Computation of the Magnetic Field and Vorticity in Heavy-Ion Collisions

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Outlook

- Electromagnetic (EM) fields in HIC

  Phys.Rev. C85 (2012) 044907 / WTD, X-G Huang

- Background Vs. CME effect

  Phys.Lett. B742 (2015) 296-302 / WTD, X-G Huang

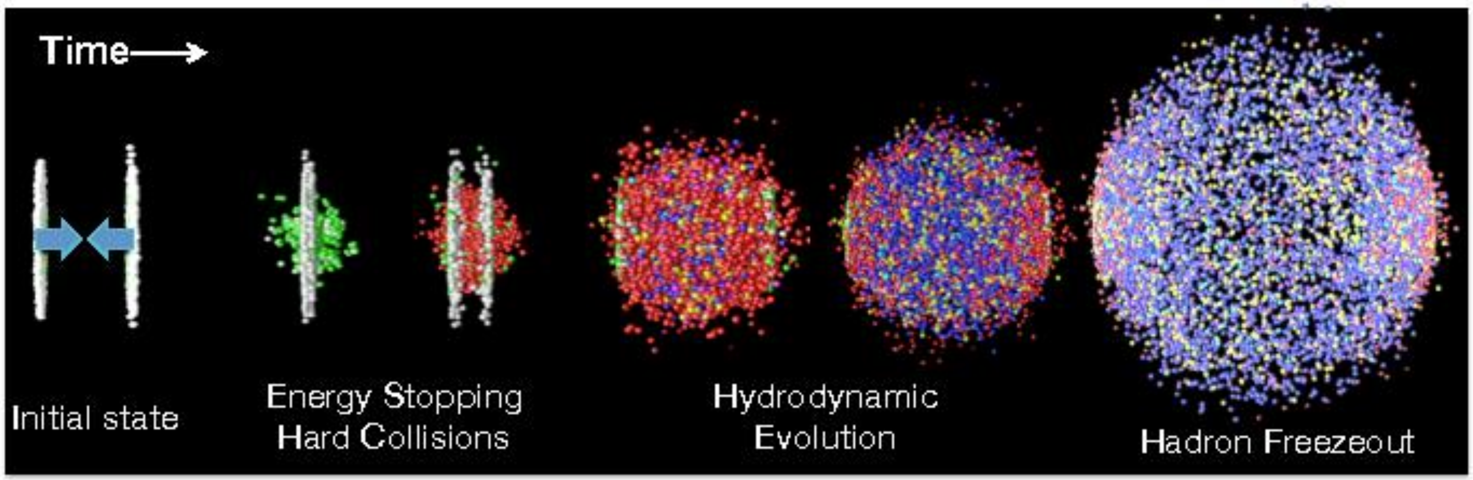
  Phys.Rev. C94 (2016) 041901 / WTD, X-G Huang, G-L Ma, G Wang

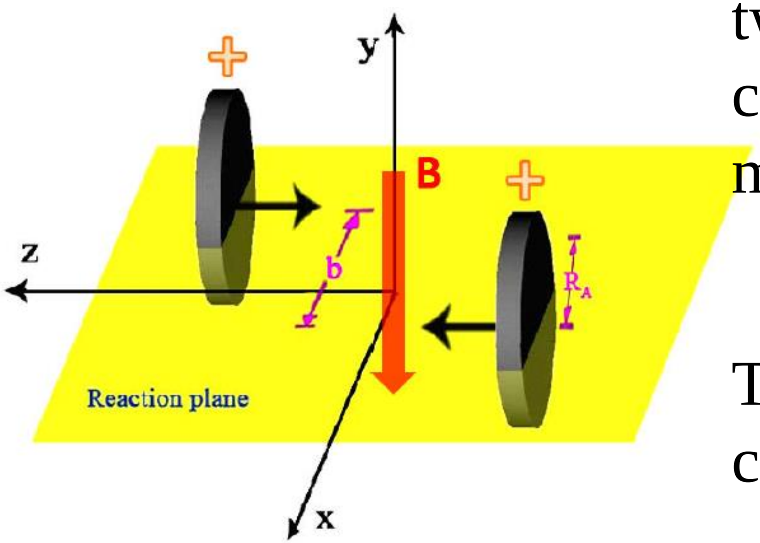
- Vorticity in HIC

  Phys.Rev. C93 (2016) no.6, 064907 / WTD, X-G Huang

Electromagnetic (EM) fields in HIC

Motivation





Due to fast, oppositely directed motion of two colliding ions, off-central heavy-ion collisions can create strong transient magnetic fields.

J.Rafelski and B.Muller, Phys.Rev.Lett.36,517

The magnetic fields generated in Au+Au collisions at RHIC can reach~ 10^{19} Gauss

D.E.Kharzeev, L.D.Mclerran, and H.J.Warringa, Nucl.Phys.A 803,227

Such a strong B field may influence the dynamics of QGP

Chirality imbalance + magnetic field = chiral magnetic effect (CME)

Kharzeev 2004, Kharzeev, Mclerran, Warringa, Fukushima 2007-2008

Chiral Magnetic Effect (CME) ⟹ Charge separation

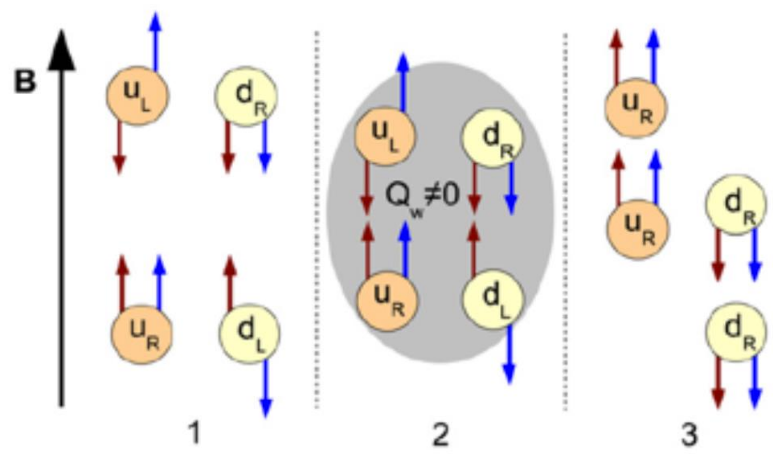
$$\vec{J} = \sigma_5 \vec{B}$$

chiral conductivity

$$\sigma_5 = N_c \sum_f \frac{q_f^2 \mu_5}{2\pi^2}$$

with aixal or chiral chemical potential

$$\mu_5 = \frac{\mu_R - \mu_L}{2}$$



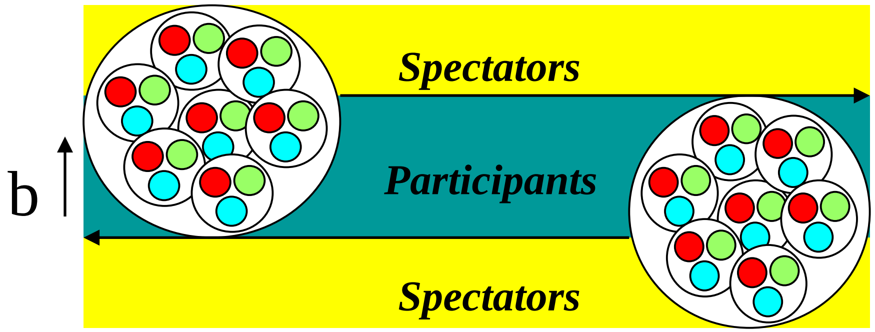
Motivation for EM in A+A

- Most study of the field strength so far are based on the averaging over events. Then only B_y remains sizable.
- However, many effects should be based on the event-by-event analysis, and depended on the space-time distribution of the field.
- So, in this work, we give a detailed study of EM field in heavy-ion collisions on the e-b-e bases using HIJING model.

HIJING Model

Binary Collision Approximation

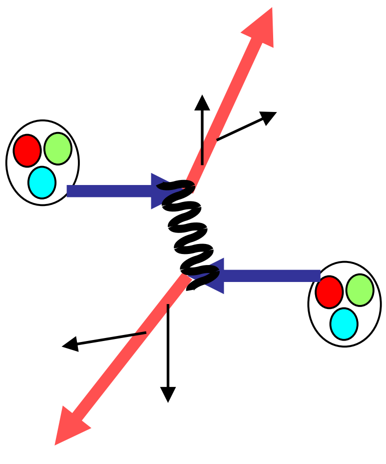
Wood-Saxon nuclear density



A+A collision

independent

Number of Binary collisions



N+N collisions

Two-Component Model in N-N Collisions

P_T

Soft Interaction | P_0 | Hard Interaction (PQCD)

σ_{soft} $\quad$ σ_{jet} $\quad \Rightarrow \sigma_{ine}, \ \sigma_{total}$

Initial Kinematics of Nucleon

In the rest frame of Nucleus:

Spherical Woods-Saxon nuclear distribution $\rho(r) = \dfrac{\rho_0}{1 + \exp\left[\left(r - R_A\right)/a\right]}$

$\Longrightarrow$ (x_N, y_N, z_N) of each nucleon (proton)

In the center-of-mass frame:

$$E = \sqrt{s}/2$$

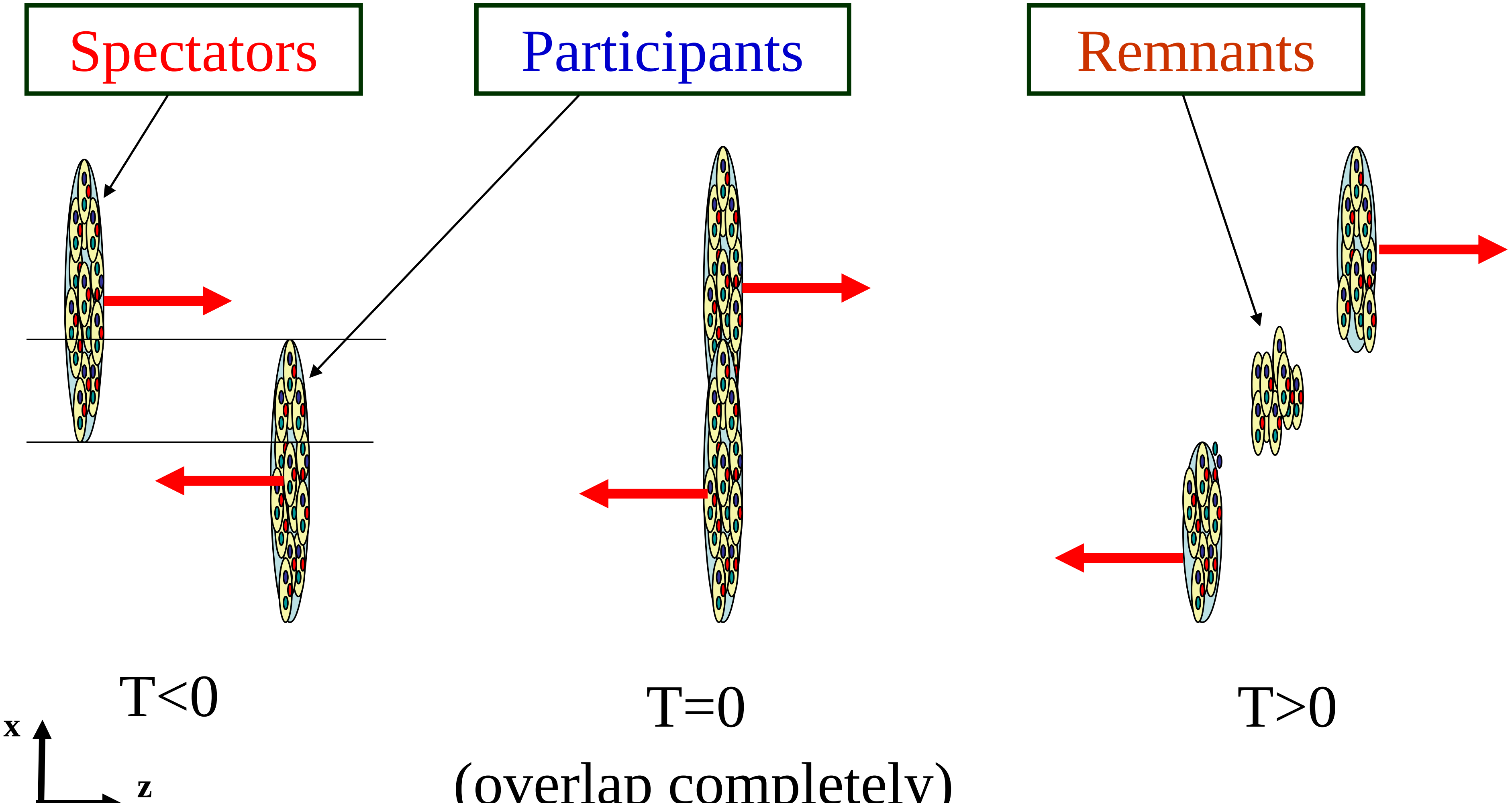
$$v_z^2 = 1 - (2m_N/\sqrt{s})^2, \;\; v_x = v_y = 0$$

$$\Rightarrow z_N^{Lorentz} = z_N 2m_N/\sqrt{s}$$

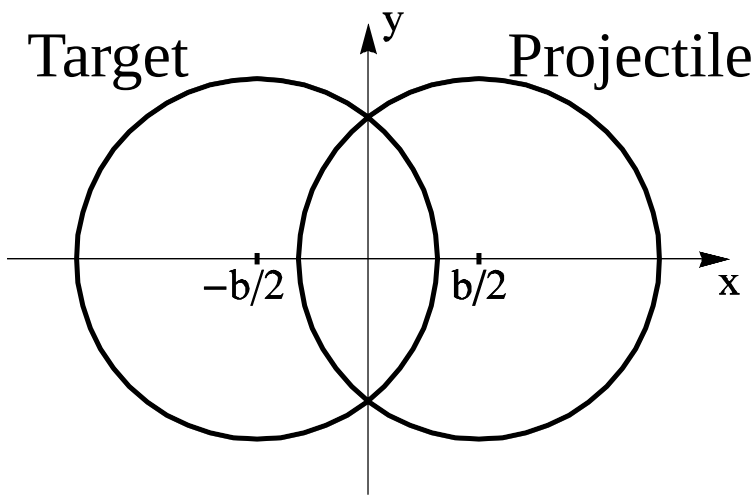
Lorentz Contraction

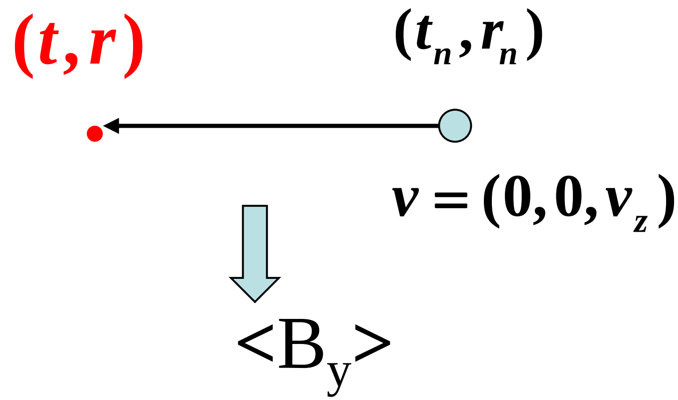
m_N is the rest mass of nucleon

Time definition in HIC



Lienard-Wiechert potential





$$e\mathbf{E}(t,\mathbf{r}) = \frac{e^2}{4\pi}\sum_n Z_n \frac{\mathbf{R}_n - R_n\mathbf{v}_n}{(R_n - \mathbf{R}_n\cdot\mathbf{v}_n)^3}(1 - v_n^2)$$

$$e\mathbf{B}(t,\mathbf{r}) = \frac{e^2}{4\pi}\sum_n Z_n \frac{\mathbf{v}_n \times \mathbf{R}_n}{(R_n - \mathbf{R}_n\cdot\mathbf{v}_n)^3}(1 - v_n^2)$$

Relative position $\mathbf{R}_n = \mathbf{r} - \mathbf{r}_n$

with retarded condition $t_n = t - |\mathbf{r} - \mathbf{r}_n|$

Event-by-Event Analysis

On event-averaged basis:
Because of the symmetry,
only B_y is non-zero.

$$\langle B_x \rangle \approx 0, \langle B_z \rangle = 0$$

$$\langle E_x \rangle \approx \langle E_y \rangle \approx \langle E_z \rangle \approx 0$$

$$< B_x >, < B_y >, < E_x >, < E_y >$$

On event-by-event basis:
because of the fluctuation of proton's position,
E_x, E_y and B_x may be non-vanished.

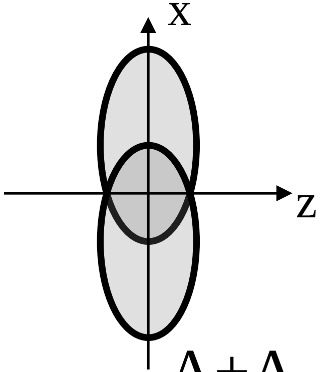
$$< |B_x| >, < |B_y| >, < |E_x| >, < |E_y| >$$

$$\langle |E_y| \rangle \approx \langle |B_x| \rangle$$

$$\langle |B_y| \rangle \geq \langle |E_x| \rangle$$

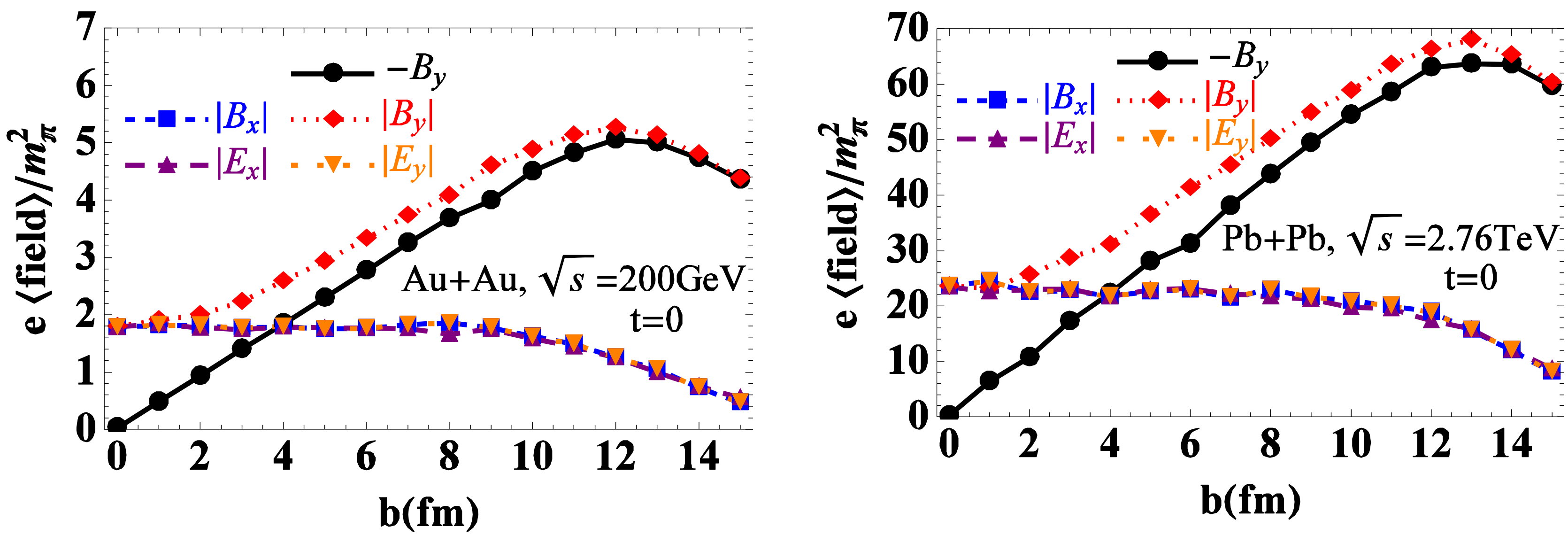
Numerical Results of EM Field

- For t=0, only Spectators and Participants contribute to the EM field.
  - b dependence
  - Energy dependence
  - Spatial distribution
- Time evolution of EM field at early stage

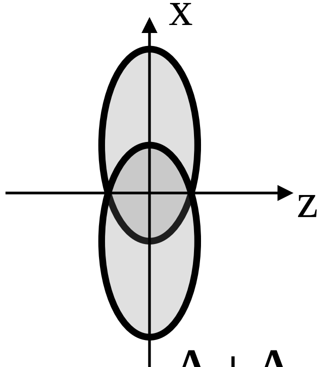


b Dependence

A+A, t=0, **r**=0

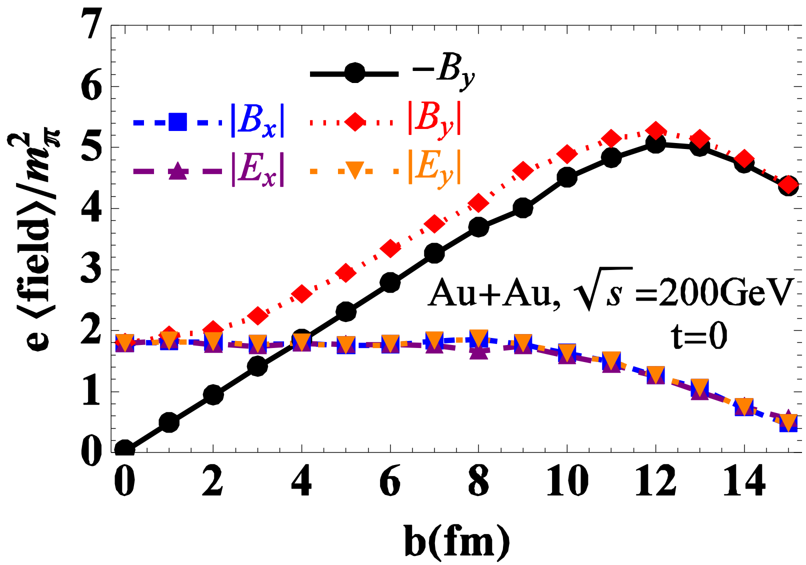


- On events-averaged basis, only B_y remains indeed
- On e-b-e basis, the B_x, E_x and E_y are approximately equivalent. And they are almost independent on the b.
- Higher energy —> higher strength of field

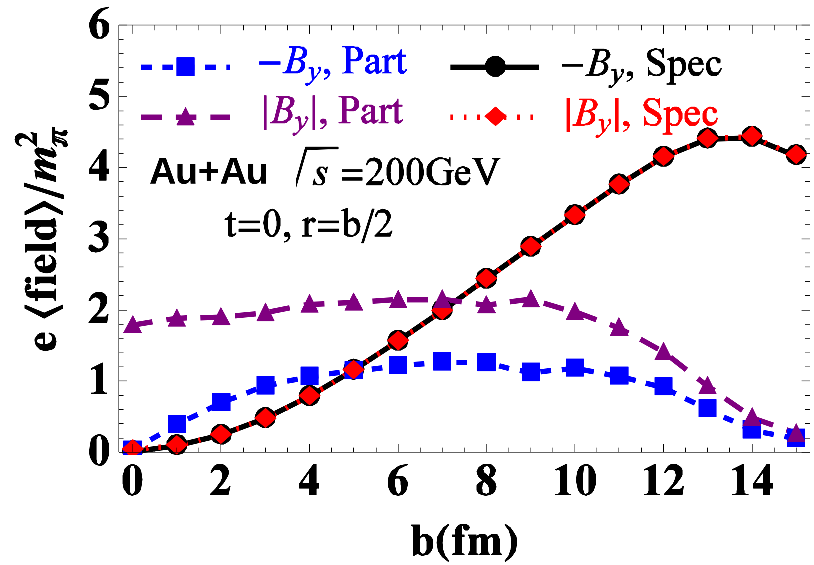


b Dependence

A+A, t=0, **r**=0



Spectator Vs. Participant



- The difference of B_y and $|B_y|$ come from Participants.
- The By produced by Spectators in each event is with the same sign, that's why $B_y=|B_y|$ for Spectators
- The net By produced by Participants may be with different sign, so $B_y<=|B_y|$ for Participants

Energy Dependence

$$e\mathbf{E}(t,\mathbf{r}) = \frac{e^2}{4\pi}\sum_n Z_n \frac{\mathbf{R}_n - R_n\mathbf{v}_n}{(R_n - \mathbf{R}_n\cdot\mathbf{v}_n)^3}(1-v_n^2)$$

$$e\mathbf{B}(t,\mathbf{r}) = \frac{e^2}{4\pi}\sum_n Z_n \frac{\mathbf{v}_n \times \mathbf{R}_n}{(R_n - \mathbf{R}_n\cdot\mathbf{v}_n)^3}(1-v_n^2)$$

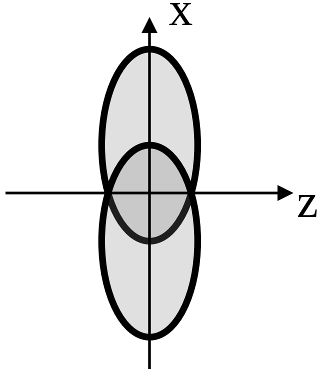
As the field at t=0 are mainly caused by spectators and participants： $v_z \to 1$

On the transverse plane:

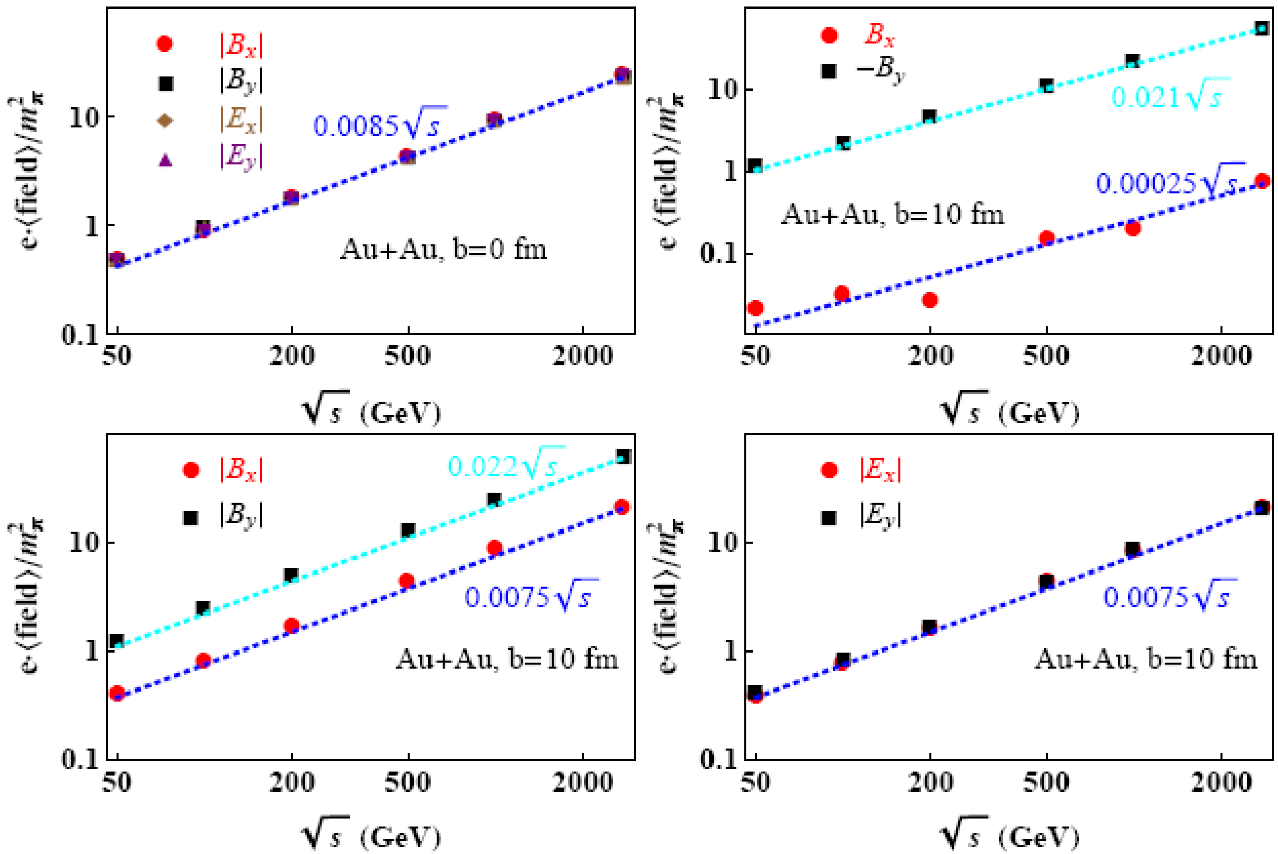
$$e\mathbf{E}_\perp(0,\mathbf{r}) \approx \frac{e^2}{4\pi}\frac{\sqrt{s}}{2m_N}\sum_n \frac{\mathbf{R}_{n\perp}}{R_{n\perp}^3},$$

$$e\mathbf{B}_\perp(0,\mathbf{r}) \approx \frac{e^2}{4\pi}\frac{\sqrt{s}}{2m_N}\sum_n \frac{\mathbf{e}_{nz}\times\mathbf{R}_{n\perp}}{R_{n\perp}^3},$$

Linear dependence on colliding energy

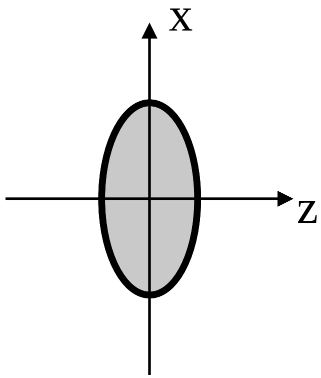


Energy Dependence



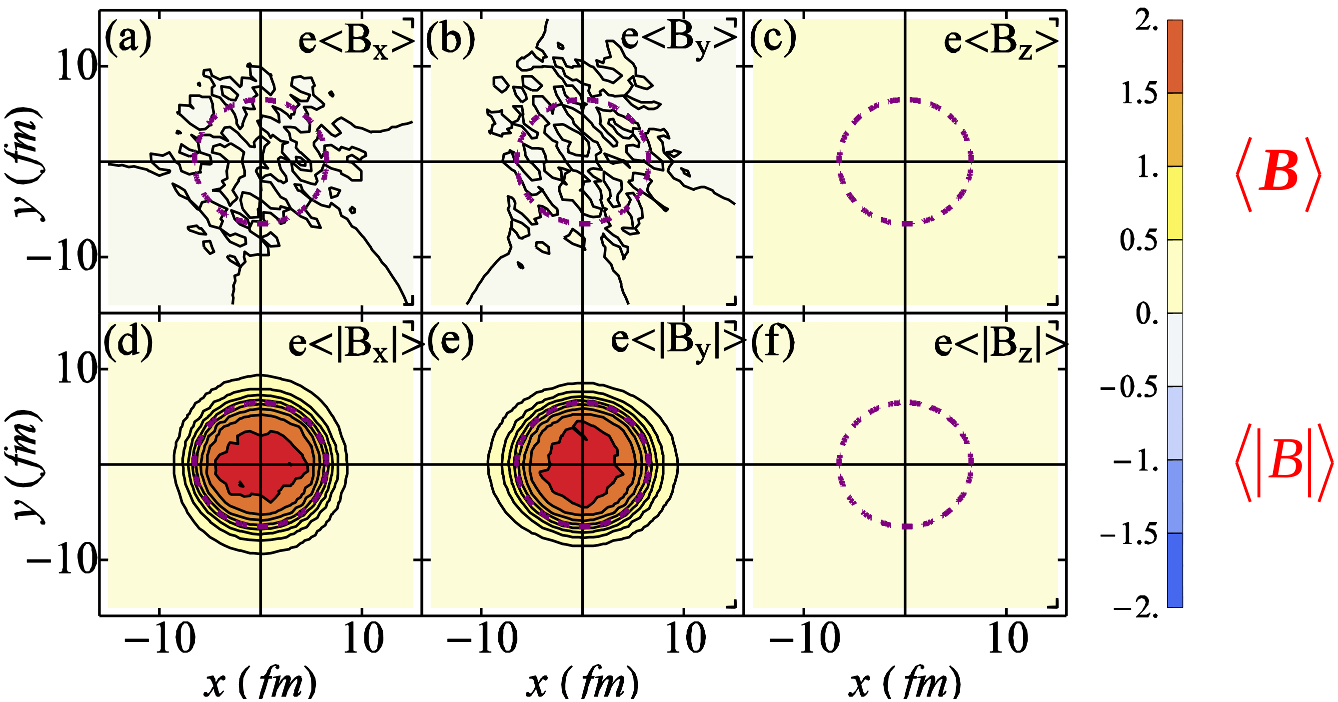
t=0, **r**=0

$$e \cdot \text{Field} \propto \sqrt{s} \cdot f(b/R_A)$$

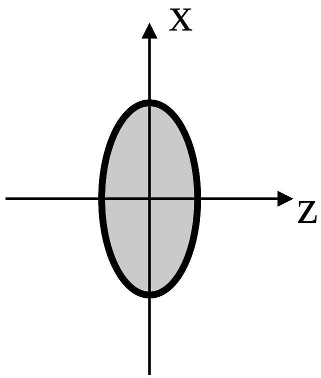


Spatial Distribution

$\sqrt{s}$=200GeV, t=0, b=0

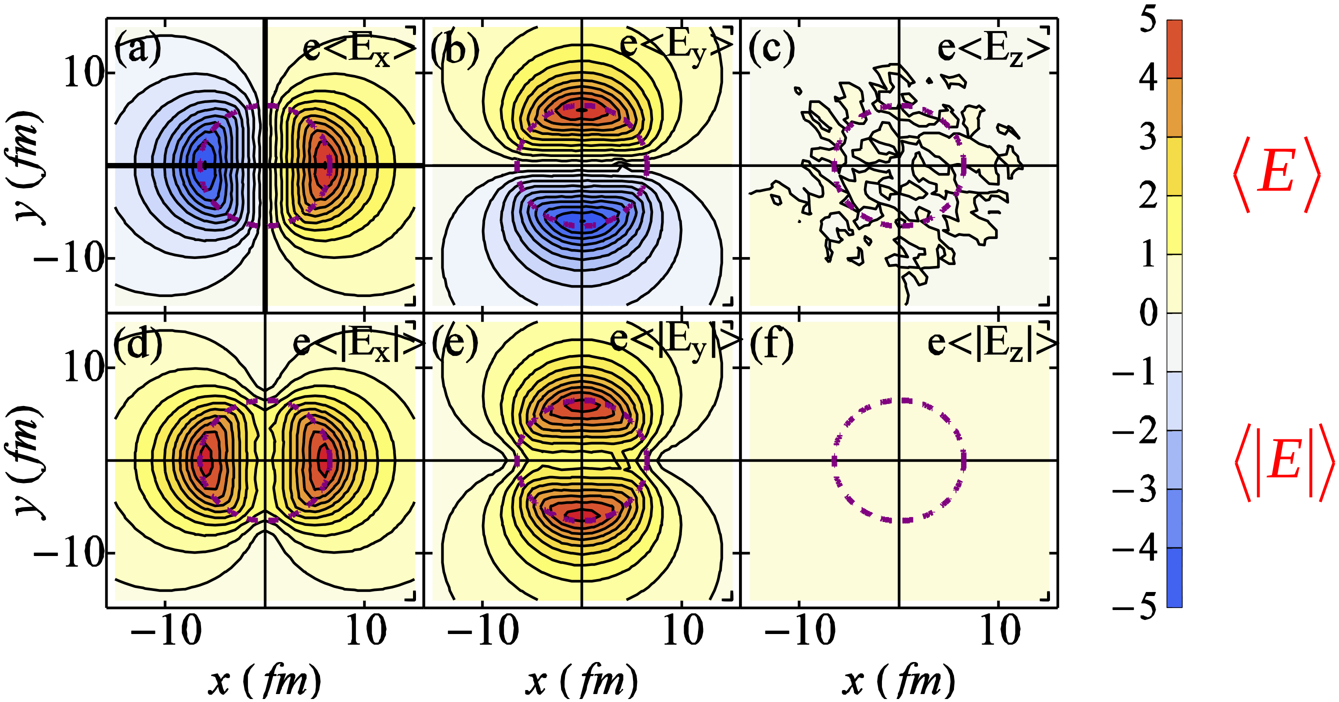


- B_x and B_y are approximately vanished at b=0 due to geometry symmetry.
- $|B_x|$ and $|B_y|$ are sizable and symmetry as a circle, and peak at r=0.

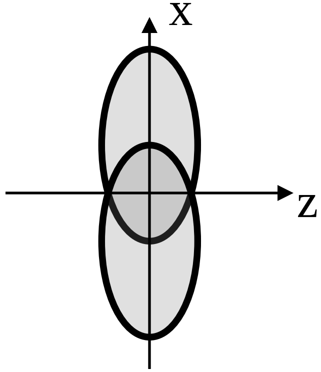


Spatial Distribution

$\sqrt{s}$=200GeV, t=0, b=0

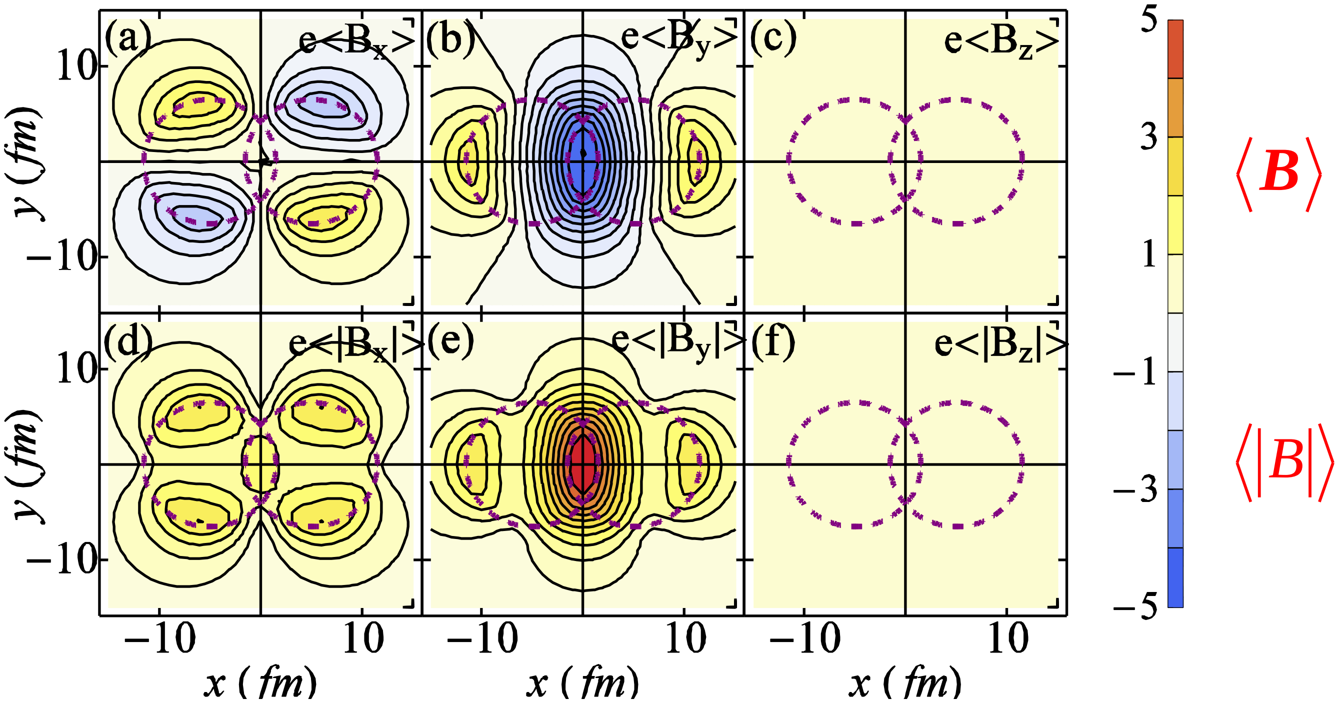


- The spatial distribution of E is quiet different with B.
- For E_x and E_y, although they are vanished at center, but still sizable at other observation points, peak at circumference

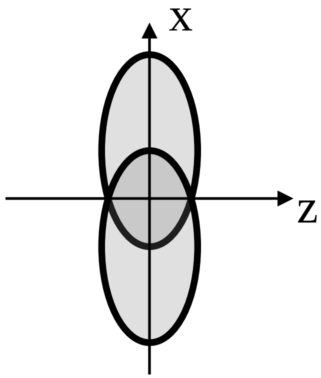


Spatial Distribution

$\sqrt{s}$=200GeV, t=0, b=10 fm

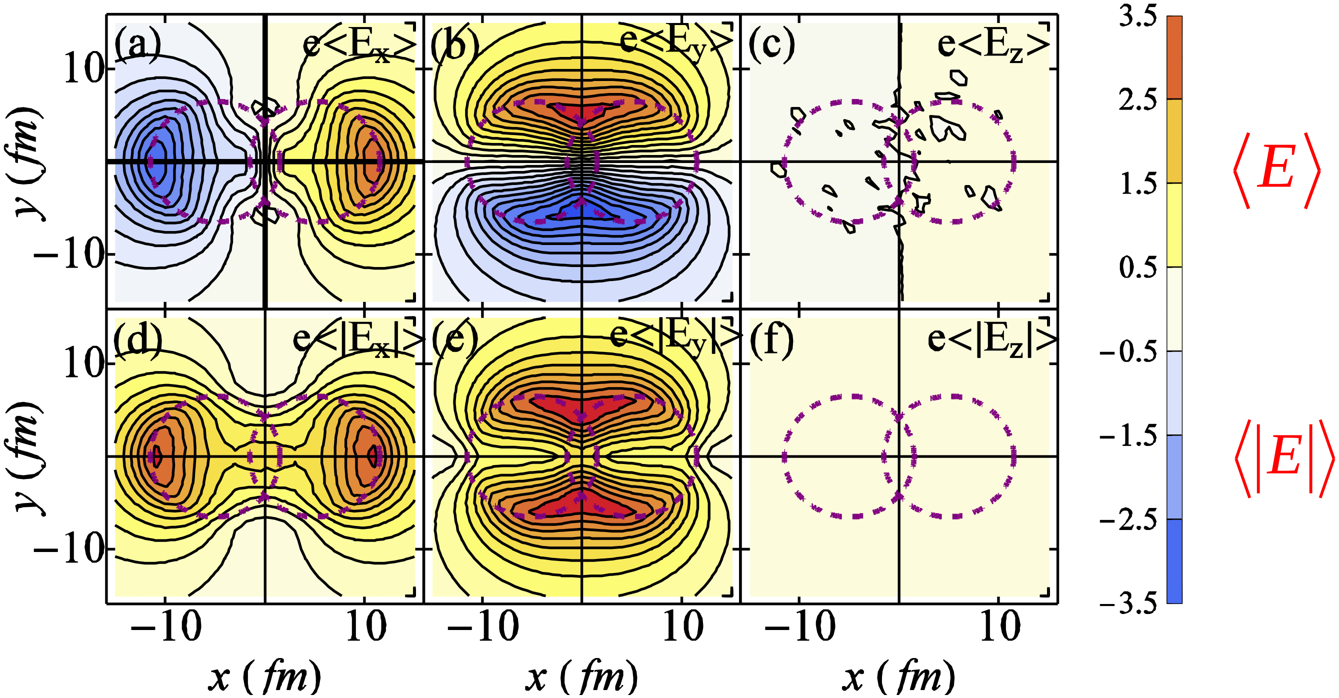


- For off-central A+A collisions, although B_x=0 at center, the spatial distribution has also structure.
- The distribution of fields distribute similarly like the fields come from two uniformly charged oppositely moving discs.



Spatial Distribution

$\sqrt{s}$=200GeV, t=0, b=10 fm



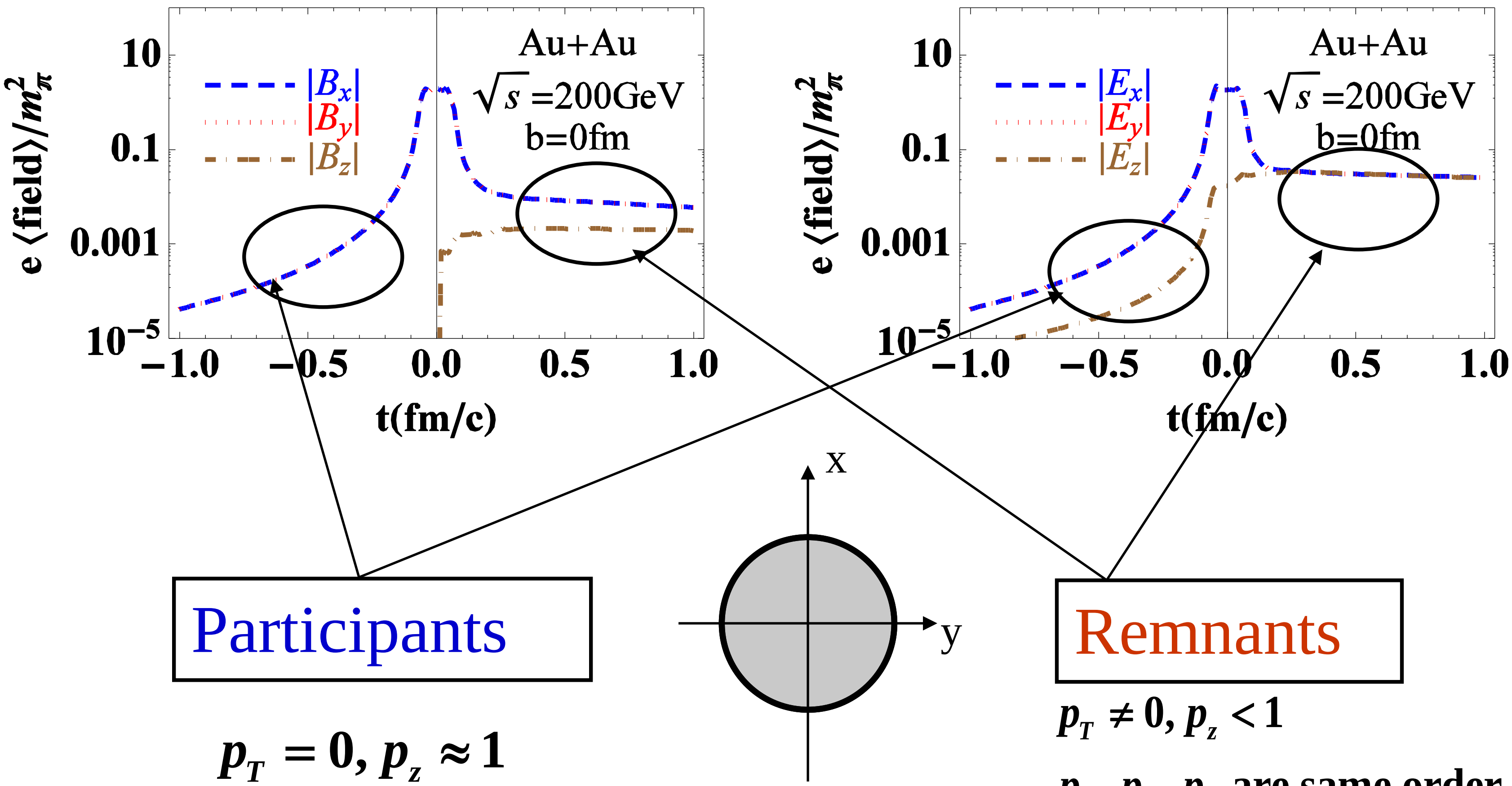
- For the events-averaged E_x and E_y, there is a large gradient in the overlap area. This maybe cause some observations, like the separation of charge parton in QGP.

Early-Stage time evolution of EM

- In our calculation, we neglect the contribution from produced partons (QGP)
  - The QGP are assumed as insulating medium
- We neglect the back response of field on the motion of nucleon.
  - So we only give the time evolution in early-stage i.e. before the formation of QGP.

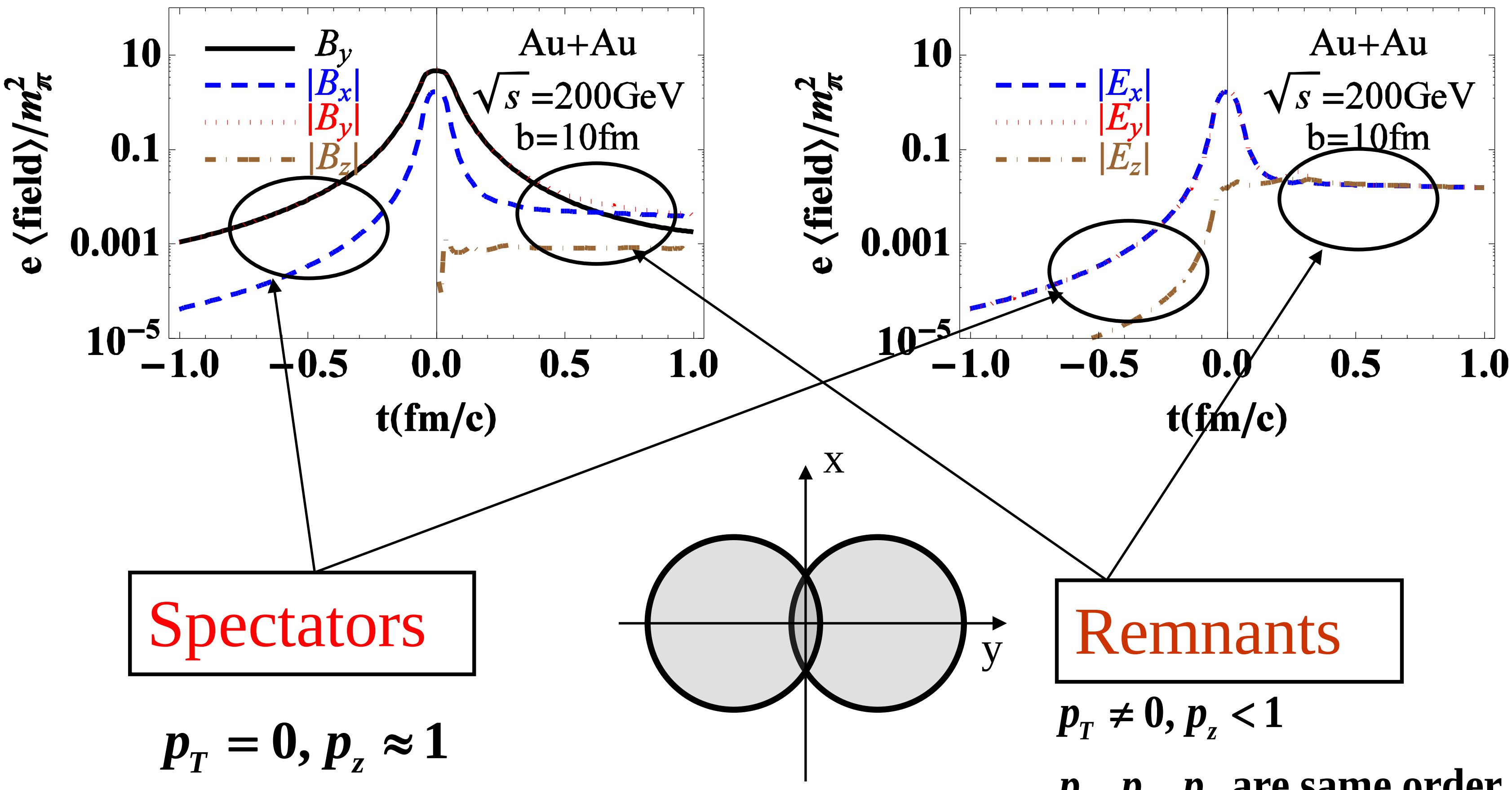
Early-Stage time evolution of EM

Au+Au, 200GeV, r=0, b=0



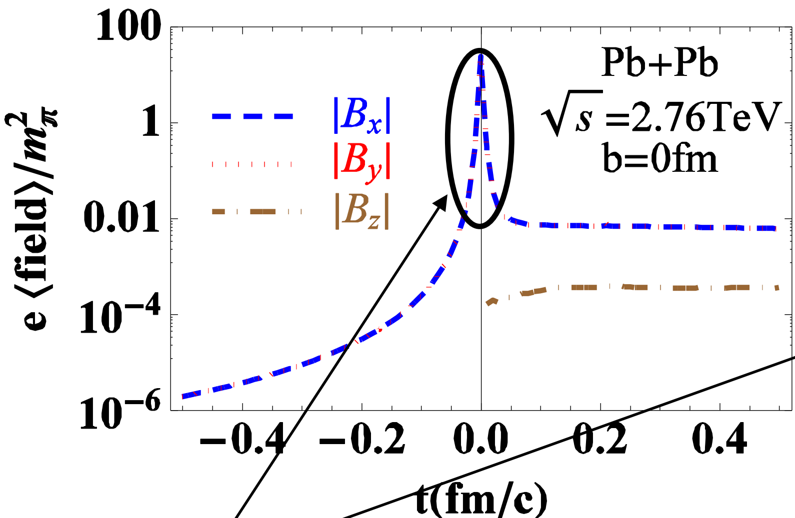
Early-Stage time evolution of EM

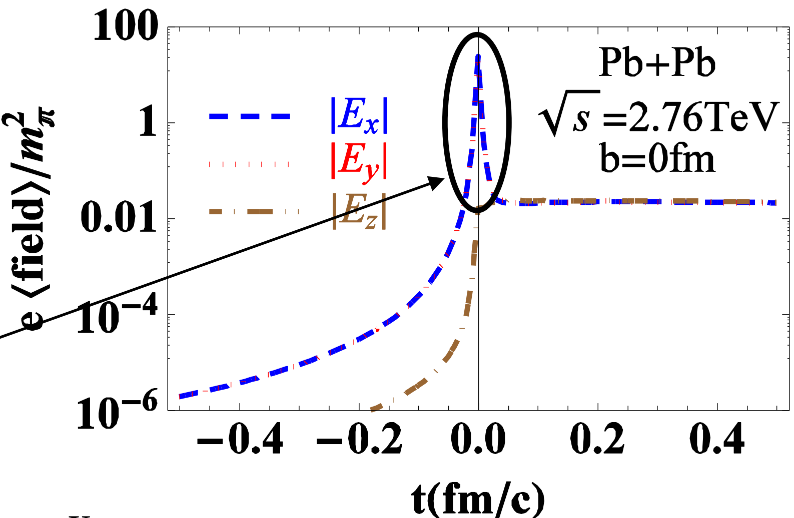
Au+Au, 200GeV, r=0, b=10fm



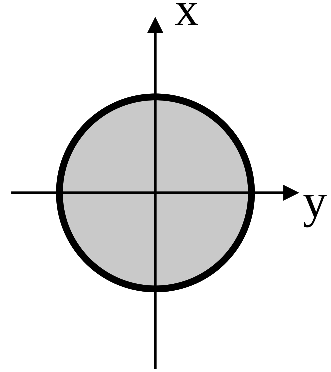
Early-Stage time evolution of EM

Pb+Pb, 2.76TeV, r=0, b=0



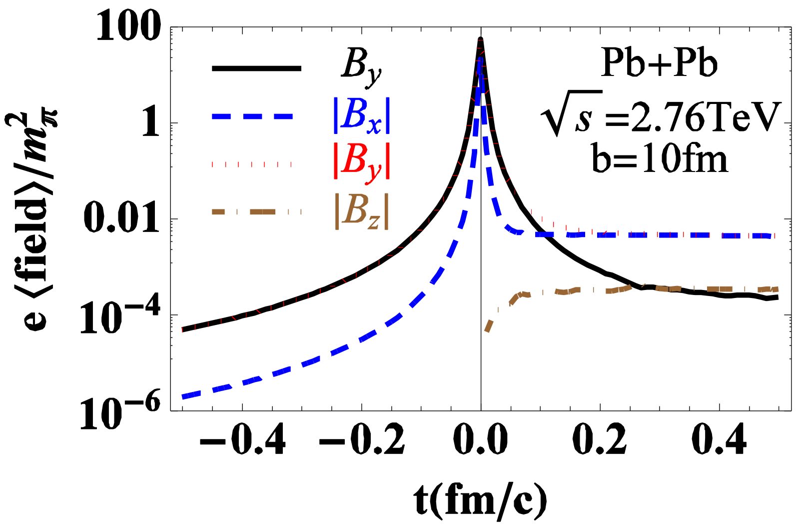


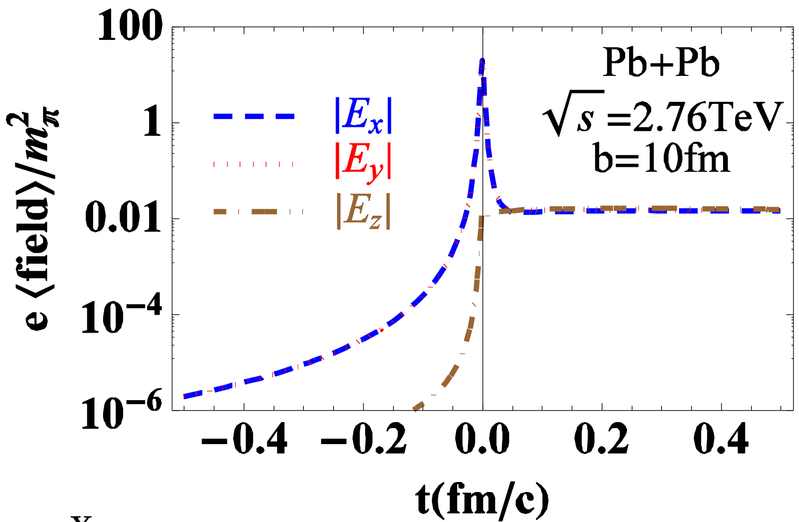
The sharp peaks are because of the larger v_z on higher energy. The nucleus approach and leave more quickly.

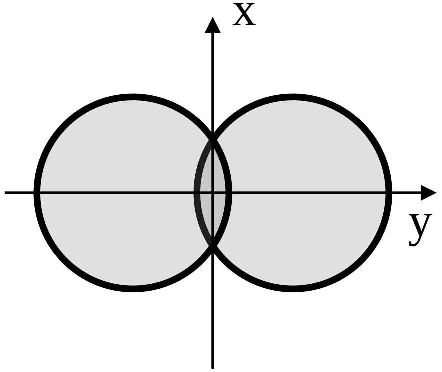


Early-Stage time evolution of EM

Pb+Pb, 2.76TeV, r=0, b=10fm

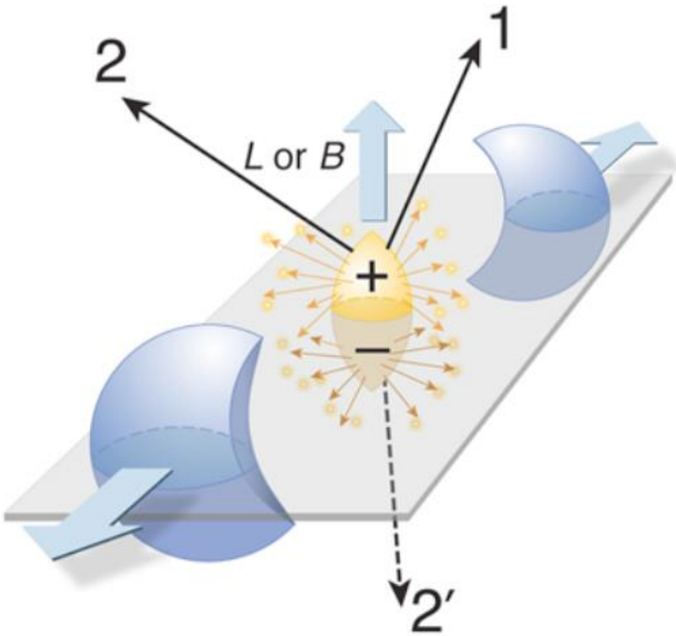




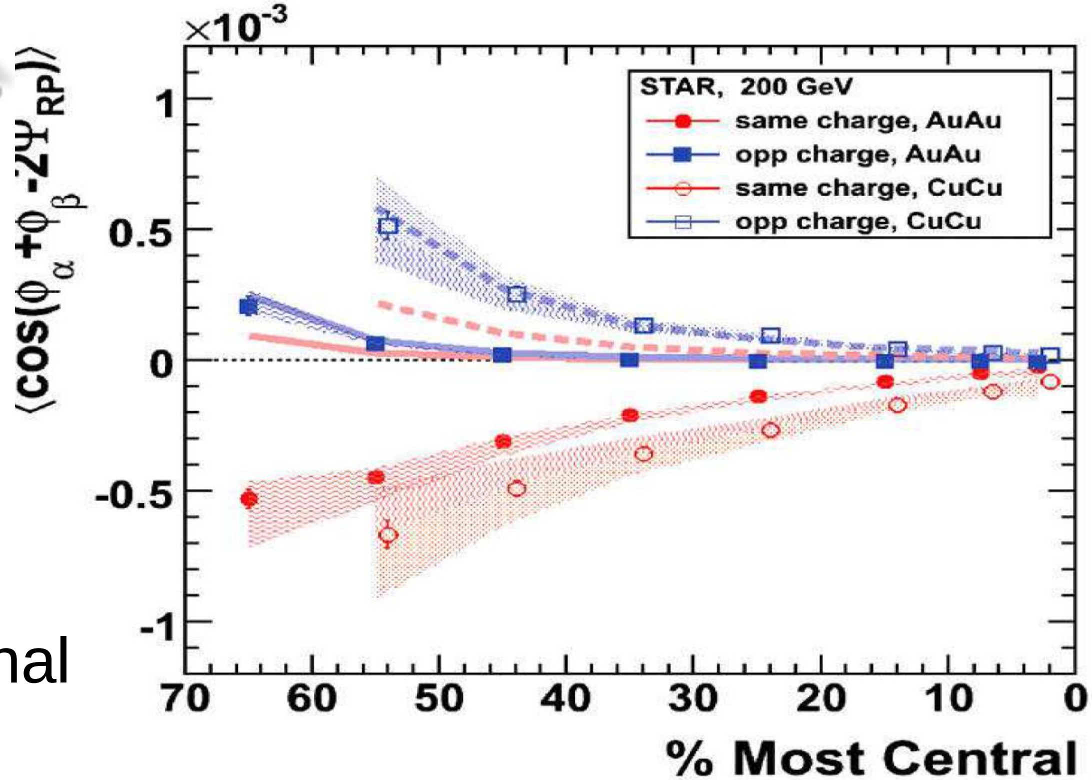


Background Vs. CME effect

Experiment searches of CME



charge correlation in Au+Au, Cu+Cu, and Pb+Pb are consistent with CME:



Two particle charge azimuthal correlation function:

$$\gamma_{q_1,q_2} = \langle \cos(\phi_1 + \phi_2 - 2\psi_{RP}) \rangle$$

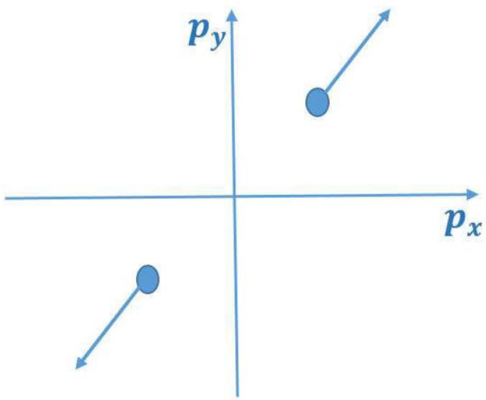
$$\gamma_{ss} = (\gamma_{++} + \gamma_{--})/2$$

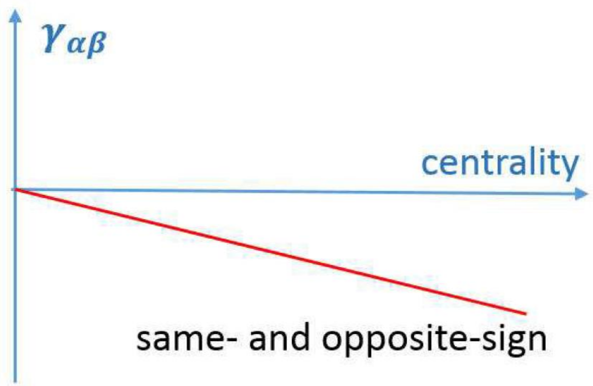
$$\gamma_{os} = \gamma_{+-}$$

Possible background of $\langle \cos(\phi_1 + \phi_2 - 2\psi_{RP}) \rangle$

- Transverse momentum conservation (TMC) $\sim -\frac{v_2}{N}$

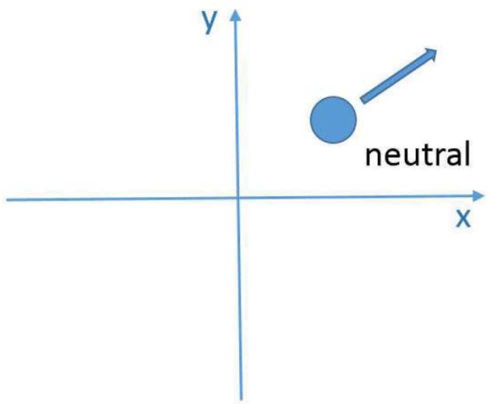
  (Pratt, Schlichting,and Gavin, 2010; Liao, Bzdak, and Koch 2010)

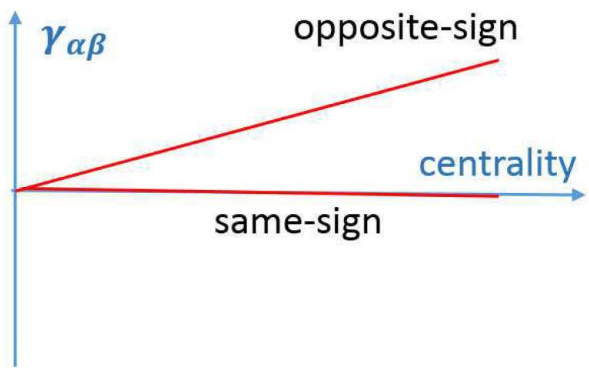




$$\Delta\gamma = \gamma_{os} - \gamma_{ss}$$

- Local charge conservation (Pratt and Schlichting, 2011) $\sim \frac{v_2}{N}$





How to determine the CME-induced signals?

1, different Size collision: **Cu+Au**

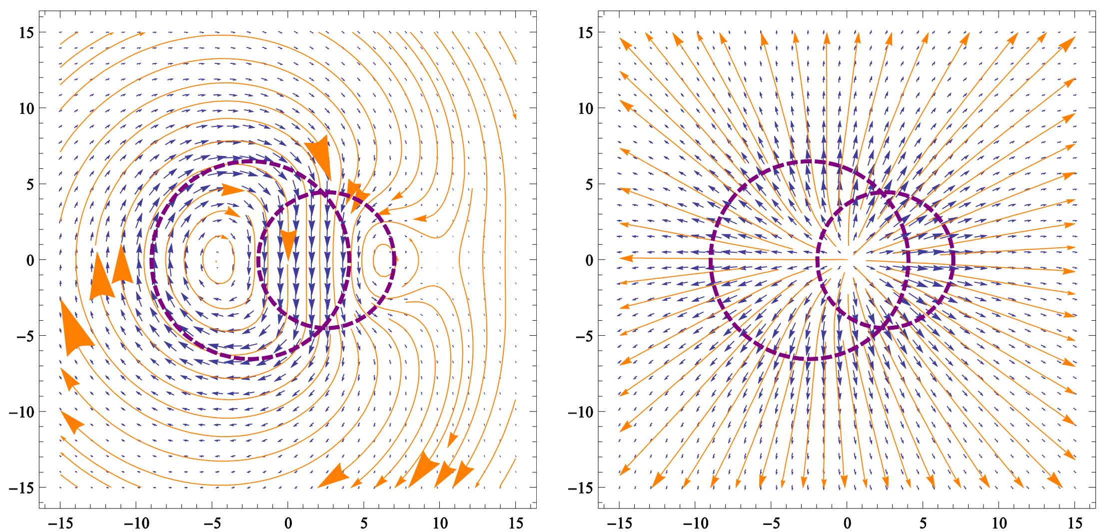
If v2-driven is dominant, the charge correlation

Au+Au ⟷ **similar** ⟷ **Cu+Au**

If CME is dominant, the charge correlation

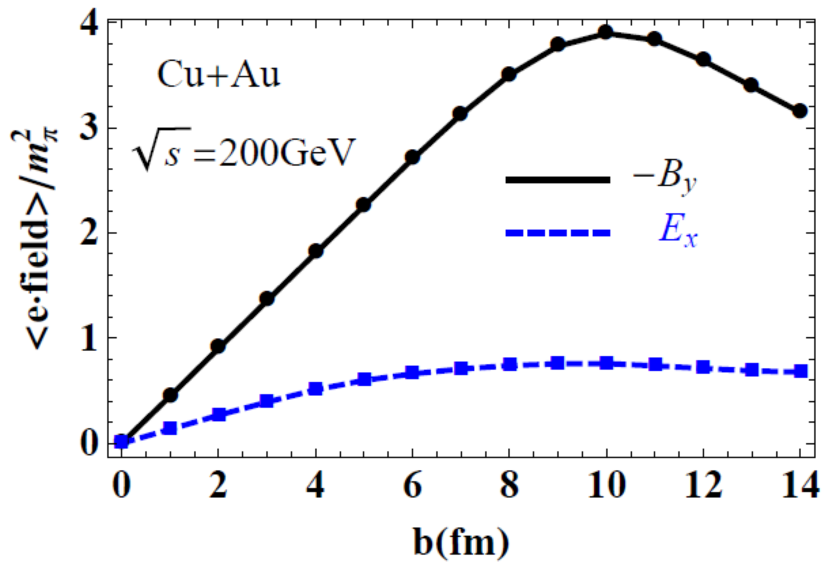
Au+Au ⟷ **different ?** ⟷ **Cu+Au**

B



E

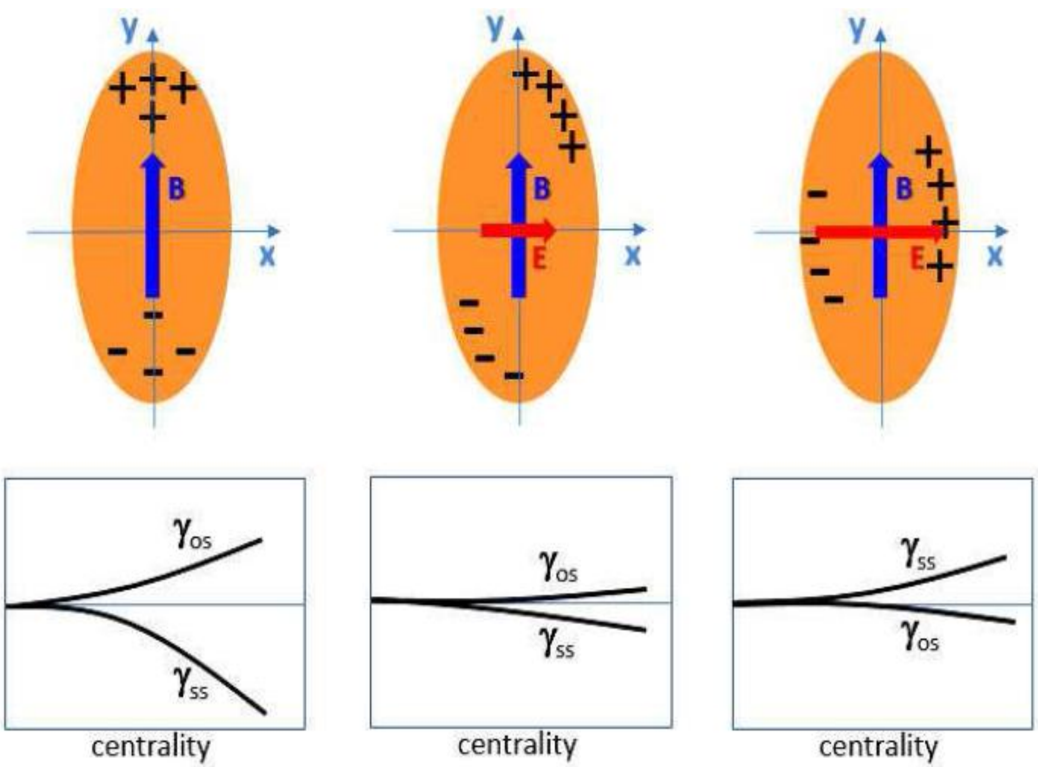
In transverse plane



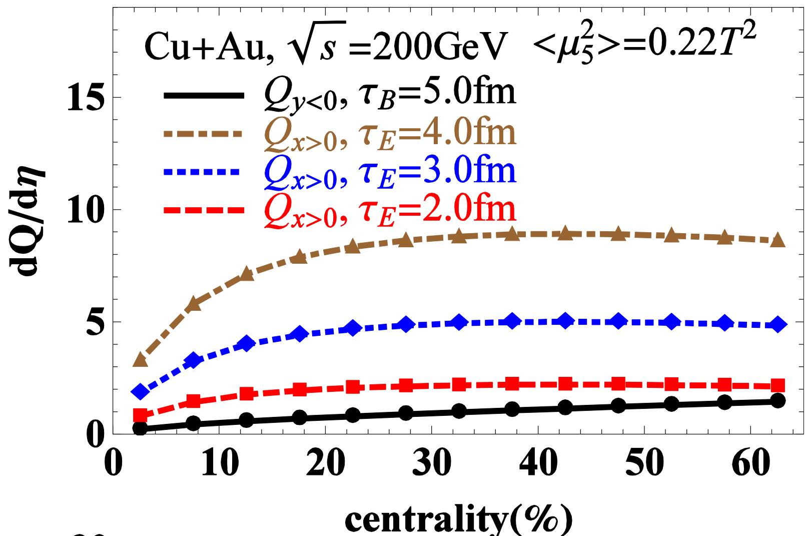
If E field is strong enough,

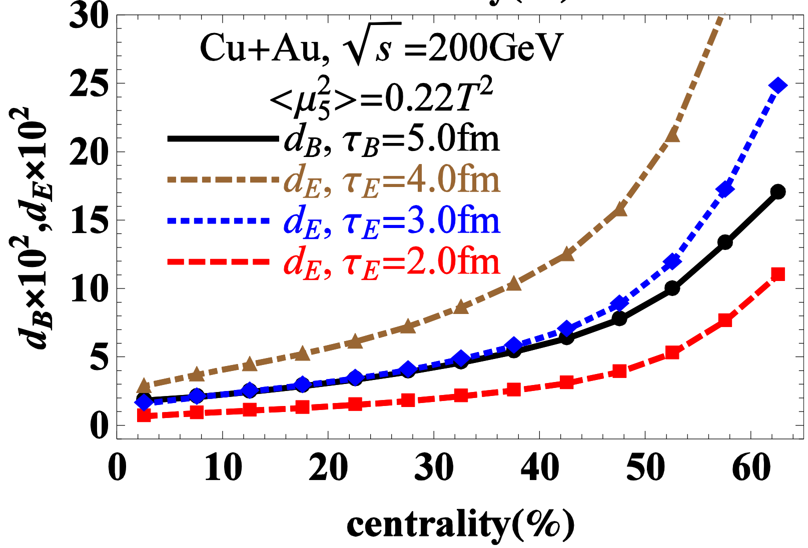
or its lifetime is long enough,

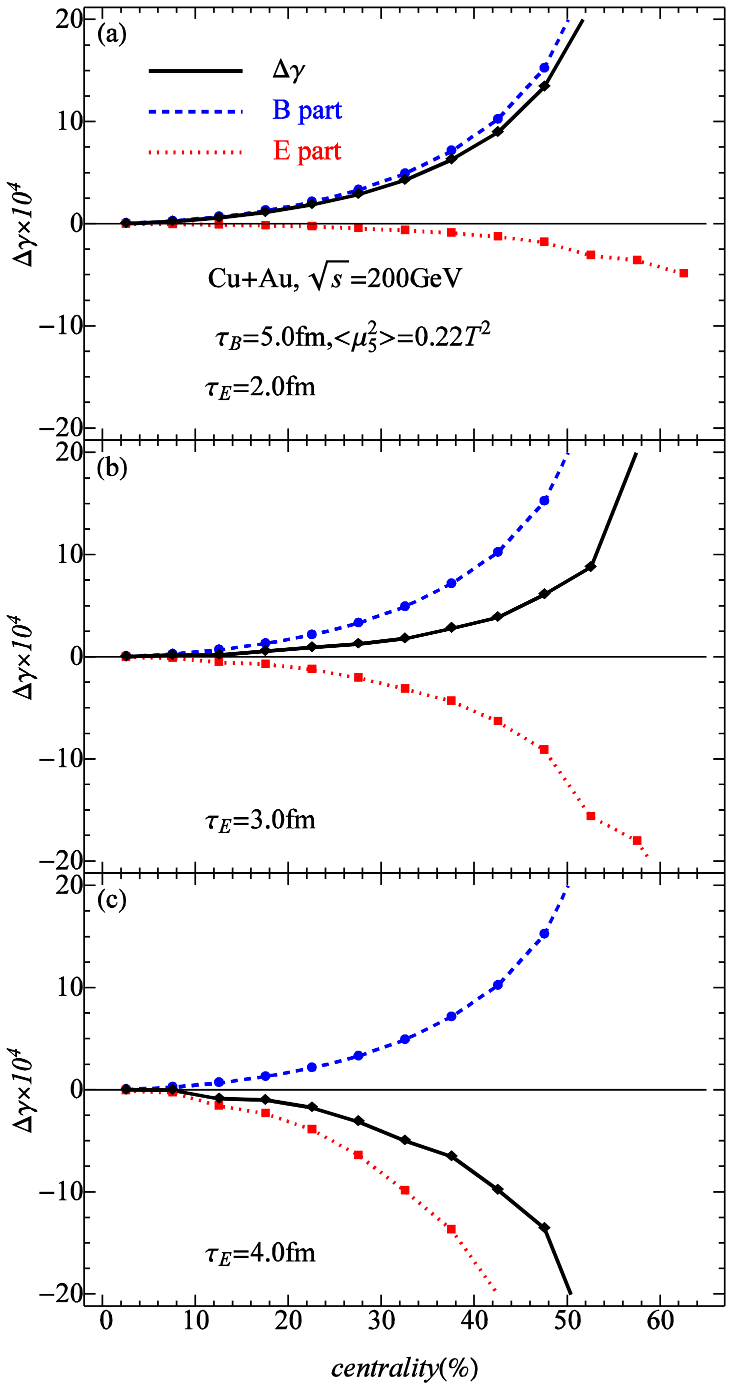
……



in general, the life time of E is shorter than B







2, Isobaric collision:

— ${}^{96}_{40}$Zirconium vs ${}^{96}_{44}$Ruthenium

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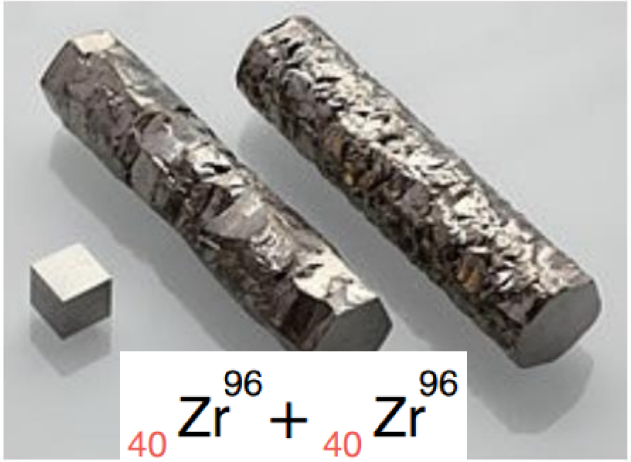

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Same A=96 → Same elliptic flow (via ε_2) → v_2-driven background fixed

Diffenent Z → different B field → CME contribution changed

Radius[4,5,6] (fm) & Deformation[3]

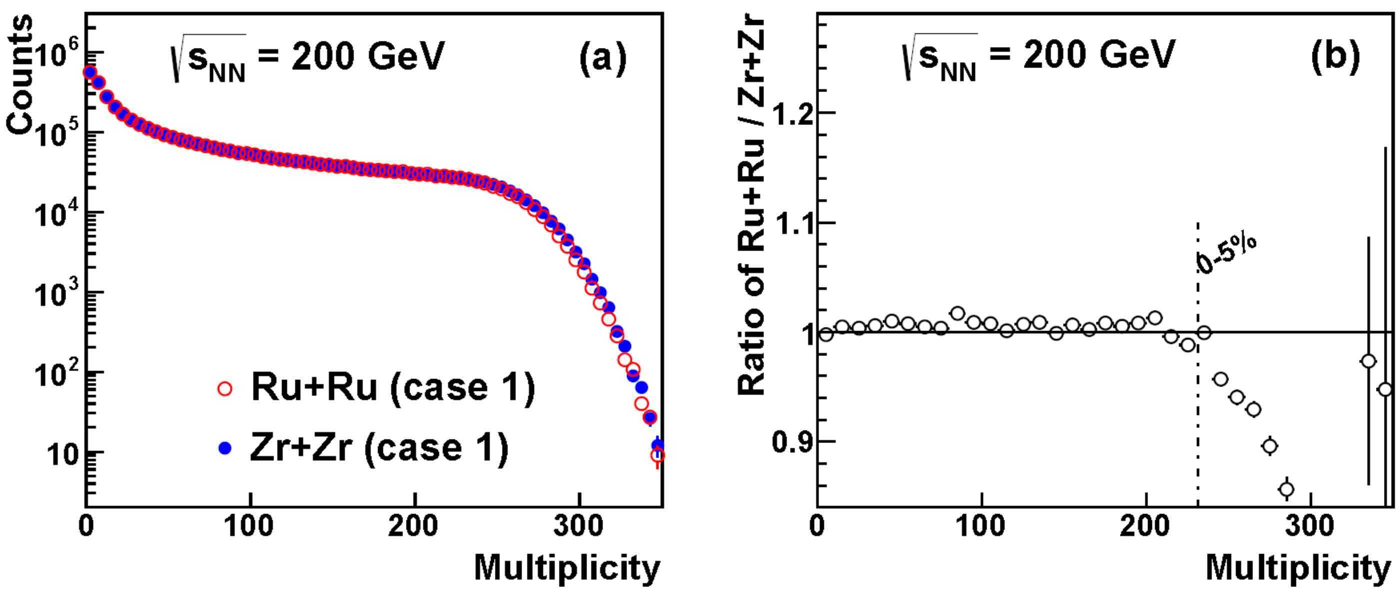
setting 1	R0	a(d)	β2	β4
Ru96	5.0845	0.567	0.1579	0.00
Zr96	5.0212	0.574	0.08	0.00

El.-Magn. properties

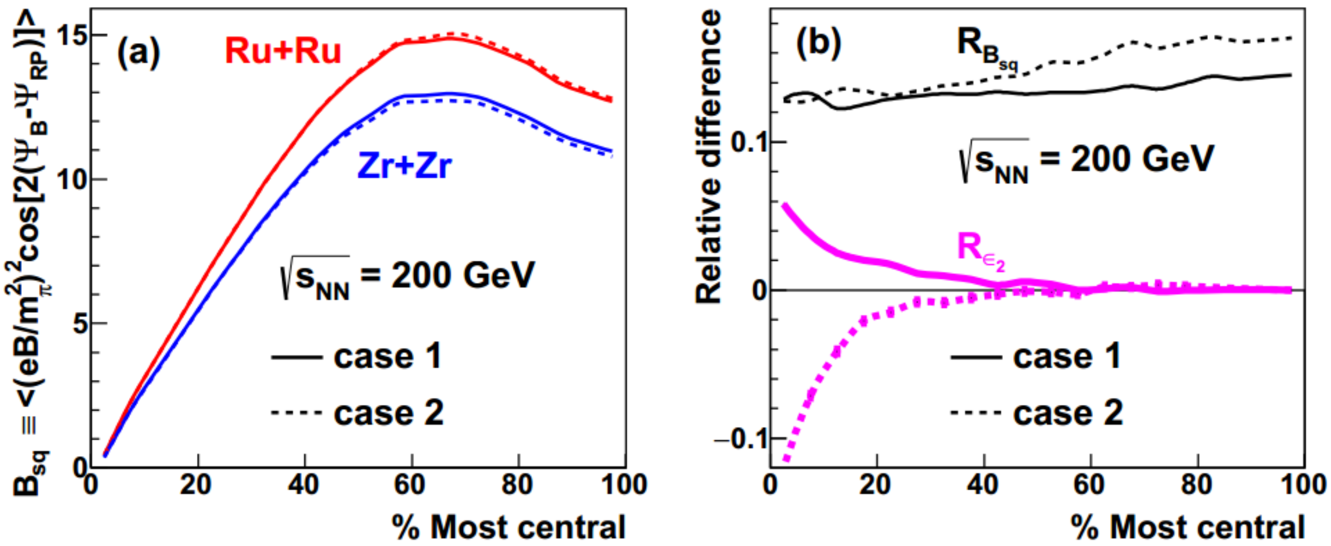
setting 2	R0	a(d)	β2	β4
Ru96	5.0845	0.567	0.053	0.009
Zr96	5.0212	0.574	0.217	0.01

Woods-Saxon density:

$$\rho(r,\theta)=\frac{\rho_0}{1+e^{(r-R_0+\beta_2 R_0 Y_2^0(\theta))/a}}$$



Initial magnetic field and initial eccentricity



B_{sq} quantifies magnetic-field fluctuation

(Blozynski, Huang, Zhang, and Liao, 2013)

R is the relative difference: 2(RuRu-ZrZr)/(RuRu+ZrZr)

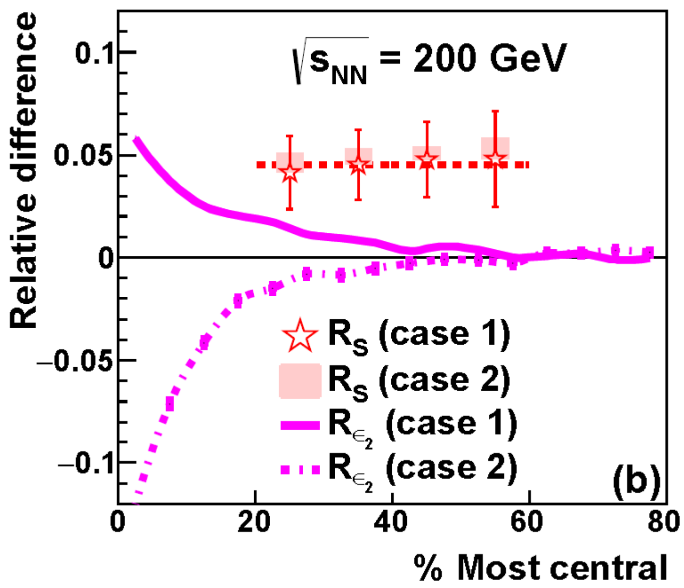
Centrality 20-60%: sizable difference in B ($R_{B_{sq}} \sim 10 - 20\%$)

but small difference in eccentricity ($R_{\varepsilon_2} < 2\%$)

Results:

From Monte-Carlo simulation and fitting experiment data, we have: $bg = 2/3$

So, we can estimate the relative difference is about 5% for Ru+Ru and Zr+Zr

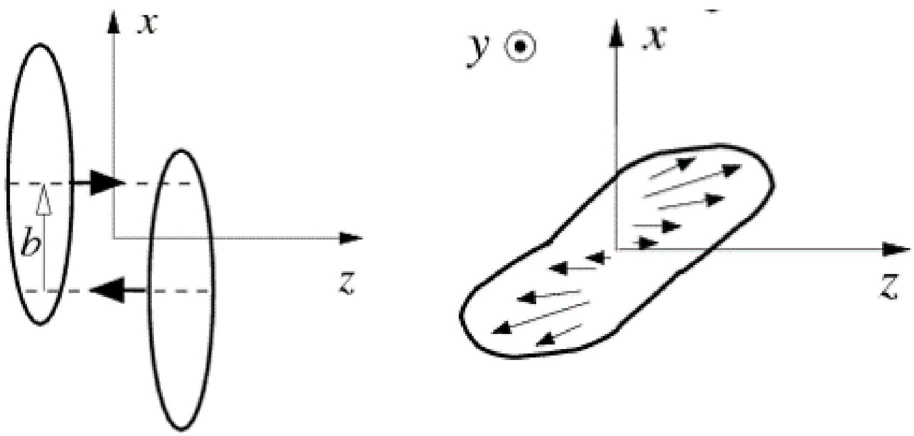


The systematic uncertainties are largely canceled out with the relative difference between Ru + Ru and Zr + Zr

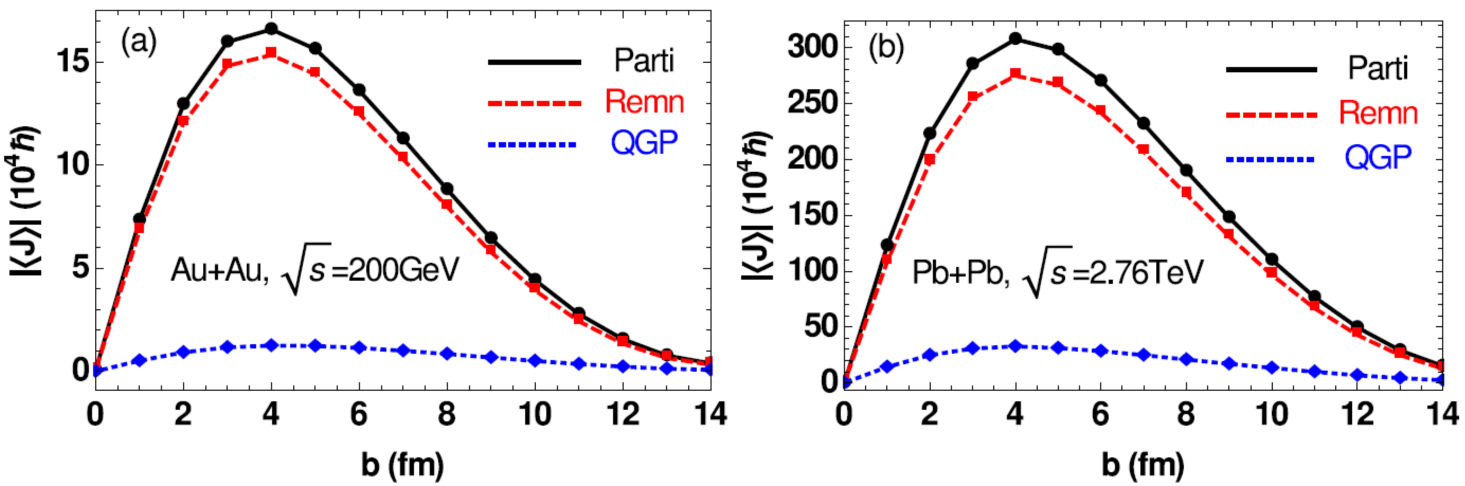
$$R_S = S^{Ru+Ru} - S^{Zr+Zr} = (N_{part}\Delta\gamma)^{Ru+Ru} - (N_{part}\Delta\gamma)^{Zr+Zr}$$

Vorticity in HIC

Total Orbital Angular Momentum in HIC



L_y



Total Orbital Angular Momentum in HIC

L_y → ?

Low energy

(nuclear binding energy)

→ Rotating compound nucleus

High energy

（RHIC, LHC）

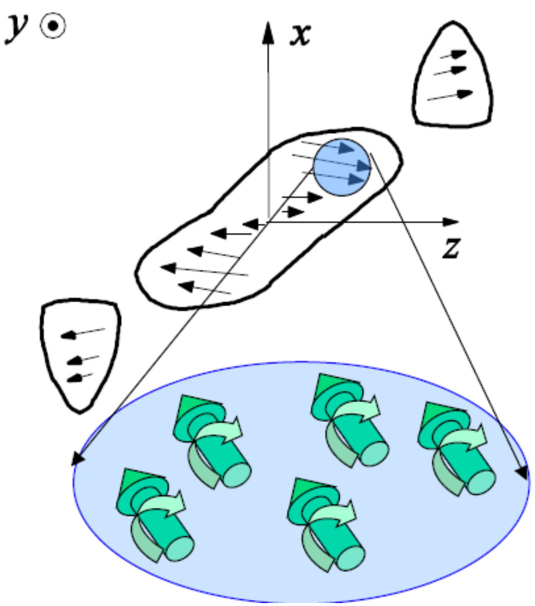
→ Shear of the longitudinal flow

↓

Local vorticity

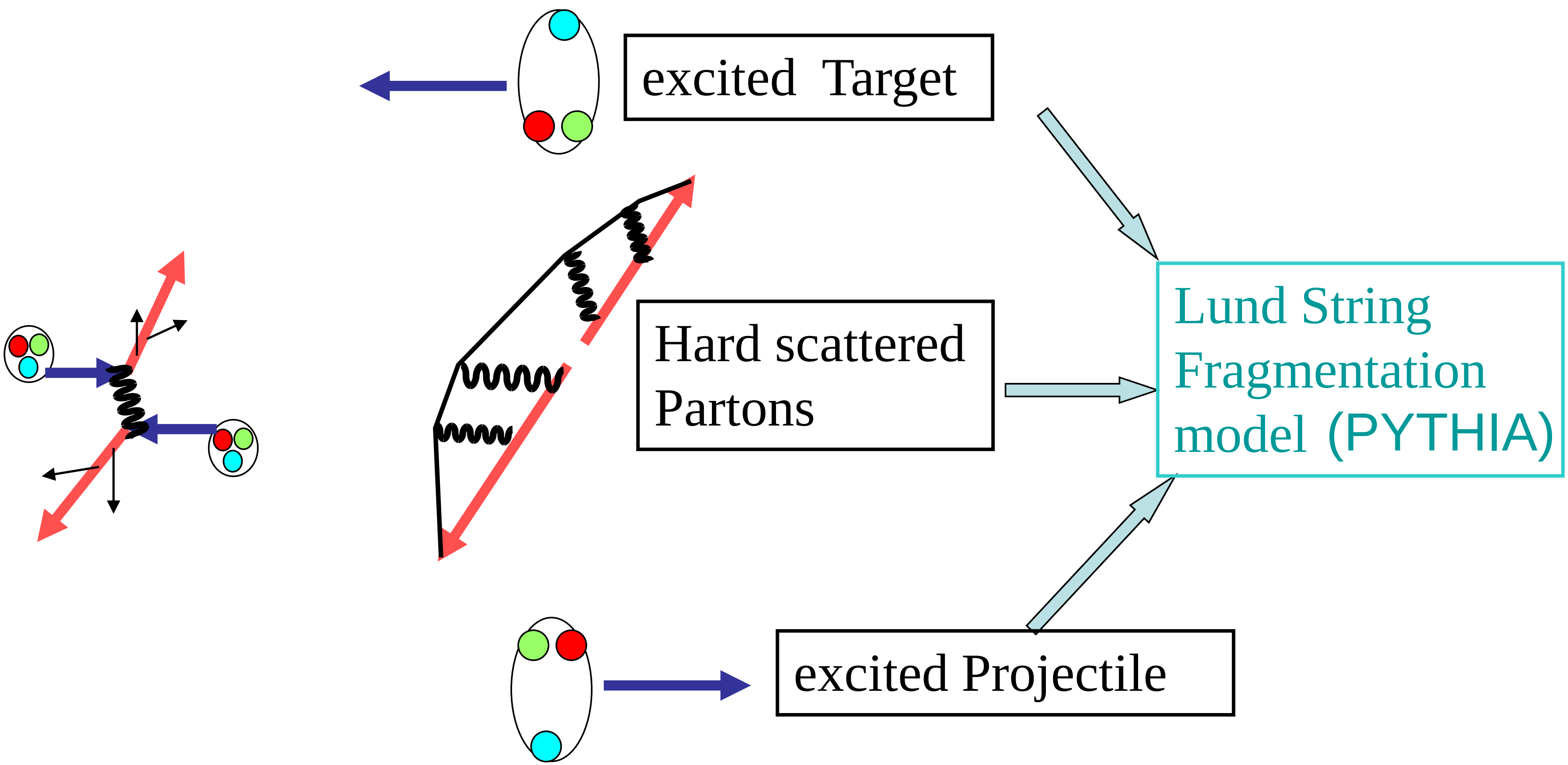
↓

Quark global polarization

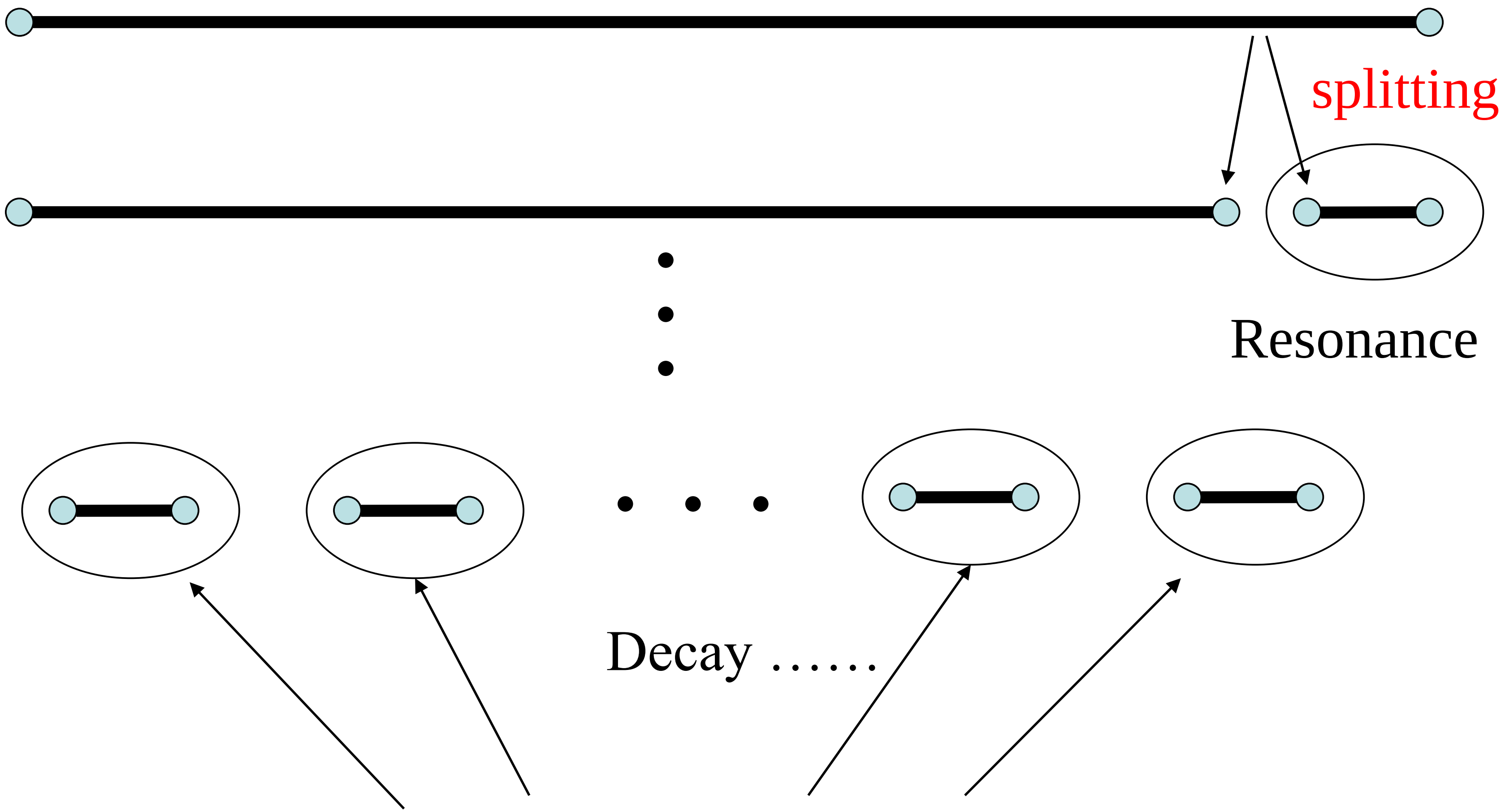


Liang and Wang 2005; Becattini etal 2013; Wang et al 2016

HIJING Model



Fragmentation process in HIJING (PYTHIA):



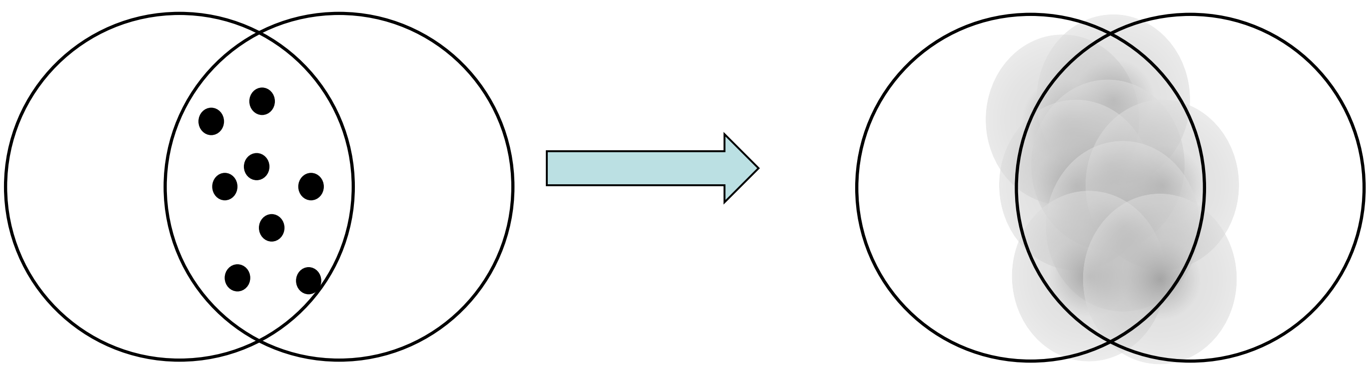
In this work: we decompose Resonances into partons

In order to get continuous distribution of velocity filed,
we introduce a smearing function:

$$\Phi_{\mathrm{G}}(x, x_i) = \frac{K}{\tau_0\sqrt{2\pi\sigma_\eta^2 2\pi\sigma_r^2}} \exp\left[-\frac{(x-x_i)^2+(y-y_i)^2}{2\sigma_r^2} - \frac{(\eta-\eta_i)^2}{2\sigma_\eta^2}\right]$$

L. Pang, Q. Wang and X. N. Wang, Phys. Rev. C 86 (2012), 024911

With this smearing density function, the energy and momentum of each parton are represented by a wave packet, smearing into whole space

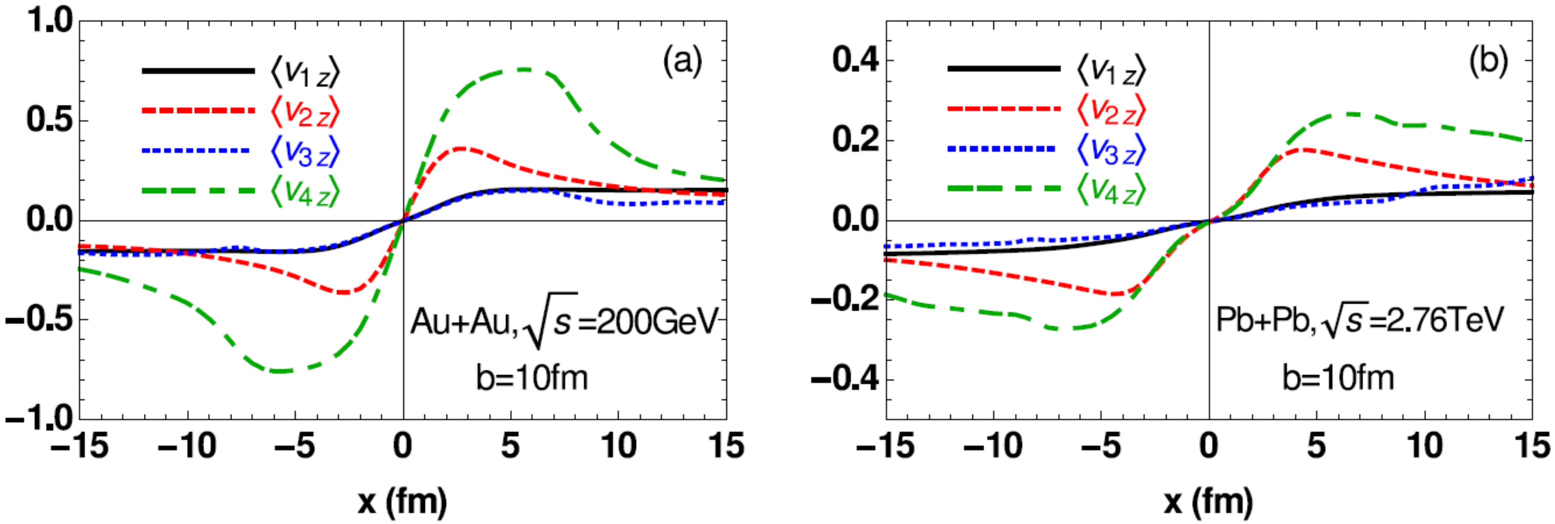


$$v_1^a(x) = \frac{1}{\sum_i \Phi(x, x_i)} \sum_i \frac{p_i^a}{p_i^0} \Phi(x, x_i),$$

——velocity of the particle flow

$$v_2^a(x) = \frac{\sum_i p_i^a \Phi(x, x_i)}{\sum_i [p_i^0 + (p_i^a)^2 / p_i^0] \Phi(x, x_i)},$$

——velocity of the energy flow

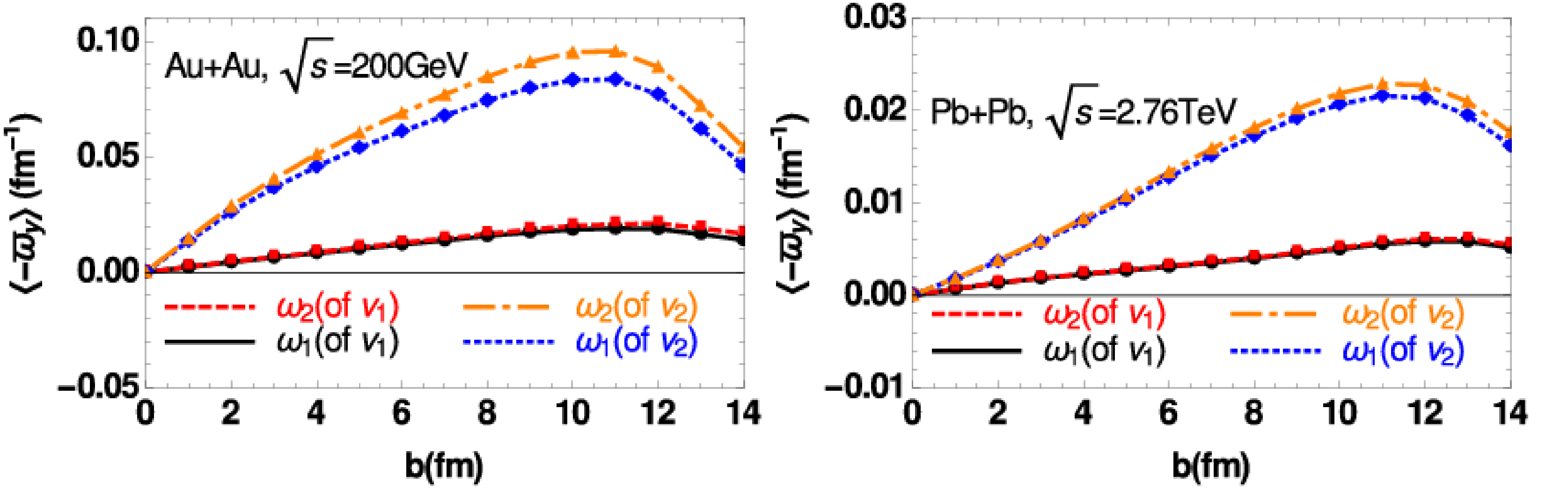


Definition of Vorticity

$$\omega_1 = \nabla \times v,$$ ——the usual nonrelativistic definition

$$\omega_2 = \gamma^2 \nabla \times v,$$ ——the spatial components of the relativistic definition

The space-averaged vorticity at τ_0 and η=0:

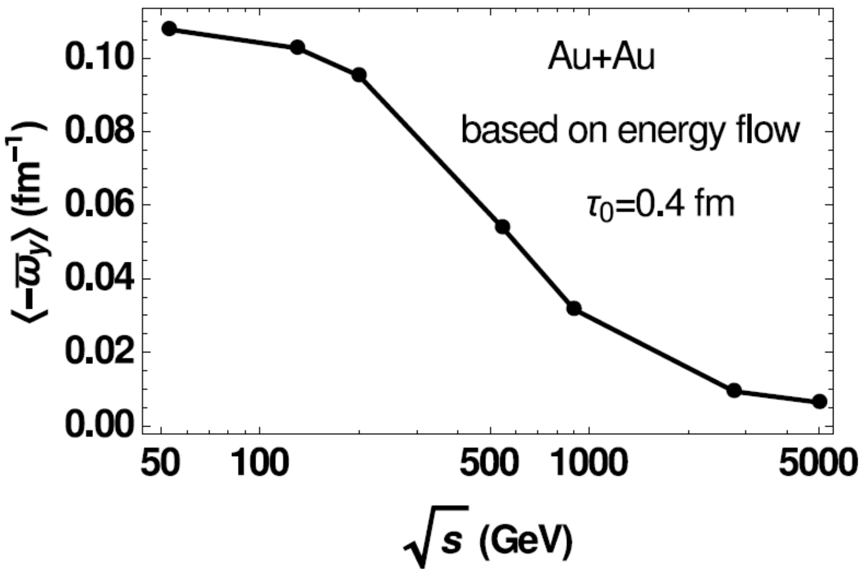


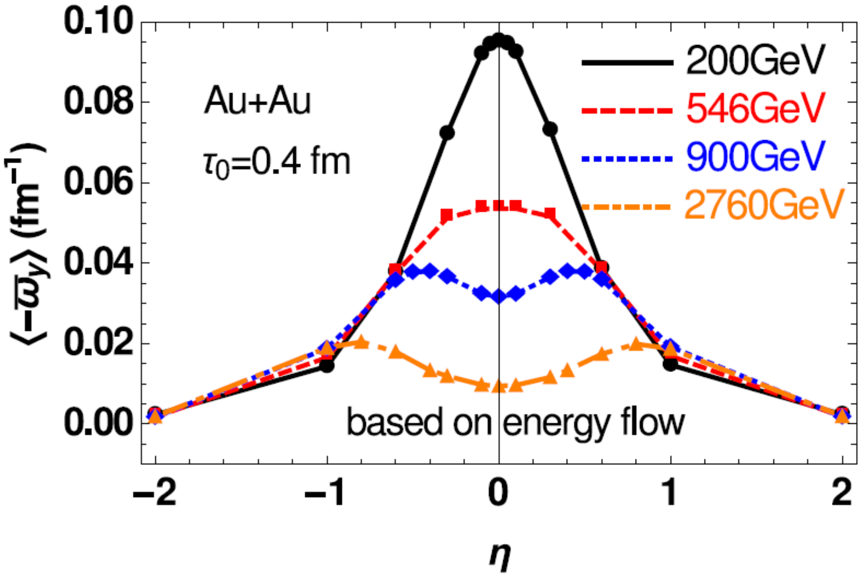
Most vortical fluid: Vorticity at 200GeV at b=10 fm is 10^{21} Hz.

(Fastest man-made rotation via laser light ~ 10^7 Hz)

Arita etal Nat.Comm. 2013

Collision energy dependence of Vorticity

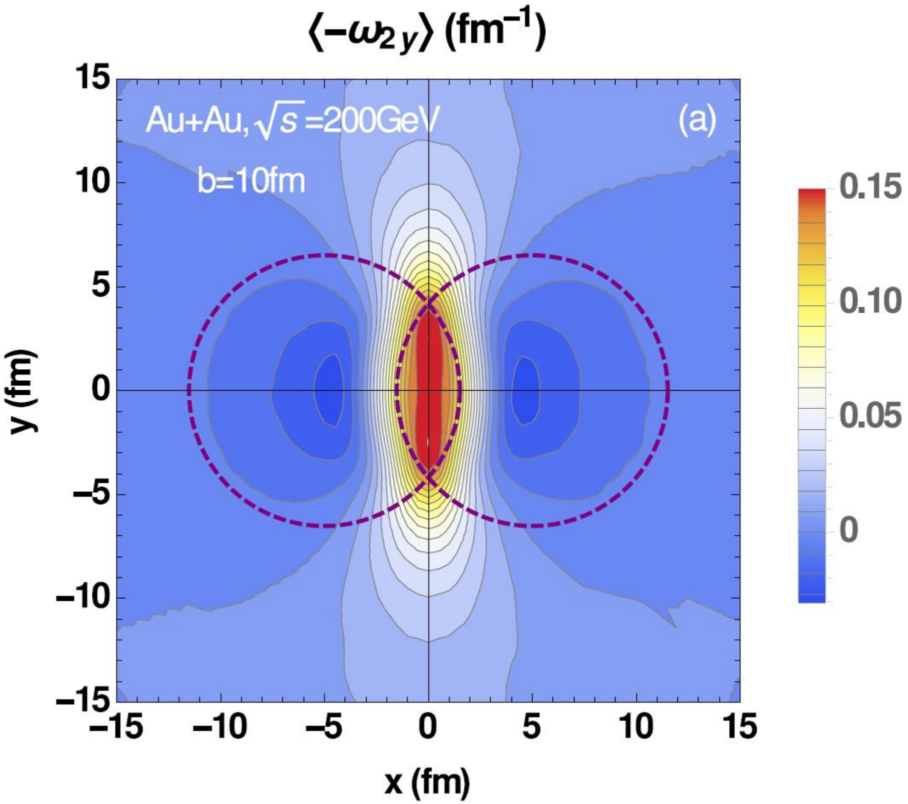


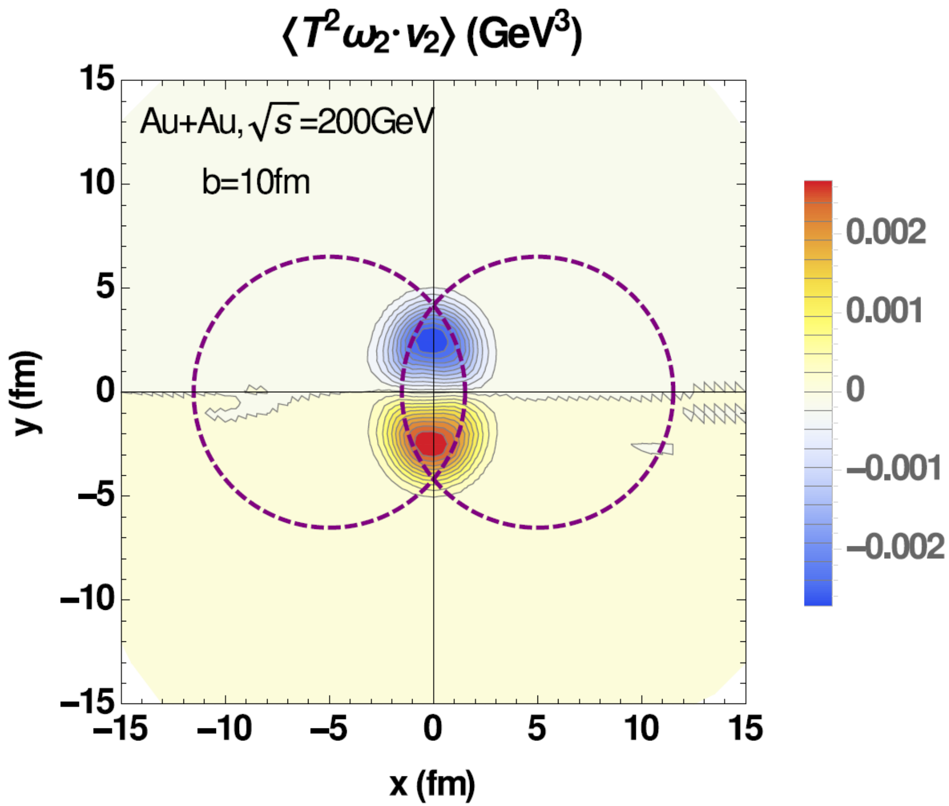


higher energy:

more AM carried by finite rapidity particles; mid-rapidity closer to Bjorken boost invariant; larger moment of inertia

Spatial distribution of Vorticity





Summary

- We have constructed a numerical framework for calculating the EM field and vorticity in HIC based on HIJING model
- We have investigated the generation and evolution of the EM fields in HIC on e-b-e events.
- Attempting to distinguish the background contribution and CME effect, we have surveyed two ways, including different size A+B collisions, and isobaric collisions.
- We have also simulated the generation of vorticity produced in HIC at initial stage.

- In the following study, we are concentrating on the evolution of this vorticity.
- The EM field produced in small system, p+p and p+A.