Quark mass effect in anomalous transports

Shu Lin

Sun Yat-Sen University

Huada Symposium, CCNU, 5/26/2017

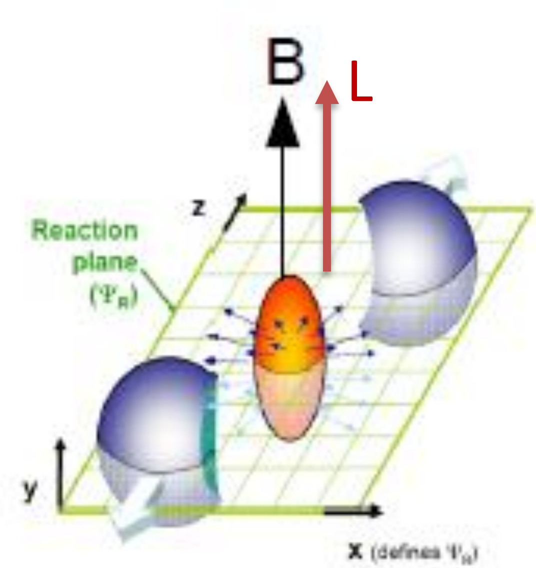
Guo, SL. PRD 2016, JHEP 2017

Outline

- Axial charge’s role in experimental search for anomalous effects
- Axial charge fluctuation/dissipation from topological transition
- Stochastic hydrodynamics for axial charge
- Axial charge fluctuation/dissipation from quark mass (dynamical)
- Quark mass effect on Chiral Separation Effect (static)
- Summary

Chiral Magnetic Effect

$$j = \frac{N_c \mu_5}{2\pi^2} eB$$



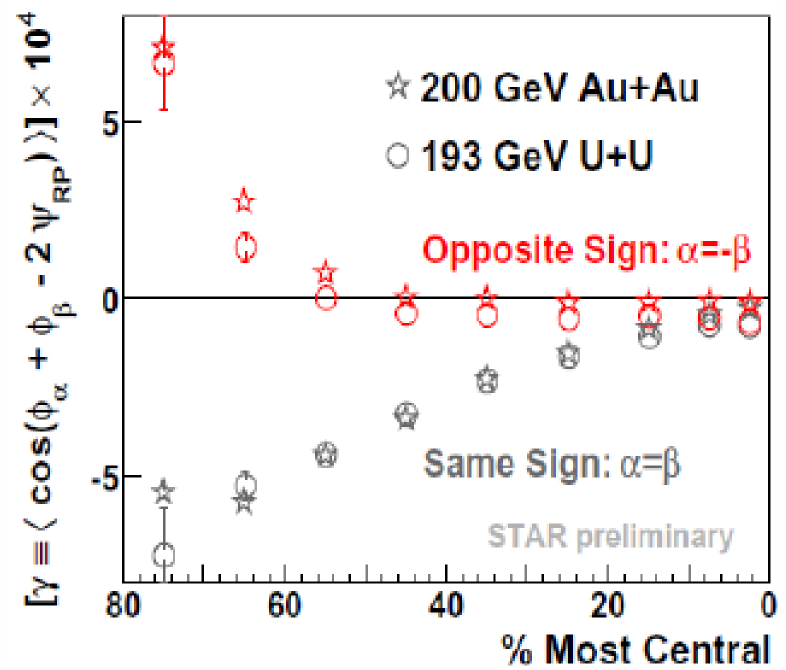
Kharzeev, Zhitnitsky, NPA 2007
Kharzeev, McLerran, Warringa, NPA 2008

Chiral Vortical Effect

$$j_B = \frac{N_c \mu_5 \mu}{2\pi^2} \omega$$

Kharzeev, Son, PRL 2011

Measurement of CME from electric charge correlation



$$j = \frac{N_c \mu_5}{2\pi^2} eB$$

Chiral imbalance characterized by μ_5, originates from fluctuation of n_5

$$\langle N_5 \rangle = 0, \langle {N_5}^2 \rangle \neq 0$$

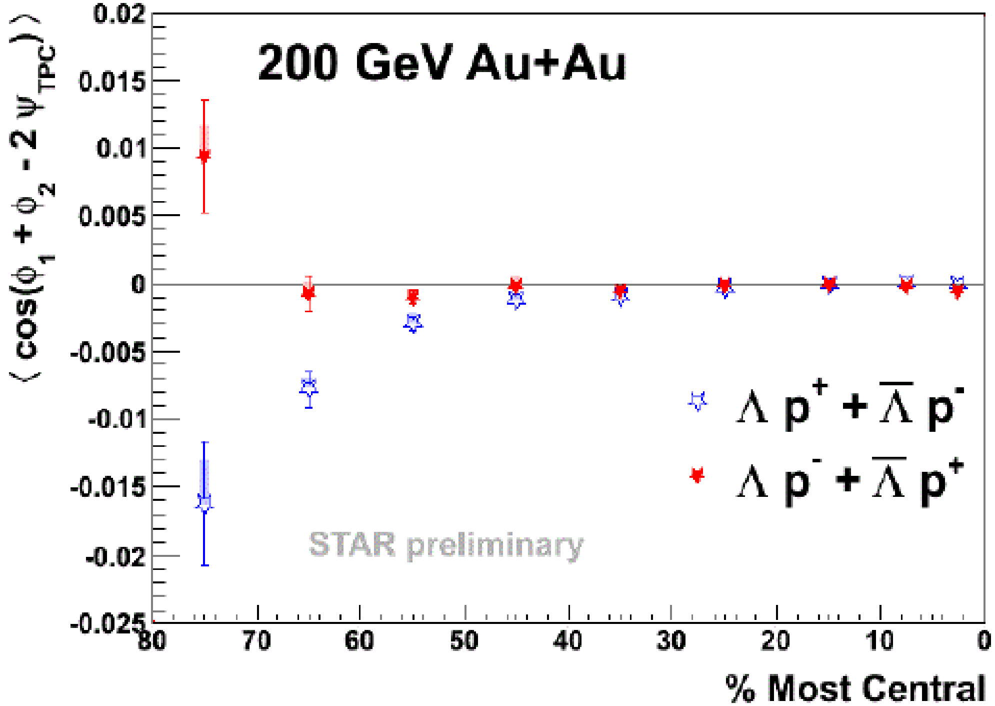
Generation of n_5: QCD anomaly

$$\partial_\mu {j_5}^\mu = -\frac{g^2 N_f}{8\pi^2} tr(G\tilde{G})$$

Same electric charge correlation enhanced than opposite electric charge correlation due to CME

STAR collaboration, PRL (2014), 1404.1433

Measurement of CVE from baryon charge correlation



$$j_B = \frac{N_c \mu_5 \mu}{2\pi^2} \omega$$

CVE also from fluctuation of n_5

Same baryon charge correlation enhanced than opposite baryon charge correlation due to CVE

Liwen Wen (STAR), RHIC AGS meeting 2015

Chiral Magnetic Wave

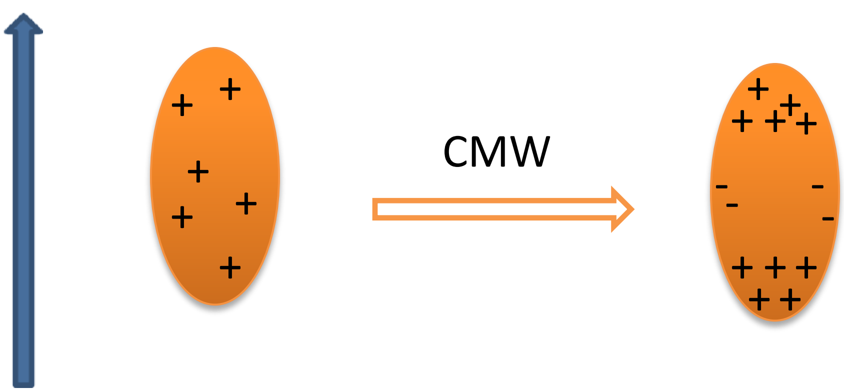
$$j = \frac{N_c \mu_5}{2\pi^2} eB \quad \xRightarrow{j \leftrightarrow j_5,\ \mu_5 \leftrightarrow \mu} \quad j_5 = \frac{N_c \mu}{2\pi^2} eB$$

CME

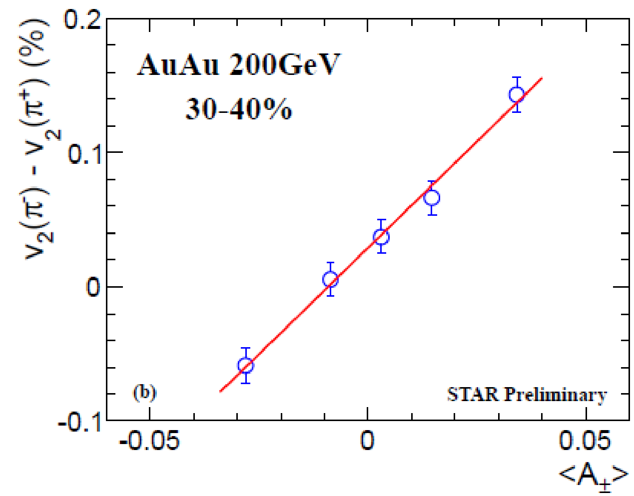
CSE

Metlitski, Zhitnitsky, PRD (2005)

Interplay of CME and CSE leads to Chiral Magnetic Wave (CMW)



Burnier, Liao, Kharzeev, Yee, PRL 2012



Hongwei Ke, J.Phys.Conf.Ser (2012)

Success of frameworks

Hydrodynamics (axial charge)

Son, Surowka, PRL (2009)
Neiman, Oz, JHEP (2011)
Landsteiner et al, PRL (2011)

Chiral kinetic theory (Berry curvature)

Son, Yamamoto, PRL (2012)
Stephanov, Yin, PRL (2012)
Gao, Liang, Pu, Q. Wang, X.-N. Wang,
PRL (2012), (2013), PRD (2014)

Possible issues with “conserved” N_5

Sources of axial charge non-conservation

$$\partial_\mu j_5{}^\mu = -\frac{e^2 N_c}{16\pi^2} F\tilde{F} - \frac{g^2 N_f}{8\pi^2} trG\tilde{G} + 2im\bar{\psi}\gamma^5\psi$$

N_5 only approximately conserved, depending on the time scale and underlying dynamics

Also field theory evidence of difficulty of spacetime dependent μ_5

Wu, Hou, Ren, 1601.06520

How is axial charge generated?

$$\partial_\mu {j_5}^\mu = -\frac{q^2 N_c}{16\pi^2} F\tilde{F} - \frac{g^2 N_f}{8\pi^2} trG\tilde{G} + 2im\bar{\psi}\gamma^5\psi$$

$F\tilde{F}$ Parallel electric and magnetic fields

$trG\tilde{G}$ Topological field configurations(instanton, sphaleron)
Parallel chromo electric and magnetic fields

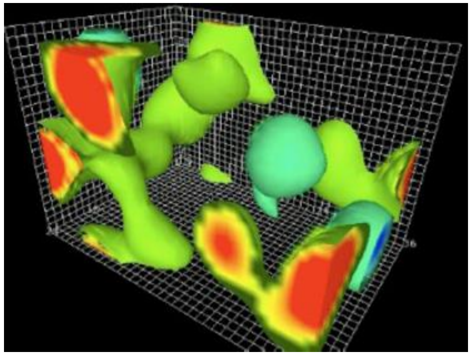
$2im\bar{\psi}\gamma^5\psi$ Explicit breaking by quark mass

In principle, all three can lead to net axial charge $N_5 = \int d^3x\, {j_5}^0$
In HIC, we usually consider $trG\tilde{G}$
In condensed matter, we can produce N_5 from $F\tilde{F}$

N_5 dynamics from topological transition

$$\partial_\mu {j_5}^\mu = -\frac{e^2 N_c}{16\pi^2} F\tilde{F} - \frac{g^2 N_f}{8\pi^2} trG\tilde{G} + 2im\bar{\psi}\gamma^5\psi$$

$trG\tilde{G}$ term: topological transition generates fluctuation of axial charge



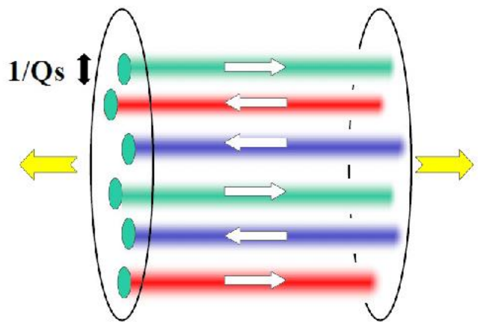
axial charge fluctuation from sphaleron: $t_{sph} \sim \frac{1}{g^4 ln g^{-1} T}$

axial charge relaxation: $t_{rel} \sim \frac{\chi T}{\Gamma_{sph}} \sim \frac{N_c}{N_f g^{10} T}$

$t_{rel} \gg t_{sph}$

Separation of time scales necessary for axial charge to build up!

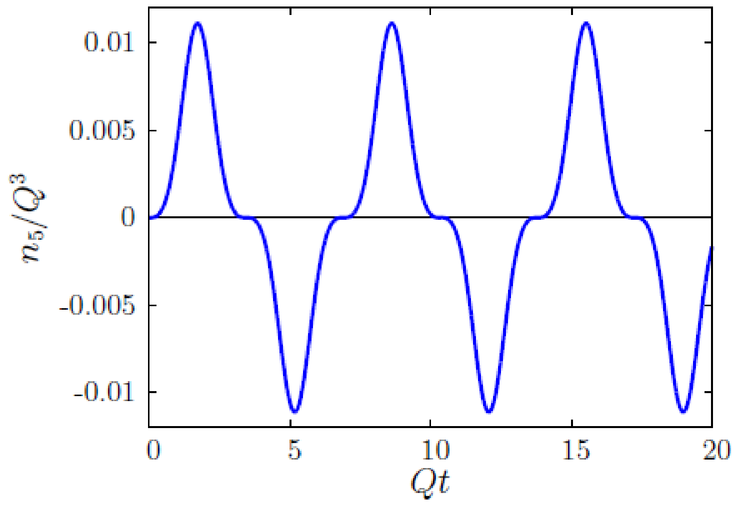
n_5 dynamics from flux tube



Fukushima, Kharzeev, Warringa, PRL 2009

axial charge generation from Glasma: $t_{glasma} \sim \frac{1}{Q_s}$

axial charge relaxation: t_{rel}?



Can axial charge survive Glasma phase?

Tanji, Mueller, Berges, PRD 2016

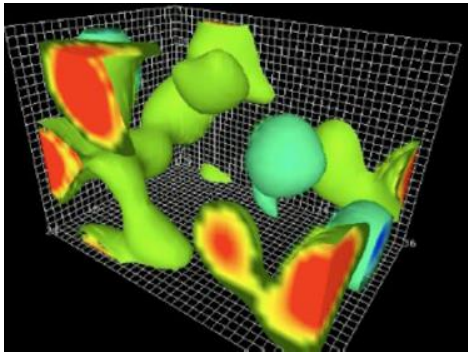
Topological fluctuation as hydro noise

Size of QGP >> fluid cell >> size of topological fluctuation

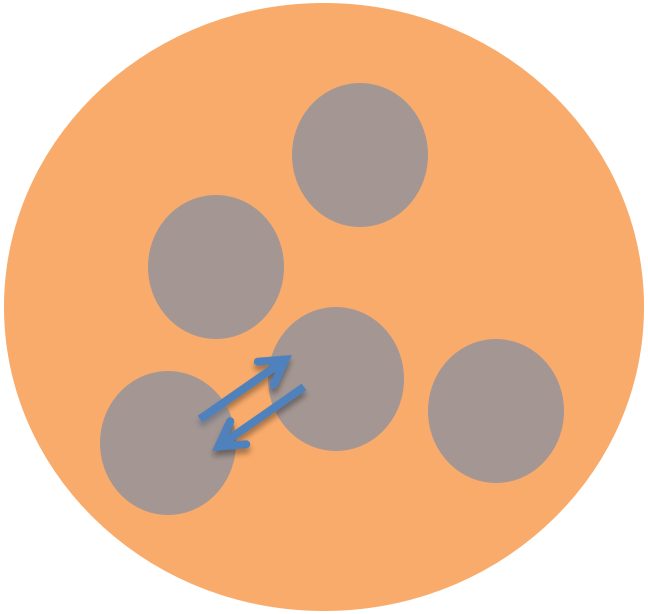
Axial charge fluctuation localized in fluid cell

Topological transition additional source of noise!

within one fluid cell



between fluid cells



Stochastic hydrodynamics for axial charge

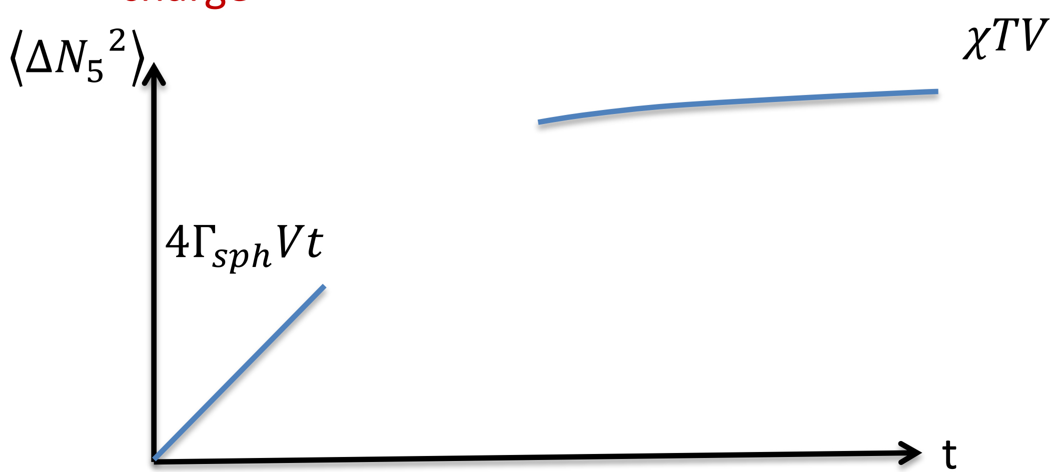
Axial charge generated from fluctuation: stochastic nature

$$\begin{cases} \partial_t n_5 + \nabla \cdot \mathbf{j}_5 = -2q \\ \mathbf{j}_5 = -D\nabla n_5 + \boldsymbol{\xi} \\ q = \dfrac{n_5}{2\tau_{sph}} + \xi_q \end{cases}$$

Usual hydrodynamic noise

Noise from topological fluctuation

Relaxation of axial charge



Iatrakis, SL, Yin, JHEP 2015 based on holographic D4/D8 model

Time evolution of axial charge from stochastic hydrodynamics

$$C_{nn}(t,x) \equiv \langle [n_5(t,x) - n_5(0,x)]^2 \rangle$$

$$C_{nn}(t,\boldsymbol{x}) = (\chi T)\left[\delta^3(\boldsymbol{x}) - \frac{1}{(8\pi D t)^{3/2}} e^{-\frac{2t}{\tau_{\rm sph}}} e^{-\frac{|\boldsymbol{x}|^2}{8Dt}}\right]$$

within cell — across cells

Early time $t \ll \tau_{\rm sph}$

$$C_{nn}(t,\boldsymbol{x}) \approx 4\Gamma_{\rm CS}\, t\, \delta^3(\boldsymbol{x})$$

Late time $t \gg \tau_{\rm sph}$

$$C_{nn}(t \to \infty, \boldsymbol{x}) \to (\chi T)\, \delta^3(\boldsymbol{x})$$

thermodynamic limit

N_5 dynamics from quark mass term

$$\partial_\mu {j_5}^\mu = -\frac{e^2 N_c}{16\pi^2} F\tilde{F} - \frac{g^2 N_f}{8\pi^2} trG\tilde{G} + 2im\bar{\psi}\gamma^5\psi$$

Quark mass term: Neglected when $m \ll T$

HIC at RHIC, $T \lesssim 350MeV$
Strange quark mass $m{\sim}100MeV$
Even current quark mass for light flavor can be significant in QGP(encouragement from Zhuang)

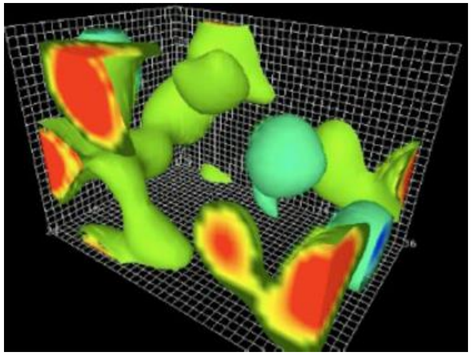
Quark mass term also provides mechanism for axial charge fluctuation as well as dissipation (relaxation)

$$t_{rel} \gg t_{fluc}?$$

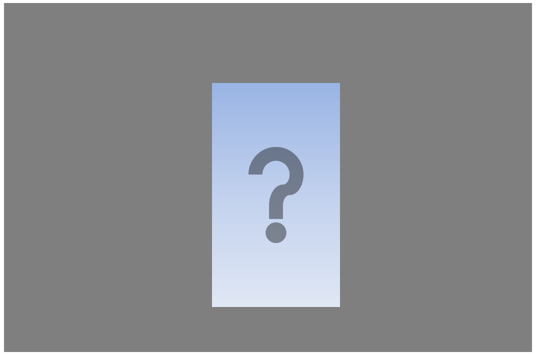
μ_5 appropriate?

Quark mass as another hydro noise?

Topological transition



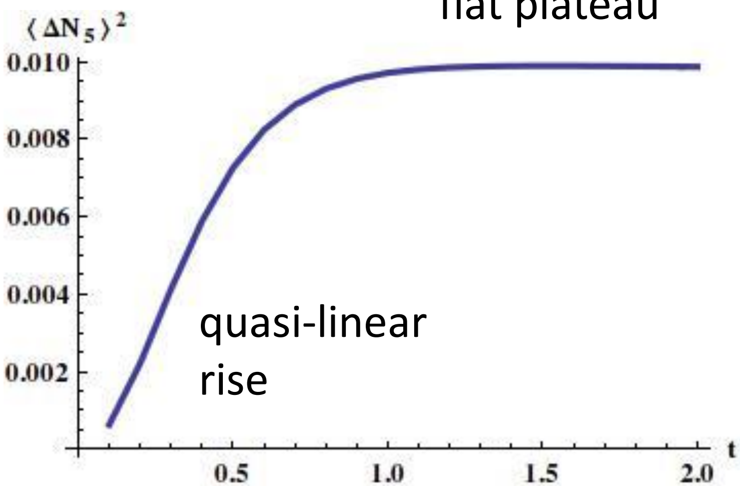
Quark mass induced fluctuation

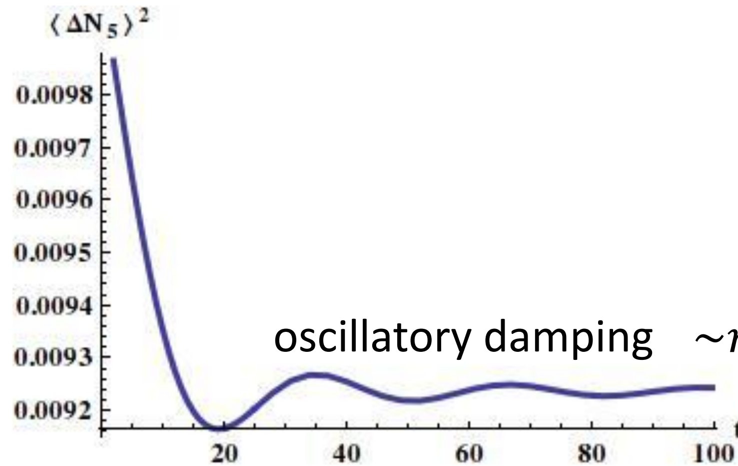


N_5 dynamics from massive field theory

Somewhat trivial example: free fermions

flat plateau



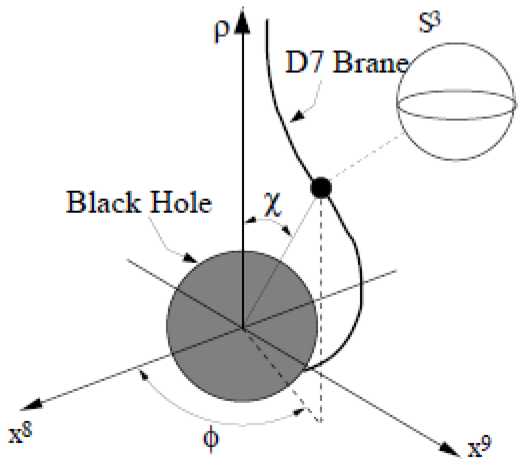


$\sim m^2 TV$

weakly coupled QGP

Hou, SL, in preparation

N_5 dynamics from holography: the D3/D7 model



axial-symmetry realized as rotation in $x8 - x9$ plane

	x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
D3	×	×	×	×						
D7	×	×	×	×	×	×	×	×		

$$S = \mathcal{N}\int d^5x\left(-\frac{1}{2}\sqrt{-G}G^{MN}\partial_M\phi\partial_N\phi - \frac{1}{4}\sqrt{-H}F^2\right) - \mathcal{N}\kappa\int d^5x\Omega\epsilon^{MNPQR}F_{MN}F_{PQ}\partial_R\phi$$

Action invariant under shift of ϕ upto boundary term

variation of boundary term gives $-\frac{e^2N_c}{16\pi^2}F\tilde{F} + 2im\bar{\psi}\gamma^5\psi$

Hoyos, Nishioka, O'Bannon, JHEP (2011)

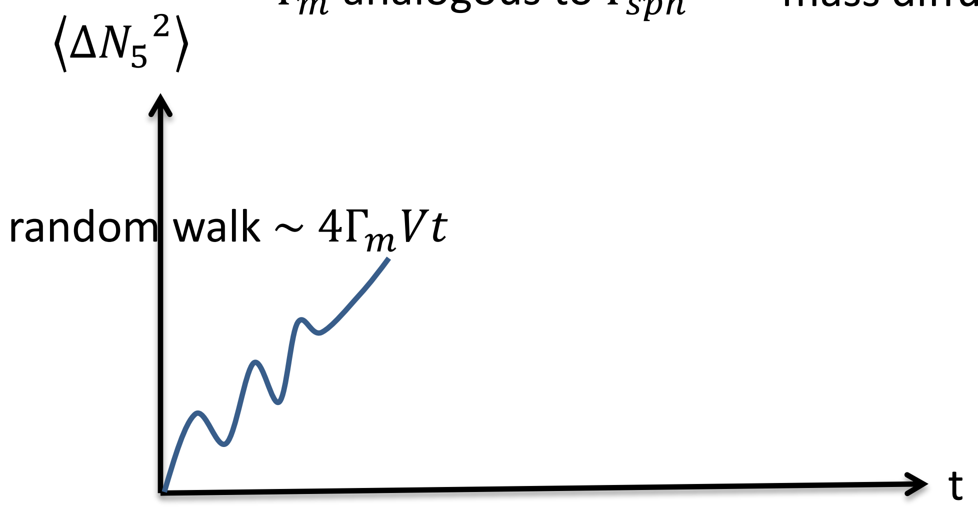
Axial charge fluctuation: mass diffusion

$$O_\eta = im\bar{\psi}\gamma^5\psi$$

$$G_{\eta\eta}(\omega) = \int dt \langle [O_\eta(t), O_\eta(0)] \rangle \Theta(t) e^{i\omega t} \sim \frac{-i\omega\Gamma_m}{2T} \qquad \text{as } \omega \to 0$$

diffusion of mass term corresponds to random walk growth of axial charge

Γ_m analogous to Γ_{sph} mass diffusion rate

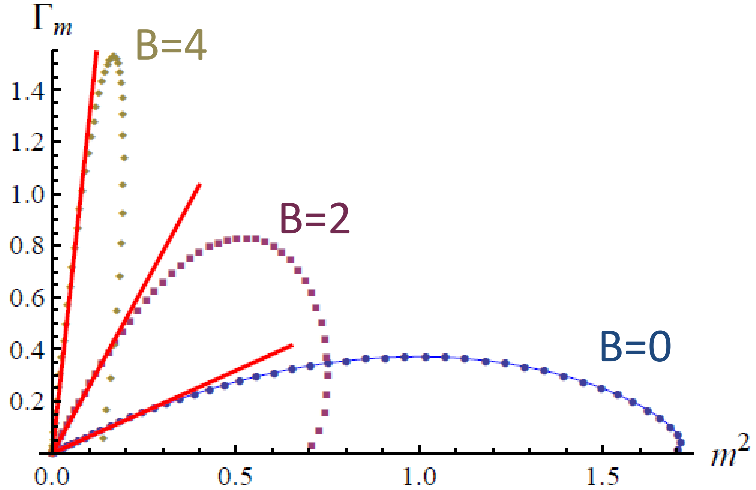


enhance axial charge fluctuation, in addition to topological transition

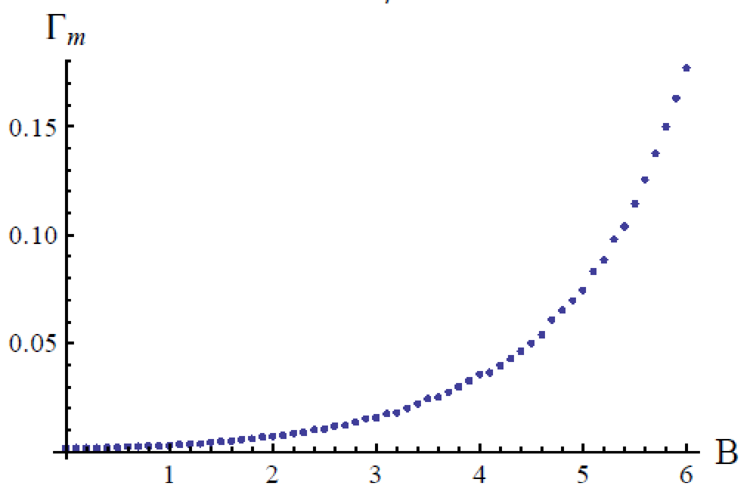
Guo, SL, PRD (2016)

Also expected in thermal field theory Hou, SL, in preparation

Mass diffusion rate: m and B dependence



m=1/20



$\Gamma_m \sim m^2 F(B)$

Measure of helicity flipping rate

Magnetic field enhances helicity flipping rate

Guo, SL, PRD (2016)

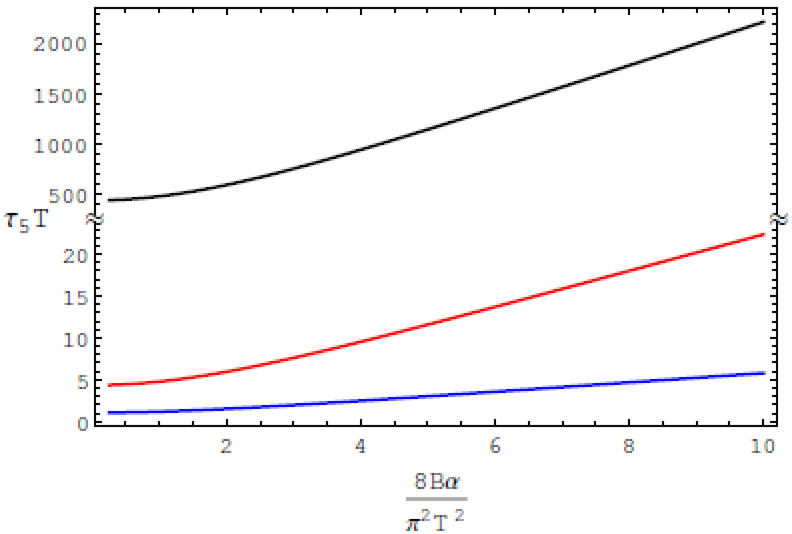
$B = {m_\pi}^2, T = 300MeV, M = M_s, N_f = 1$

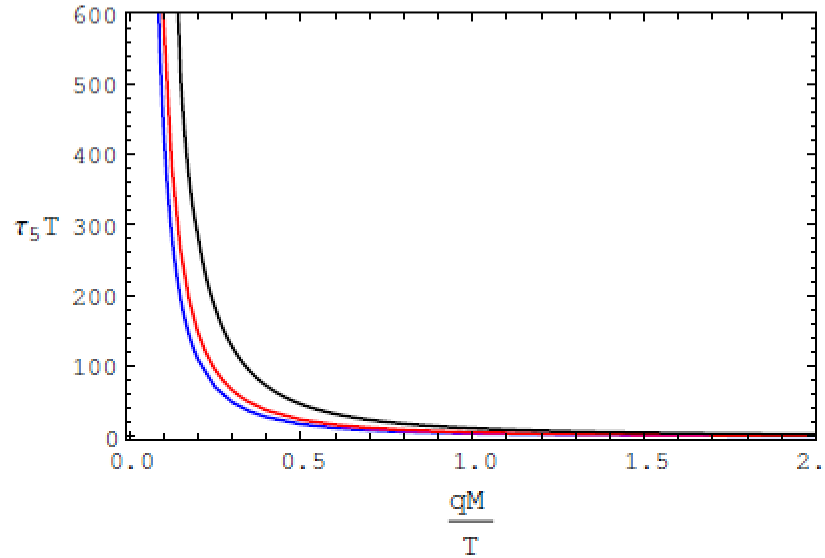
$\Gamma_m \sim \Gamma_{sph}$

Mass diffusion can be significant compared to sphaleron diffusion

Axial charge relaxation: mass dissipation

Relaxation time approximation





τ_{rel} increases with B, decreases with m

Landsteiner et al, JHEP 2015

Large N theory, susceptibility well-defined

other holographic model

True in reality?

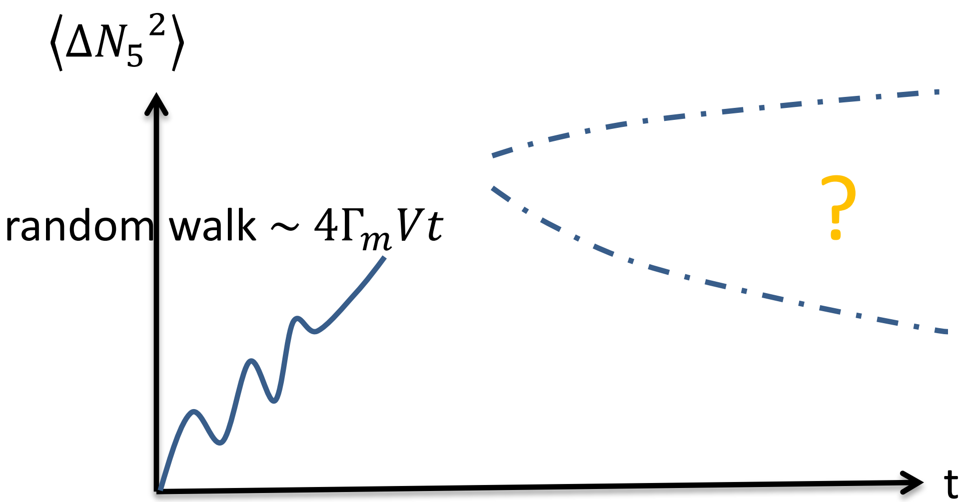
Dynamical susceptibility from CME

Define dynamical axial chemical potential using CME $J(\omega) = C\mu_5(\omega)B(\omega)$

Dynamical susceptibility $$\chi(\omega) = \frac{n_5(\omega)}{\mu_5(\omega)}$$

$\chi \sim O(\omega^{-1})$ as $\omega \to 0$ divergent susceptibility Guo, SL, PRD (2016)

while m=0 has a finite χ as $\omega \to 0$



Phenomenology? finite m versus ω

What other modification quark mass brings in?

Implicit modification

CME $$j = \frac{N_c \mu_5}{2\pi^2} eB$$

CVE $$j_B = \frac{N_c \mu_5 \mu}{2\pi^2} \omega$$

Expression not corrected by quark mass, but μ_5 might not be appropriate

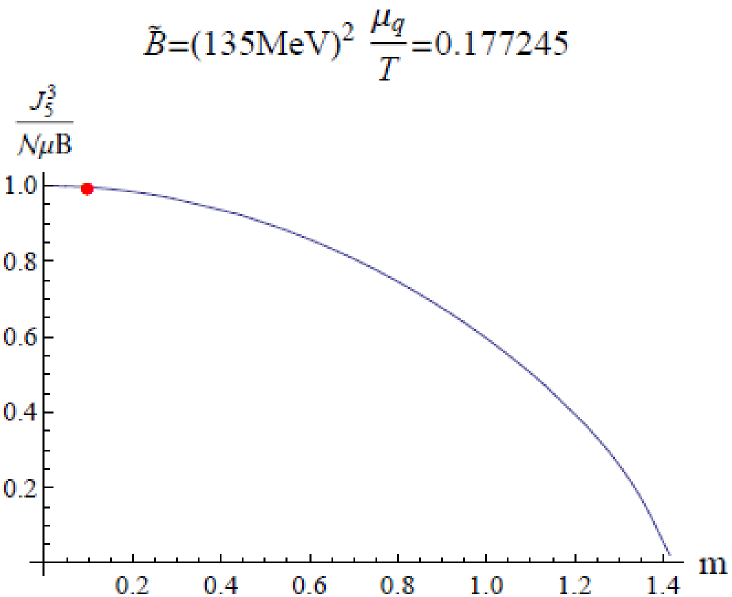
Explicit modification

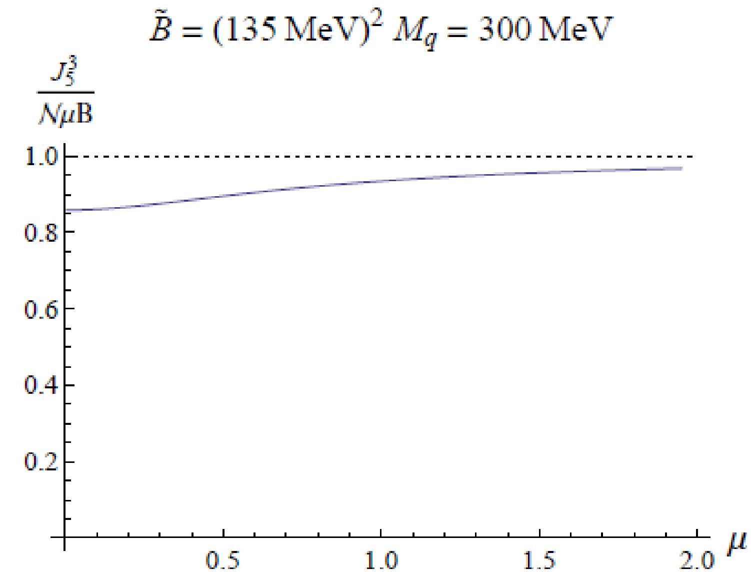
CSE $$j_5 = \frac{N_c \mu}{2\pi^2} eB + O(m^2)$$

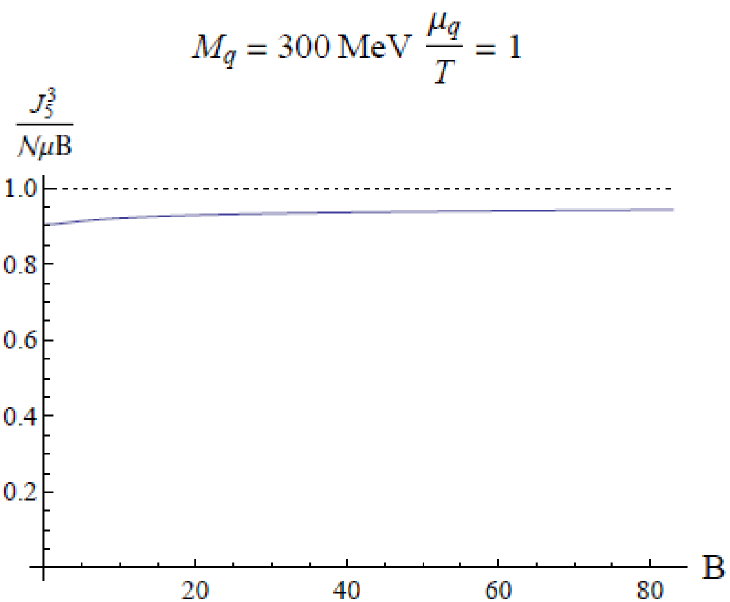
$O(m^2)$ known in confined phase

Metlitski, Zhitnitsky, PRD (2005)
in confined phase

Quark mass correction to CSE in QGP







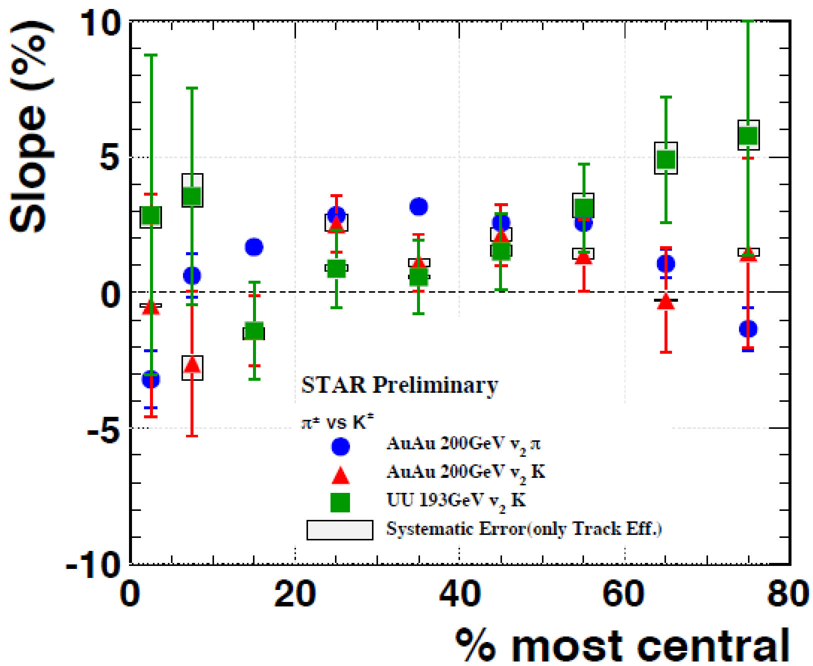
m suppresses CSE

μ and B enhance CSE

$$\Delta j_5 = -\#\frac{m^2}{T^2}\mu B + o(m^2)$$

Correction numerically small

Quark mass dependence of CMW



consistent with small quark mass effect?

Need to incorporate mass diffusion and dissipation in hydrodynamic framework

Anomalous Viscous Fluid Dynamics(Jiang，Shi，Liao，Yin, 2016)

Qiye Shou, J.Phys.Conf.Ser (2014)

Summary

- Quark mass can modify anomalous effect in different ways

- Quark mass enhances fluctuation of axial charge in addition to topological fluctuation

- Quark mass dissipation consistent with relaxation time approximation in the large N.
- Dynamical susceptibility might not be well-defined, need quantitative answer

- Quark mass correction to CSE

Thank you!

How do we study quark mass effect?

- Holographic model: D3/D7, D4/D8 etc

pros: easy to treat transport, non-perturbative dynamics

cons: large N limit suppresses quark backreaction

- Thermal field theory

pros: arbitrary N, treat fluctuation/dissipation in a unified way

cons: treatment of long time limit not as easy, usually need kinetic theory

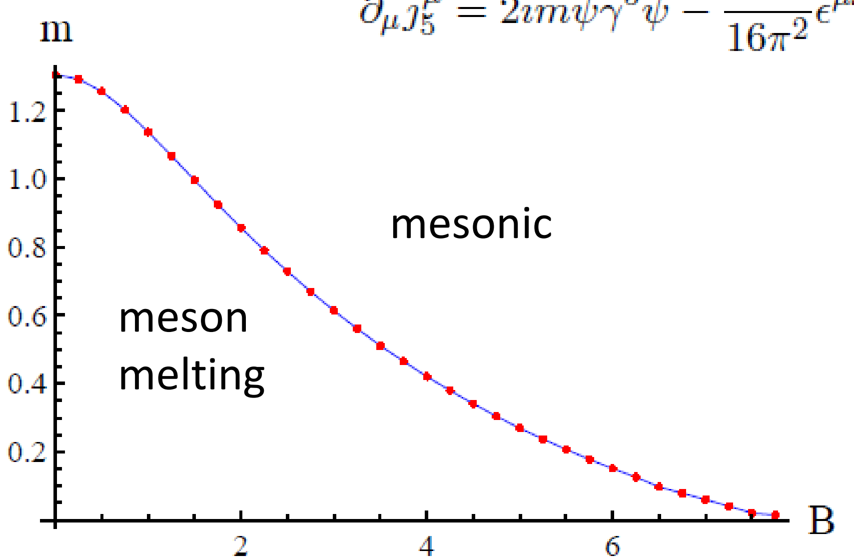
The D3/D7 model

N_c D3 branes + N_f D7 branes

gluons   quarks

Pro: quark mass explicit (as compare to D4/D8)
Con: w/o QCD anomaly, but with QED anomaly and mass term

$$\partial_\mu j_5^\mu = 2im\bar{\psi}\gamma^5\psi - \frac{e^2}{16\pi^2}\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma} - \frac{g^2}{16\pi^2}\mathrm{tr}\epsilon^{\mu\nu\rho\sigma}G_{\mu\nu}G_{\rho\sigma};$$



Mateos et al, PRL 2006
Filev et al, JHEP 2007
Erdmenger et al, JHEP 2007

Correction to CSE in QGP

$$\nabla \cdot \mathbf{j}_5 = C\mathbf{E} \cdot \mathbf{B} + 2M_q i\bar{\psi}\gamma^5\psi \quad \equiv \sigma_5$$

massless case: $\nabla \cdot \mathbf{j}_5 = -\nabla \cdot (C\mu\mathbf{B}) \Rightarrow \mathbf{j}_5 = -C\mu_q\mathbf{B}$

massive case: σ_5 P odd, T odd, while B P even, T odd, μ_q P even, T even

$$\sigma_5 = g(M_q^2, T, \mu, B)\mathbf{B} \cdot \nabla\mu_q \quad \Longrightarrow \quad \mathbf{j}_5 = -C\mu\mathbf{B} + g(M_q^2, T, \mu_q, B)\mu\mathbf{B}.$$

$g = \#\frac{M_q^2}{T^2} + o(M_q^2)$ when $\mu_q \ll T, B \ll T^2$

Guo, SL. JHEP 2017

Relativistic hydrodynamics for HIC

$$\partial_\mu T^{\mu\nu} = F^{\nu\lambda} j^V{}_\lambda$$
$$\partial_\mu {j_V}^\mu = 0$$
$$\partial_\mu {j_A}^\mu = C E^\mu B_\mu$$

Son, Surowka, PRL (2009)

with QED anomaly,
without QCD anomaly

$${j_V}^\mu = n_V u^\mu + {\nu_V}^\mu$$
$${j_A}^\mu = n_A u^\mu + {\nu_A}^\mu$$

talks by Jiang, Huang and Liao

CVE

CME

Anomalous part:

$${\nu_V}^\mu = C\mu_V\mu_A\omega^\mu + C\mu_A B^\mu$$
$${\nu_5}^\mu = C/2({\mu_V}^2 + {\mu_A}^2 + \cdots)\omega^\mu + C\mu_V B^\mu$$

CSE

However, in HIC we need QCD anomaly to generate axial charge!

$\langle N_A \rangle = 0, \langle {N_A}^2 \rangle \neq 0$

Axial charge stochastic,
hydrodynamic noise necessary!

How hydro noise is included

Conserved charge as an example

$$\partial_\mu J^\mu = 0$$

w/o noise

$J^0 = n$ charge density $J_k = -D\partial_k n$ diffusive current

with noise

$J^0 = n$ $J_k = -D\partial_k n + r_k$

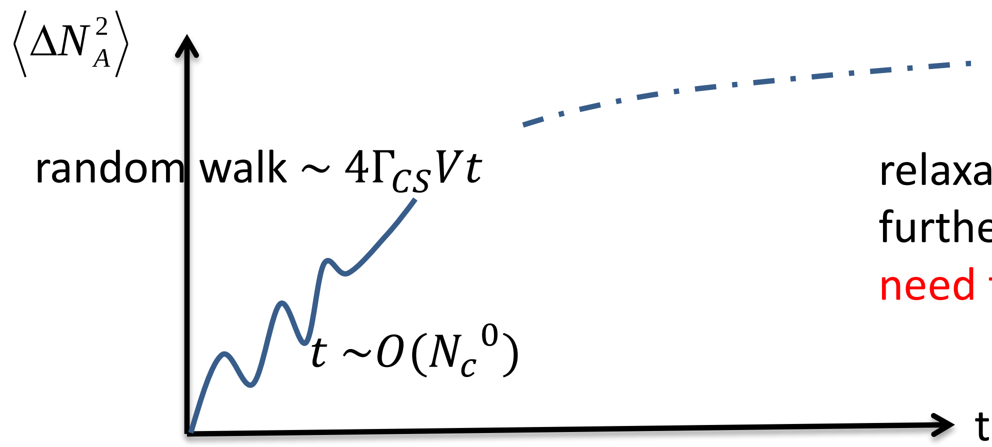
dissipation ($-D\partial_k n$) fluctuation (r_k)

$$\langle r_i(\mathbf{x},t)\, r_k(\mathbf{x}',t')\rangle = C\delta_{ik}\delta(\mathbf{x}-\mathbf{x}')\delta(t-t')$$

Yan's talk

Kovtun, 1205.5040

Axial charge from topological fluctuation



$$t \sim \tau = \frac{\chi T}{2\Gamma_{CS}} \sim O(N_c)$$

relaxation: presence of n_A affects further topological charge fluctuation.
need fermion dynamics

Chern-Simon diffusion rate $\Gamma_{CS} = \int d^4x \langle q(x)q(0) \rangle$

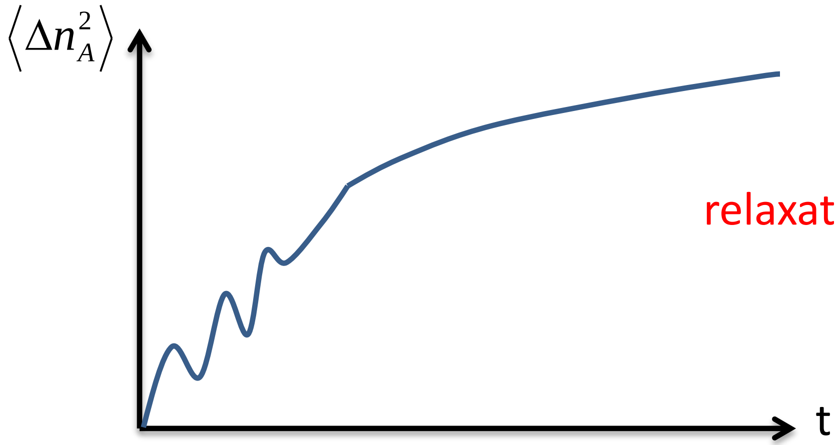
$q \sim trG\tilde{G}$ topological charge density

weak coupling extrapolation: $\Gamma_{CS} \sim 30\alpha_s{}^4T^4$ — Moore, Tassler, JHEP 2011

strong coupling: $\Gamma_{CS} = \alpha_s{}^2N_c{}^2T^4/16\pi$ — Son, Starinets, JHEP 2002

strong coupling w/B: $\Gamma_{CS} \sim \alpha_s{}^2N_c{}^2BT^2$ — Basar, Kharzeev, PRD 2012

Axial charge relaxation



relaxation, explicit in our model with fermions

Response of q to n_A

$$q = \frac{\Gamma_{CS}}{\chi T} n_A \quad \Longrightarrow \quad \frac{dn_A}{dt} = -2q = -\frac{2\Gamma_{CS}}{\chi T} n_A = -\frac{n_A}{\tau_{sph}}$$

χ: static susceptibility $\quad \tau_{sph} = \frac{\chi T}{2\Gamma_{CS}}$: relaxation time

consistent with early statistical argument

Also work by Akamatsu, Rothkopf, Yamamoto, JHEP 2016

Iatrakis, SL, Yin, JHEP 2015

Stochastic hydrodynamics for axial charge

Dynamical equation

$$\partial_t n_A(t,x) + \nabla \cdot j_A(t,x) = -2q(t,x)$$

Constitutive equations

$$j_A(t,x) = -D\nabla n_A(t,x) + \xi(t,x)$$

$$q(t,x) = \frac{n_A(t,x)}{2\tau_{\mathrm{sph}}} + \xi_q(t,x)$$

Non-topological fluctuation $\langle \xi_i(t,x)\xi_j(t,x')\rangle = 2\sigma T\delta_{ij}\delta(t-t')\delta^3(x-x')$

topological fluctuation $\langle \xi_q(t,x)\xi_q(t,x')\rangle = \Gamma_{\mathrm{CS}}\delta(t-t')\delta^3(x-x')$

Iatrakis, SL, Yin, JHEP 2015

Chiral magnetic wave: an interplay of CME and CSE

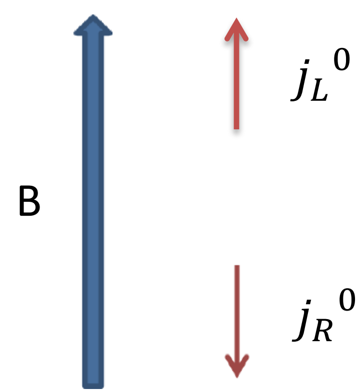
$$j_V{}^1 = \frac{N_c \mu_A}{2\pi^2} eB - D_L \partial_1 j_V{}^0$$ Chiral magnetic effect + diffusion

$$j_A{}^1 = \frac{N_c \mu_V}{2\pi^2} eB - D_L \partial_1 j_A{}^0$$ Chiral separation effect + diffusion

$$\partial_\mu j_V{}^\mu = 0$$
$$\partial_\mu j_A{}^\mu = 0$$

$\Longrightarrow$

$$\left(\partial_0 \mp \frac{N_c e B \alpha}{2\pi^2} \partial_1 - D_L \partial_1^2 \right) j^0_{L,R} = 0$$



$$v_\chi = \frac{N_c e B \alpha}{2\pi^2} = \frac{N_c e B}{4\pi^2} \left(\frac{\partial \mu_L}{\partial j^0_L} \right) = \frac{N_c e B}{4\pi^2} \left(\frac{\partial \mu_R}{\partial j^0_R} \right)$$

Kharzeev, Yee, PRD (2011)