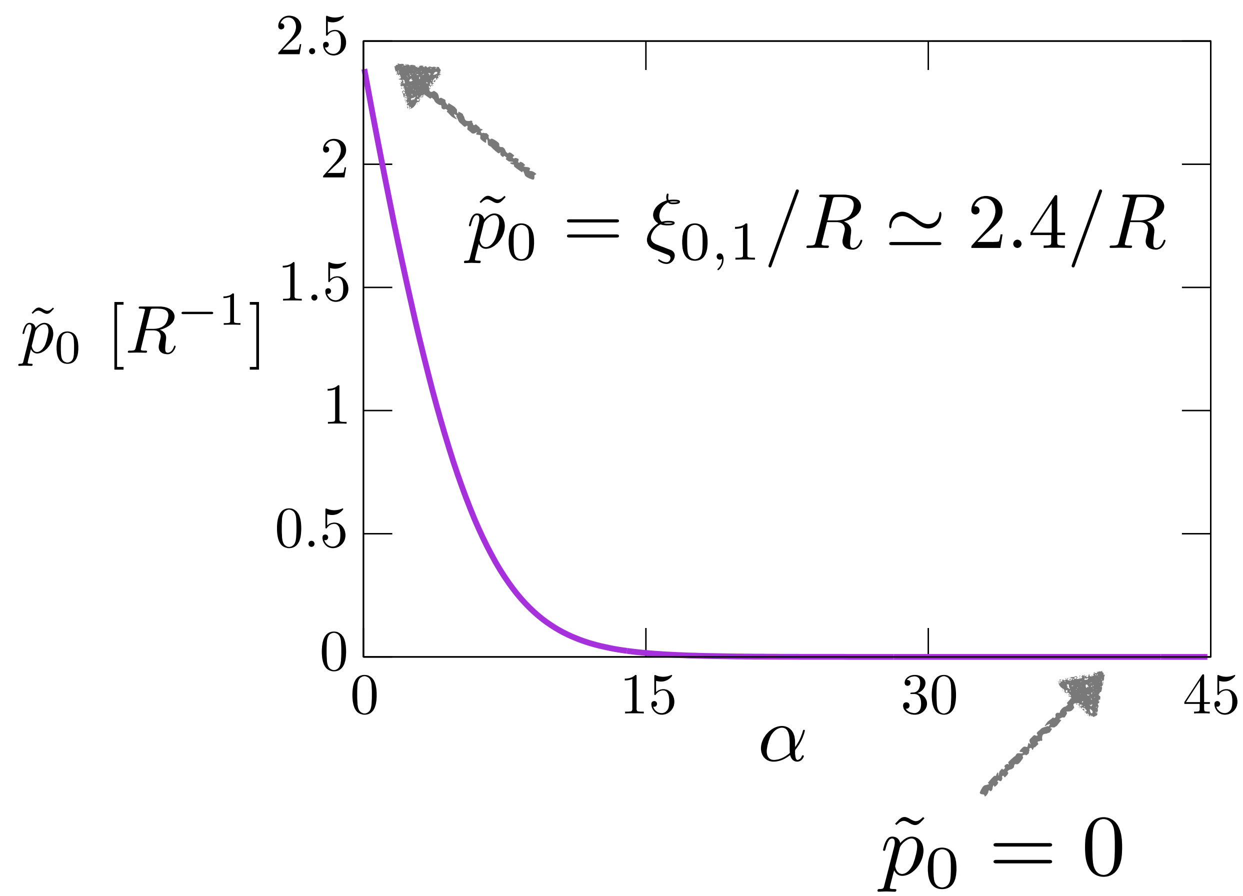
Incomplete Landau Level

$$[i\gamma^\mu(\partial_\mu + ieA_\mu) - m]\psi = 0$$ with

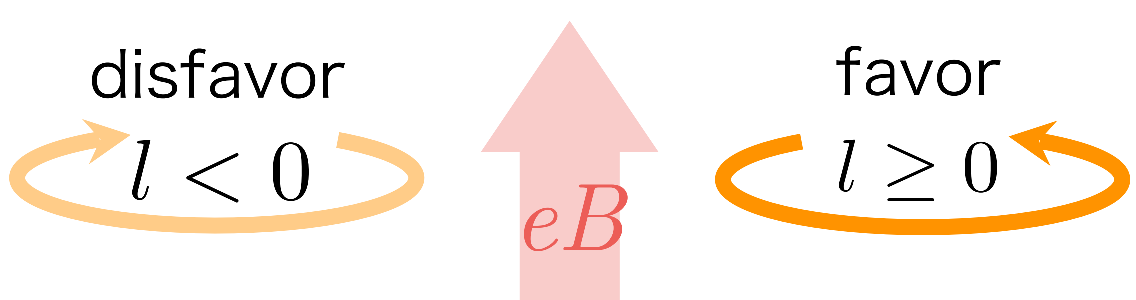


$\tilde{p}_l$ = lowest transverse momentum for l

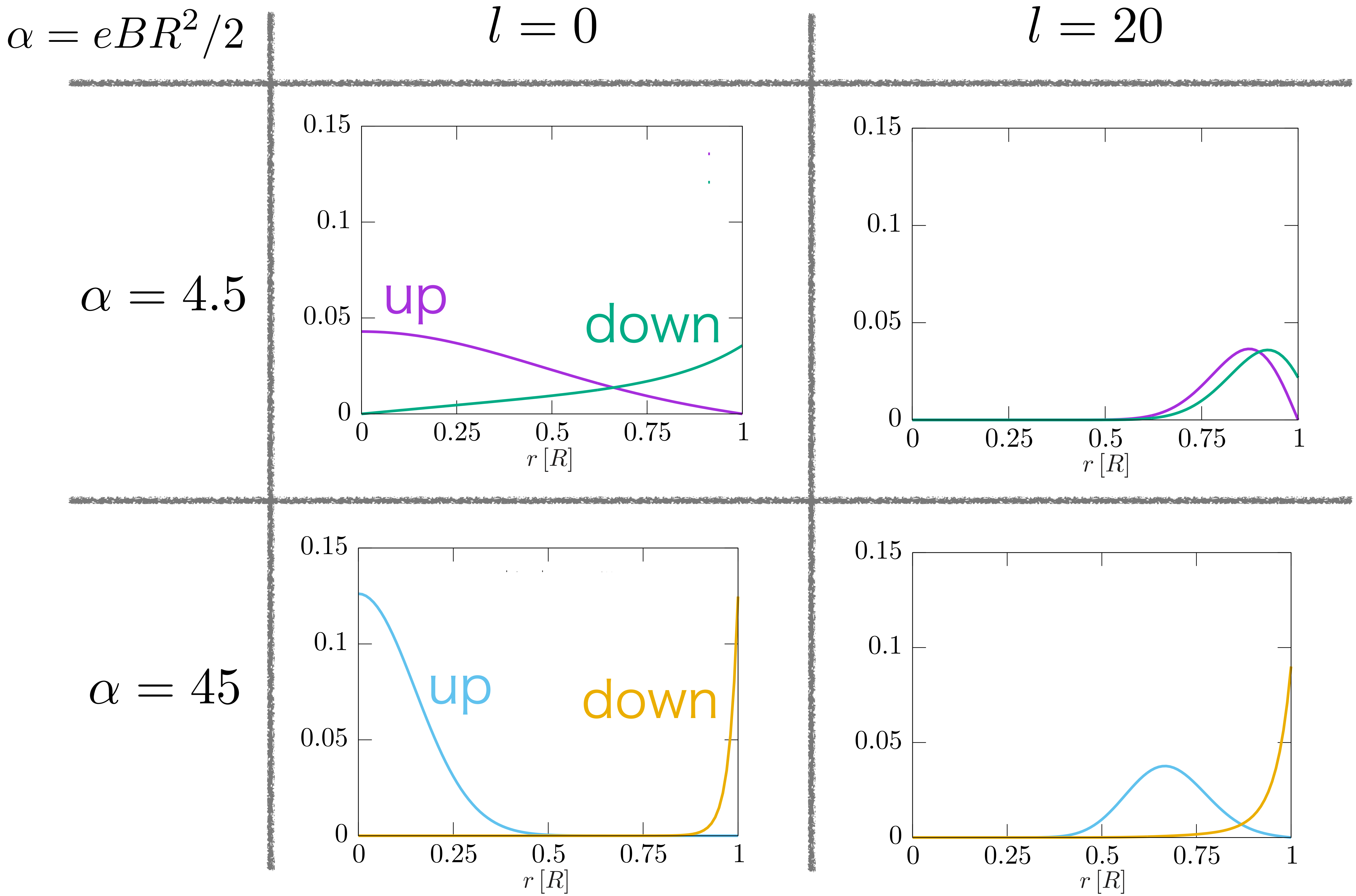




Landau zero mode



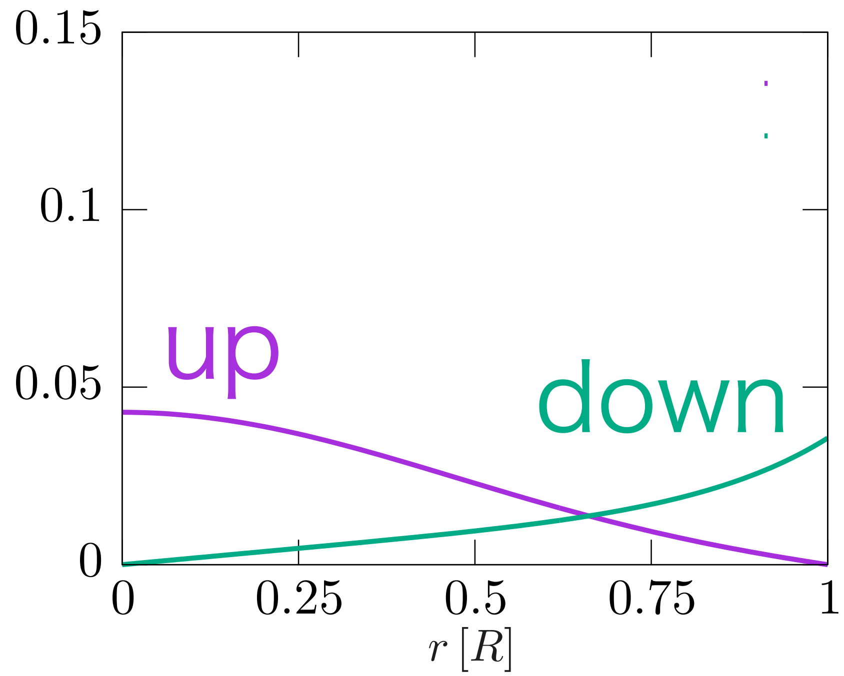
Wave Functions of Lowest Modes



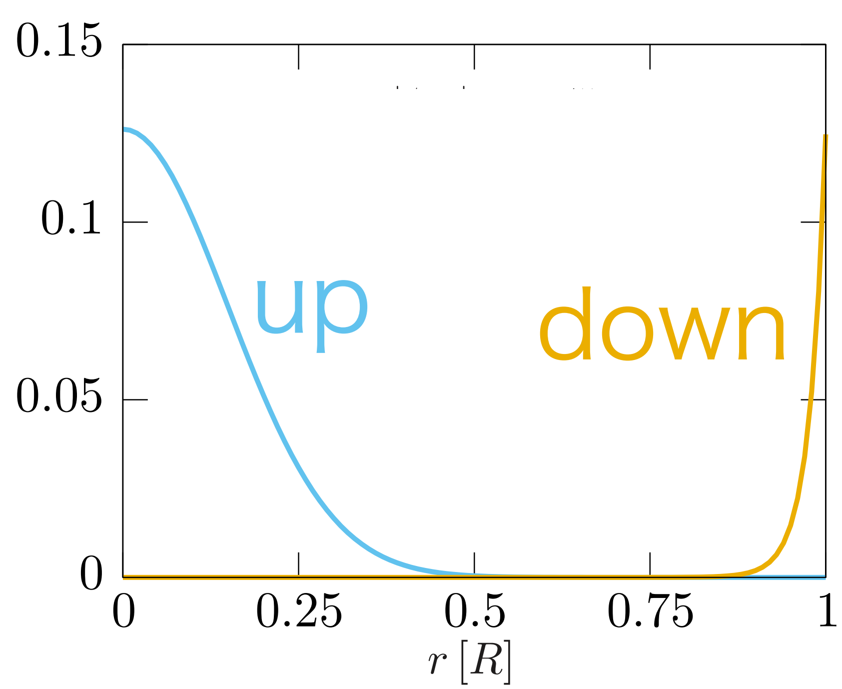
Wave Functions of Lowest Modes

$\alpha = eBR^2/2$ | $l = 0$ | $l = 20$

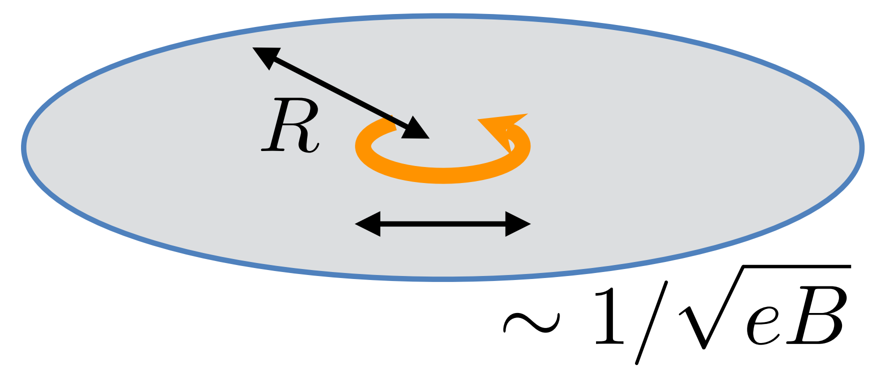
$\alpha = 4.5$



$\alpha = 45$



Landau localization



$\delta(r - R)$ for $R \to \infty$

No LLL for down-spins

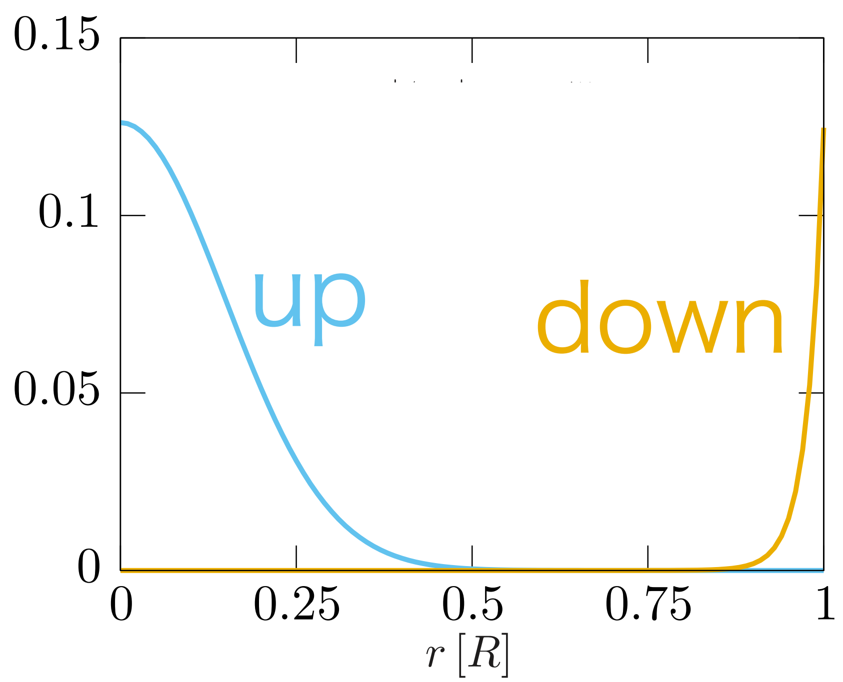
Wave Functions of Lowest Modes

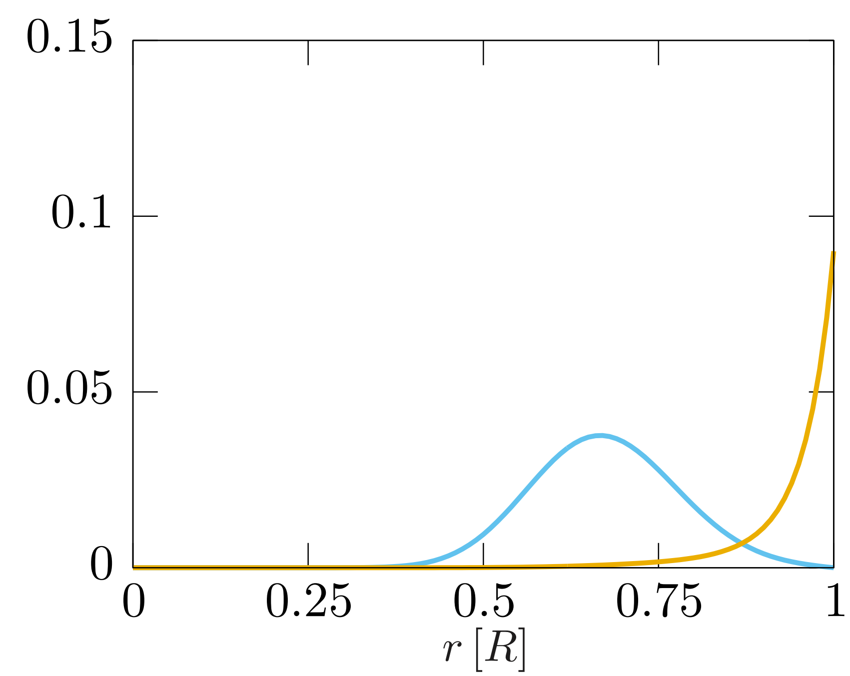
$\alpha = eBR^2/2$ | $l = 0$ | $l = 20$

many localized down-spin modes $(l = 0, \cdots, 20)$

→ strong magnetic effects around $r = R$

$\alpha = 45$



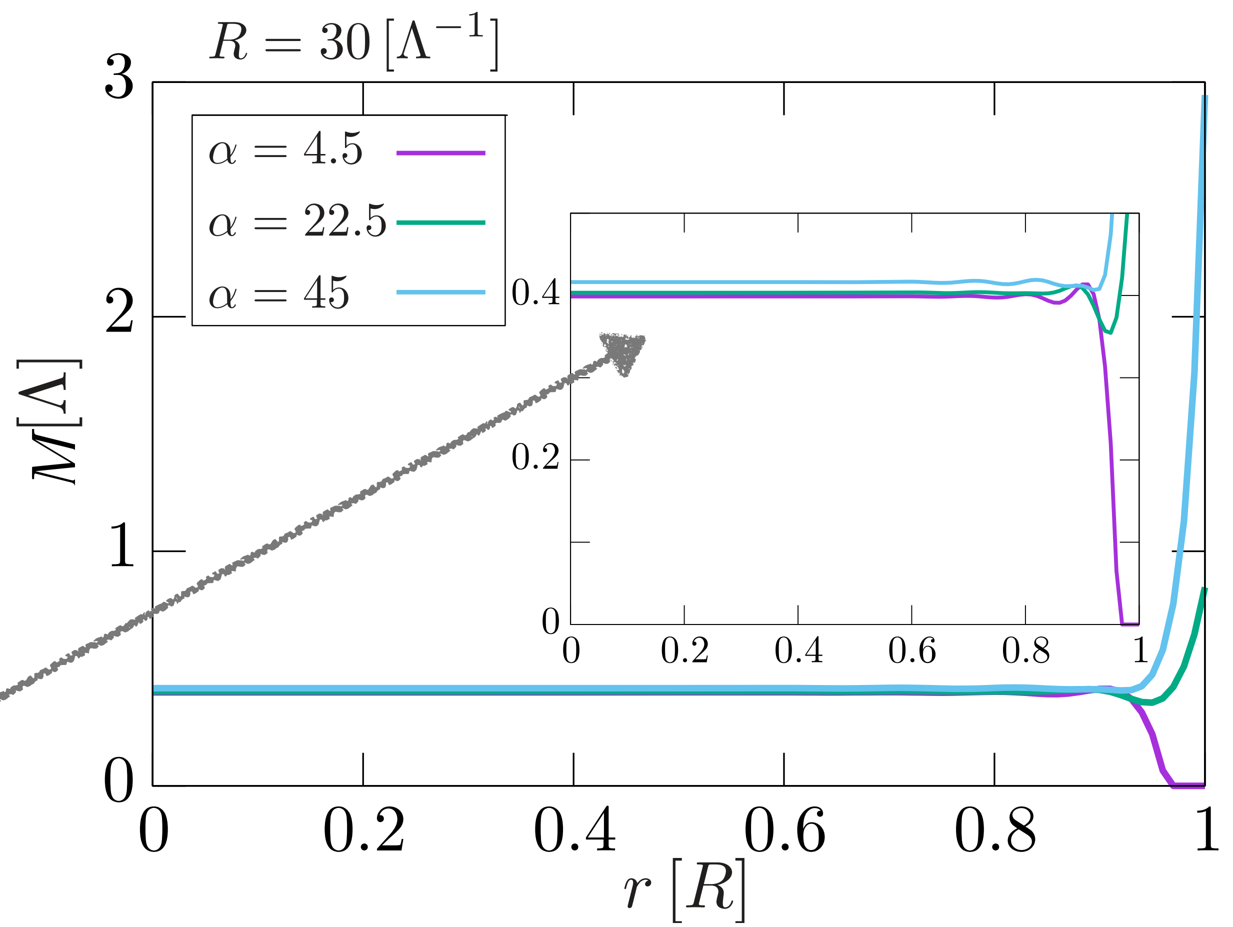


Dynamical Mass at $T = 0$

$$\mathcal{L}_{\mathrm{NJL}} = \bar{\psi}\, i\gamma^{\mu}(\partial_{\mu} + ieA_{\mu})\psi + \frac{G}{2}\left[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma^5\psi)^2\right]$$

NJL model (mean field approx.) + Local density approx.

$\partial_r M \ll M^2$ Jiang, Liao (2016)



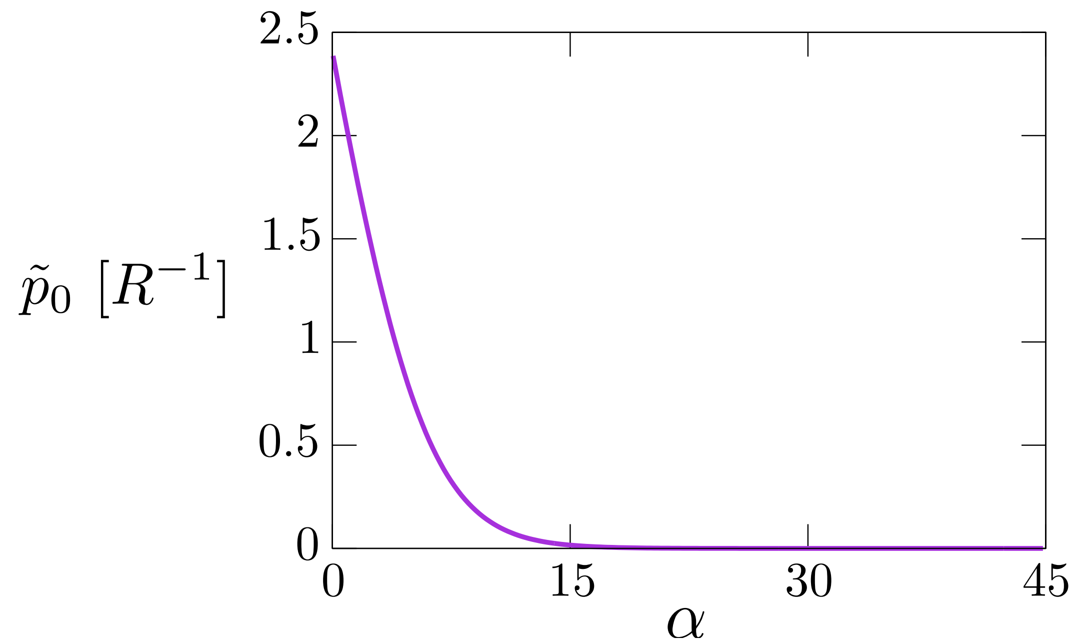
usual
magnetic catalysis

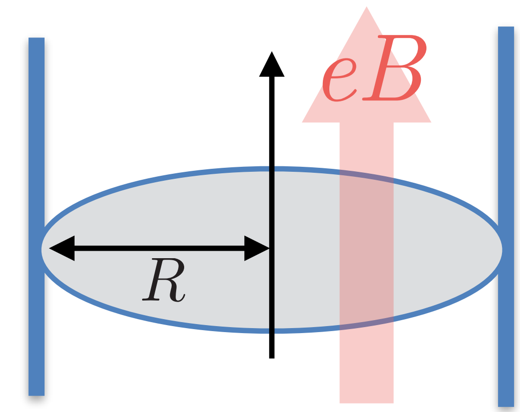
much stronger
enhancement

many localized modes

Part III Finite-size system with Ω and eB

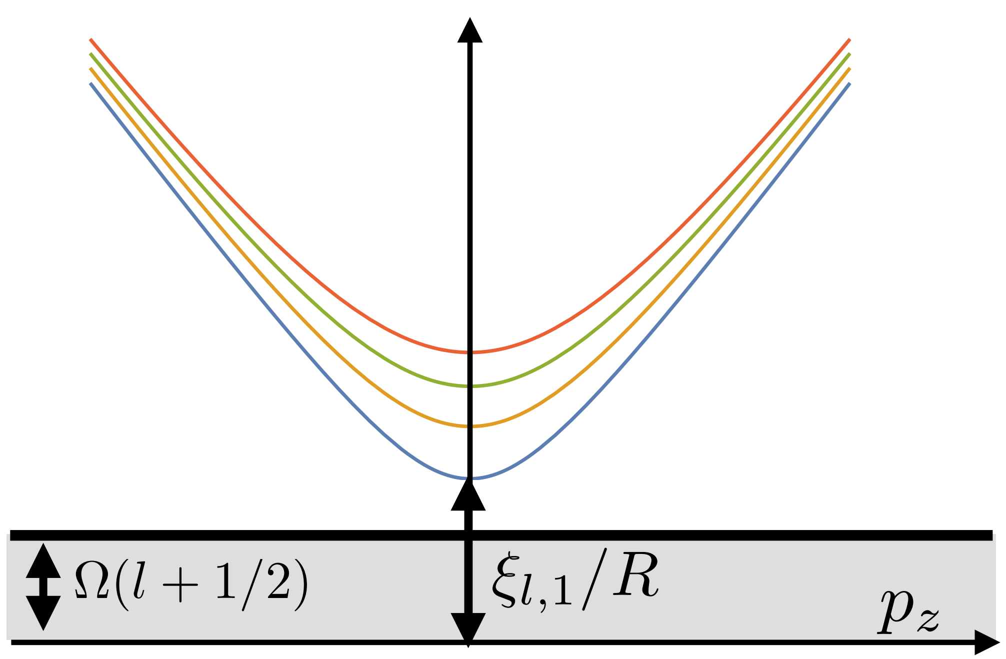
Gapped to Gapless

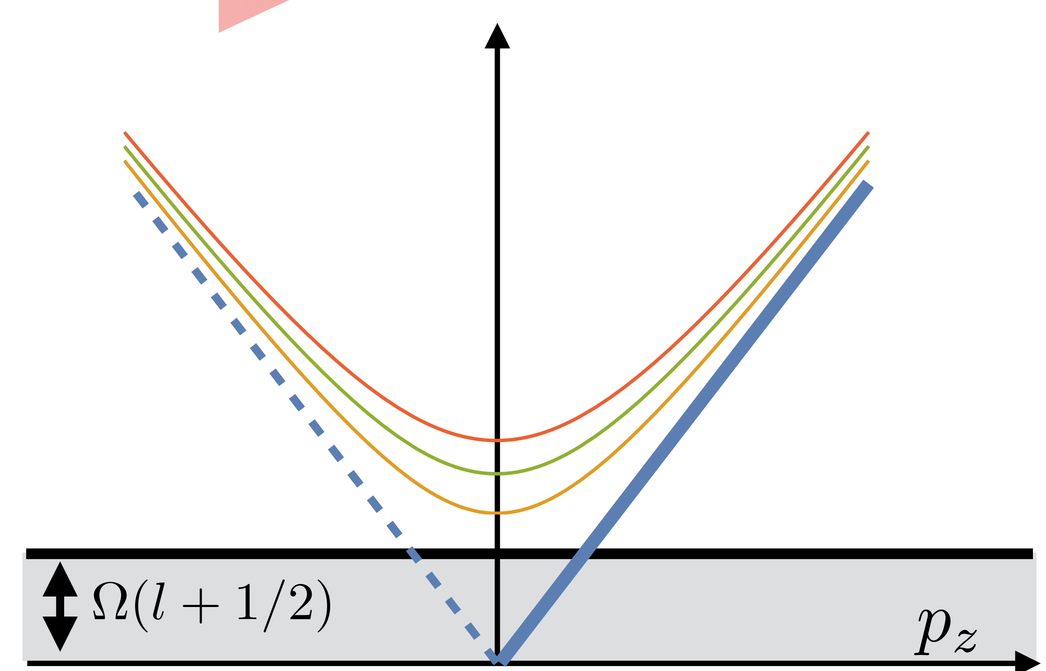




$\alpha = eBR^2/2$

weak magnetic field strong



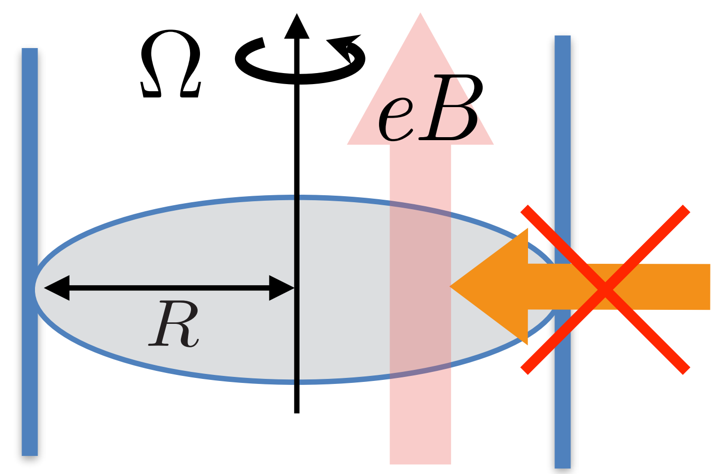


visible rotational effect due to magnetic field

Ex.1) Density induced by Rotation

Huang, KM (in preparation)

$$[i\gamma^\mu(\partial_\mu + ieA_\mu + \Gamma_\mu)]\psi = 0$$ with



magnetic field

rotation

Ωj = effective chemical potential $\quad f_\pm(\varepsilon) = \dfrac{1}{e^{\beta(\varepsilon \mp \Omega j)} + 1}$

$$n(r) = \langle \psi^\dagger(x)\psi(x)\rangle$$
$$= \sum_{p_z, p_\perp} \left[f_+(\varepsilon) - f_-(\varepsilon)\right] \times \left(r\text{-dependence}\right)$$

$$n(r=0) \xrightarrow[\sqrt{eB} \gg \Omega]{} \frac{eB\Omega}{4\pi^2}$$

temperature independent

Cf. Hattori, Yin (2016)
hydrodynamical description

A similar discussion is applicable?

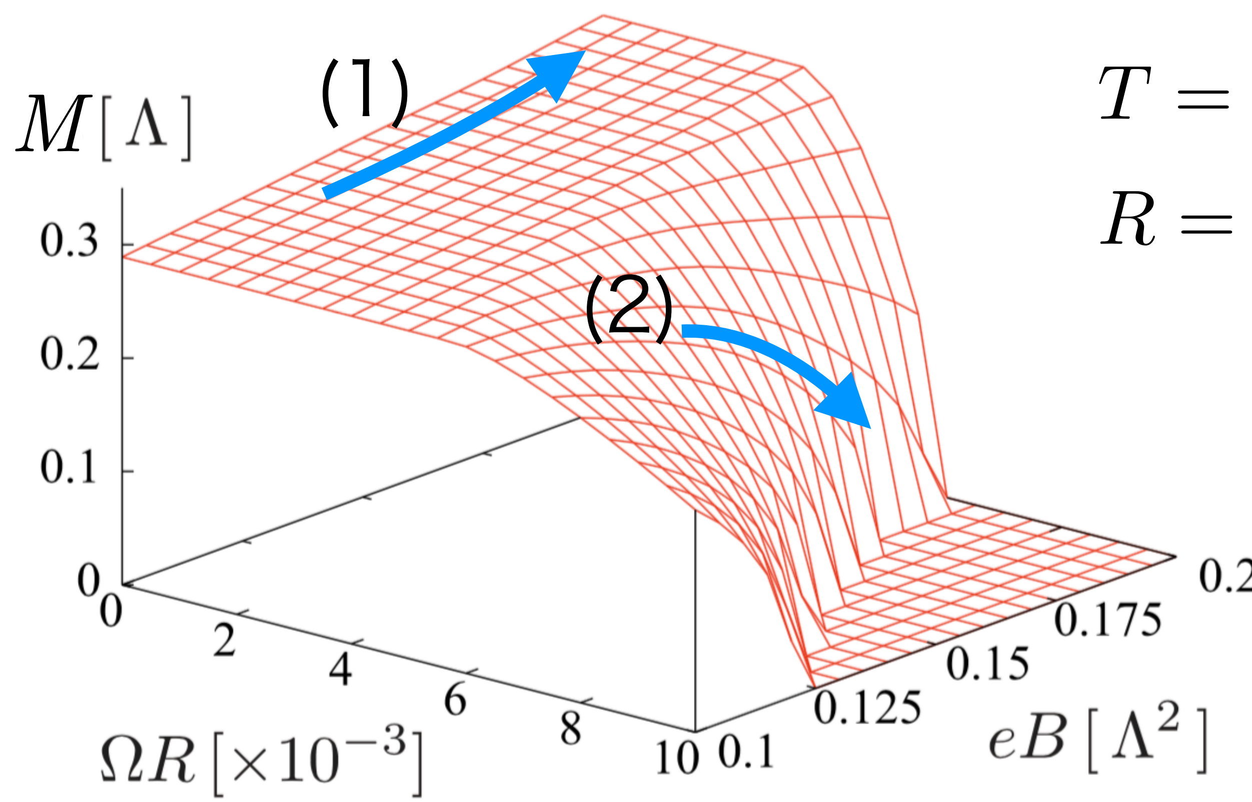
$$T^{ij} = \#\, eB^i \Omega^j$$

Hermandez, Kovtun (2017)
hydrodynamical description

Ex.2) Chiral Symmetry Breaking

Chen, Huang, Fukushima, KM (2016)

NJL model (mean field approx.) + neglecting inhomogeneity



$T = 0$

$R = 10^3\ [\Lambda^{-1}]$

(1) eB increases → M increases　　Magnetic Catalysis

(2) eB increases → M decreases　　**Inverse MC by Rotation**

Summary

[1] Rotating Systems

- No rotational effect at zero temperature
- Chiral phase transition   Critical Point?

[2] Magnetized Systems

- Strong magnetic response around the boundary (not only in a cylinder)

MC on the boundary in Dirac/Weyl semimetals?

[3] Rotating Magnetized Systems

- Visible rotational effect even at zero temperature
- Anomalous transport : electric current   energy-momentum tensor?
- Chiral structure : inverse phenomenon for the MC

spacial dependence?

Distribution in Rotating Systems

$$p_{\ell k} = \xi_{\ell k}/R \longrightarrow E - \Omega\ell > 0 \longrightarrow n_{\mathrm{BE}} = \frac{1}{e^{\beta(E-\Omega\ell)} - 1} > 0$$

IR gap

(Proof) Davies et. al (1996)

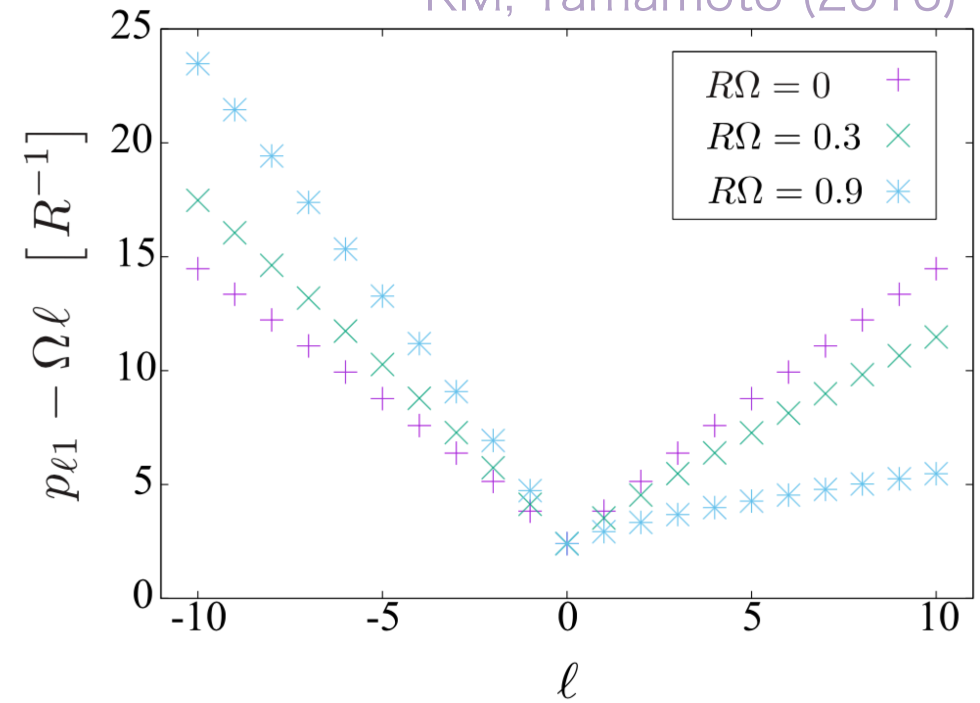
causality

$\Omega R \leqq 1$

$$\begin{aligned} E - \Omega\ell &\geq \frac{\xi_{\ell 1}}{R} - \Omega\ell \\ &= \frac{1}{R}\left[\xi_{\ell 1} - \Omega R \ell\right] \\ &\geq \frac{1}{R}\left[\xi_{\ell 1} - \ell\right] > 0 \end{aligned}$$

$$\boxed{\begin{aligned} &\xi_{01} = 2.40483 > 0 \quad (\ell = 0) \\ &\xi_{\ell 1} > \ell + 1.855757\ell^{1/3} + 0.5\ell^{-1/3} \quad (\ell \geq 1) \end{aligned}}$$

KM, Yamamoto (2016)



Boundary effect (= IR gapped mode) is important