Transport phenomena in strong magnetic fields

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6th Huada QCD School @ CCNU, May, 2017

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KH and Xu-Guang Huang (Fudan), arXiv:1609.00747 [nucl-th]

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K. Fukushima (Tokyo), KH, H.-U. Yee (UIC), Yi Yin (MIT),
PRD. [arXiv:1512.03689 [hep-ph]]

Cf.) **KH and Xu-Guang Huang (Fudan)**, arXiv:1609.00747 [nucl-th]

2. Electrical conductivity in QGP

KH, Shiyong Li (UIC), Daisuke Satow (Frankfurt), Ho-Ung Yee,
PRD. arXiv:1610.06839 [hep-ph]
KH and Daisuke Satow, PRD. [arXiv:1610.06818 [hep-ph]]

3. New magneto-hydrodynamic instability in a chiral fluid with CME

KH, Yuji Hirono (BNL), Ho-Ung Yee, and Yi Yin, In preparation.

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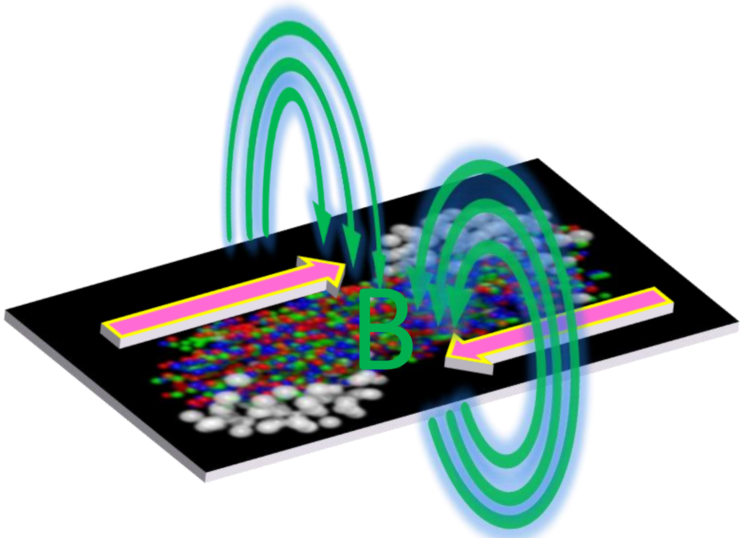
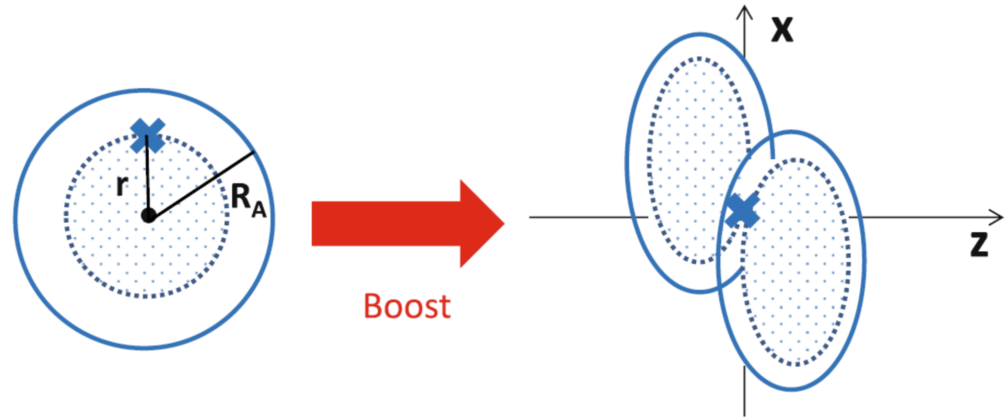
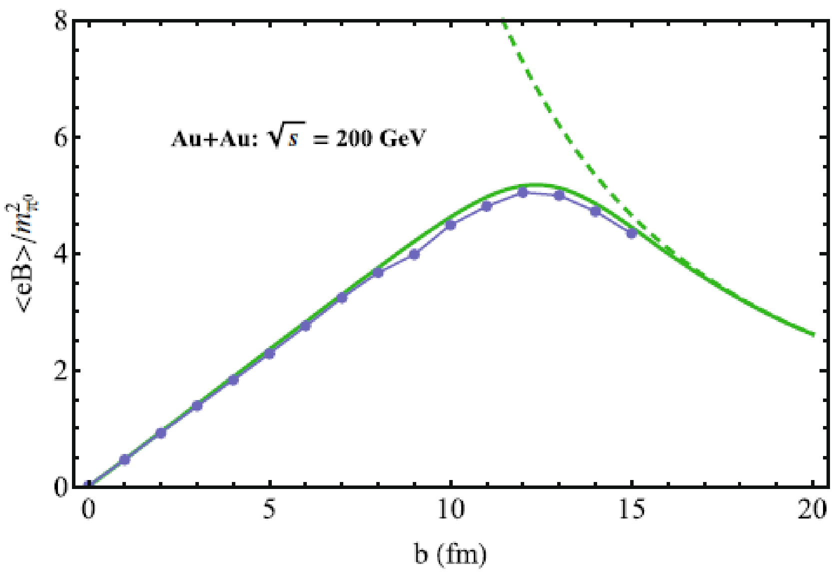
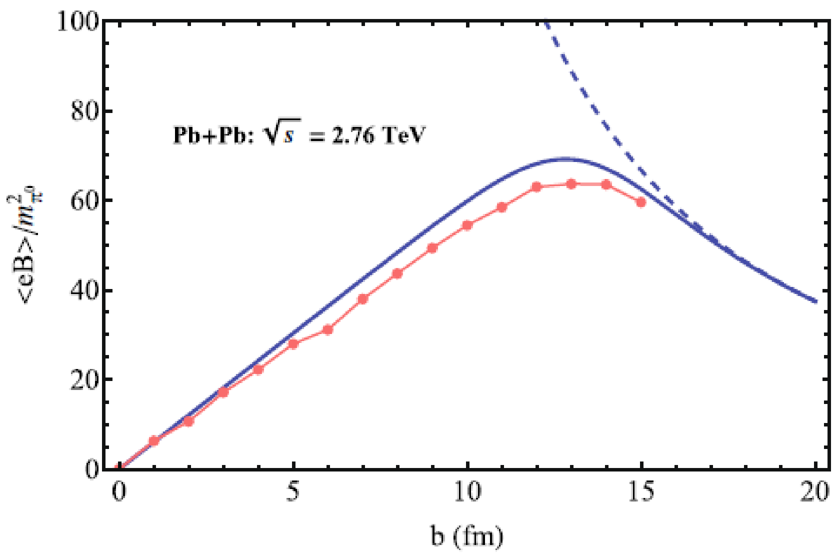


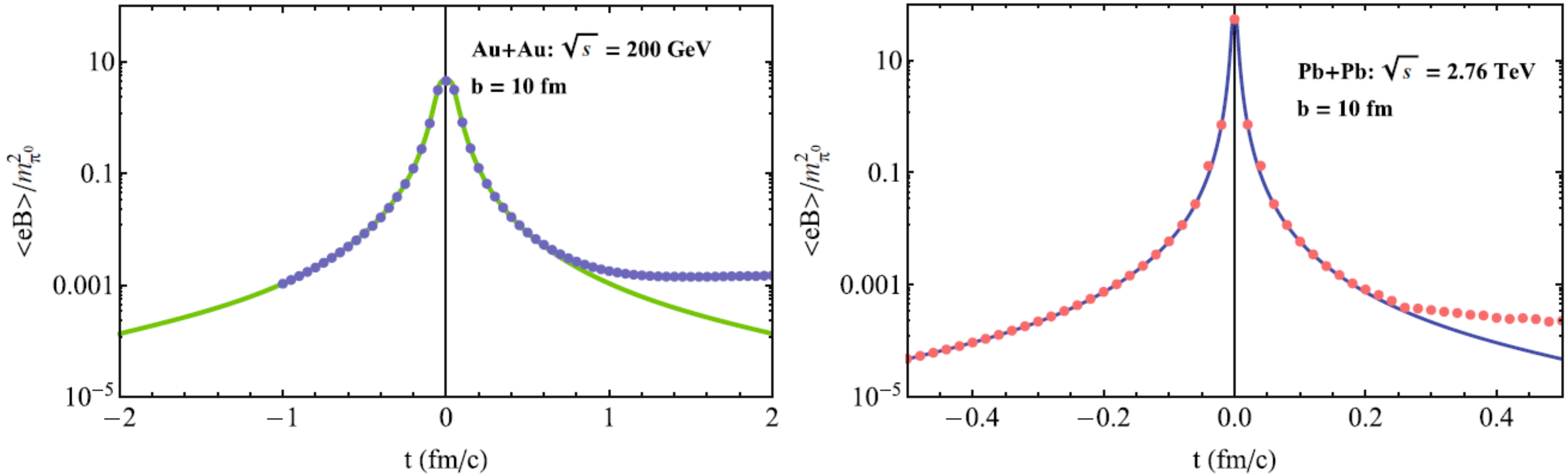






W.-T. Deng & X.-G. Huang, KH and X.-G. Huang

Time dependences



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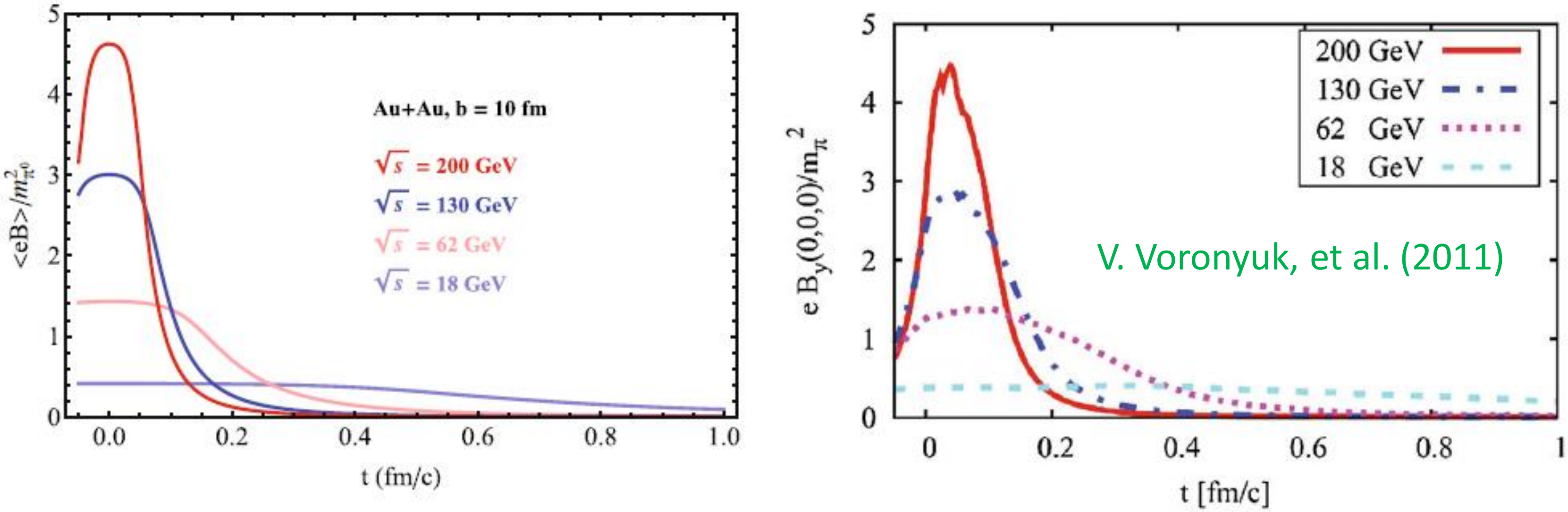
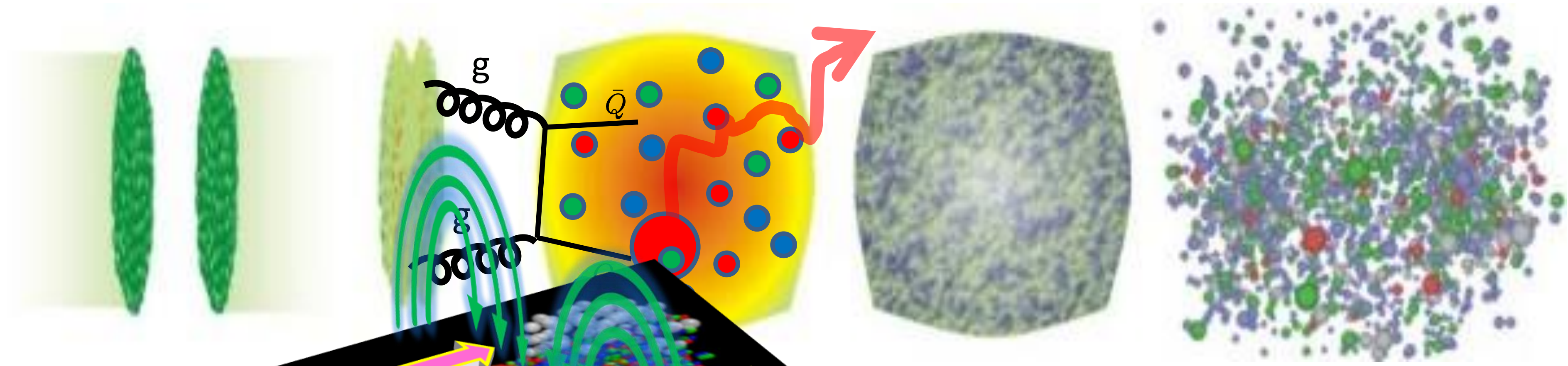
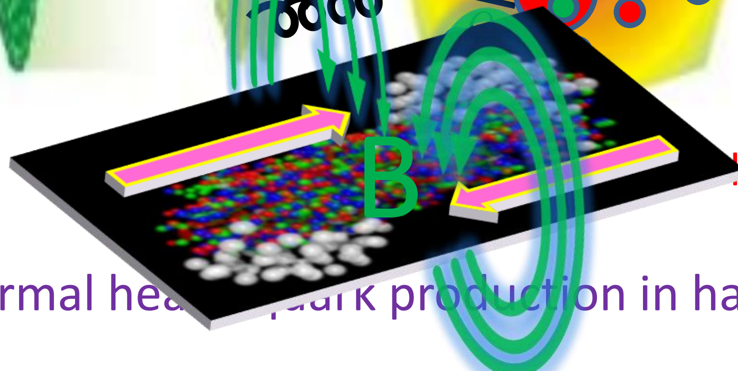


1. Heavy quark diffusion in magnetic fields

K. Fukushima (Tokyo), KH, H.-U. Yee (UIC), Yi Yin (BNL→MIT),
Phys. Rev. D 93 (2016) 074028. [arXiv:1512.03689 [hep-ph]]

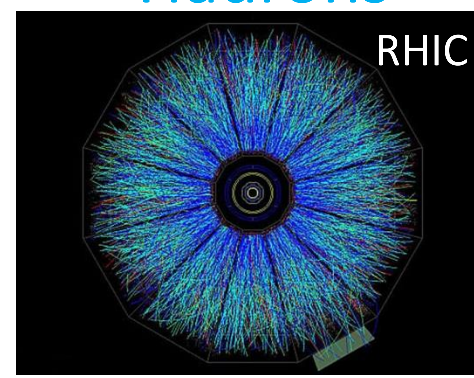
Heavy quarks as a probe of QGP





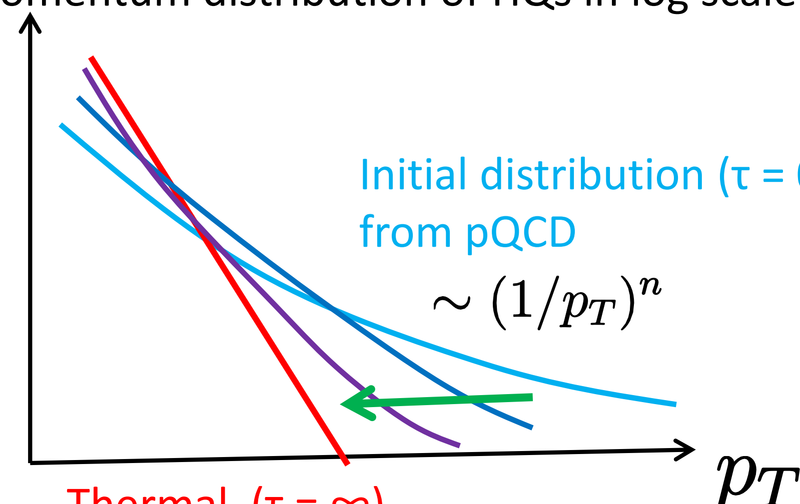
lasma (QGP)

Hadrons



Non-thermal hea... quark production in hard scatterings

Momentum distribution of HQs in log scale



Relaxation time is controlled by transport coefficients (Drag force, diffusion constant)

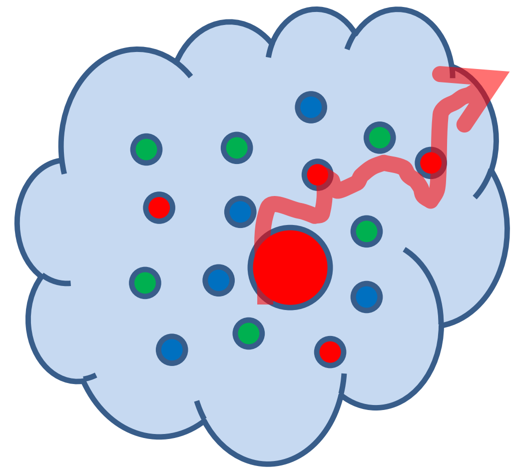
Heavy quark (HQ) dynamics in the QPG -- In soft regime

Langevin equation

$$\frac{d\boldsymbol{P}}{dt} = \boldsymbol{\xi}(t) - \eta_D \boldsymbol{P}$$

Random kick (white noise)

$$\langle \xi_i(t)\xi_j(t')\rangle = \kappa\delta_{ij}\delta(t-t')$$



Drag force coefficient: η_D
Diffusion constant: κ

Einstein relation

$$\eta_D = \frac{\kappa}{2MT}$$

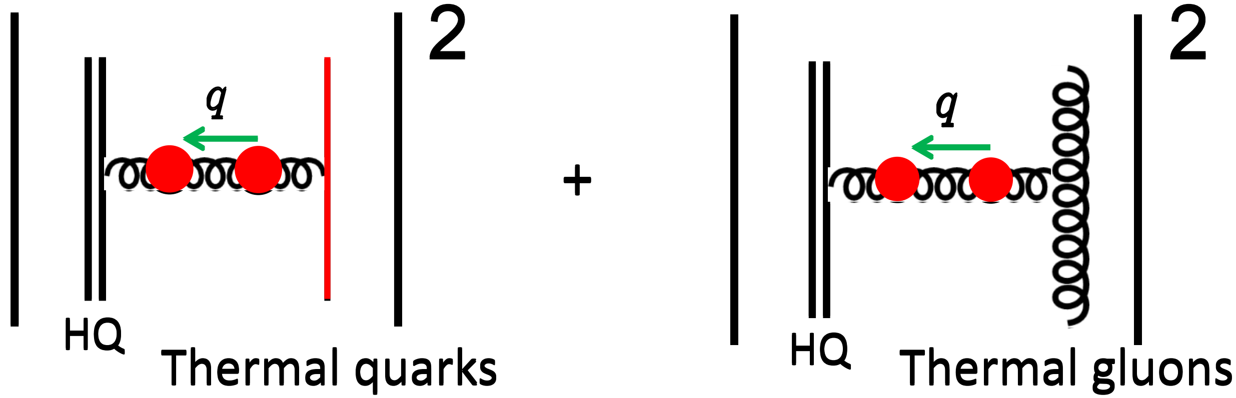
Perturbative calculation by finite-T field theory (Hard Thermal Loop resummation)
LO and NLO without B are known (Moore & Teaney, Caron-Huot & Moore).

Perturbative computation of momentum diffusion constant

$$\kappa_i = \int d^3\boldsymbol{q}\, q_i^2 \frac{d\Gamma}{d^3\boldsymbol{q}}$$

Momentum transfer rate in the LO Coulomb scatterings

$$\frac{d\Gamma}{d^3\boldsymbol{q}} =$$

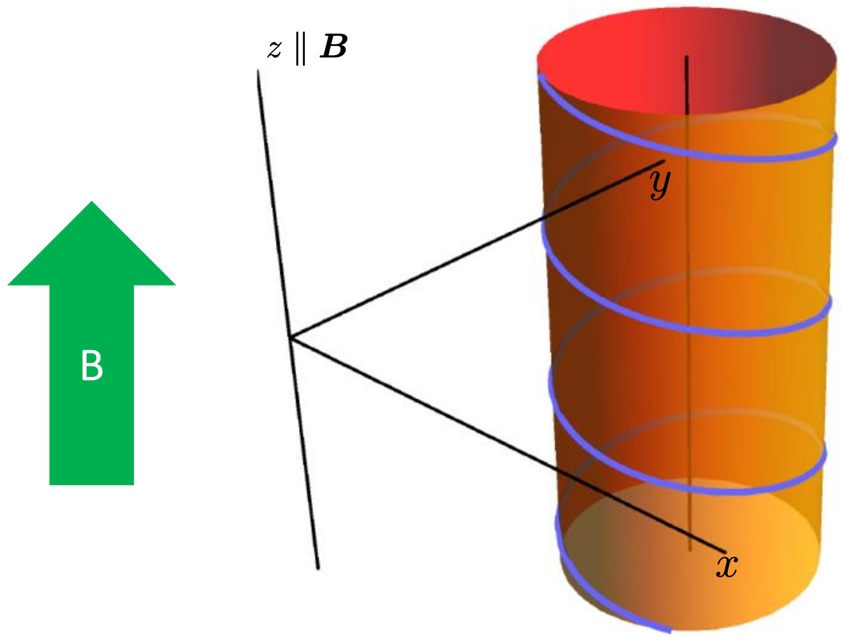


c.f.) LO and NLO without B (Moore & Teaney, Caron-Huot & Moore)

Effects of a strong magnetic field (eB >> T^2)

1. Modification of the dispersion relation of thermal quarks
2. Modification of the Debye screening mass

Landau level discretization due to the cyclotron motion



"Harmonic oscillator" in the transverse plane

Nonrelativistic: $\epsilon_n = \dfrac{p_z^2}{2m^2} + (n + \dfrac{1}{2})\boxed{\dfrac{eB}{m^2}}$ Cyclotron frequency

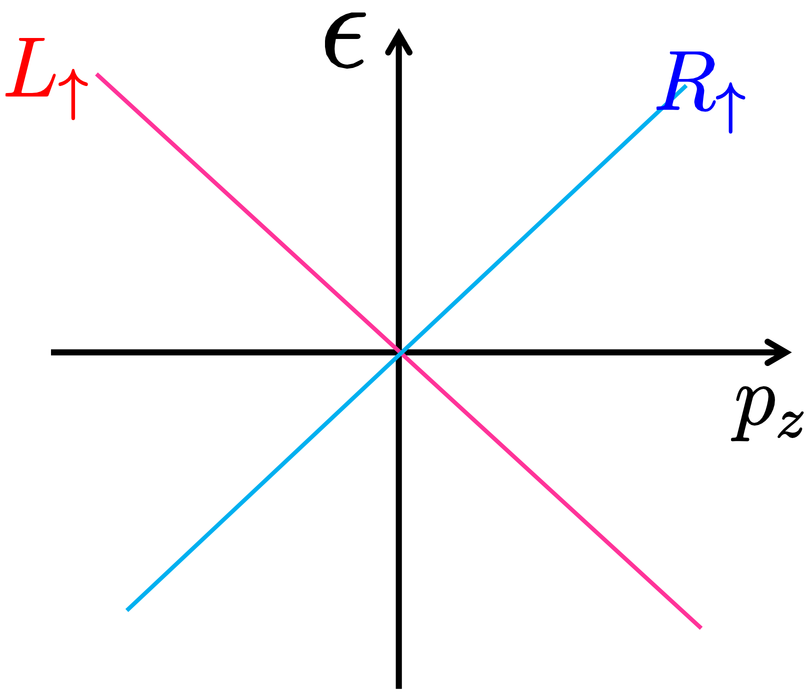
Relativistic: $\epsilon_n = \sqrt{p_z^2 + (2n+1)eB + m^2}$

In addition, there is the Zeeman effect.

Schematic picture of the lowest Landau levels

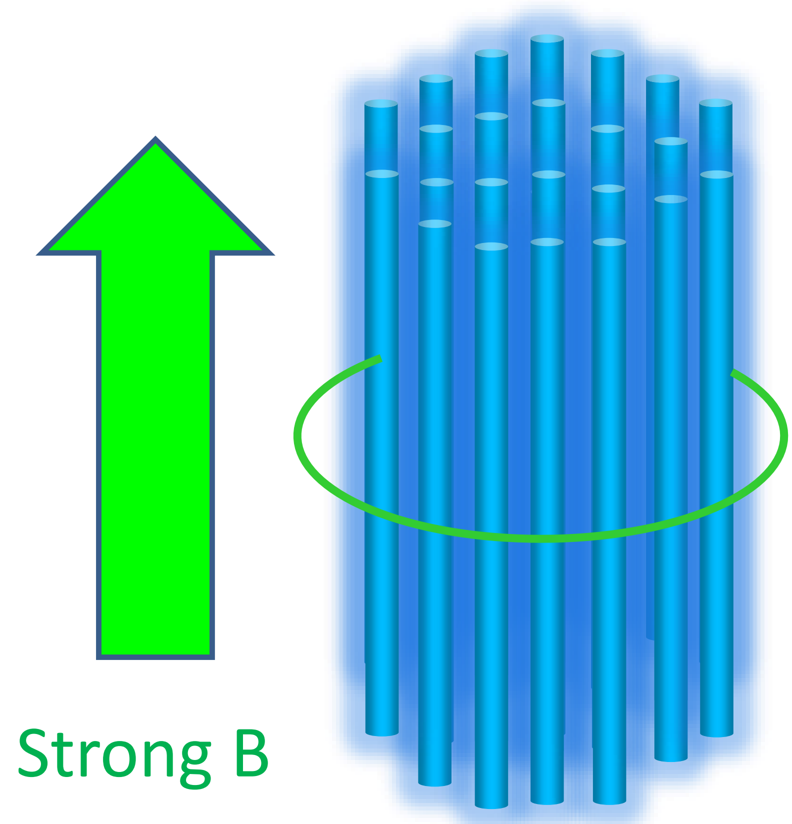
(1+1)-D dispersion relation

$$\varepsilon = \pm p_z$$



Squeezed wave function

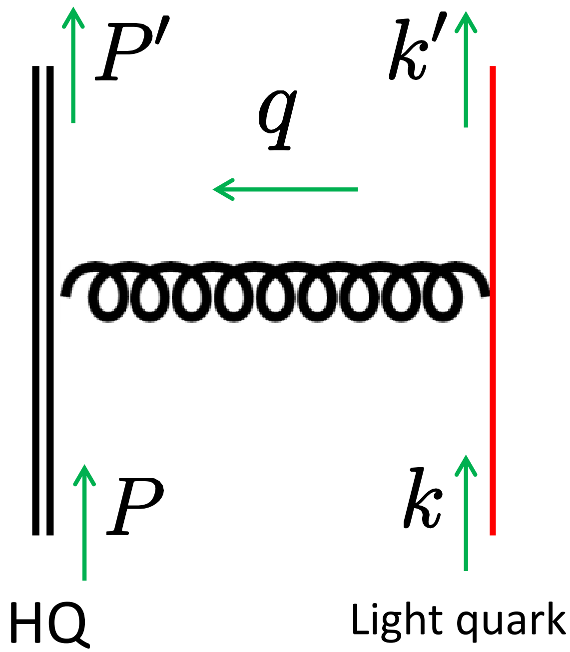
$$\psi(\boldsymbol{x}) \propto e^{-\frac{\boldsymbol{x}_\perp^2}{4\ell_B^2}} \quad (\ell_B^2 = 1/eB)$$



$$\rho = \frac{N_{\mathrm{state}}}{S} = \frac{eB}{2\pi}$$

Prohibition of the longitudinal momentum transfer

Massless limit



Linear dispersion relation $k^0 = \pm k_z$

Energy and momentum transfers in the direction of B

$$q^0 = k'^0 - k^0 \,, \quad q_z = k'_z - k_z$$

From the chirality conservation

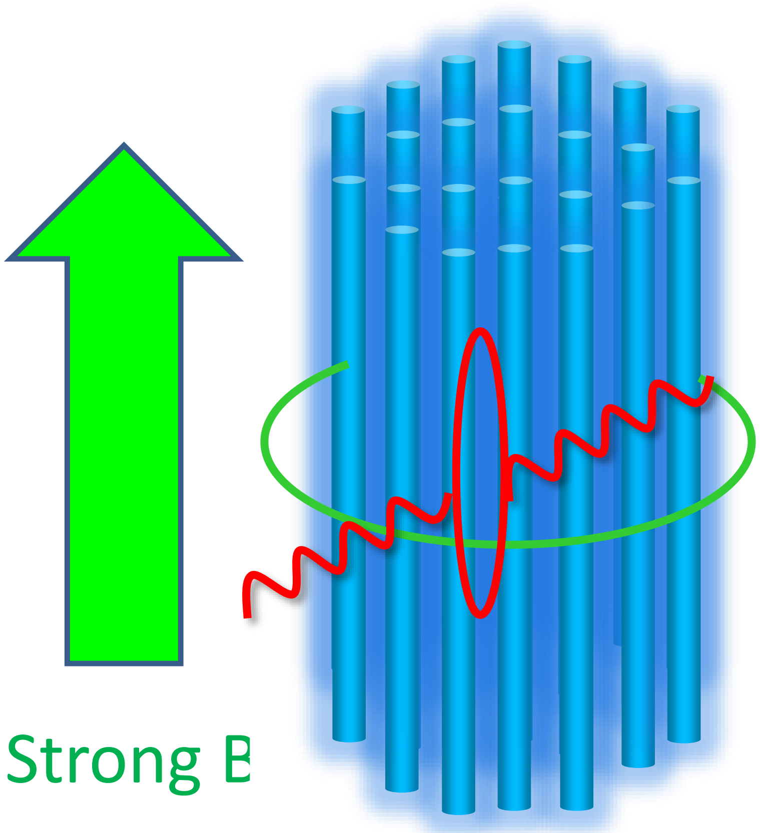
$$q^0 = \pm(k'_z - k_z) = \pm q_z$$

In the static limit (or HQ limit) $q^0 \to 0$

$$q_z \to 0.$$

$$\kappa_\parallel = 0 \text{ in massless limit, while } \kappa_\perp \neq 0.$$

Screening effect in a strong B



Gluon self-energy $$\Pi^{\mu\nu}(q) = \frac{eB}{2\pi}\Pi^{\mu\nu}_{1+1}$$

Schwinger model

$$\Pi^{\mu\nu}_{1+1} = \mathrm{tr}[t^a t^a]\frac{g^2}{\pi} f(q_\parallel^2)(q_\parallel^2 g_\parallel^{\mu\nu} - q_\parallel^\mu q_\parallel^\nu)$$

$$m_D^2 \sim \frac{eB}{2\pi}\cdot\frac{g^2}{\pi} \gg (gT)^2$$

Transverse diffusion constant in massless limit

$$\kappa_\perp \;=\; \alpha_s \lim_{q^0\to 0} \frac{T}{q^0} \int d^3\boldsymbol{q}\, q_\perp^2 \frac{\mathrm{Im}\Pi(\boldsymbol{q})}{[\,\boldsymbol{q}^2 + m_D^2\,]^2}$$

Distribution of the quark scatterers $n(q^0) \sim \dfrac{T}{q^0}$

Screened Coulomb scattering amplitude (squared)

$$m_D^2 \sim \alpha_s eB$$

Spectral density

$$2\mathrm{Im}\Pi(\boldsymbol{q}) = \rho(\boldsymbol{q}) \sim m_D^2\, q^0 \delta(q_z)$$

$$\kappa_\perp \;\sim\; \alpha_s T \int d^2\boldsymbol{q}_\perp q_\perp^2 \frac{m_D^2}{[\,\boldsymbol{q}_\perp^2 + m_D^2\,]^2} \sim \alpha_s T m_D^2 \log 1/\alpha_s$$

Longitudinal diffusion constant

1. Quark contribution to the longitudinal diffusion constant

$$\kappa_{\parallel}^{\text{quark}} = 0$$

2. Gluon contribution to the longitudinal diffusion constant

$$\kappa^{\text{gluon}} \sim \alpha_s^2 T^3 \log 1/\alpha_s$$

Same as Moore & Teaney up to constants

Anisotropic momentum diffusion constant

$$\kappa_{\perp}^{\text{quark}} \sim \alpha_s^2 T \times eB \times \log 1/\alpha_s$$

$$\kappa_{\parallel}^{\text{gluon}} \sim \alpha_s^2 T \times T^2 \times \log 1/\alpha_s$$

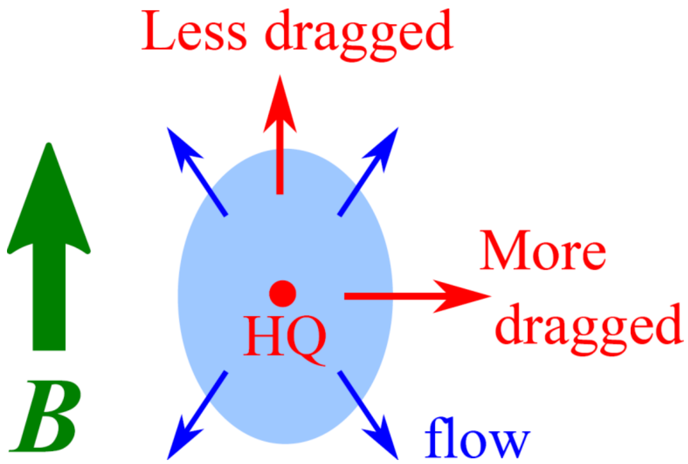
Remember the density of states in B-field, $\rho = \dfrac{N_{\text{state}}}{S} = \dfrac{eB}{2\pi}$

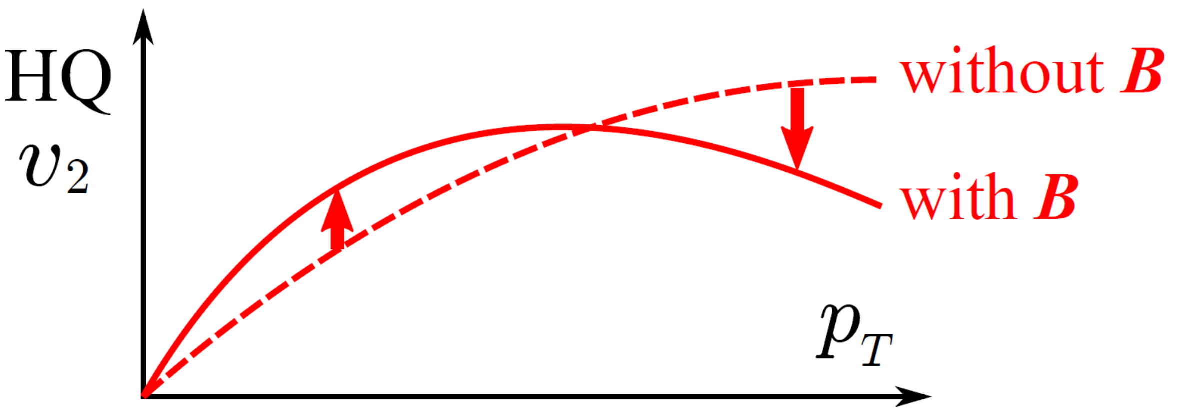
In the strong field limit,

$$\frac{\kappa_{\parallel}}{\kappa_{\perp}} \sim \frac{\kappa_{\text{gluon}}}{\kappa_{\text{quark}} + \kappa_{\text{gluon}}} \sim \frac{T^2}{eB} < 1$$

Implication for v2 of heavy flavors

$$\kappa_{\parallel} < \kappa_{\perp}$$





Magnetic anisotropy gives rise to v2 of HQs
even without the v2 of medium.
→ Possible to generate v2 of HQs in the early QGP stage.

2. *Electrical conductivity in strong magnetic field*

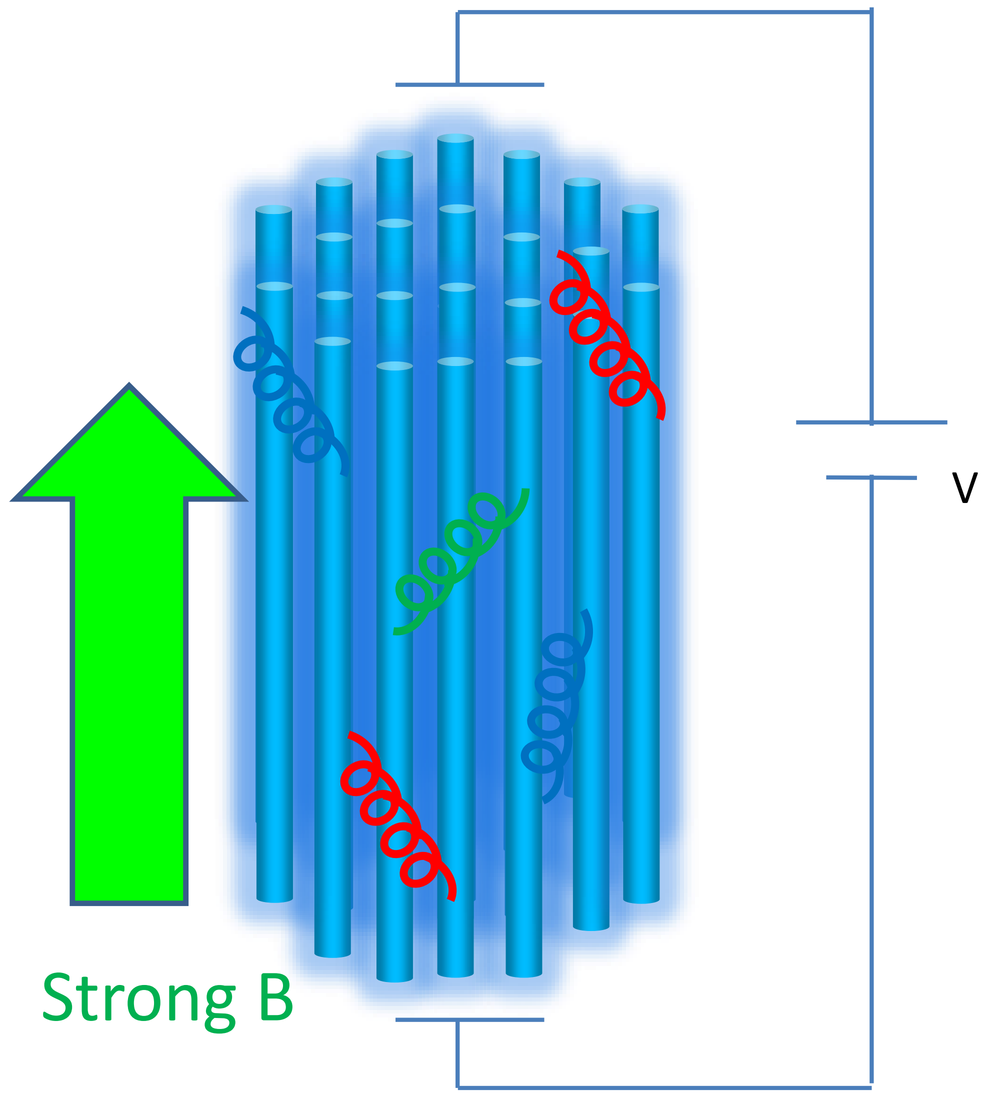
KH, Shiyong Li, Daisuke Satow, and Ho-Ung Yee, arXiv:1610.06839 [hep-ph].
KH and Daisuke Satow, arXiv:1610.06818 [hep-ph].

$$j^\mu = \underbrace{\cancel{n_e u^\mu}}_{\text{Ideal}} + \underbrace{\sigma_{\mathrm{Ohm}} E^\mu}_{\text{Dissipation}}$$

$$T \neq 0, \quad \mu = 0$$

σ_{Ohm} needs to be computed with the microscopic theory (QCD)

Quarks live in (1+1) D
Gluons live in (3+1) D



"Longitudinal conductivity" in the strong B

$$j^i = \sigma^{ij} E^j$$

Current only in z direction

$$\sigma_{zz}$$

(1+1)-D effective Boltzmann eq.

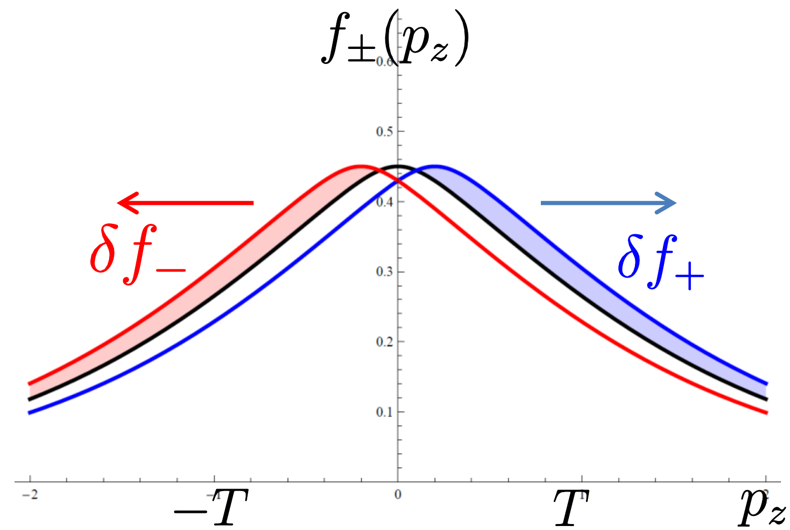
$$\frac{\partial f_\pm}{\partial t} + \dot{z}\frac{\partial f_\pm}{\partial z} + \dot{p}_z \frac{\partial f_\pm}{\partial p_z} = C[f_\pm]$$

$$\dot{p}_z = \pm q_f E_z$$

Total current integrated over p_z

$$j_z = \frac{|q_f B|}{2\pi} \cdot q_f \int \frac{dp_z}{2\pi} v_z \underbrace{(f_+ - f_-)}_{2\delta f}$$

$$\sigma_{zz} = \frac{j_z}{E_z}$$



$$f_\pm = f^{\mathrm{eq}} + \delta f_\pm$$

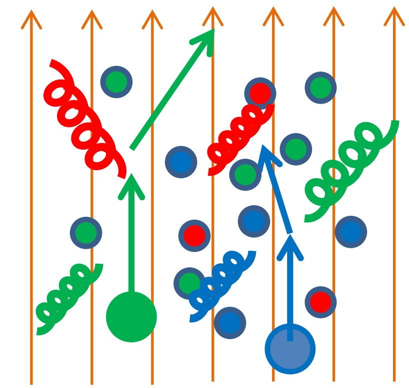
$$\delta f_+ = -\delta f_- (\equiv \delta f)$$

How linearized Boltzmann eq. works

Stationary and homogeneous limit

$$\cancel{\frac{\partial f_\pm}{\partial t}} + \dot{z}\cancel{\frac{\partial f_\pm}{\partial z}} + \dot{p}_z \frac{\partial f_\pm}{\partial p_z} = C[f_\pm]$$

External driving force v.s. Relaxation



E field

Collision term: e.g., relaxation time approximation

$$C[f_\pm] = -\frac{1}{\tau_R}\delta f$$

Linearized Boltzmann eq.

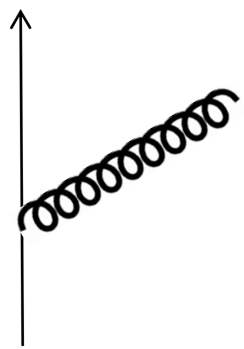
$$q_f E_z v_z f_\pm^{\mathrm{eq}}(1 - f_\pm^{\mathrm{eq}}) = \frac{1}{\tau_R}\delta f$$

$$v_z = \frac{\partial \epsilon_p}{\partial p_z} = \frac{p_z}{\epsilon_p}$$

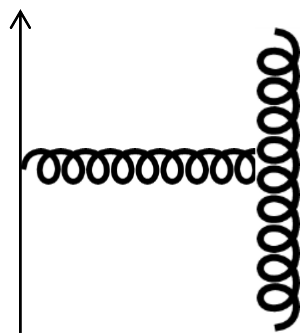
Solution

$$\sigma_{zz} = \frac{j_z}{E_z} = q_f \frac{|q_f B|}{2\pi} \frac{2\tau_R q_f \cancel{E_z}}{\cancel{E_z}} \int \frac{dp_z}{2\pi} v_z^2 f^{\mathrm{eq}}(1 - f^{\mathrm{eq}})$$

Quark-damping mechanism in magnetic fields

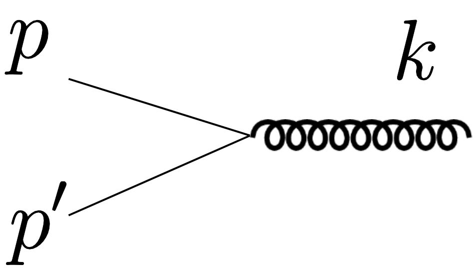


$$|\mathcal{M}|^2 \sim \mathcal{O}(\alpha_s)$$



$$|\mathcal{M}|^2 \sim \mathcal{O}(\alpha_s^2)$$

Finite B opens 1-2 processes



$$\epsilon^2_{\text{quark}} = p_z^2 + m_f^2$$
$$\epsilon^2_{\text{gluon}} = k_z^2 + |\boldsymbol{k}_\perp|^2$$

$|\boldsymbol{k}_\perp|$ works as a gluon mass for 2D kinematics.

Analogue of a massive weak boson production from $q\bar{q}$ annihilation in 4D.

Chirality selection in the massless limit

$$(\bar{\psi}\gamma^{\mu}\psi)A_{\mu} = (\bar{\psi}_R\gamma^{\mu}\psi_R + \bar{\psi}_L\gamma^{\mu}\psi_L)A_{\mu}$$

$$\epsilon_p = \pm p_z$$

Chirality conservation at the vertex

Right

$p'_z - p_z)^2$

Right

Gluo

The quark damping rate is

i) order g^2

ii) proportional to the small mass.

$$\gamma \propto g^2 m_f^2$$

0

Transverse polarizations

$$A_\mu \propto (0, 1, \pm i, 0)$$

unless $|\boldsymbol{k}_\perp|$ is finite.

Results

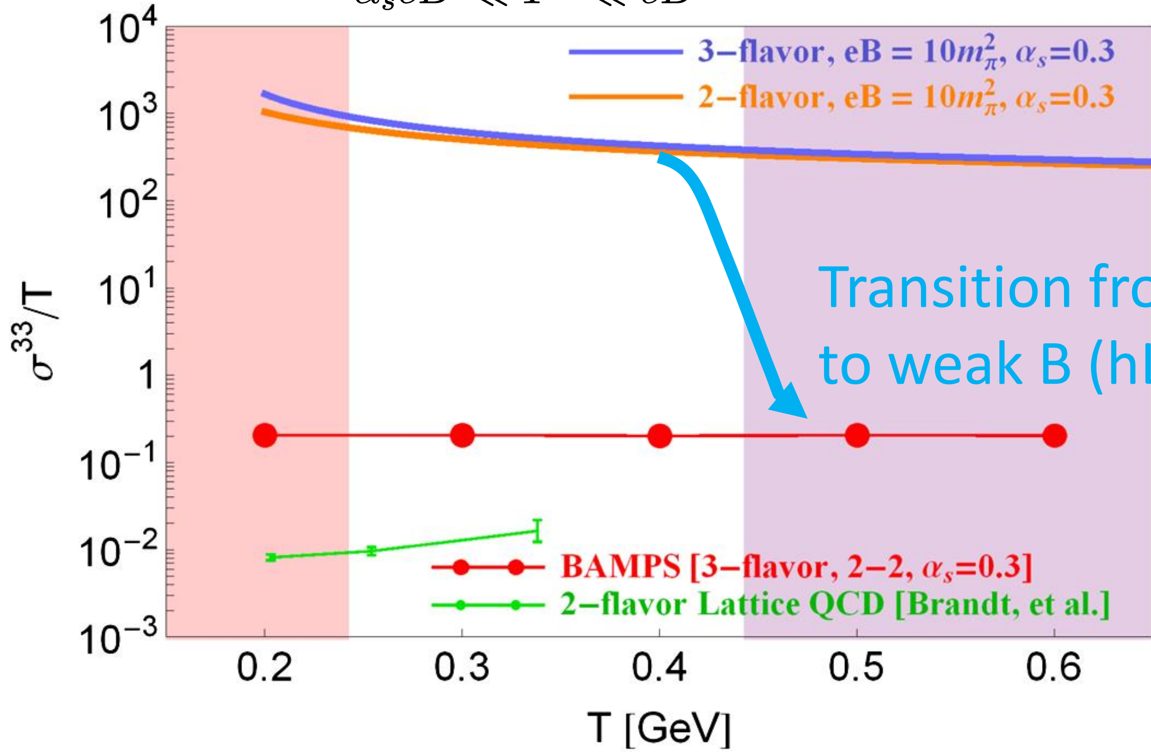
$$\sigma^{zz} = q_f^2 \frac{|q_f B|}{2\pi} \frac{4T}{g^2 m_f^2 \ln(T/M)}$$

+ Density of states
+ Chirality conservation in 1-2 (2-1) processes
+ IR cutoff $M^2 \sim \alpha_s$*eB

while $\sigma_{B=0} \sim q_f^2 \frac{T}{g^4 \ln \frac{T}{gT}}$

AMY (Arnold, Moore, Yaffe)

$\alpha_s eB \ll T^2 \ll eB$

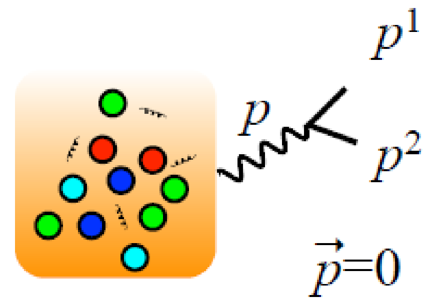


$$eB = 10m_\pi^2$$

From Daisuke Satow's slides

Possible Phenomenological Implications

2. Soft Dilepton Production



G. D. Moore and J. -M. Robert, hep-ph/0607172.

$$\frac{d\Gamma}{d^4p} = \frac{\alpha}{12\pi^4\omega^2} T\sigma^{33}$$

$\because$(virtual photon emission rate)$\sim n_B(\omega)\mathrm{Im}\Pi^{\mu}{}_{\mu}\sim T\sigma^{33}$

(photon interaction energy w leptons) (quark mean free path)$^{-1}$

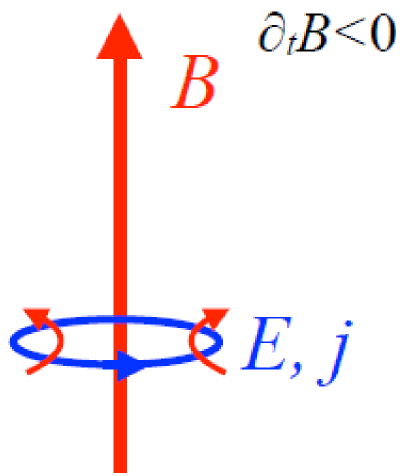
$$e\sqrt{eB} \ll \omega \ll \frac{g^2m^2}{T}\ln\left(\frac{T}{M}\right)$$

σ^{33} is large

Soft dilepton production is enhanced by B?

Possible Phenomenological Implications

3. Back Reaction to EM Fields



Bad news:

In LLL approximation, we have **no current in transverse plane**, so **Lenz's law does NOT work!**

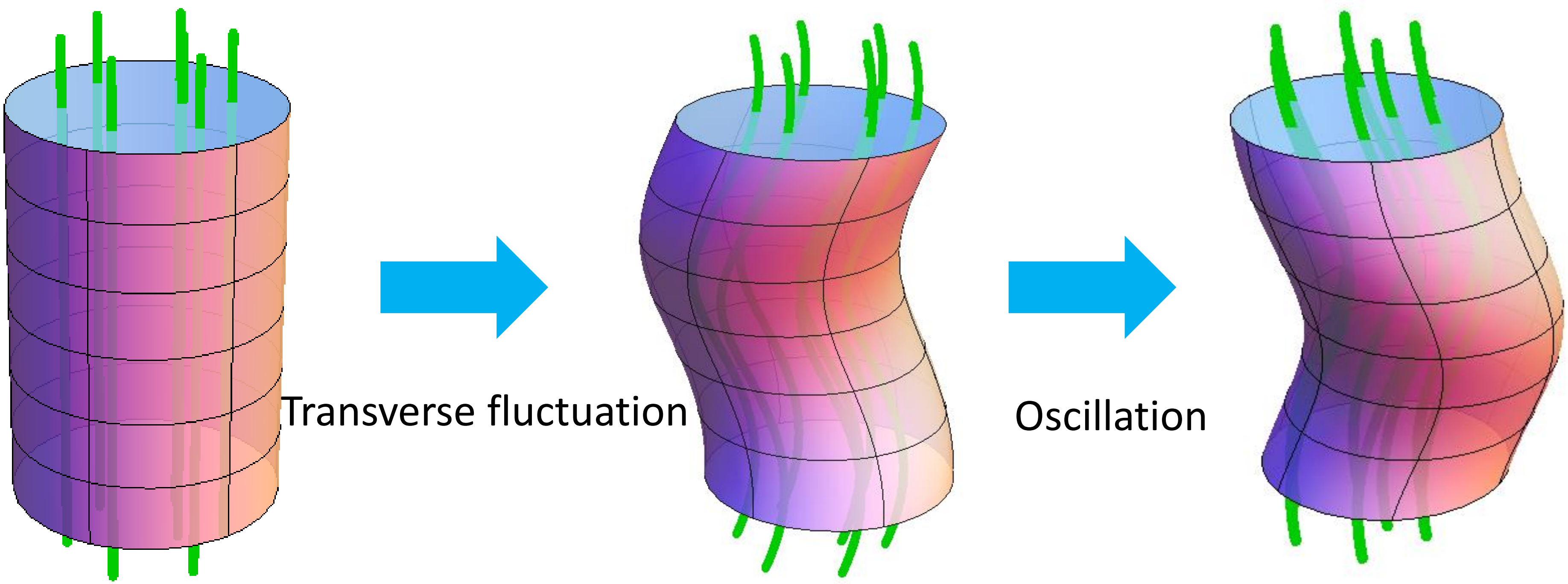
The lifetime of B does not increase…

3. New magnetohydrodynamic instability in a chiral fluid with CME

KH, Yuji Hirono, Ho-Ung Yee, and Yi Yin, In preparation.

Magneto-hydrodynamics (MHD) in conducting plasmas

Alfven's theorem = "Frozen-in" condition:
Magnetic flux is frozen in a fluid volume
and moves together with the fluid.



Tension of B-field = Restoring force
Fluid mass density = Inertia

Transverse Alfven wave

Alfven wave from a linear analysis

$$\boldsymbol{B}_0 \neq 0\,,\ T > 0\,,\ \mu_V = 0$$

Navier-Stokes + Maxwell eqs.

$$\partial_\mu T^{\mu\nu} = F^{\nu\alpha} j_\alpha$$

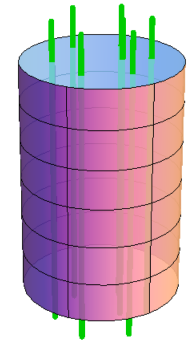
$$\partial_\mu F^{\mu\nu} = j^\nu,\ \partial_\mu \tilde{F}^{\mu\nu} = 0$$

$$j^\mu = \sigma_{\mathrm{Ohm}} E^\mu$$

$$\boldsymbol{B}_0 \to \boldsymbol{B}_0 + \delta \boldsymbol{B}$$

Stationary solutions

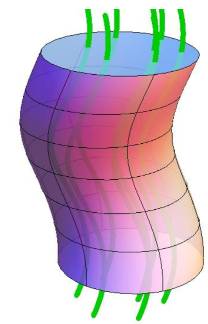
$$u^\mu = (1, \boldsymbol{0}),\ B^\mu = (0, \boldsymbol{B}_0),\ j^\mu = (0, \boldsymbol{0})$$



Transverse perturbations

$$\boldsymbol{B}_0 \perp \delta \boldsymbol{v}, \delta \boldsymbol{B}, \delta \boldsymbol{j}(z)$$

as functions of z.



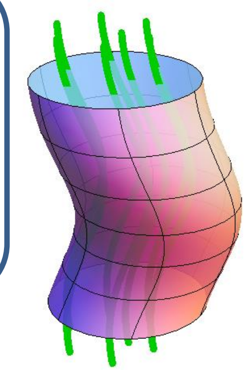
Wave equation

$$\partial_t^2 \delta \boldsymbol{B}(t,z) = \frac{B_0^2}{\epsilon + p} \partial_z^2 \delta \boldsymbol{B}(t,z)$$

Alfven wave velocity

Transverse wave propagating along background B_0

$$\boldsymbol{B}_0 \parallel \boldsymbol{k}$$



Same wave equation for δv

→ Fluctuations of B and v propagate together.

Anomalous MHD in conducting plasmas

$$\mu_A \neq 0\,,\ \boldsymbol{B}_0 \neq 0\,,\ T > 0\,,\ \mu_V = 0$$

A finite CME current without CVE: $j^\mu = \sigma_{\rm Ohm} E^\mu + \sigma_{\rm CME} B^\mu$

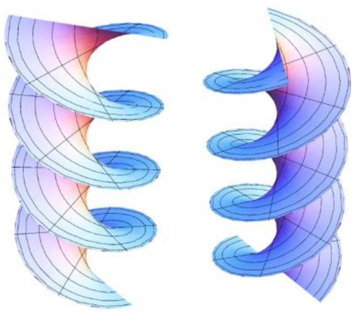
Wave equation

$$\partial_t^2 \delta \boldsymbol{B}(t,z) = v_{\rm Alf}^2\, \partial_z^2 \delta \boldsymbol{B}(t,z) + \sigma_{\rm CME} \nabla \times \partial_t \delta \boldsymbol{B}$$

$$v_{\rm Alf}^2 = \frac{B_0^2}{\epsilon + p}$$

and a similar wave equation for δv.

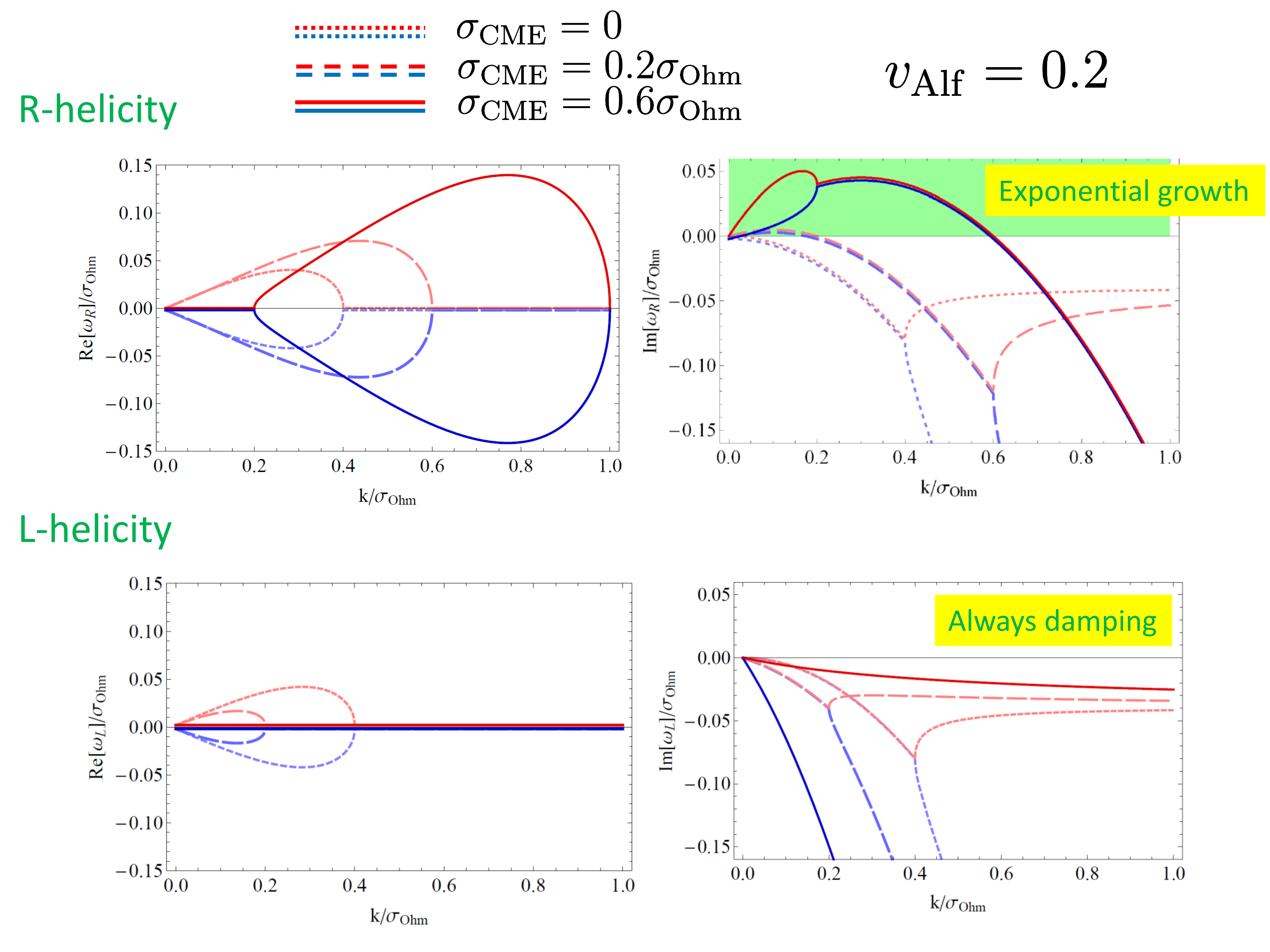
Helicity decomposition (Circular R/L polarizations)



$$\nabla \times \boldsymbol{e}_{R/L} = \pm \boldsymbol{e}_{R/L}$$

Two modes for each helicity propagating in parallel to B AND in antiparallel to B.

In total, there are 4 modes.

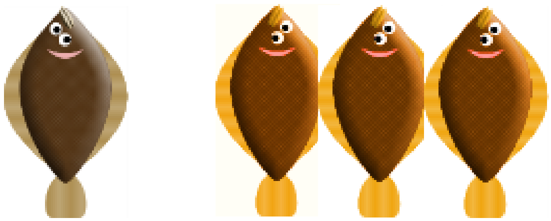


$\sigma_{\rm CME} = 0$
$\sigma_{\rm CME} = 0.2\sigma_{\rm Ohm}$
$\sigma_{\rm CME} = 0.6\sigma_{\rm Ohm}$
$v_{\rm Alf} = 0.2$
R-helicity
$\mathrm{Re}[\omega_R]/\sigma_{\rm Ohm}$
$\mathrm{Im}[\omega_R]/\sigma_{\rm Ohm}$
Exponential growth
$k/\sigma_{\rm Ohm}$
L-helicity
$\mathrm{Re}[\omega_L]/\sigma_{\rm Ohm}$
$\mathrm{Im}[\omega_L]/\sigma_{\rm Ohm}$
Always damping
$k/\sigma_{\rm Ohm}$

Helicity conversions

Akamatsu, Yamamoto
Hirono, Kharzeev, Yin
Xia, Qin, Wang

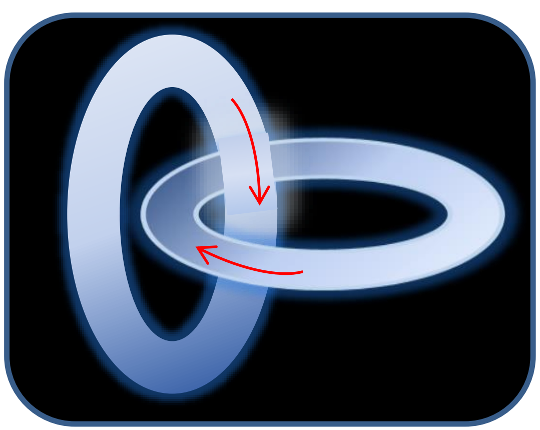
Chiral imbalance
btw R and L fermions



Chiral Plasma Instability (CPI)

Magnetic helicity

Fluid helicity (structures of vortex strings)



Summary 1

The chirality selection plays crucial roles in the non-anomalous transports
as well as in the anomalous transports.

Consequences of the chirality selections

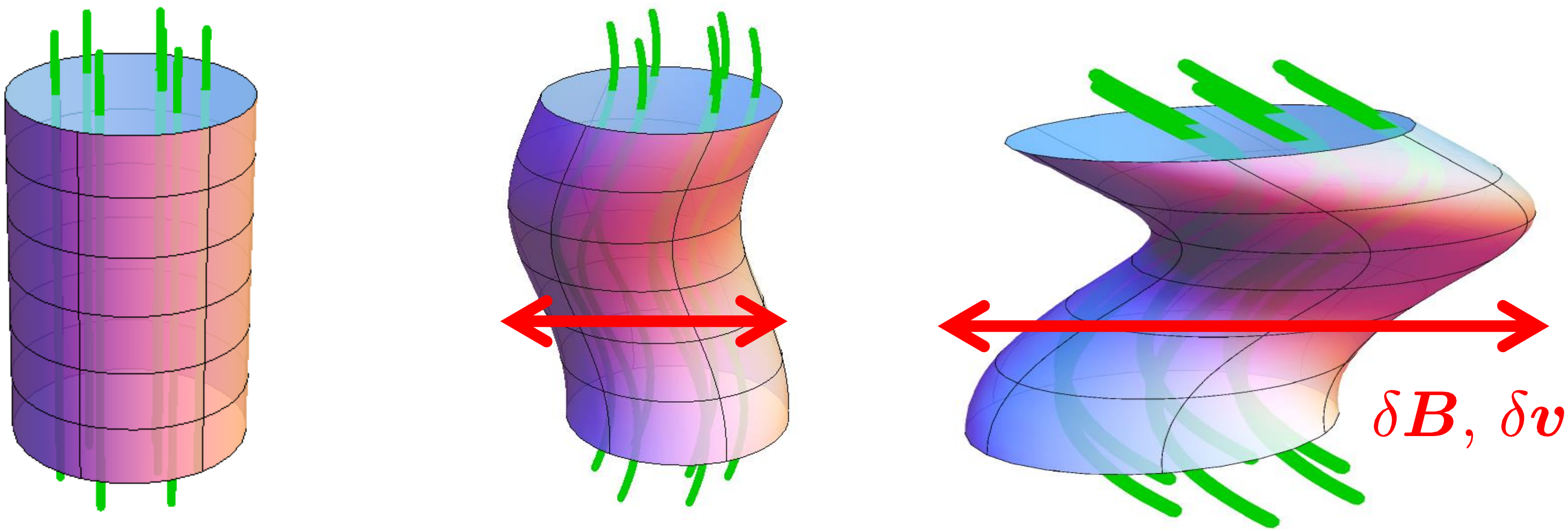
HQ diffusion: $\kappa_{\parallel}^{\rm quark} = 0 \quad \Rightarrow \quad \kappa_{\parallel} \ll \kappa_{\perp}$

Electrical conductivity: $\sigma_{zz} \propto g^{-2} \dfrac{eB}{m_f^2}$

Summary 2

We observed an onset of instabilities in both B and fluid velocity v in anomalous hydrodynamics, when a chiral imbalance induces a CME current.

Amplitudes of both B and v grow exponentially in time.



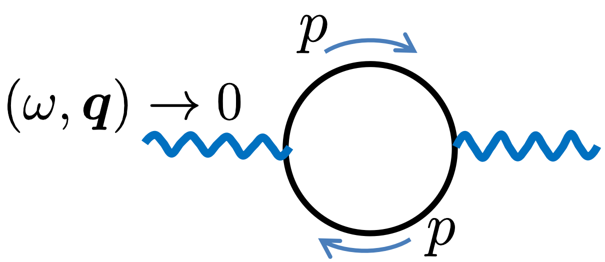
Stay tuned for more results in publication.

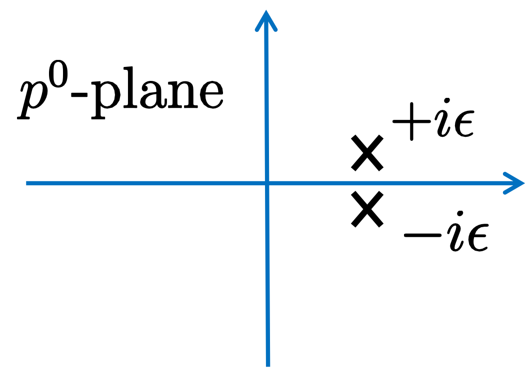
Consistent result from diagrammatic calculation

KH, D. Satow

Response function $\longrightarrow$ Kubo formula

$$j^\mu = \Pi_R^{\mu\nu} A_\nu(q) \qquad \boldsymbol{E} = i\omega \boldsymbol{A} \qquad j^i = \frac{\Pi_R^{ij}}{i\omega} \boldsymbol{E}^j$$

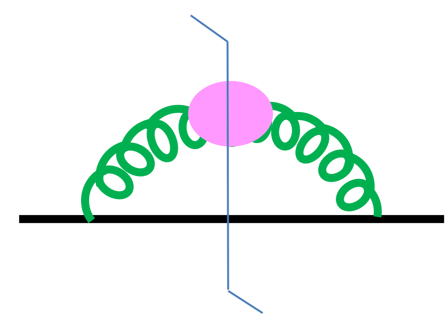




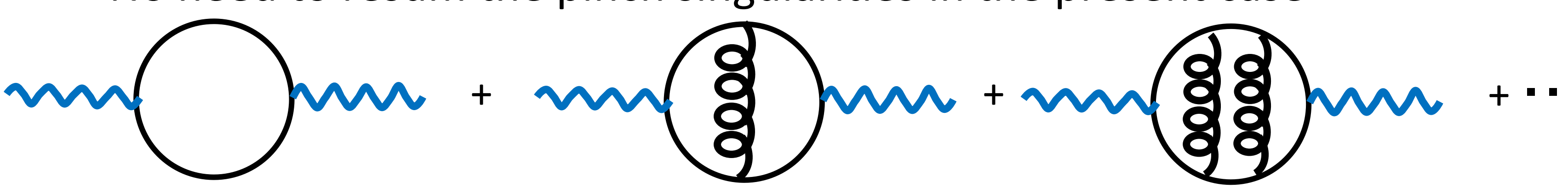
Divergence in free theory $\quad \Pi_R^{33} \sim \int \frac{dp_z}{2\pi} \frac{1}{i\epsilon}$

$$i\epsilon \to i\gamma_{\mathrm{damp}} \propto g^2 m_f^2 \ln(T/m_f)$$

Regularized by quark damping rate $\quad \Pi_R^{33} \sim \int \frac{dp_z}{2\pi} \frac{1}{i\gamma}$



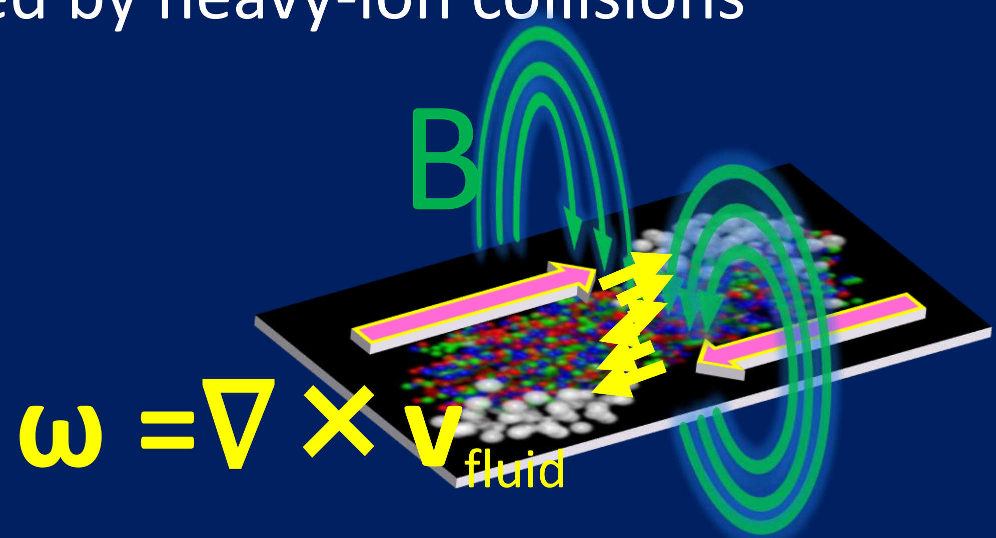
No need to resum the pinch singularities in the present case



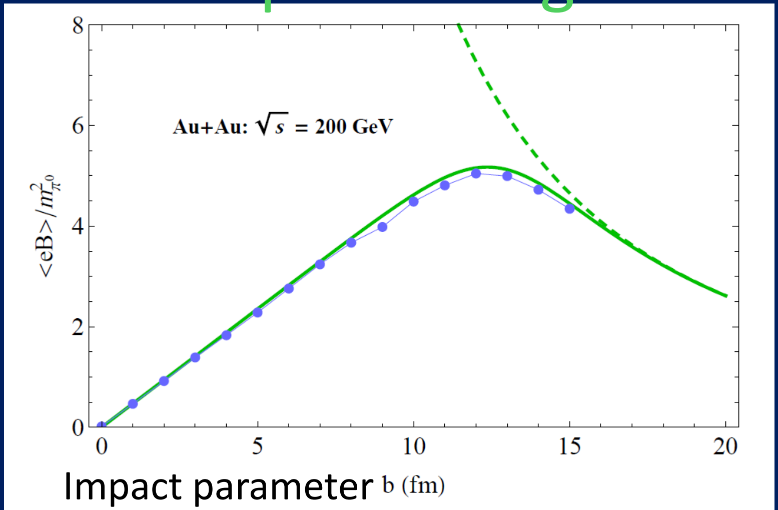
3. Anomalous transports from magneto-vorticity coupling

KH and Yi Yin, Phys. Rev. Lett. 117 (2016) 15. [arXiv:1607.01513 [hep-th]]

Strong magnetic field & vorticity/angular momentum induced by heavy-ion collisions

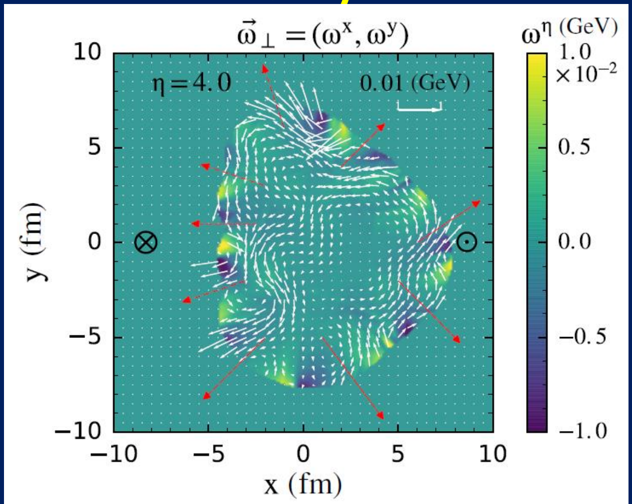


Super-strong B



Deng & Huang (2012), KH & Huang (2016)
Skokov et al. (2009), Voronyuk et al. (2011),
Bzdak, Skokov (2012) McLerran, Skokov (2014)

Vorticity in HIC



Pang, Petersen, Wang, Wang (2016)
Becattini et al., Csernai et al., Huang, Huovinen, Wang
Jiang, Lin, Liao(2016) Deng, Huang (2016)

Anomaly-induced transports in a magnetic OR vortex field

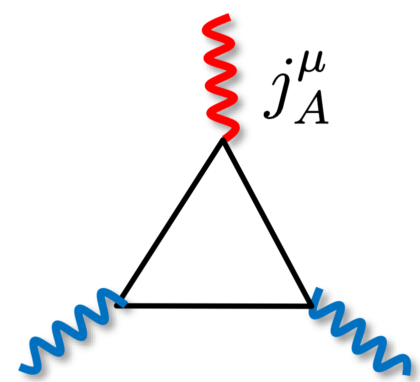
$$\begin{pmatrix} j_V^\mu \\ j_A^\mu \end{pmatrix} = C_A \begin{pmatrix} q_f \mu_A & q_f \mu_V \\ \mu_V \mu_A & (\mu_V^2 + \mu_A^2)/2 + C_A^{-1} T^2/12 \end{pmatrix} \begin{pmatrix} B^\mu \\ \omega^\mu \end{pmatrix}$$

$$B^\mu = \tilde{F}^{\mu\nu} u_\nu \qquad \omega^\mu = \frac{1}{2}\epsilon^{\mu\alpha\beta\gamma} u_\alpha \partial_\beta u_\gamma$$

Coefficients are time-reversal even.
Non-dissipative transport phenomena is due to topology in quantum anomaly and is nonrenormalizable.

Anomaly relation: $\partial_\mu j_A^\mu = q_f^2 C_A \boldsymbol{E} \cdot \boldsymbol{B}$

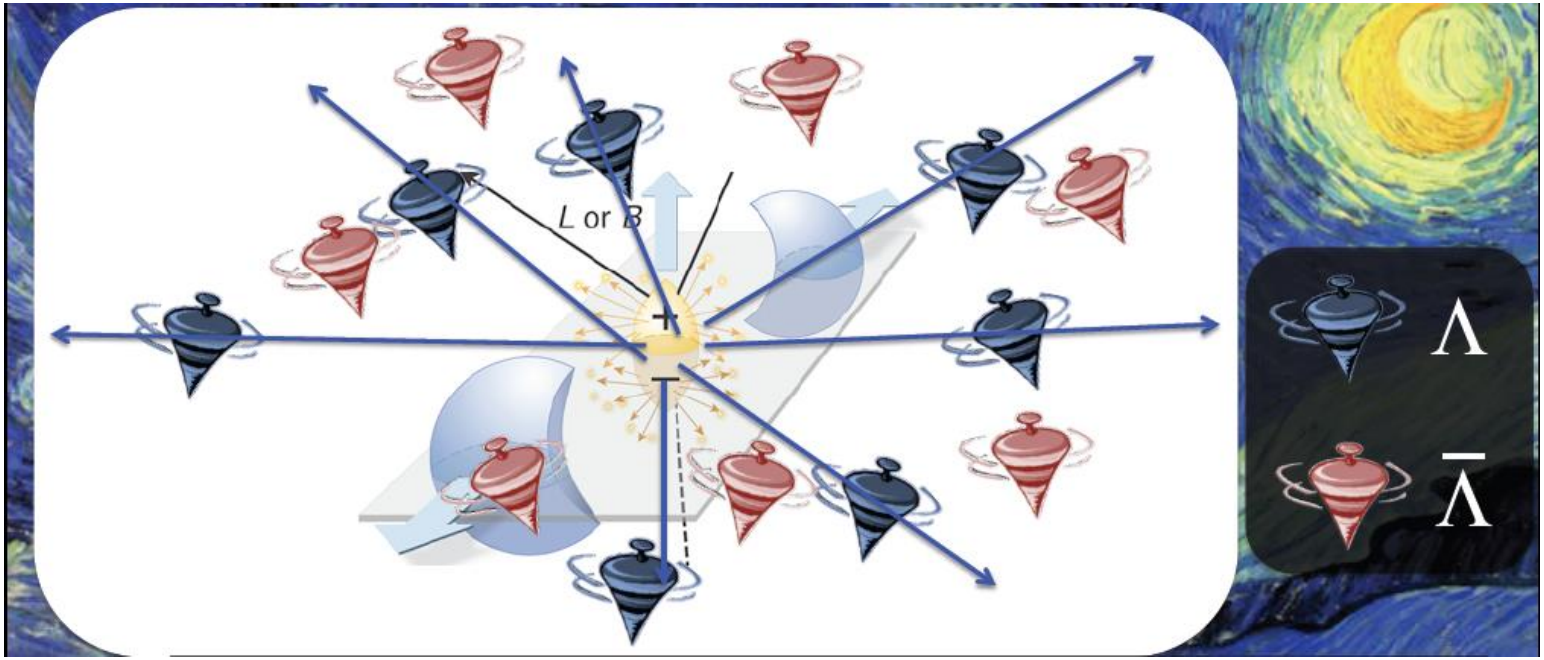
$$C_A = \frac{1}{2\pi^2}$$



Spin polarizations from spin-rotation coupling

$$f^{\pm}(\epsilon, \omega) = f_0^{\pm}(\epsilon - \boldsymbol{S} \cdot \boldsymbol{\omega}) \qquad f_0^{\pm}(\epsilon) = \frac{1}{e^{\pm\beta(\epsilon-\mu)} + 1}$$

Λ polarization



Slide by M. Lisa

See talks by Becattini, Niida, Konyushikhin, Li

Becattini et al., Glastad & Csernai, Gyulassy & Torrieri, Xie,,,,

An interplay B ⊗ ω

For dimensional reason, one would get

$$j \sim \# \, \boldsymbol{B} \cdot \boldsymbol{\omega}$$

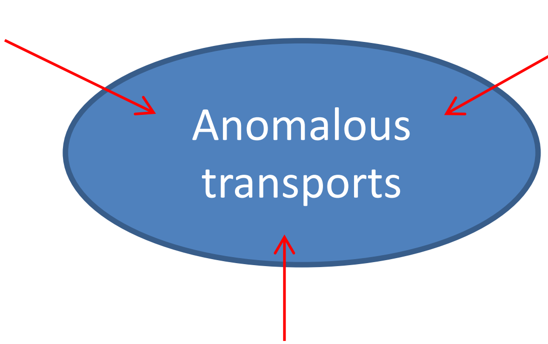
Cf., In CVE, it was

$$j^{\mu}_{R/L} = C_A \mu^2_{R/L} \omega^{\mu}$$

Could the magneto-vorticity coupling be important ??

QFT with Kubo formula

Anomalous hydrodynamics due to Son and Surowka



$$j^{\mu} = \sigma_{\mathrm{Ohm}} E^{\mu} + \xi_B B^{\mu} + \xi_{\omega} \omega^{\mu}$$

The first-order derivative expansion $[A^{\mu} \sim \mathcal{O}(\partial^0),\ v^{\mu} \sim \mathcal{O}(\partial^0)]$

$$E^{\mu} \sim B^{\mu} \sim \omega^{\mu} \sim \mathcal{O}(\partial^1)$$

Chiral kinetic theory

→ Yes, it is important when B is so strong that B >> O(∂^1).

Q1. Is the coefficient related to any quantum anomaly?
Q2. How is T and/or μ dependence?

Consequences of a magneto-vorticity coupling

Shift of thermal distribution functions by the spin-vorticity coupling

Spin-vorticity coupling

$$f^{\pm}(\epsilon, \omega) = f_0^{\pm}(\epsilon - \boldsymbol{S} \cdot \boldsymbol{\omega}) \qquad f_0^{\pm}(\epsilon) = \frac{1}{e^{\pm\beta(\epsilon-\mu)} + 1}$$

Landau & Lifshitz, Becattini et al.

In the LLL, the spin direction is aligned along the magnetic field .

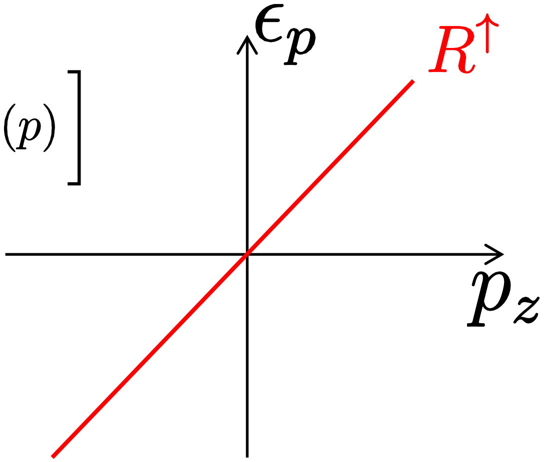
$$\Delta\epsilon^{\pm} \equiv -\boldsymbol{S} \cdot \boldsymbol{\omega} = \mp \mathrm{sgn}(q_f)\frac{1}{2}\hat{\boldsymbol{B}} \cdot \boldsymbol{\omega}$$

- for particle

+ for antiparticle

Number density

$$n_R = \frac{|q_f B|}{2\pi}\left[\int_0^{\infty} \frac{dp_z}{2\pi} f^{+}(p) + \int_{-\infty}^{0} \frac{dp_z}{2\pi} f^{-}(p)\right]$$



At the LO in the energy shift Δε

$$\Delta n_R = \frac{|q_f B|}{2\pi}\left[\Delta\epsilon^{+} \int_0^{\infty} \frac{dp_z}{2\pi} \frac{\partial f_0^{+}(p_z)}{\partial p_z} + \Delta\epsilon^{-} \int_{-\infty}^{0} \frac{dp_z}{2\pi} \frac{\partial f_0^{-}(p_z)}{\partial p_z}\right]$$

$$\Delta n_R = q_f \frac{C_A}{4} \boldsymbol{B} \cdot \boldsymbol{\omega} [f_0^+(0) + f_0^-(0)]$$
$$= q_f \frac{C_A}{4} \boldsymbol{B} \cdot \boldsymbol{\omega}$$

$f_0^+(0) + f_0^-(0) = 1$ identically for any T and μ.

The shift is independent of the chirality, and depends only on the spin direction.

$$\Delta n_L = \Delta n_R$$

In the V-A basis,

$$\Delta n_V = \Delta n_R + \Delta n_L = q_f \frac{C_A}{2} \boldsymbol{B} \cdot \boldsymbol{\omega}$$
$$\Delta n_A = \Delta n_R - \Delta n_L = 0$$

½ from the size of the spin

Spatial components of the current

$$\Delta j_R^3 = v_R \Delta n_R$$

$j^1 = j^2 = 0$ for the LLL

Velocity: $v_{R/L} = \pm \mathrm{sgn}(q_f B)$

The shift depends on the chirality through the velocity.

$$\Delta j_R^3 = -\Delta j_L^3$$

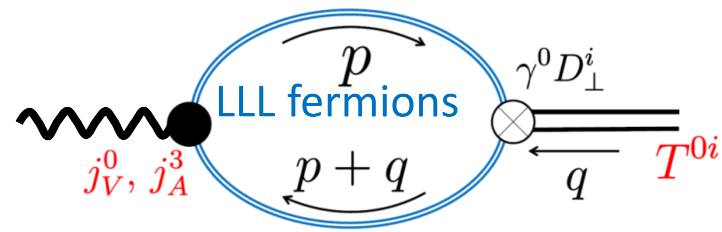
In the V-A basis,

$$\Delta j_V^3 = 0$$
$$\Delta j_A^3 = |q_f| \mathrm{sgn}(B) \frac{C_A}{2} \boldsymbol{B} \cdot \omega$$

Field-theoretical computation by Kubo formula

Perturbative ω in a strong B

$$\lambda = -2i \lim_{q_x \to 0} \frac{\partial}{\partial q_x} \langle n_V(x) T^{02}(x') \rangle \theta(t - t')$$

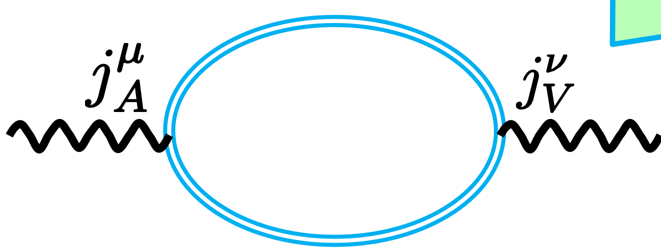


Similar to the Kubo formula used to get the T[2] term in CVE (Landsteiner, Megias, Pena-Benitez)

We confirm

1. the previous results obtained from the shift of distributions.
2. a relation of <n_V T^{02}> to the chiral anomaly diagram in the (1+1) dim.

$\Pi_{AV}^{\mu\nu} =$



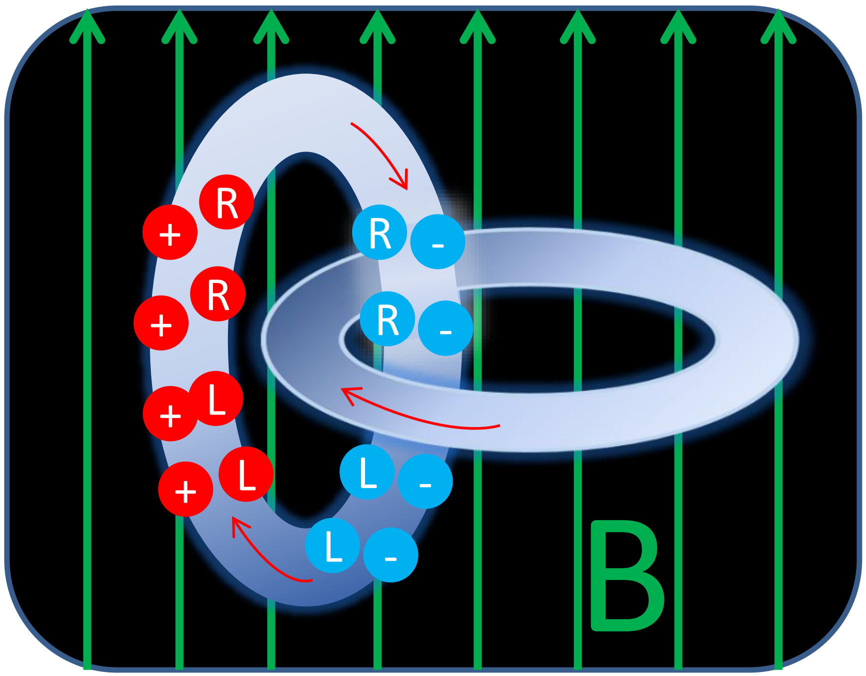
$$q_\mu \Pi_{AV}^{\mu\nu} \neq 0 \;!!!$$

There is no T or μ correction in the massless limit, since it is related to the chiral anomaly!

Summary 2

A magneto-vorticity coupling B ⊗ ω induces charge redistributions without μ_A.

- Related to the chiral anomaly in the (1+1) dimensions.
- No T or μ correction.



When $\boldsymbol{B}\cdot\boldsymbol{\omega}\neq 0$,

$$j^0_{EM,V} = q_f^2 \frac{C_A}{2}(\boldsymbol{B}\cdot\boldsymbol{\omega})$$

$$j^3_{EM,A} = \mathrm{sgn}(q_f) q_f^2 \frac{C_A}{2}(\boldsymbol{B}\cdot\omega)\hat{\boldsymbol{B}}$$

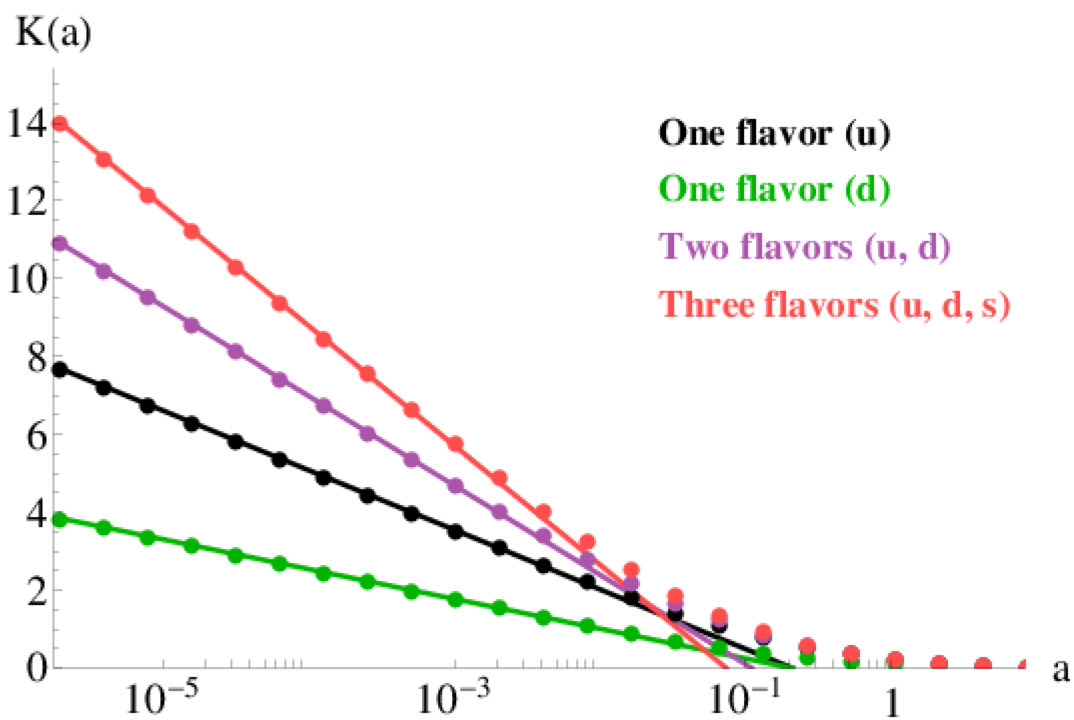
Emerges even without μ_A.

Coupling between the CME and fluid velocity induces a new instability in MHD. Take by Y. Hiron. KH, Hirono, Yee, Yin, In preparation.

$$\int_0^\infty dx\, \frac{x\, \sum_f |q_f| e^{-x/|q_f|}}{[x + a\, \sum_f |q_f| e^{-x/|q_f|}]^2}$$

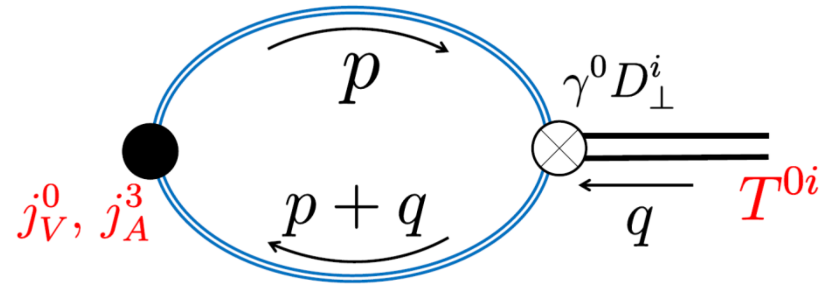
$$= Q_{\rm em}\left[\log\left(\frac{1}{\alpha_s}\right) - \log\left(\frac{T_R}{\pi}\right) - \gamma_E - 1 + \sum_f \frac{|q_f|}{Q_{\rm em}} \log(\frac{|q_f|}{Q_{\rm em}})\right]$$

$$Q_{\rm em} = \sum_f |q_f|$$



Field-theoretical computation by Kubo formula

$$\lambda = -2i \lim_{q_x \to 0} \frac{\partial}{\partial q_x} \langle n_V T^{02} \rangle$$



$$n_V(x) = \bar{\psi}(x)\gamma^0\psi(x)$$

$$T^{0i}(x) = \frac{i}{2}\bar{\psi}(x)(\gamma^0 D^i + \gamma^i D^0)\psi(x)$$

$$S_{LLL} = 2e^{-\frac{|\boldsymbol{p}_\perp|^2}{q_f B}} \frac{i}{\not{p}_\parallel + m_f}\mathcal{P}_+ \qquad \mathcal{P}_+ = (1 + i\mathrm{sgn}(q_f B)\gamma^1\gamma^2)/2$$

$$\langle n_V T^{02} \rangle \propto \frac{|q_f B|}{2\pi} q_x \Pi^{00}_{1+1}$$

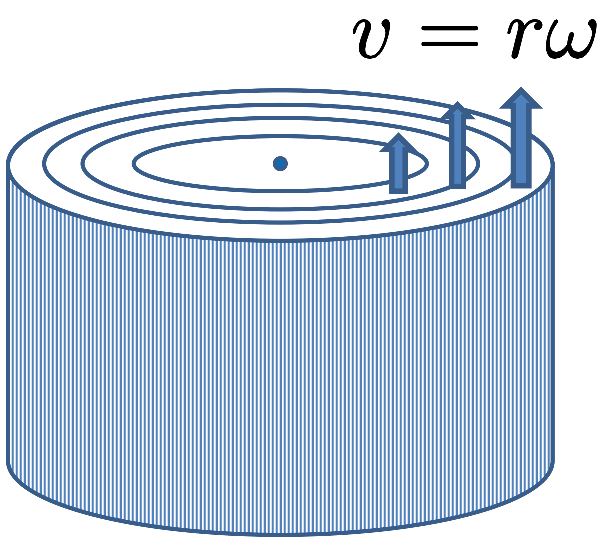
$$\Pi^{\mu\nu}_{1+1} = \int \frac{d^2 p_\parallel}{(2\pi)^2} \mathrm{tr}[\gamma^\mu_\parallel S_{1+1}(p_\parallel + q_\parallel)\gamma^\nu_\parallel S_{1+1}(p_\parallel)] = \frac{1}{\pi}\frac{1}{q^2_\parallel}(q^2_\parallel g^{\mu\nu}_\parallel - q^\mu_\parallel q^\nu_\parallel)$$

There is no T or μ correction in the massless limit!

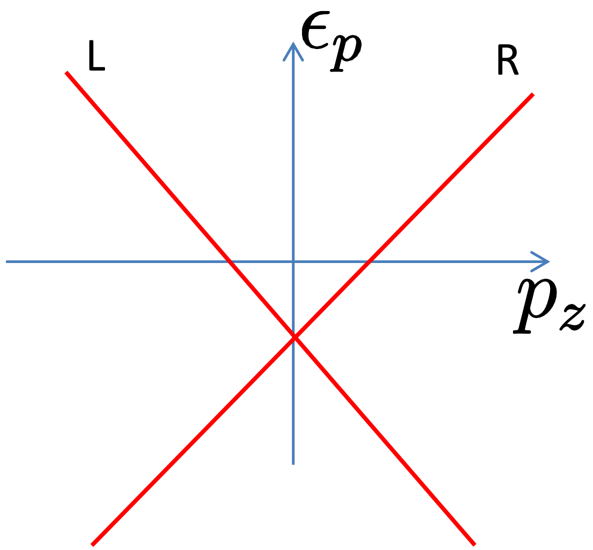
→ Consistent with the previous observation from the shift of distributions.

Causality problem:
A rigid rotation of an infinite-size system breaks causality in the peripheral region.

One must have a finite-size system with an exterior boundary or a local vortex field.



Local shift of the charge density
→ Redistribution of charge in the system



E.g., when $v \to 0$ sufficiently fast,

$$\Delta n_{\rm net} \propto \int d^3x \boldsymbol{B}\cdot\boldsymbol{\omega} = \frac{1}{2}\int d^3x \nabla\cdot(\boldsymbol{B}\times\boldsymbol{v}) = \frac{1}{2}\int_{\partial S} dS\cdot(\boldsymbol{B}\times\boldsymbol{v}) = 0$$