# Synergy between measurements of the GW and the triple Higgs coupling

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[K.H, S.Kanemura and Y.Orikasa, Phys. Lett. B 752, 217 (2016)]

[K.H, M. Kakizaki, S. Kanemura and T. Matsui, Phys. Rev. D 94, no. 1, 015005 (2016)]

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# Contents

1. Introduction

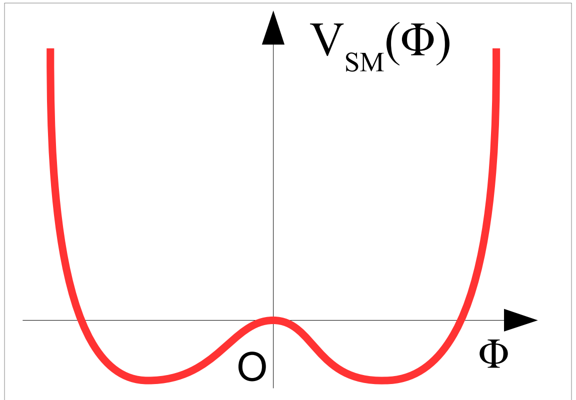
2. O(N) singlet model with and without CSI

3. GWs from 1$^{st}$ order phase transition

4. Summary

# Introduction

- We discovered a Higgs boson which is predicted in the Standard Model(SM)
- But the structure of the Higgs sector is still vague.
  - The number of the Higgs fields?
  - The Higgs field is elementary or composite?
  - Dynamics of the electroweak symmetry breaking(EWSB)?




$$V_{SM}(\phi)=\underline{\mu^2|\phi|^2}+\lambda|\phi|^4$$

What's the origin of the EWSB ?

( Is it natural to suppose that the mass term is negative? )

- We discuss the model based on classical scale invariance (CSI).

# The model for EWSB based on CSI

- CSI prohibits the mass term. $V_{SM}(\phi) = \cancel{\mu^2 |\phi|^2} + \lambda |\phi|^4$
- EWSB can radiatively happen by the Coleman and Weinberg mechanism. [S.R. Coleman and E.J. Weinberg, Phys.Rev.D7,1888(1973)]
- The minimal model with one field cannot explain the Higgs mass.

  → We have to consider the extended Higgs model.
- We analyze the model by the Gildener and Weinberg method.

  [E. Gildener and S. Weinberg, Phys. Rev. D 13, 3333(1976)]
- The effective potential along the flat direction is obtained by

$$V_{eff}(\varphi) = A\varphi^4 + B\varphi^4 \ln\frac{\varphi^2}{Q^2} \quad \begin{cases} A = \dfrac{1}{64\pi^2 v^4}\left[3\,\mathrm{Tr}\left(M_V^4 \ln\dfrac{M_V^2}{v^2}\right) - 4\,\mathrm{Tr}\left(M_f^4 \ln\dfrac{M_f^2}{v^2}\right) + \mathrm{Tr}\left(M_S^4 \ln\dfrac{M_S^2}{v^2}\right)\right] \\ B = \dfrac{1}{64\pi^2 v^4}\left[3\,\mathrm{Tr}\left(M_V^4\right) - 4\,\mathrm{Tr}\left(M_f^4\right) + \mathrm{Tr}\left(M_S^4\right)\right] \end{cases}$$

# The model for EWSB based on CSI

- The triple Higgs boson coupling of the models $\Gamma_{hhh}^{CSI}$ is universally

$$\Gamma_{hhh}^{CSI} \equiv \frac{\partial^3 V_{eff}}{\partial \varphi^3}\bigg|_{\varphi=v} = 40\, vB = \frac{5 m_h^2}{v} = \frac{5}{3} \times \Gamma_{hhh}^{SM\,tree}.$$

$$\left( \Gamma_{hhh}^{SM\,tree} = \frac{3 m_h^2}{v} \right)$$

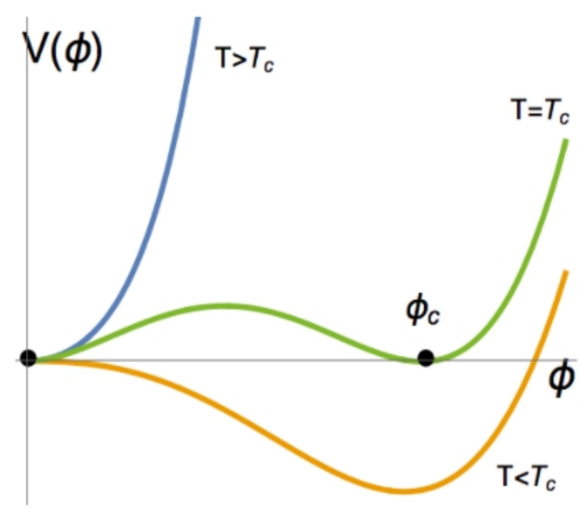
[K.H, S.Kanemura and Y.Orikasa, Phys. Lett. B 752, 217 (2016)]

- If other models have the same deviation in the hhh coupling from the SM value, can we distinguish those?

- By extending the Higgs sector with additional scalar fields, strongly 1st order phase transition(1stOPT) for EWSB can be realized.

$$(\phi_c / T_c \gtrsim 1)$$

- When electroweak phase transition(EWPT) is 1st OPT, gravitational waves(GWs) occur.



# The model for EWSB based on CSI

- The triple Higgs boson coupling of the models $\Gamma^{CSI}_{hhh}$ is universally

$$\Gamma^{CSI}_{hhh} \equiv \left.\frac{\partial^3 V_{eff}}{\partial \varphi^3}\right|_{\varphi=v} = 40\, vB = \frac{5\mathrm{m}_h^2}{v} = \frac{5}{3} \times \Gamma^{SM\ tree}_{hhh}.$$

$$\left( \Gamma^{SM\ tree}_{hhh} = \frac{3\mathrm{m}_h^2}{v} \right)$$

[K.H, S.Kanemura and Y.Orikasa, Phys. Lett. B 752, 217 (2016)]

- If other models have the same deviation in the hhh coupling from the SM value, can we distinguish those?

- We will focus on the CSI models where N extra isospin singlet scalars obey a global O(N) symmetry.

- We discuss how these models can be differentiated from similar extended models by the measurements of

**the hhh coupling** and **the GW spectrum**.

# Contents

# $\Phi$ + O(N) singlet model with CSI

- Tree-level potential

$$V_0(\Phi, \vec{S}) = \frac{\lambda}{2}|\Phi|^4 + \frac{\lambda_S}{4}|\vec{S}|^4 + \frac{\lambda_{\Phi S}}{2}|\Phi|^2|\vec{S}|^2$$

$\Phi$ : SM-like Higgs doublet

$\vec{S} = (S_1, S_2, \ldots, S_N)^T$

( We suppose that there is the flat direction in the tree-level potential. )

- Effective potential (T=0)

$$V_{eff}(\varphi) = A\varphi^4 + B\varphi^4 \ln\frac{\varphi^2}{Q^2}$$

$$\begin{cases} A = \frac{1}{64\pi^2 v^4}\left[3\,\mathrm{Tr}\left(M_V^4 \ln\frac{M_V^2}{v^2}\right) - 4\,\mathrm{Tr}\left(M_f^4 \ln\frac{M_f^2}{v^2}\right) + \mathrm{Tr}\left(M_S^4 \ln\frac{M_S^2}{v^2}\right)\right] \\ B = \frac{1}{64\pi^2 v^4}\left[3\,\mathrm{Tr}(M_V^4) - 4\,\mathrm{Tr}(M_f^4) + \mathrm{Tr}(M_S^4)\right] \end{cases}$$

- Singlet scalal boson masses $m_S$

$$N m_S^4 = 8\pi^2 v^2 m_h^2 - 3m_Z^4 - 6m_W^4 + 12m_t^4$$

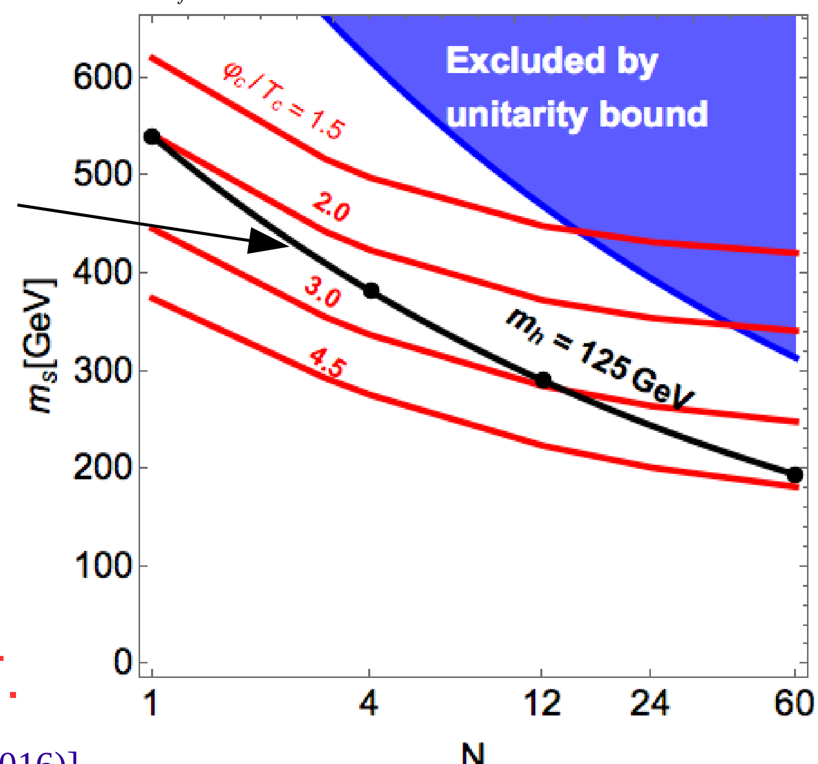
- The triple Higgs boson coupling

$$\Gamma_{hhh}^{CSI\,O(N)} = \frac{5m_h^2}{v} = \frac{5}{3} \times \Gamma_{hhh}^{SM\,tree}$$

- 3 Independent parameters

$N,\ m_S,\ \lambda_S$

- In the model, EWPT is storongly 1st OPT.




[K.H, M.Kakizaki, S.Kanemura and T.Matsui, Phys. Rev. D 94, no. 1, 015005 (2016)]

# $\Phi$ + O(N) singlet model without CSI

$\Phi$ : SM-like Higgs doublet

$\vec{S} = (S_1, S_2, ..., S_N)^T$

- Tree-level potential

$$V_0(\Phi, \vec{S}) = V_{\rm SM}(\Phi) + \frac{\mu_S^2}{2}|\vec{S}|^2 + \frac{\lambda_S}{4}|\vec{S}|^4 + \frac{\lambda_{\Phi S}}{2}|\Phi|^2|\vec{S}|^2$$

- Effective potential (T=0)

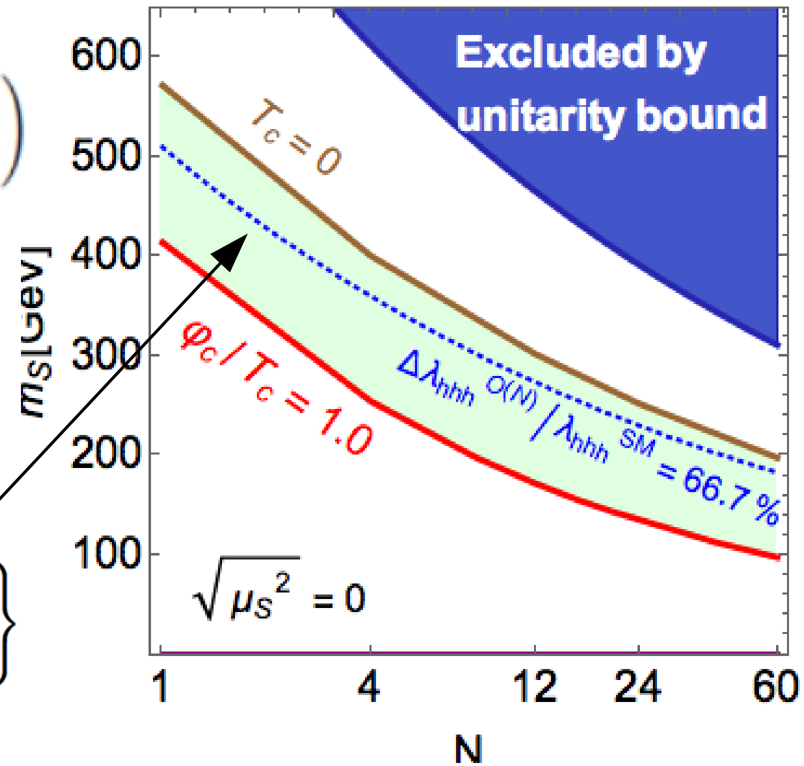
$$V_{\rm eff}(\varphi) = -\frac{\mu^2}{2}\varphi^2 + \frac{\lambda}{4}\varphi^4 + \sum_i \frac{n_i}{64\pi^2} M_i^4(\varphi)\left(\ln\frac{M_i^2(\varphi)}{Q^2} - \frac{3}{2}\right)$$

- Singlet scalal boson masses $m_S$

$$m_S^2 = \mu_S^2 + \frac{\lambda_{\Phi S}}{2}v^2$$

- The triple Higgs boson coupling

$$\lambda_{hhh}^{O(N)} = \frac{3m_h^2}{v}\left\{1 - \frac{1}{\pi^2}\frac{m_t^4}{v^2 m_h^2} + \frac{N}{12\pi^2}\frac{m_S^4}{v^2 m_h^2}\left(1 - \frac{\mu_S^2}{m_S^2}\right)^3\right\}$$




[ M. Kakizaki et al, Phys. Rev. D 92, no.11, 115007 (2015)]

- 4 Independent parameters

$\mu_S$, N, $m_S$, $\lambda_S$

- If the model has $\Delta\lambda_{hhh}^{O(N)}/\lambda_{hhh}^{SM} \simeq 67\ \%$, EWPT is storongly 1stOPT.

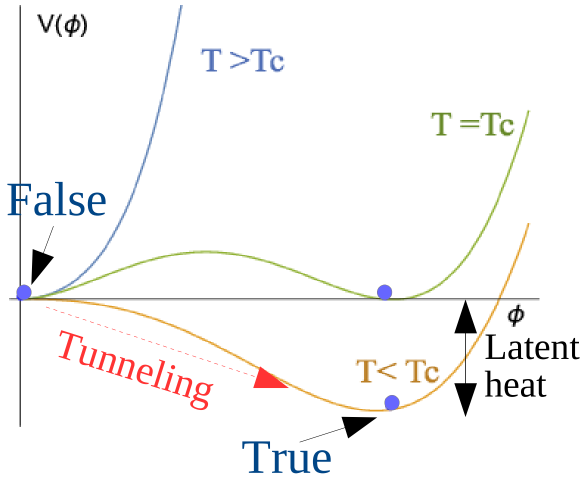
# Contents

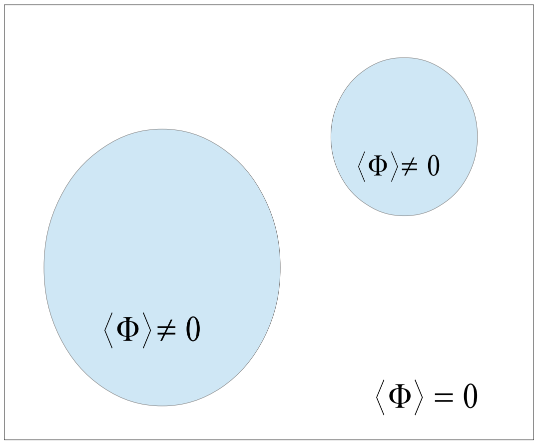
# GWs from 1$^{st}$ order phase transition

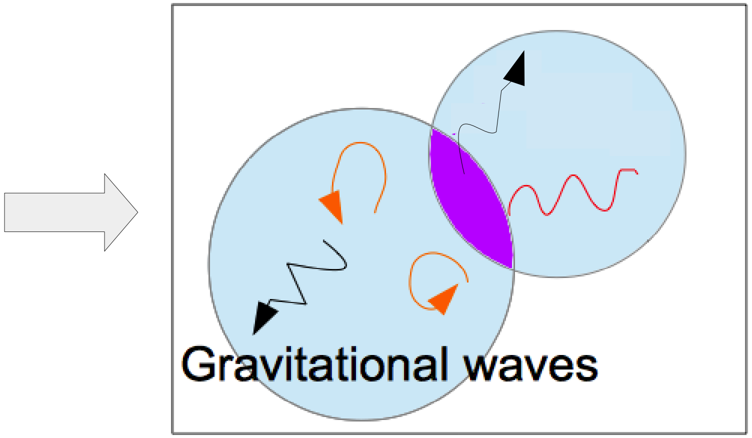
- How do we evaluate the GWs from 1stOPT?

Bubbles nucleation

Collision of bubbles wall




- Sources of GWs :

  1.Collision of wall  2.Compression wave of plasma  3. Plasma turbulence

- The GWs from 1$^{st}$OPT spectrum is characterized by

  $\alpha \simeq$ Latent heat released by PT,  $\beta \simeq$ 1/(The duration of PT)

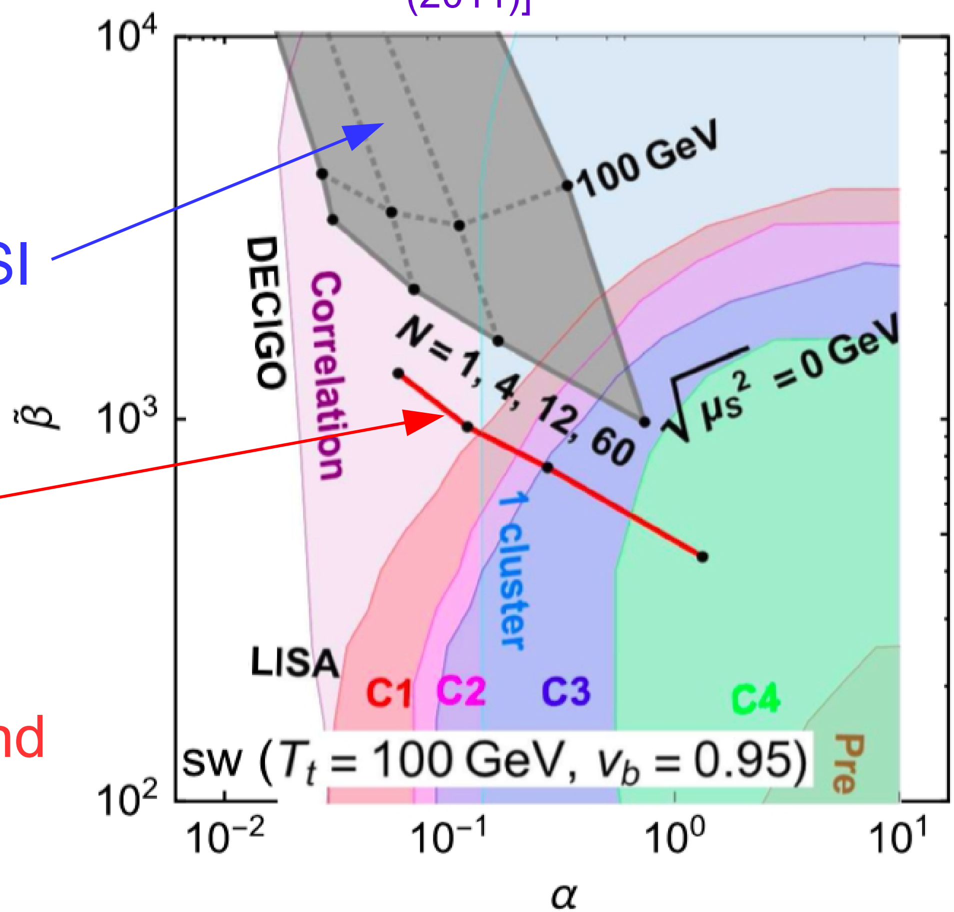
- α, β are determined by calculating the effective potential.

# GW spectra for O(N) singlet model with CSI and O(N) singlet model without CSI

LISA: [JCAP 1604, no. 04, 001 (2016)]
DECIGO: [Class. Quant. Grav. 28, 094011 (2011)]

- The peaks of GW spectra for compression wave of plasma are described by (α, $\tilde{\beta}(\equiv \beta/H_T)$) plane.

  - O(N) singlet model without CSI
    $$\left(\Delta\lambda_{hhh}^{O(N)}/\lambda_{hhh}^{SM} = 2/3 \simeq 67\,\%\right)$$
  - O(N) singlet model with CSI
    $$\left(\Delta\lambda_{hhh}^{CSI}/\lambda_{hhh}^{SM} = 2/3 \simeq 67\%\right)$$




- We can distinguish the model with and without CSI, and it will be possible to observe GW spectra in the future.

Vb: Velocity of bubble
Tt : Transition temperature
$H_T$ : Hubble parameter at T

[K.H, M.Kakizaki, S.Kanemura and T.Matsui, Phys. Rev. D 94, no. 1, 015005 (2016)]

# Summary

- We focused on the CSI models where N extra isospin singlet scalars obey a global O(N) symmetry.
- CSI models for EWSB predict $\Gamma_{hhh}^{CSI} = \frac{5}{3} \times \Gamma_{hhh}^{SM\ tree}.$
- Even if other models have the same deviation, we could distinguish the models by GWs from 1st OPT.

(For example: O(N) singlet model with CSI and O(N) singlet model without CSI.)

- Synergy between the future measurements of the hhh coupling and the GW signals provides us important hint about narrowing down the dynamics behind the EWSB.

# Backup

# The model for electroweak symmetry breaking based on classical scale invariance

Scale transformation

$$x \to e^{-\alpha} x, \quad \partial_\mu \to e^{\alpha} \partial_\mu, \quad \Phi \to e^{\alpha} \Phi, \quad \int d^4x \sqrt{-g} \to e^{-4\alpha} \int d^4x \sqrt{-g}$$

For example : $V_{SM}(\phi) = \mu^2 |\phi|^2 + \lambda |\phi|^4$

$$\int d^4x \sqrt{-g}\phi^4 \to \int d^4x \sqrt{-g'}\phi'^4 = e^{-4\alpha+4\alpha} \int d^4x \sqrt{-g}\phi^4 = \int d^4x \sqrt{-g}\phi^4$$

$$\int d^4x \sqrt{-g}\phi^2 \to \int d^4x \sqrt{-g'}\phi'^2 = e^{-4\alpha+2\alpha} \int d^4x \sqrt{-g}\phi^2 = e^{-2\alpha} \int d^4x \sqrt{-g}\phi^2$$

The kinetic term

$$\int d^4x \sqrt{-g}(\partial^\mu \phi)^2 \to \int d^4x \sqrt{-g'}(\partial'^\mu \phi')^2 = \int d^4x \sqrt{-g} e^{-4\alpha+4\alpha} (\partial^\mu \phi)^2 = \int d^4x \sqrt{-g}(\partial^\mu \phi)^2$$

# Landau pole $\Lambda$ (CSI O(N) models)

- We calculate the Landau pole Λ of the CSI O(N) models.

$$V_0(\Phi, \vec{S}) = \frac{\lambda}{2}|\Phi|^4 + \frac{\lambda_S}{4}|\vec{S}|^4 + \frac{\lambda_{\Phi S}}{2}|\Phi|^2|\vec{S}|^2 \qquad \vec{S} = (S_1, S_2, ..., S_N)^T$$

| $N$ | 1 | 4 | 12 | 60 |
|---|---|---|---|---|
| $Q$ | 381 GeV | 257 GeV | 188 GeV | 119 GeV |
| $\Lambda(\lambda_S = 0)$ | 5.4 TeV | 17 TeV | 28 TeV | 33 TeV |
| $\Lambda(\lambda_S = 0.1)$ | 5.3 TeV | 16 TeV | 23 TeV | 13 TeV |
| $\Lambda(\lambda_S = 0.2)$ | 5.2 TeV | 15 TeV | 19 TeV | 5.4 TeV |
| $\Lambda(\lambda_S = 0.3)$ | 5.0 TeV | 14 TeV | 15 TeV | 2.7 TeV |

TABLE : The energy scale of the Landau pole Λ in the CSI O(N) models for N = 1,4,12 and 60.

[K. Hashino, M. Kakizaki, S. Kanemura and T. Matsui, Phys. Rev. D 94, no. 1, 015005 (2016)]

- The renormalization scale Q is decided by the stationary condition.
- The cutoff scale Λ is defined as the scale where any of the scalar couplings diverges.

# Discriminative phenomenological features for the models

The models have three discriminative features.

[K. Hashino, S. Kanemura and Y. Orikasa, Phys. Lett. B 752, 217 (2016)]

1) A general upper bound on the mass $m_1^{CSI}$ of the lightest of the scalar bosons is

$$m_1^{CSI} \leq 543\mathrm{GeV}$$

2) The scaling factor $\kappa_\gamma^{CSI}$ of the $h\gamma\gamma$ coupling is

$$\kappa_\gamma^{CSI} \simeq 1 - \frac{n}{16} - \frac{m}{4}$$

where n and m are the numbers of singly- and doubly- charged scalar bosons, respectively.

3) The triple Higgs boson coupling $\Gamma_{hhh}^{CSI}$ is universally predicted at the leading order.

$$\Gamma_{hhh}^{CSI} = \frac{5}{3} \times \Gamma_{hhh}^{SM\ tree}$$

## 1) A general upper bound on the mass $m_1^{CSI}$

$$V_{\rm eff}(\varphi) = A\varphi^4 + B\varphi^4 \ln\frac{\varphi^2}{Q^2}$$

$$A = \frac{1}{64\pi^2 v^4}\left[3{\rm Tr}\left(M_V^4 \ln\frac{M_V^2}{v^2}\right) - 4{\rm Tr}\left(M_f^4 \ln\frac{M_f^2}{v^2}\right) + {\rm Tr}\left(M_S^4 \ln\frac{M_S^2}{v^2}\right)\right]$$

$$B = \frac{1}{64\pi^2 v^4}\left[3{\rm Tr}\left(M_V^4\right) - 4{\rm Tr}\left(M_f^4\right) + {\rm Tr}\left(M_S^4\right)\right]$$

- The Higgs mass is

$$m_h^2 \equiv \left.\frac{\partial^2 V_{\rm eff}}{\partial\varphi^2}\right|_{\varphi=v} = 8Bv^2 \simeq (125{\rm GeV})^2$$

$$\boxed{{\rm Tr}M_S^4 = 8\pi^2 v^2 m_h^2 - 3m_Z^4 - 6m_W^4 + 12m_t^4 \ (\equiv C^4 \simeq (543\,{\rm GeV})^4)}$$

- We consider a case including N extra scalar bosons and the masses can be written as

$$m_1^{CSI} \leq m_2^{CSI} \cdots \leq m_N^{CSI}.$$

$$\boxed{{\rm Tr}\, M_s^4 = \sum_{n=1}^{N} (m_n^{CSI})^4 \geq N\,(m_1^{CSI})^4}$$

## 1) A general upper bound on the mass $m_1^{CSI}$

$$V_{\rm eff}(\varphi) = A\varphi^4 + B\varphi^4 \ln\frac{\varphi^2}{Q^2}$$

$$A = \frac{1}{64\pi^2 v^4}\left[3{\rm Tr}\left(M_V^4 \ln\frac{M_V^2}{v^2}\right) - 4{\rm Tr}\left(M_f^4 \ln\frac{M_f^2}{v^2}\right) + {\rm Tr}\left(M_S^4 \ln\frac{M_S^2}{v^2}\right)\right]$$

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- We consider a case including N extra scalar bosons and the masses can be written as

$$m_1^{CSI} \le m_2^{CSI} \cdots \le m_N^{CSI}.$$

$$\boxed{{\rm Tr}\, M_s^4 = \sum_{n=1}^{N} (m_n^{CSI})^4 \ge N\,(m_1^{CSI})^4}$$

$$\boxed{m_1^{CSI} \le \frac{C}{\sqrt[4]{N}} \le 543\,(GeV)}$$

- $m_1^{CSI}$ is generally less than 543 GeV !

## 2) The scaling factor $\kappa_\gamma^{CSI}$ of the $h\gamma\gamma$ coupling

$$\kappa_\gamma^{CSI} \equiv \sqrt{\frac{\Gamma^{(n,m)}_{h\to\gamma\gamma}}{\Gamma^{\rm SM}_{h\to\gamma\gamma}}} \sim \sqrt{\left|1+\frac{1}{2}\frac{\sum_{i=1}^{n}(v/m^2_{\phi_i^\pm})\lambda_{h\phi_i^+\phi_i^-}A_0(\tau_{\phi_i})+4\sum_{j=1}^{m}(v/m^2_{\phi_j^{\pm\pm}})\lambda_{h\phi_j^{++}\phi_j^{--}}A_0(\tau_{\phi_j})}{A_1(\tau_W)+\frac{4}{3}A_{\frac{1}{2}}(\tau_t)}\right|^2}$$

n (m) is the number of singly- (doubly- ) charged scalar bosons and $\tau_x = 4m_x^2/m_h^2$.

The loop effect of ...

| | |
|---|---|
| top quark | $A_{1/2}(\tau_t) = -1.4$ |
| W boson | $A_1(\tau_W) = 8.4$ |
| Charged scalar boson ($m_h \ll m_i$) | $A_0(\tau_i) = -1/3$ |

$$m^2_{\phi_i^\pm} = \frac{1}{2}\left(\frac{\lambda_{h\phi_i^+\phi_i^-}}{v}\right)v^2$$

$$m^2_{\phi_i^{\pm\pm}} = \frac{1}{2}\left(\frac{\lambda_{h\phi_i^{++}\phi_i^{--}}}{v}\right)v^2$$

The characteristics of the model for EWSB based on CSI

$$\kappa_\gamma^{CSI} \simeq 1 - \frac{n}{16} - \frac{m}{4}$$

Non-decoupling effects

## 2) The scaling factor $\kappa_\gamma^{CSI}$ of the $h\gamma\gamma$ coupling

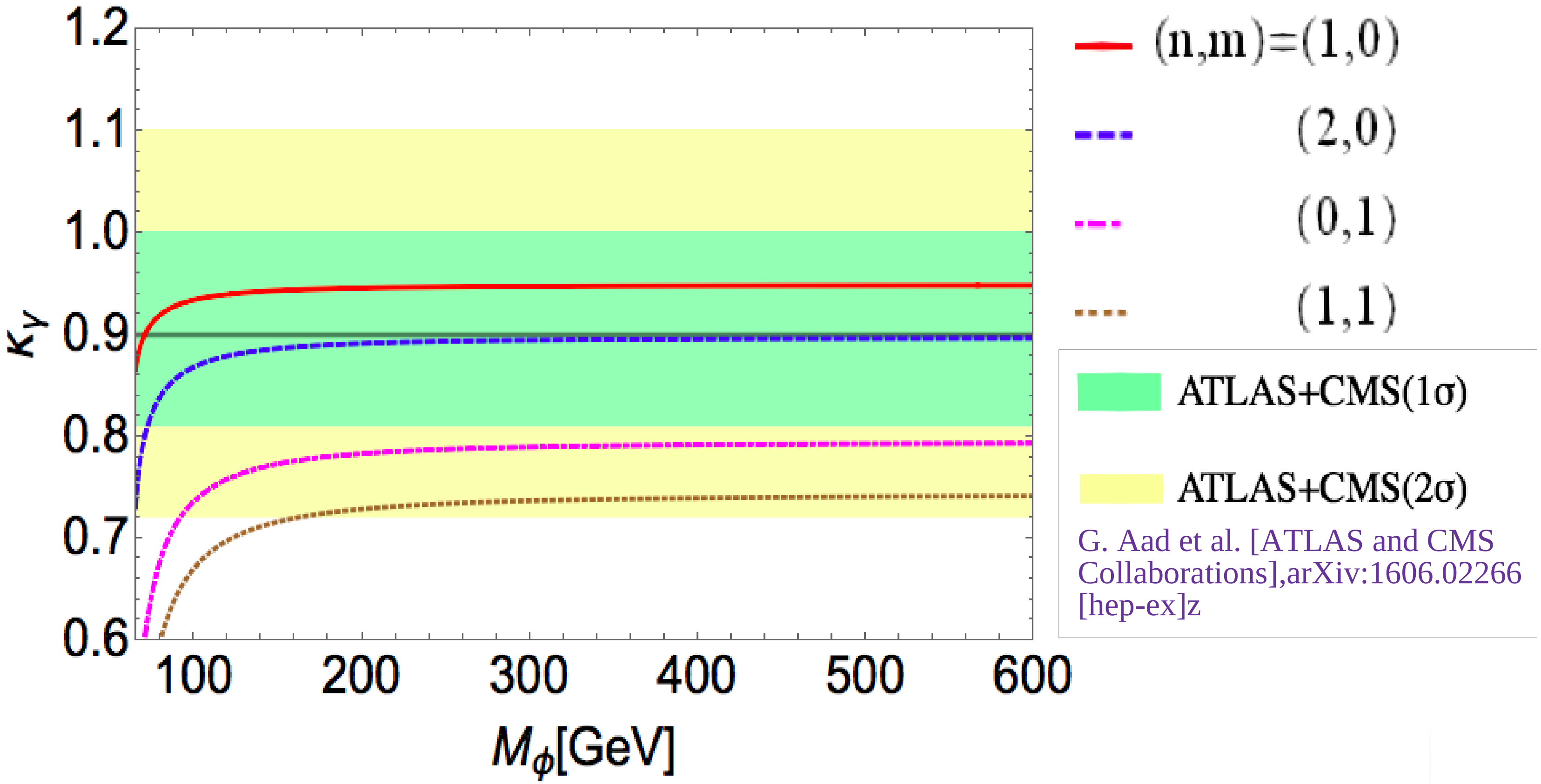



FIG : Behavior of $\kappa_\gamma$ in specific (n,m) is expressed by charged scalar boson mass $M_\phi$ of the horizontal axis.

- $\kappa_\gamma^{CSI}$ will be measured with the 5-7% accuracy at the LHC Run-2.

[S. Dawson et al. arXiv:1310.8361]

- We expect that the number of the charged scalar bosons in the model will be determined by LHC Run-2 !

## 3) The triple Higgs boson coupling $\Gamma_{hhh}^{CCI}$

- All models for EWSB based on CSI is universally

$$\boxed{\Gamma_{hhh}^{CSI} \equiv \left.\frac{\partial^3 V_{\mathrm{eff}}}{\partial \varphi^3}\right|_{\varphi=v} = \frac{5 m_h^2}{v} = \frac{5}{3} \times \Gamma_{hhh}^{SM\ tree}}$$

[K. Hashino, S. Kanemura and Y. Orikasa, Phys. Lett. B 752, 217 (2016)]

- O(N) extended Higgs model that does not based on CSI, the triple Higgs boson coupling is

$$\lambda_{hhh}^{O(N)} = \frac{3m_h^2}{v}\left\{1 - \frac{1}{\pi^2}\frac{m_t^4}{v^2 m_h^2} + \frac{N}{12\pi^2}\frac{m_S^4}{v^2 m_h^2}\left(1 - \frac{\mu_S^2}{m_S^2}\right)^3\right\}$$

[ M. Kakizaki, S. Kanemura and T. Matsui, Phys. Rev. D 92, no. 11, 115007 (2015)]

- The deviation of $\Gamma_{hhh}^{CSI}$ from $\Gamma_{hhh}^{SM\ tree} = \frac{3m_h^2}{v}$ is universally about 67% !

- The deviation will be measured with the 10% accuracy at the ILC.

[ T. Barklow, J. Brau, K. Fujii, J. Gao, J. List, N. Walker and K. Yokoya., arXiv:1506.07830]

- We able to check whether the model is true in the future !!

# Upper bound on the mass $m_1^{CSI}$ in 2HDM

## For a specific model

- We rewrite N as $N_{I,Y}$ which is the number of scalar fields with isospin I and hypercharge Y.

$$N = N_{0,0} + 2\mathrm{N}_{0,1} + 4\mathrm{N}_{\frac{1}{2},\frac{1}{2}} + 3\mathrm{N}_{1,0} + 6\mathrm{N}_{1,1} + s \quad s \quad s$$

$$m_1^{\mathrm{CCI}} \leq \frac{C}{\sqrt[4]{N_{0,0} + 2N_{0,1} + 4N_{\frac{1}{2},\frac{1}{2}} + 3N_{1,0} + 6N_{1,1} + \cdots}}$$

- When we consider the extensions for doublets( $I = \frac{1}{2}, Y = \frac{1}{2}$ ), this upper bound is stronger!

$$\boxed{m_1^{\mathrm{CCI}} \leq \frac{C}{\sqrt[4]{4N_{\frac{1}{2},\frac{1}{2}}}} \sim \frac{1}{\sqrt[4]{N_{\frac{1}{2},\frac{1}{2}}}} \times 383\mathrm{GeV}}$$

# Gildener - Weinberg method

The Gildener and Weinberg method supposes that there is the flat direction in the tree-level potential $V_0(\Phi)$.

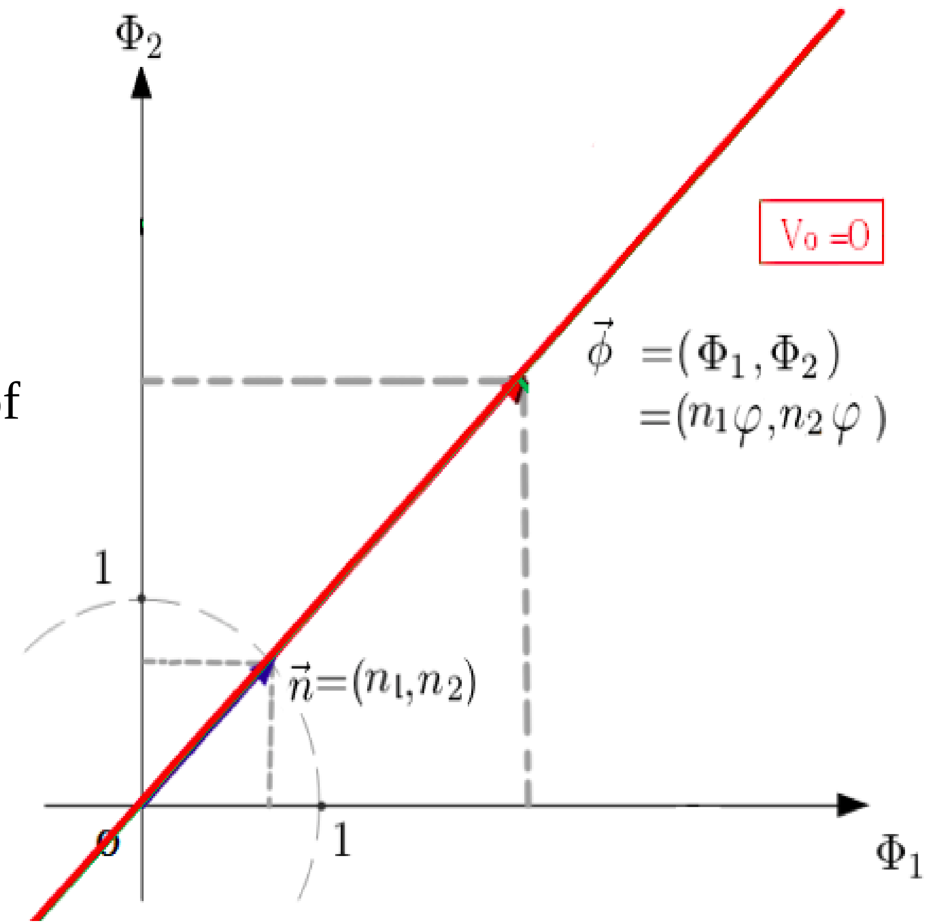
$$V_0(\Phi)=\frac{1}{24}f_{ijkl}\Phi_i\Phi_j\Phi_k\Phi_l\ ,\quad f_{ijkl}\equiv\frac{\partial^4 V_0(\Phi)}{\partial\Phi_i\partial\Phi_j\partial\Phi_k\partial\Phi_l}$$

The flat direction is decided by

$$\Phi_i=n_i\varphi\,.$$

The unit vector $n_i$ represents the direction of flat direction and $\varphi$ is order parameter.

On the flat direction, $V_0(n_i\varphi)=0$,
EWSB occurs radiatively by CWM.



$$V_{\mathrm{eff}}(\varphi) = A\varphi^4 + B\varphi^4 \ln\frac{\varphi^2}{Q^2}$$

$$A = \frac{1}{64\pi^2 v^4}\left[3\,\mathrm{Tr}\left(M_V^4 \ln\frac{M_V^2}{v^2}\right) - 4\,\mathrm{Tr}\left(M_f^4 \ln\frac{M_f^2}{v^2}\right) + \mathrm{Tr}\left(M_S^4 \ln\frac{M_S^2}{v^2}\right)\right]$$

$$B = \frac{1}{64\pi^2 v^4}\left[3\,\mathrm{Tr}\left(M_V^4\right) - 4\,\mathrm{Tr}\left(M_f^4\right) + \mathrm{Tr}\left(M_S^4\right)\right]$$

$$\left.\frac{\partial V_{\mathrm{eff}}(\varphi)}{\partial\varphi}\right|_{\varphi=v} = \ln\frac{v^2}{Q^2} + \frac{1}{2} + \frac{A}{B} = 0, \qquad m_h^2 \equiv \left.\frac{\partial^2 V_{\mathrm{eff}}}{\partial\varphi^2}\right|_{\varphi=v} = 8Bv^2 \simeq (125\mathrm{GeV})^2$$

$$\boxed{V_{\mathrm{eff}}(\varphi) = \frac{m_h^2}{8v^2}\varphi^4\left(\ln\frac{\varphi^2}{v^2} - \frac{1}{2}\right)}$$

$$\lambda_{hhh} = \left.\frac{\partial^3 V_{\mathrm{eff}}(\varphi)}{\partial\varphi^3}\right|_{\varphi=v} = 40vB = \frac{5m_h^2}{v} = \frac{5}{3}\lambda_{hhh}^{\mathrm{SM(tree)}}$$

## The triple Higgs boson coupling $\Gamma_{hhh}^{CCI}$

- All models for EWSB based on CSI is universally

$$\Gamma_{hhh}^{CSI} \equiv \left.\frac{\partial^3 V_{\mathrm{eff}}}{\partial \varphi^3}\right|_{\varphi=v} = \frac{5 m_h^2}{v} = \frac{5}{3} \times \Gamma_{hhh}^{SM\, tree}$$

[K. Hashino, S. Kanemura and Y. Orikasa, Phys. Lett. B 752, 217 (2016)]

- O(N) extended Higgs model that does not based on CSI, the triple Higgs boson coupling is

$$\lambda_{hhh}^{O(N)} = \frac{3m_h^2}{v}\left\{1 - \frac{1}{\pi^2}\frac{m_t^4}{v^2 m_h^2} + \frac{N}{12\pi^2}\frac{m_S^4}{v^2 m_h^2}\left(1 - \frac{\mu_S^2}{m_S^2}\right)^3\right\}$$

[ M. Kakizaki, S. Kanemura and T. Matsui, Phys. Rev. D 92, no. 11, 115007 (2015)]

- The deviation of $\Gamma_{hhh}^{CSI}$ from $\Gamma_{hhh}^{SM\, tree} = \frac{3 m_h^2}{v}$ is universally about 67% !

- The deviation will be measured with the 10% accuracy at the ILC.

[ T. Barklow, J. Brau, K. Fujii, J. Gao, J. List, N. Walker and K. Yokoya., arXiv:1506.07830]

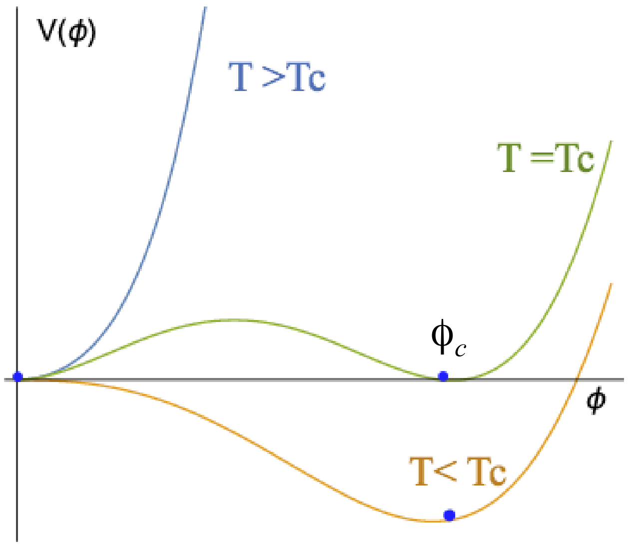
- We able to check whether the model is true in the future !!

# Electroweak baryogenesis

- One of the scenarios explaining Baryon asymmetry of the Universe is Electroweak baryogenesis (EWBG)

- Sakharov's conditions

   1. Baryon number violation
      - Sphaleron process
   2. C and CP violation
      - Extended Higgs sector
   3. Departure from thermal equilibrium
      - Strongly 1st order phase transition




$\longrightarrow$ $\boxed{\phi_c / T_c \geq 1}$

The SM doesn't satisfy $\phi_c / T_c \geq 1$ . (It needs $m_h$<60GeV)

# High temperature expansion

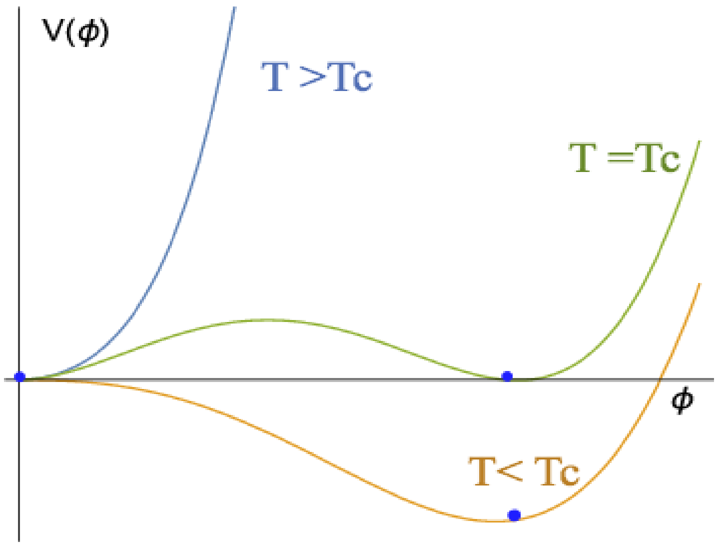
The loop effect of bosons and fermions

$$V_{\rm eff}(\varphi, T) = D(T^2 - T_0^2)\varphi^2 - E\,T\,\varphi^3 + \frac{\lambda(T)}{4}\varphi^4$$

The loop effect of bosons
( Non-decoupling effect )

$$\phi_c / T_c = 2E/\lambda(T) \geq 1$$

Strongly 1st OPT is possible by non-decoupling effect from the loop of additional bosons.

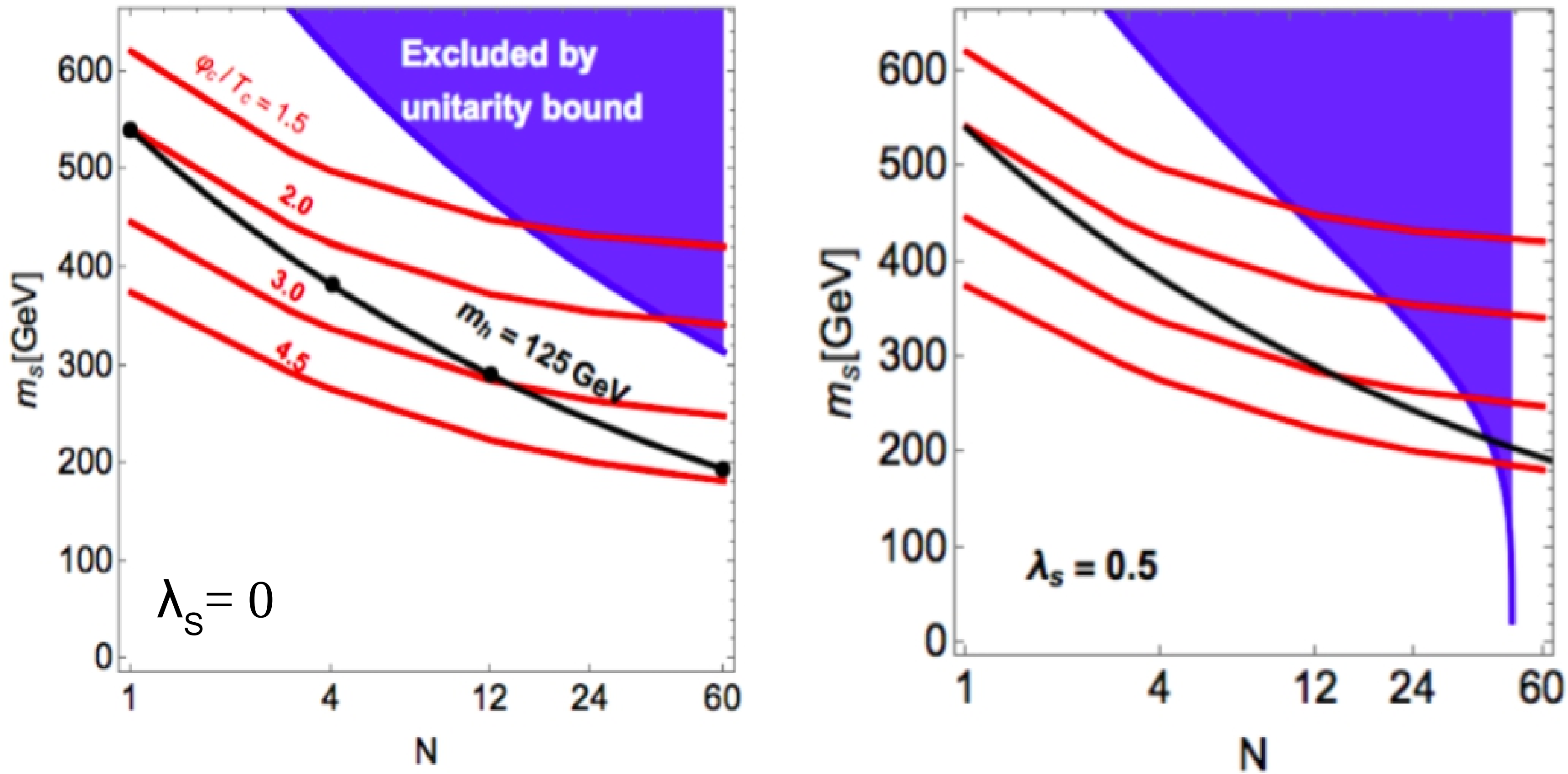



$$E = \frac{1}{12\pi v^3}(6m_W^3 + 3m_Z^3)$$

$$\lambda(T) = \frac{m_h^2}{2v^2}\left[1 - \frac{1}{8\pi^2 v^2 m_h^2}\left\{6m_W^4 \ln\frac{m_W^2}{\alpha_B T^2} + 3m_Z^4 \ln\frac{m_Z^2}{\alpha_B T^2} - 12 m_t^4 \ln\frac{m_t^2}{\alpha_F T^2}\right\}\right]$$

In the SM

# Strongly 1st order phase transition

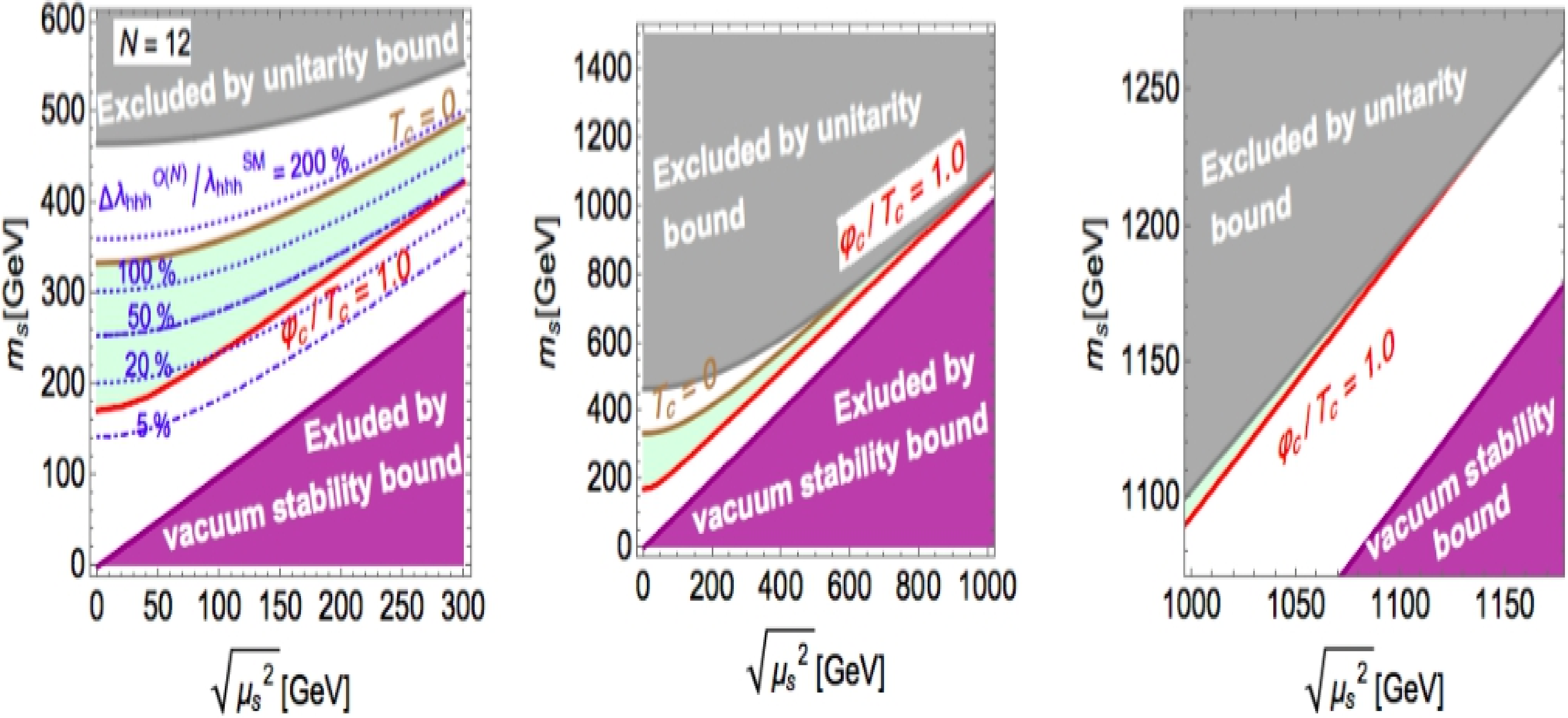



Unitarity bound

$$\frac{1}{32\pi}\left[3\lambda + (N+2)\lambda_S + \sqrt{\{3\lambda - (N+2)\lambda_S\}^2 + 4N\lambda_{\Phi S}^2}\right] < \frac{1}{2}$$

# O(N) singlet model without classical scale invariance

[K. Hashino, M. Kakizaki, S. Kanemura and T. Matsui, Phys. Rev. D 94, no. 1, 015005 (2016)]



# Gravitational waves

- The ground-based experiments

LIGO, KAGURA, VIRGO,...

The main target of the ground-based experiments is GWs from astronomical phenomena.

(For example: Binary systems of neutron stars or black holes. )

These can detect the gravity wave of frequency bands less than 10-$10^3$ Hz.

- The (future) space-based experiments

LISA, DECIGO,..

These experiments have the sensitivity to investigate some cosmological phenomena.

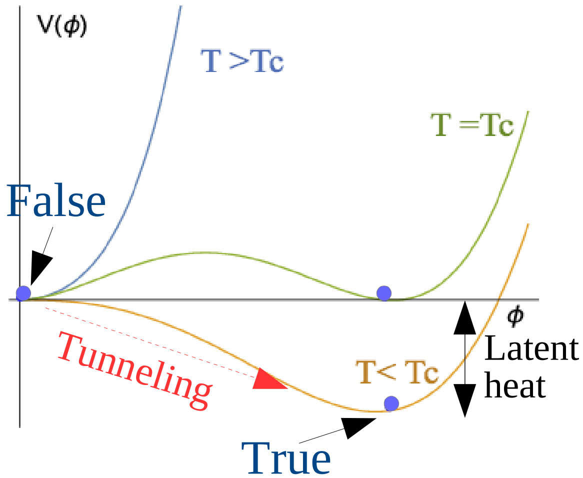
(For example: strongly 1stOPTs and cosmic inflation at the early Universe.)

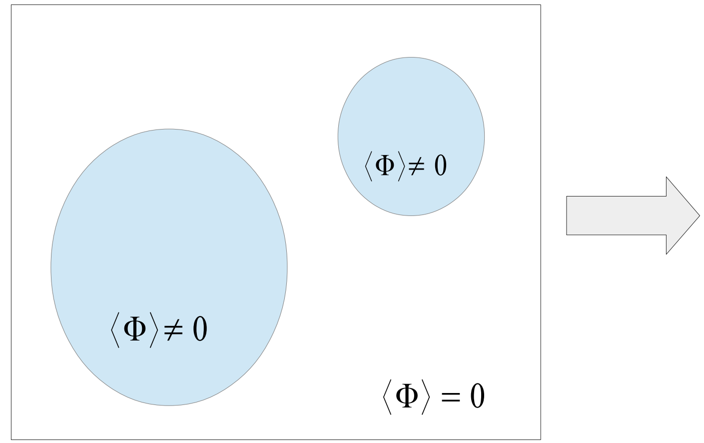
These can detect the gravity wave of frequency bands less than 1 Hz.

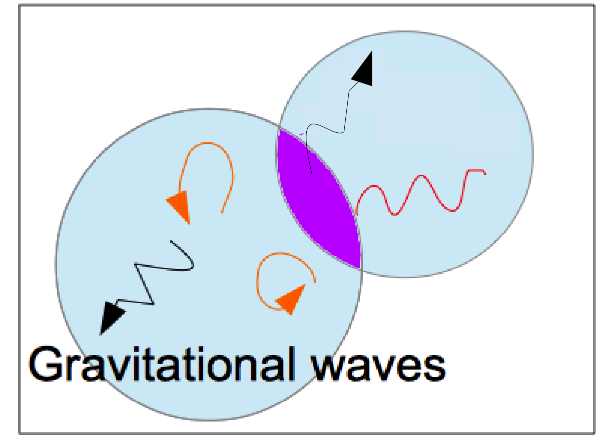
# Gravitational waves

Bubble nucleation

Collision of bubble walls




- Bubble nucleation rate per unit volume per unit time Γ : $\Gamma(T) \simeq T^4 e^{-\frac{S_3(T)}{T}}$
- The three dimensional Euclidean action $S_3$ :

$$S_3(T) = \int dr^3 \left\{ \frac{1}{2} \left( \vec{\nabla}\varphi \right)^2 + V_{\text{eff}}(\varphi, T) \right\}$$

- Transition temperature $T_t$ : $\left. \frac{\Gamma}{H^4} \right|_{T=T_t} \simeq 1 \longrightarrow \frac{S_3(T_t)}{T_t} = 4 \ln(T_t/H_t) \simeq 140$

$$\alpha = \frac{\epsilon(T_t)}{\rho_{rad}(T_t)}, \quad \beta \simeq \frac{1}{\Gamma} \frac{d\Gamma}{dT}$$

Latent heat : $\epsilon(T) = -V_{\text{eff}}(\varphi_B(T), T) + T \frac{\partial V_{\text{eff}}(\varphi_B(T), T)}{\partial T}$

Radiative energy dencity : $\rho_{rad}$

TABLE I. Predictions of the four benchmark points $N = 1, 4, 12$, and 60 in the CSI $O(N)$ models (top). For comparison, the predictions of $O(N)$ models without CSI with $\Delta\lambda_{hhh}/\lambda_{hhh}^{\mathrm{SM}} = 2/3(\simeq 70\%)$ are also shown for $\sqrt{\mu_S^2} = 0$ GeV (middle) and $\sqrt{\mu_S^2} =$ 100 GeV (bottom).

| $N$ | 1 | 4 | 12 | 60 |
|---|---|---|---|---|
| $m_S$ [GeV] | 540 | 382 | 290 | 194 |
| $\varphi_c/T_c$, $T_c$ [GeV] | 2.01, 102 | 2.40, 90.1 | 2.91, 76.8 | 4.11, 56.1 |
| $(\alpha, \tilde{\beta})$, $T_t$ [GeV] | (0.0593, 1320), 88.5 | (0.120, 956), 74.3 | (0.273, 705), 59.7 | (1.33, 438), 38.4 |
| ($\sqrt{\mu_S^2}$ [GeV], $N$) | (0, 1) | (0, 4) | (0, 12) | (0, 60) |
| $m_S$ [GeV] | 510 | 361 | 274 | 183 |
| $\varphi_c/T_c$, $T_c$ [GeV] | 1.62, 119 | 2.03, 102 | 2.54, 85.6 | 3.65, 61.5 |
| $(\alpha, \tilde{\beta})$, $T_t$ [GeV] | (0.0303, 3320), 111 | (0.0695, 2180), 92.5 | (0.164, 1600), 74.8 | (0.739, 1090), 50.3 |
| ($\sqrt{\mu_S^2}$ [GeV], $N$) | (100, 1) | (100, 4) | (100, 12) | (100, 60) |
| $m_S$ [GeV] | 524 | 380 | 299 | 219 |
| $\varphi_c/T_c$, $T_c$ [GeV] | 1.56, 121 | 1.89, 106 | 2.25, 92.1 | 2.89, 71.6 |
| $(\alpha, \tilde{\beta})$, $T_t$ [GeV] | (0.0272, 4380), 115 | (0.0552, 3480), 99.5 | (0.111, 3210), 85.7 | (0.334, 4082), 67.2 |

[K. Hashino, M. Kakizaki, S. Kanemura and T. Matsui, Phys. Rev. D 94, no. 1, 015005 (2016)]

# Gravitational spectrum

[C. Caprini et al., arXiv:1512.06239 [astro-ph.CO]]

- Collision of wall

$$\tilde{\Omega}_{\rm env} h^2 \simeq 1.67 \times 10^{-5} \times \left(\frac{0.11 v_b^3}{0.42 + v_b^2}\right) \tilde{\beta}^{-2} \left(\frac{\kappa_\varphi \alpha}{1+\alpha}\right)^2 \left(\frac{100}{g_*^t}\right)^{1/3}$$

the peak of energy dencity

$$\tilde{f}_{\rm env} \simeq 1.65 \times 10^{-5}\ {\rm Hz} \times \left(\frac{0.62}{1.8 - 0.1 v_b + v_b^2}\right) \tilde{\beta} \left(\frac{T_t}{100\ {\rm GeV}}\right) \left(\frac{g_*^t}{100}\right)^{1/6}$$

the peak frequency

- Compression wave of plasma

$$\tilde{\Omega}_{\rm sw} h^2 \simeq 2.65 \times 10^{-6} v_b \tilde{\beta}^{-1} \left(\frac{\kappa_v \alpha}{1+\alpha}\right)^2 \left(\frac{100}{g_*^t}\right)^{1/3}$$

$$\tilde{f}_{\rm sw} \simeq 1.9 \times 10^{-5}\ {\rm Hz} \frac{1}{v_b} \tilde{\beta} \left(\frac{T_t}{100\ {\rm GeV}}\right) \left(\frac{g_*^t}{100}\right)^{1/6}$$

- Plasma turbulence

$$\tilde{\Omega}_{\rm turb} h^2 \simeq 3.35 \times 10^{-4} v_b \tilde{\beta}^{-1} \left(\frac{\epsilon \kappa_v \alpha}{1+\alpha}\right)^{3/2} \left(\frac{100}{g_*^t}\right)^{1/3}$$

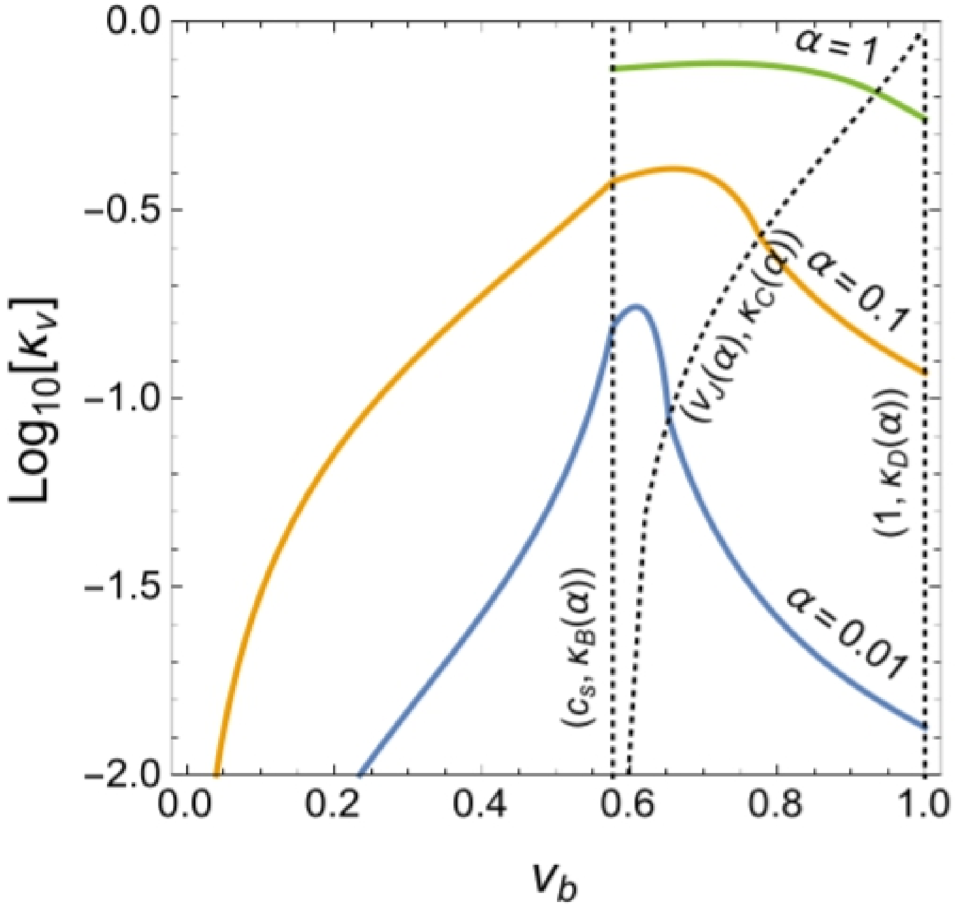
$$\tilde{f}_{\rm turb} \simeq 2.7 \times 10^{-5}\ {\rm Hz} \frac{1}{v_b} \tilde{\beta} \left(\frac{T_t}{100\ {\rm GeV}}\right) \left(\frac{g_*^t}{100}\right)^{1/6}$$

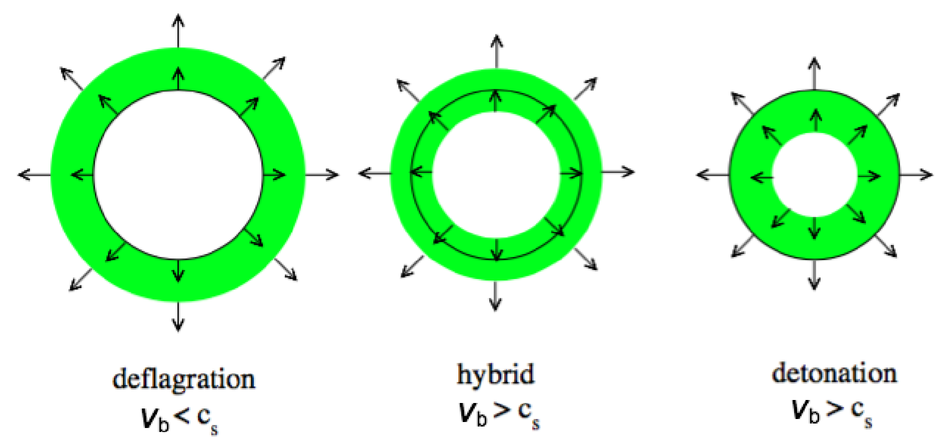
$\kappa_\varphi, \kappa_v, \epsilon$ : efficiency factors

$v_b$ : wall velocity

# Efficiency factors

[J. R. Espinosa, T. Konstandin, J. M. No and G. Servant, JCAP 1006, 028 (2010)]




The black circle is bubble wall.
In green we show the region of non-zero fluid velocity.

$\kappa_\varphi$ : $\kappa_\varphi = 1 - \dfrac{\alpha_\infty}{\alpha}$ , $\alpha_\infty \simeq \dfrac{30}{24\pi^2 g_*^t T_t^2} \sum_i c_i \left[M_i^2(\varphi_t) - M_i^2(0)\right]$ .

[C. Caprini et al., arXiv:1512.06239 [astro-ph.CO], J. R. Espinosa et al.,JCAP 1006, 028 (2010)]

$\epsilon$ : 5-10%. ( In our numerical analysis, we set $\epsilon$ = 0.05. )

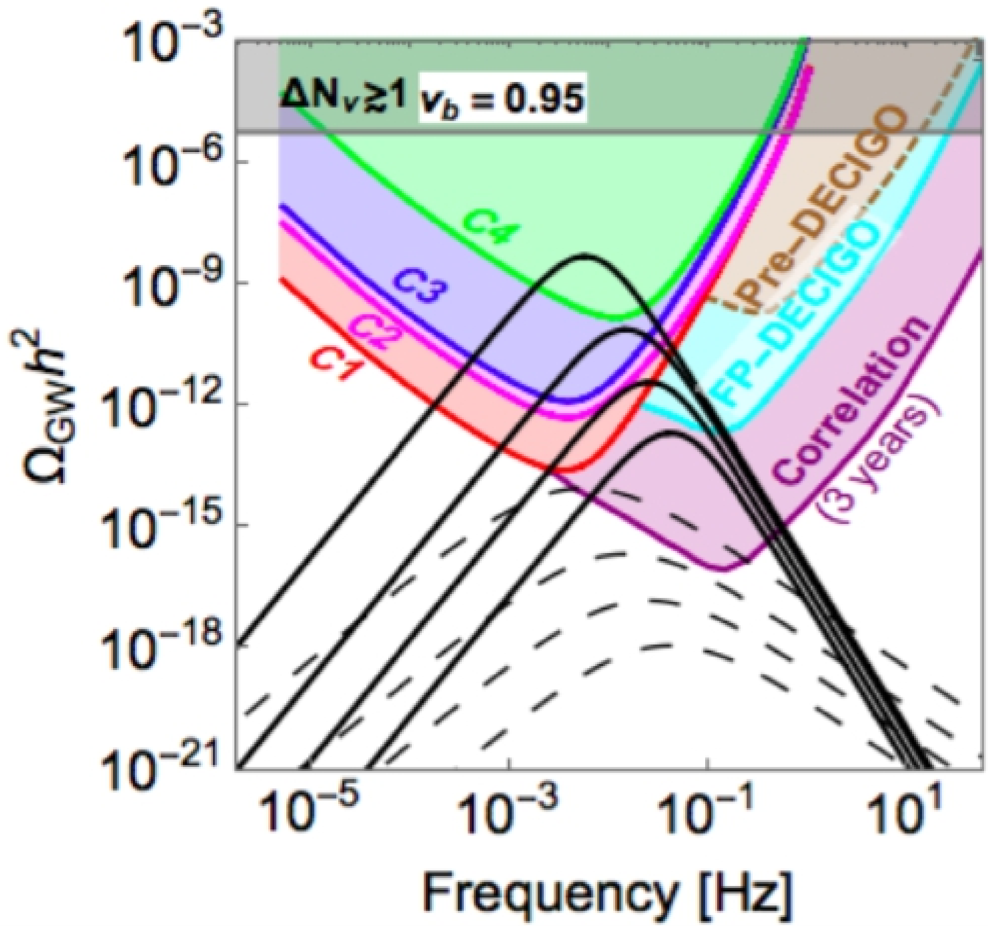
[M.Hindmarsh et al.,Phys. Rev. D92, no. 12, 123009 (2015) ]

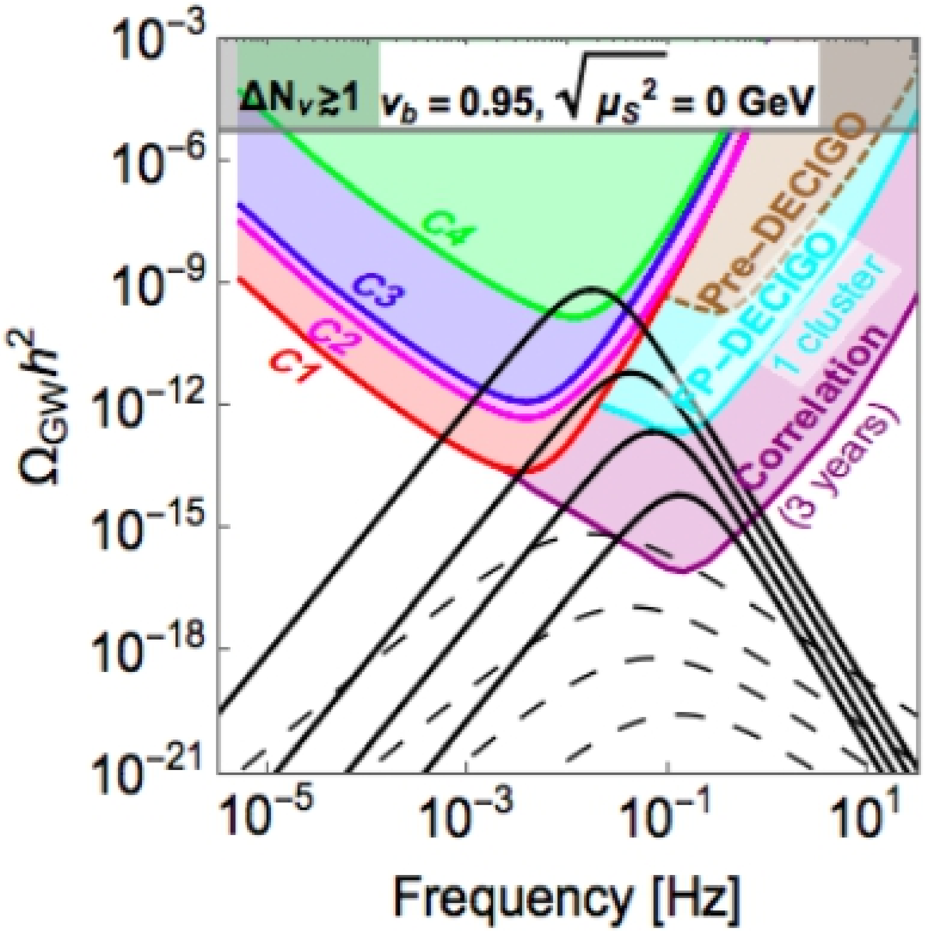
Sound wave
Turblence

# GW spectrum

N=1,4,12,60 from the bottom.

eLISA [arXiv:1512.06239 [astro-ph.CO]]
DECIGO [Class. Quant. Grav. 28, 094011 (2011)]




O(N) singlet model based on classical scale invariance.

When $\Gamma_{hhh}^{O(N)} = \frac{5m_h^2}{v} = \frac{5}{3} \times \Gamma_{hhh}^{SM\,tree}$,
O(N) singlet model without classical scale invariance.



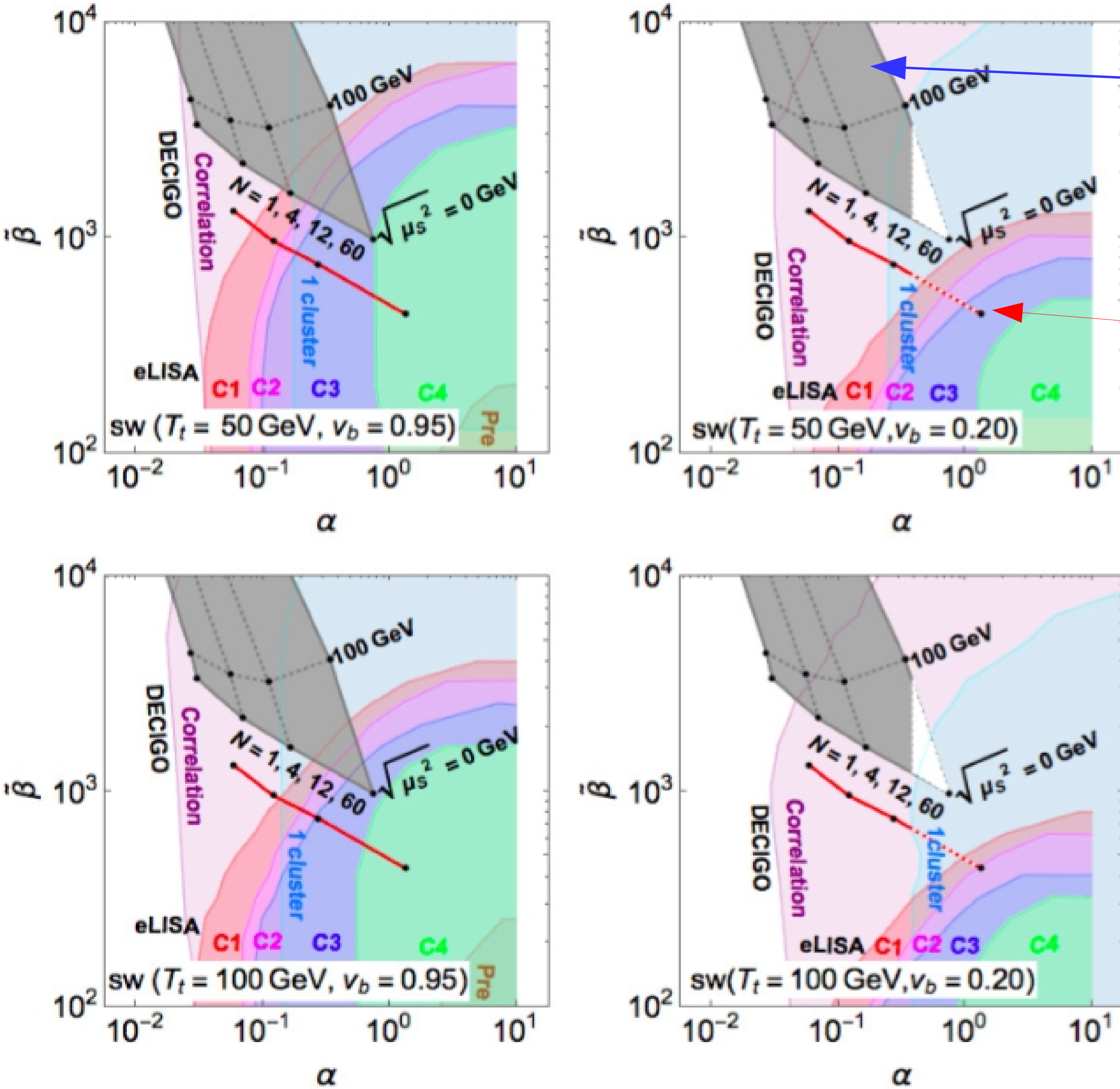
[K. Hashino, M. Kakizaki, S. Kanemura and T. Matsui, Phys. Rev. D 94, no. 1, 015005 (2016)]

[K. Hashino, M. Kakizaki, S. Kanemura and T. Matsui, Phys. Rev. D 94, no. 1, 015005 (2016)]

Vb=0.95 Vb=0.2

Tt=50GeV

Tt=100GeV




O(N) singlet model without CSI

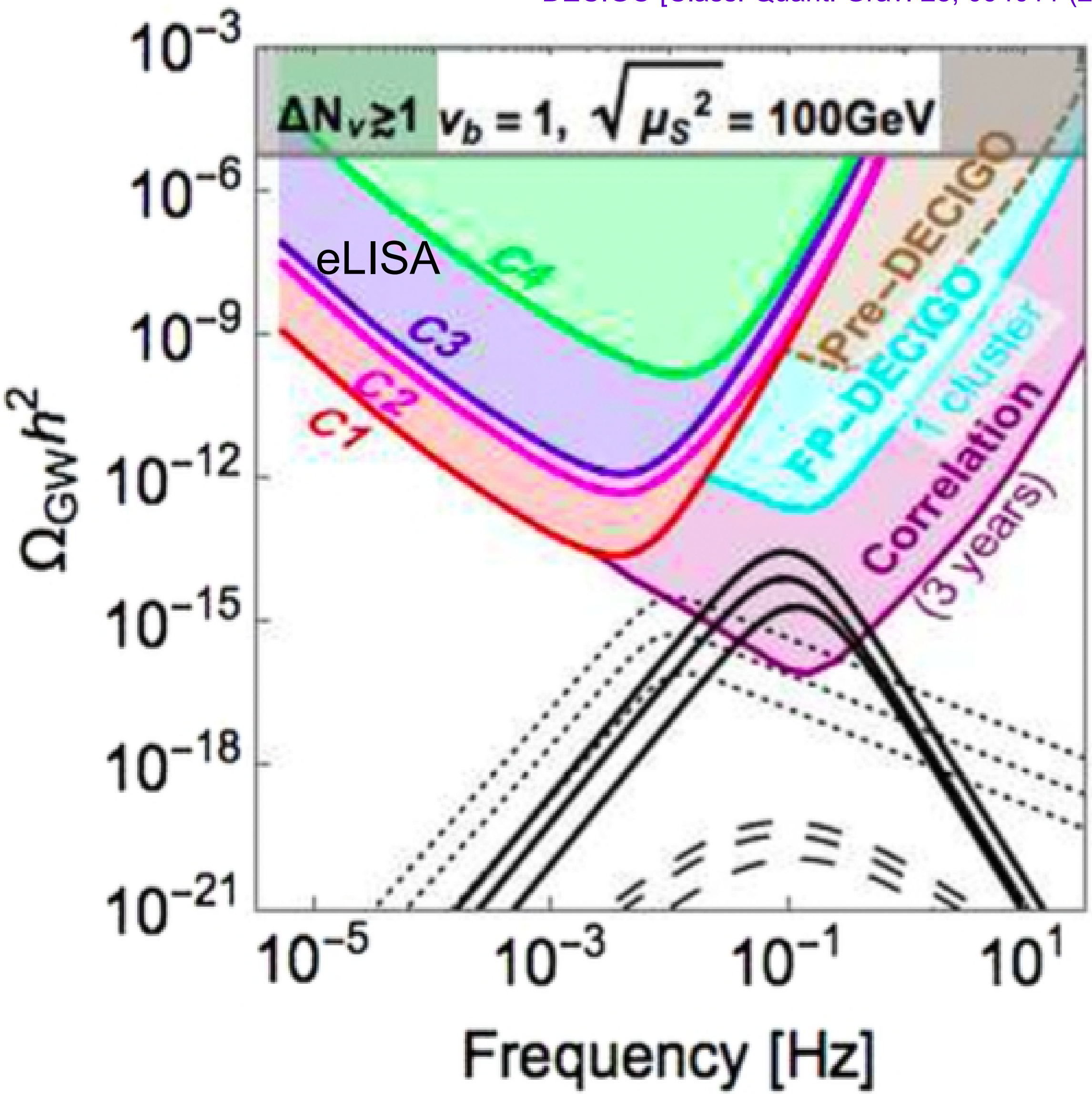
$(\Delta\lambda_{hhh}^{O(N)}/\lambda_{hhh}^{SM} = 2/3 \simeq 67\,\%)$

O(N) singlet model with CSI

The experimenttal sensitivities expected at several designs of eLISA and DECIGO are set by using the sound wave contribution for Tt = 50 GeV and Tt = 100 GeV. The upper bound on ( = 0.39) is delineated for ɣ = 0.2 .

# $\Phi$ + O(N) singlet model without classical scale invariance $\left(\Delta\lambda_{hhh}^{O(N)}/\lambda_{hhh}^{SM} = 2/3 \simeq 67\%\right)$

eLISA [arXiv:1512.06239 [astro-ph.CO]]
DECIGO [Class. Quant. Grav. 28, 094011 (2011)]




Collision

Sound wave

Turblence

(N=4,12,60 from the bottom)

[K. Hashino, M. Kakizaki, S. Kanemura and T. Matsui, Phys. Rev. D 94, no. 1, 015005 (2016)]