H-COUP: A Tool for Precise Calculation of the Higgs Boson Couplings

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3rd RISE Meeting

7th March, University of Toyama

H-COUP

Kanemura, Kikuchi, Sakurai, KY

Fortran code to calculate the h couplings at 1-loop level in non-minimal Higgs sectors based on the (modified) on-shell renormalization scheme.

	hVV	htt	hbb	hττ	hhh	hγγ	hZγ	hgg
HSM	✓	✓	✓	✓	✓	✓	✓	✓
Type-I	✓	✓	✓	✓	✓	✓	✓	✓
Type-II	✓	✓	✓	✓	✓	✓	✓	✓
Type-X	✓	✓	✓	✓	✓	✓	✓	✓
Type-Y	✓	✓	✓	✓	✓	✓	✓	✓
IDM	✓	✓	✓	✓	✓	✓	✓	✓
HTM	✓				✓	✓	✓	✓

HSM:
Kanemura, Kikuchi, KY, NPB907 (2016)
Kanemura, Kikuchi, KY, NPB917 (2017)

Type-I, Type-II, Type-X, Type-Y:
Kanemura, Okada, Senaha, Yuan, PRD70 (2004)
Kanemura, Kikuchi, KY, PLB731 (2014)
Kanemura, Kikuchi, KY, NPB896 (2015)

IDM:
Kanemura, Kikuchi, Sakurai, PRD94 (2016)

HTM:
Aoki, Kanemura, Kikuchi, KY, PLB714 (2012)
Aoki, Kanemura, Kikuchi, KY, PRD87 (2022)

H-COUP Ver. 1.0 (will be public soon)

Structure of H-COUP

1. Define relevant tree level couplings
2. Compute 1PI graphs (1p, 2p and 3p)
3. Compute counter-terms
4. 1-loop corrected Higgs couplings

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Structure of H-COUP

Input a set of parameters (model, masses, mixing angles...)

1. Define relevant tree level couplings
2. Compute 1PI graphs (1p, 2p and 3p)
3. Compute counter-terms
4. 1-loop corrected Higgs couplings

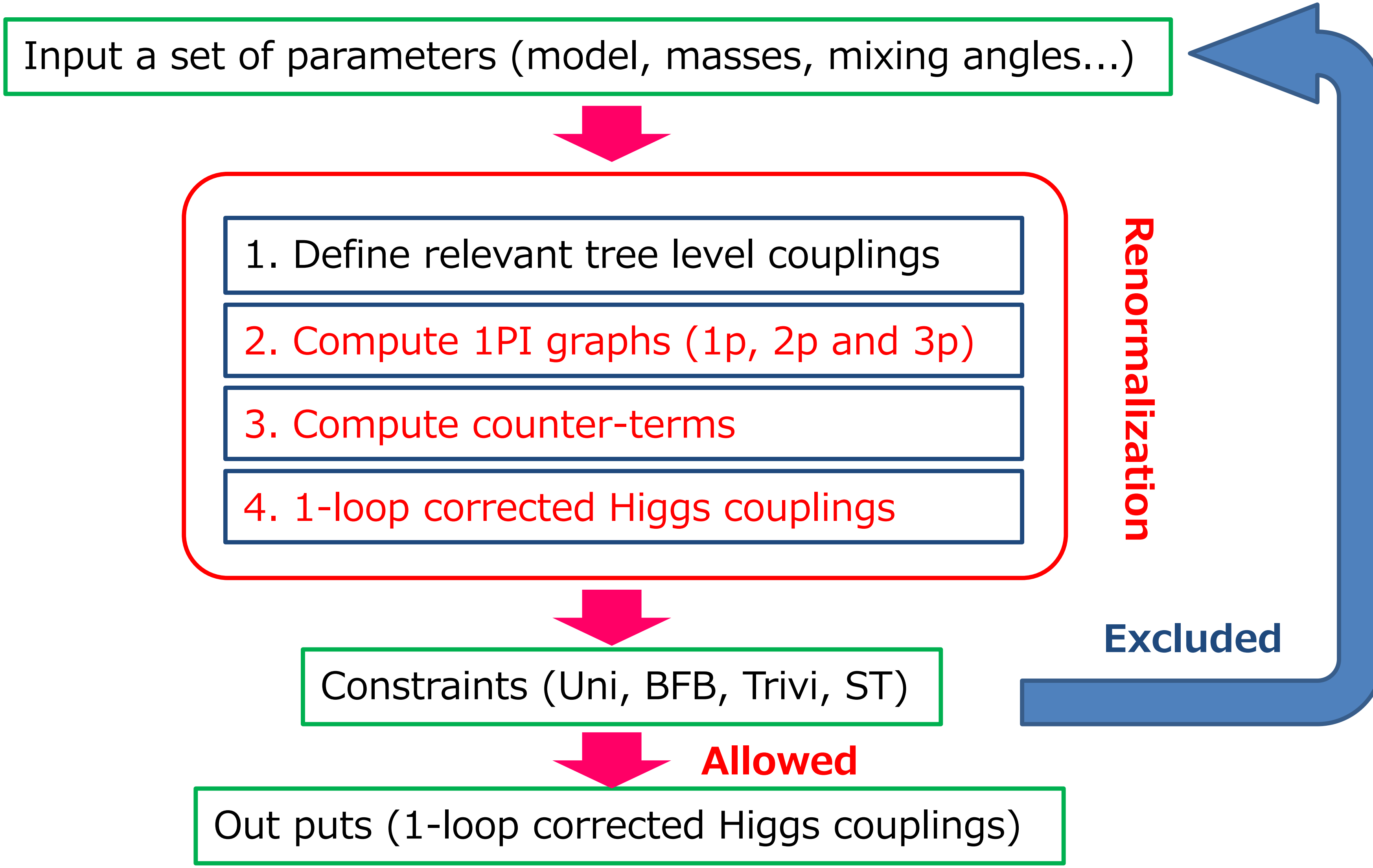
Constraints (Uni, BFB, Trivi, ST)

Excluded

Allowed

Out puts (1-loop corrected Higgs couplings)

Structure of H-COUP



Higgs potential of 2HDM (CPC + Z_2)

- Higgs potential with softly-broken Z_2 symmetry and CP-conservation

$$V = m_1^2|\Phi_1|^2 + m_2^2|\Phi_2|^2 - m_3^2[\Phi_1^\dagger\Phi_2 + \text{h.c.}]$$
$$+\frac{1}{2}\lambda_1|\Phi_1|^4 + \frac{1}{2}\lambda_2|\Phi_2|^4 + \lambda_3|\Phi_1|^2|\Phi_2|^2 + \lambda_4|\Phi_1^\dagger\Phi_2|^2 + \frac{1}{2}\lambda_5\left[(\Phi_1^\dagger\Phi_2)^2 + \text{h.c.}\right]$$

- 8 parameters

v (=246 GeV), m_h (=125 GeV),
m_H, m_A, m_{H+}, **sin(β-α), tanβ**, and **M^2**

$$M^2 = m_3^2/(\sin\beta\cos\beta)$$

- Mass parameters [sin(β-α) ~1]

$m_h^2 \sim \lambda v^2$, $m_\Phi^2 \sim M^2 + \lambda' v^2$

$\Phi = H^\pm, A, H$

Process of Renormalization

1. Count the # of parameters in the Lagrangian.

$$\mathcal{L}_B = \mathcal{L}_B(g_1^B, g_2^B, \dots)$$

2. Prepare the same # of counter terms by shifting the parameters.

$$\mathcal{L}_B(g_1^B, g_2^B, \dots) \to \mathcal{L}_R(g_1^R, g_2^R, \dots) + \delta\mathcal{L}(\delta g_1, \delta g_2, \dots)$$

3. Set the same # of ren. conditions to determine the CT's.

4. Calculate the renormalized quantities.

Renormalization in the Higgs sector

1. Count the # of parameters in the Lagrangian.

- Parameters in the potential (8) : m_h, m_H, m_A, m_{H+}, α, β, v, M^2
- Tadpoles (2) : T_h, T_H
- Wave functions (12) : $Z_{even}(2\times 2)$, $Z_{odd}(2\times 2)$, $Z_{\pm}(2\times 2)$
- Total (22)

2. Prepare the same # of counter terms by shifting the parameters.

- Parameter shift : $m_\varphi \to m_\varphi + \delta m_\varphi$, $\alpha \to \alpha + \delta\alpha, \cdots$
- Tadpole shift : $T_h \to 0 + \delta T_h$, $T_H \to 0 + \delta T_h$
- Field shift :

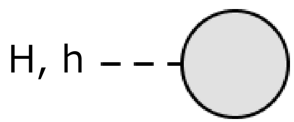
$$\begin{pmatrix} H \\ h \end{pmatrix} \to Z_{\text{even}} \begin{pmatrix} H \\ h \end{pmatrix} \qquad Z_{\text{even}} = \begin{pmatrix} 1 + \frac{1}{2}\delta Z_H & \delta C_{Hh} \\ \delta C_{hH} & 1 + \frac{1}{2}\delta Z_h \end{pmatrix}$$

Renormalization in the Higgs sector

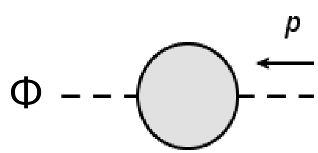
3. Set the same # of ren. conditions.

δv : Ren. in EW sector *Hollik*

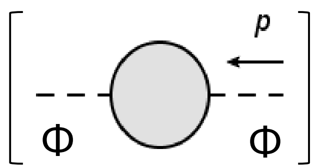
δM^2 : Minimal subtraction *Kanemura, Okada, Senaha, Yuan*

Tadpole condition: 

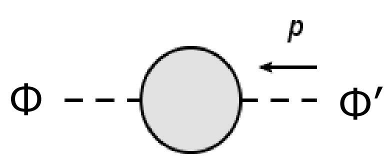
= 0 ⇒ δT_h, δT_H (2)

OS condition I: 

= 0 @ $p^2 = m_\Phi{}^2$ ⇒ δm_φ (4) (φ = h, H, A, $H^\pm$)

OS condition II: $\frac{d}{dp^2}$ 

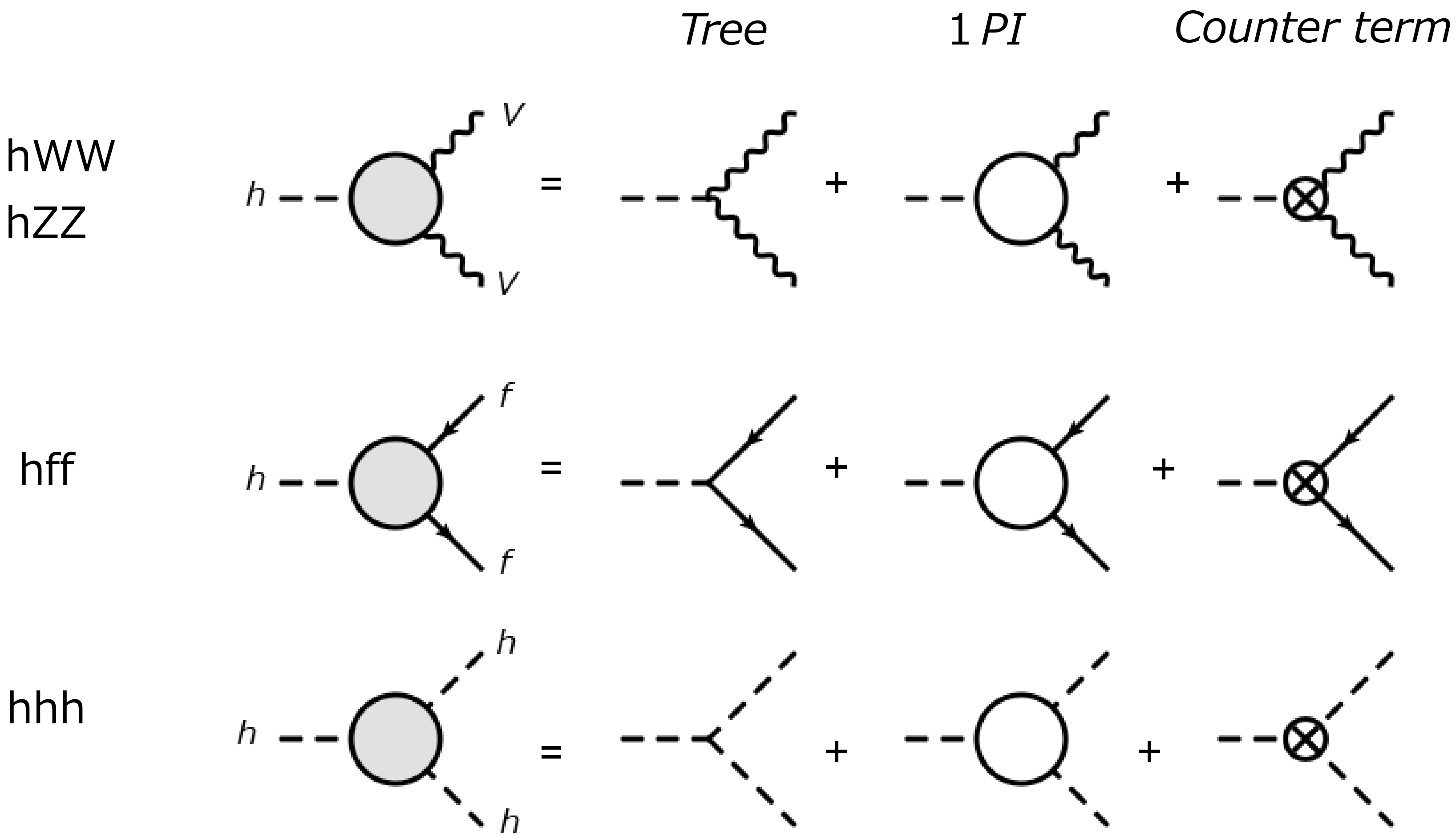
= 0 @ $p^2 = m_\Phi{}^2$ ⇒ δZ_φ (6) (φ = h, H, A, $H^\pm$, G^0, $G^\pm$)

OS condition III: 

= 0 @ $p^2 = m_\Phi{}^2$ and $p^2 = m_{\Phi'}{}^2$ ⇒ $\delta\alpha$, $\delta\beta$, δC_h, δC_A, δC_{H+} (8) ($\delta C_{ij} = \delta C_{ji}$)

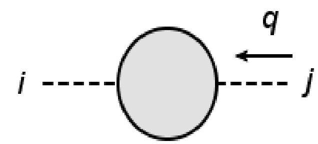
Renormalized Higgs Couplings

4. Calculate the renormalized quantities.



Gauge Dependence (Introduction)

N. K. Nielsen, Nucl. Phys. B 101, 173 (1975)

Nielsen Identity: $\partial_\xi \Big[$ 

$\Big] = (q^2 - m_i^2)\Lambda_i(q^2) + (q^2 - m_j^2)\Lambda_j(q^2)$

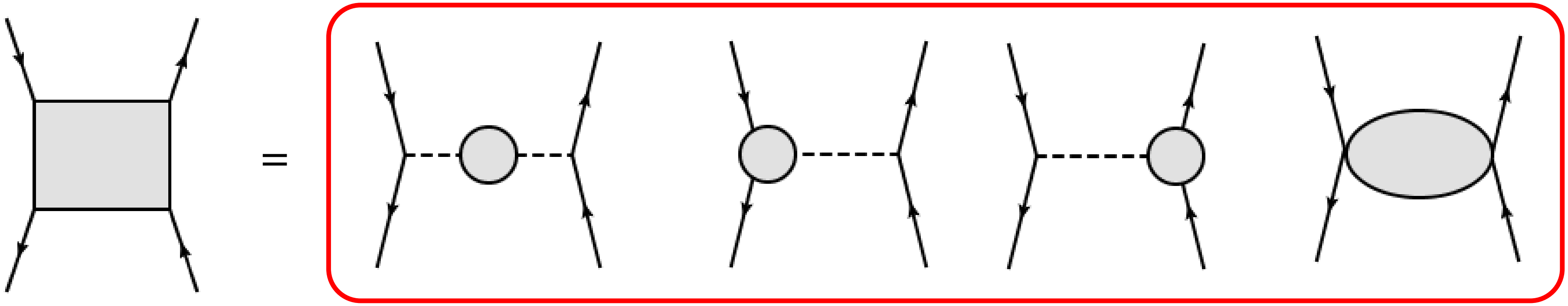
On-shell condition: $\hat{\Pi}_{ij}(q^2 = m_i^2) = \hat{\Pi}_{ij}(q^2 = m_j^2) = 0$

➡ Gauge dependence reminds in renormalized mixing angles, e.g., δα, δβ.

Q: Why does it happen?

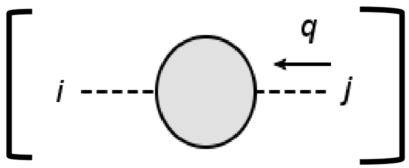
A: We calculate only the **PART** of the S-matrix amplitude!

Bruit force way



Gauge Dependence (Introduction)

N. K. Nielsen, Nucl. Phys. B 101, 173 (1975)

Nielsen Identity: ∂_ξ 

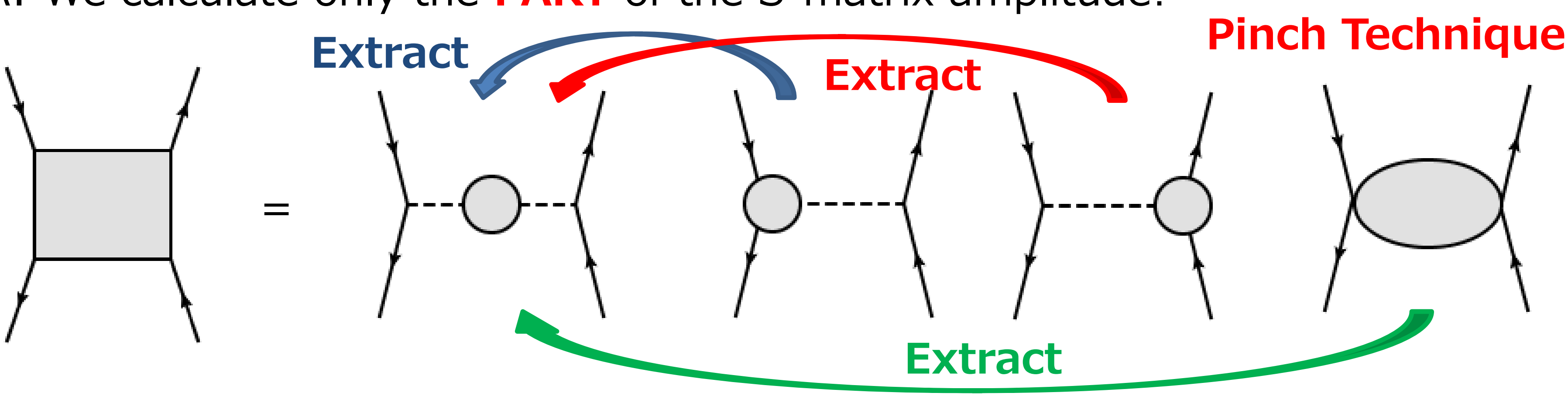
$= (q^2 - m_i^2)\Lambda_i(q^2) + (q^2 - m_j^2)\Lambda_j(q^2)$

On-shell condition: $\hat{\Pi}_{ij}(q^2 = m_i^2) = \hat{\Pi}_{ij}(q^2 = m_j^2) = 0$

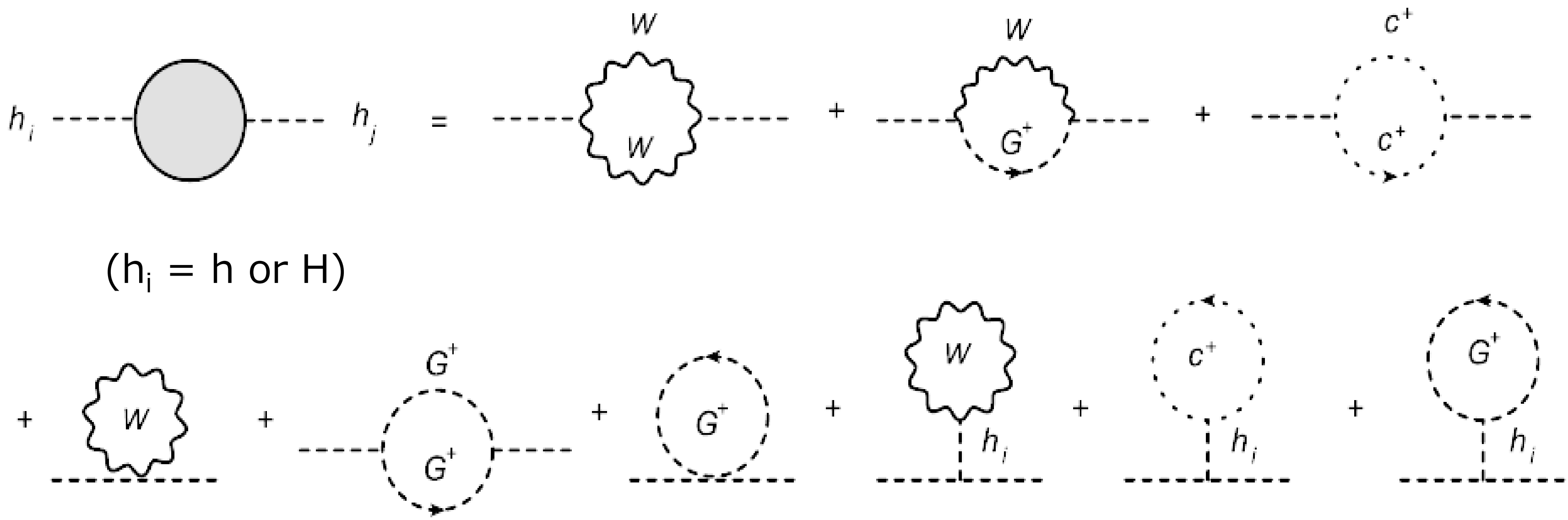
→ Gauge dependence reminds in renormalized mixing angles, e.g., $\delta\alpha$, $\delta\beta$.

Q: Why does it happen?

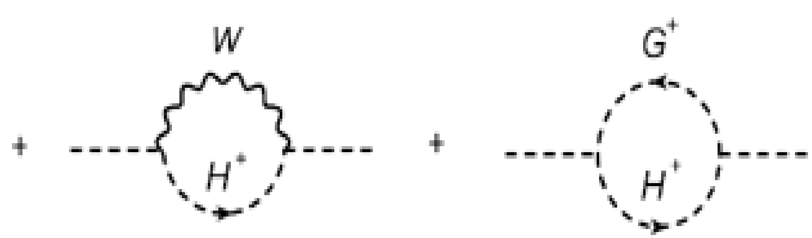
A: We calculate only the **PART** of the S-matrix amplitude!



Gauge Dep. in CP-even Sector in R_ξ Gauge



(h_i = h or H)



Gauge boson propagator :

$$\Delta_V^{\mu\nu} = \frac{1}{k^2 - m_V^2}\left[g^{\mu\nu} - (1 - \xi_V)\frac{k^\mu k^\nu}{k^2 - \xi_V m_V^2}\right]$$

Mass of NGB and FPG: $\xi_V \times m_V^2$

ξ_Z contribution can be extracted by $(W, H^\pm, G^\pm, c^\pm) \to (Z, A, G^0, c_Z)$

Gauge Dep. in CP-even Sector in R_ξ Gauge

Gauge dependent part of the H-h mixing:

Yamada, PRD64, 0103064

Espinosa, Yamada, PRD67, 036003

$$\Delta_\xi \Pi_{Hh}(p^2) \equiv \Pi_{Hh}(p^2) - \Pi_{Hh}(p^2)\Big|_{\xi=1}$$

$$= \frac{g^2}{64\pi^2} s_{\beta-\alpha} c_{\beta-\alpha} (1-\xi_W)$$

$$\times [g_{Hh}(p^2,0) C_0(p^2; W, G^\pm) - 2 g_{Hh}(p^2, m^2_{H^\pm}) C_0(p^2; W, G^\pm, H^\pm)]$$

$$C_0(p^2; A, B) \equiv \frac{1}{m_A^2 - m_B^2} [B_0(p^2; A, A) - B_0(p^2; B, B)],$$

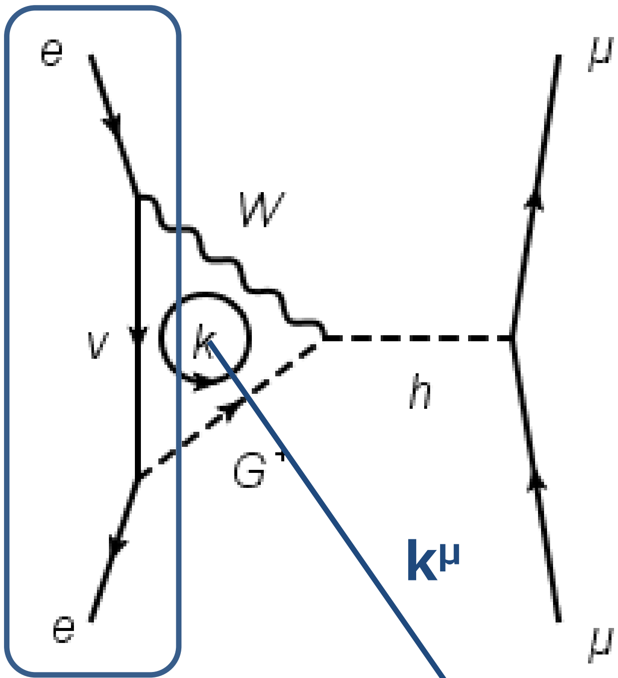
$$C_0(p^2; A, B, C) \equiv \frac{1}{m_A^2 - m_B^2} [B_0(p^2; A, C) - B_0(p^2; B, C)].$$

$$g_{ij}(p^2, m^2) \equiv (p^2 - m^2)(2p^2 - m_i^2 - m_j^2) - (p^2 - m_i^2)(p^2 - m_j^2)$$

Nielsen identity is satisfied!

Essence of Pinch Technique

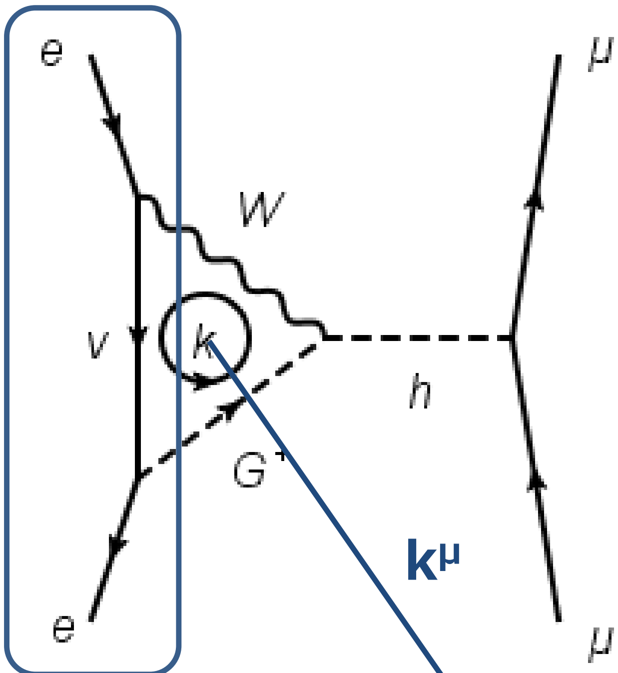
Essence of Pinch Technique

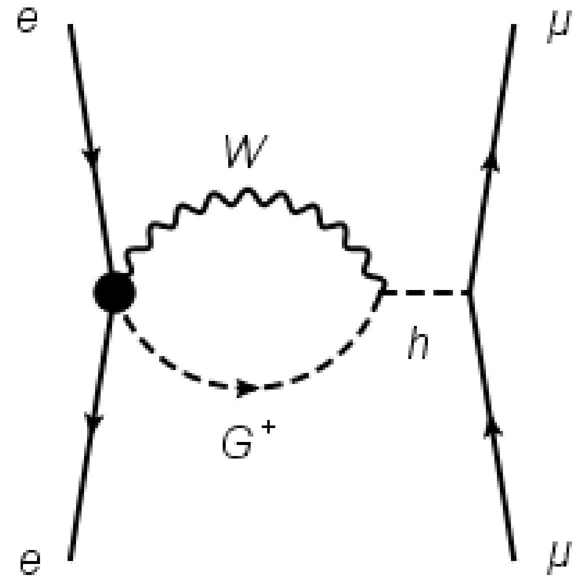


$$-\frac{g^2}{2}\bar{e}(p')(m_e P_L)D_\nu^{-1}\gamma^\mu P_L e(p)$$

$$-\frac{g^2}{2}\bar{e}(p')(m_e P_L)e(p)$$

Essence of Pinch Technique



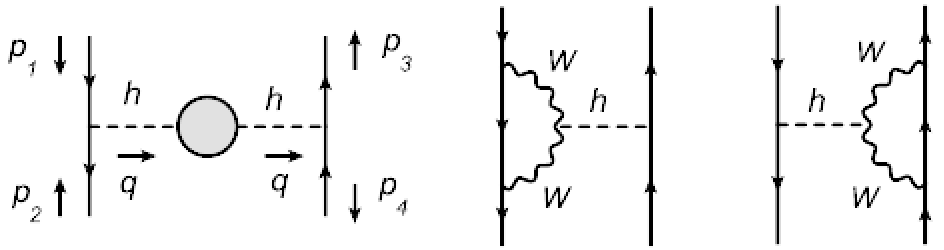


$$-\frac{g^2}{2}\bar{e}(p')(m_e P_L)D_\nu^{-1}\gamma^\mu P_L e(p)$$

$$-\frac{g^2}{2}\bar{e}(p')(m_e P_L)e(p)$$

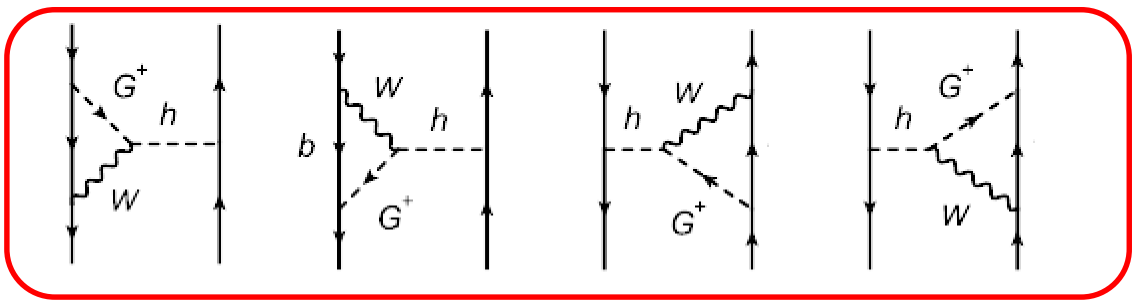
Pinching

Pinch technique for the CP-even sector



(S-1) (V-1) (V-2)

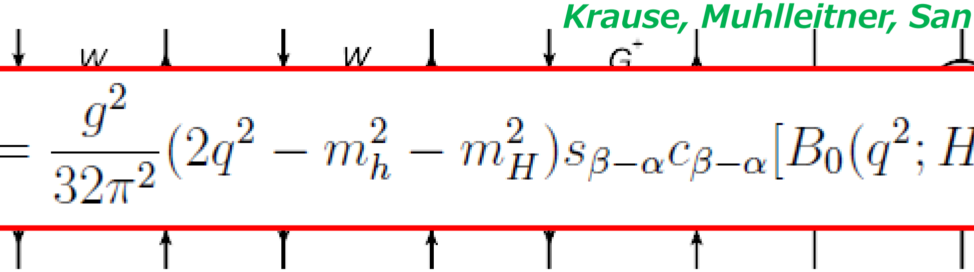
There are the other graphs obtained by $G^+ \to H^+$ and $h \to H$.



(V-3) (V-4) (V-5) (V-6)

Feynman-gauge

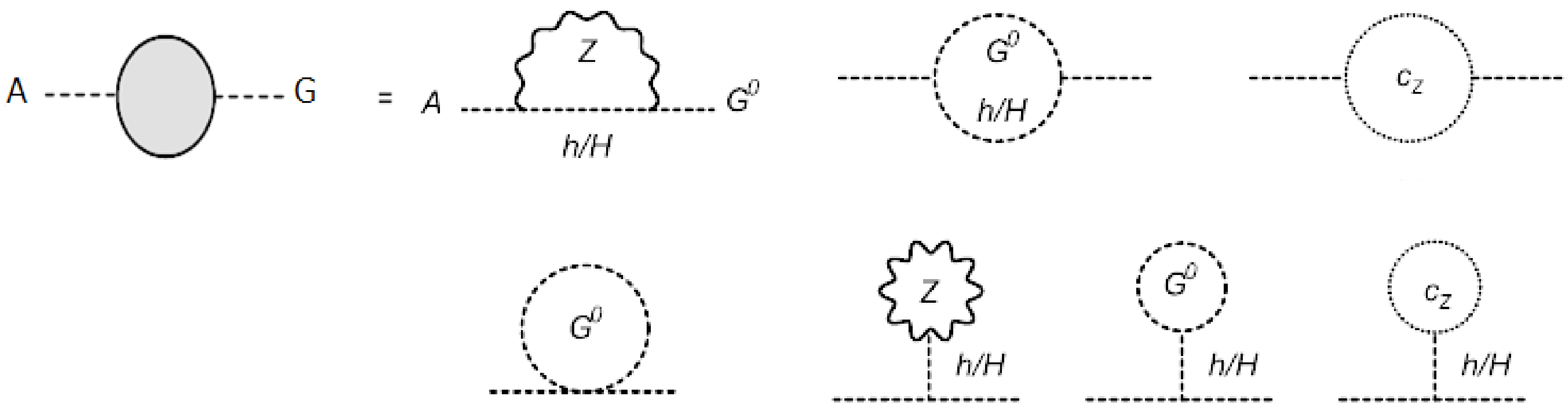
Krause, Muhlleitner, Santos, Ziesche, JHEP1609 (2016)



$$\Pi_{Hh}^{\mathrm{PT}}(q^2) = \frac{g^2}{32\pi^2}(2q^2 - m_h^2 - m_H^2) s_{\beta-\alpha} c_{\beta-\alpha} [B_0(q^2; H^\pm, W) - B_0(q^2; W, W)]$$

(B-1) (B-2) (B-3) (W-1)

Gauge Dep. in A-G mixing in R_ξ Gauge

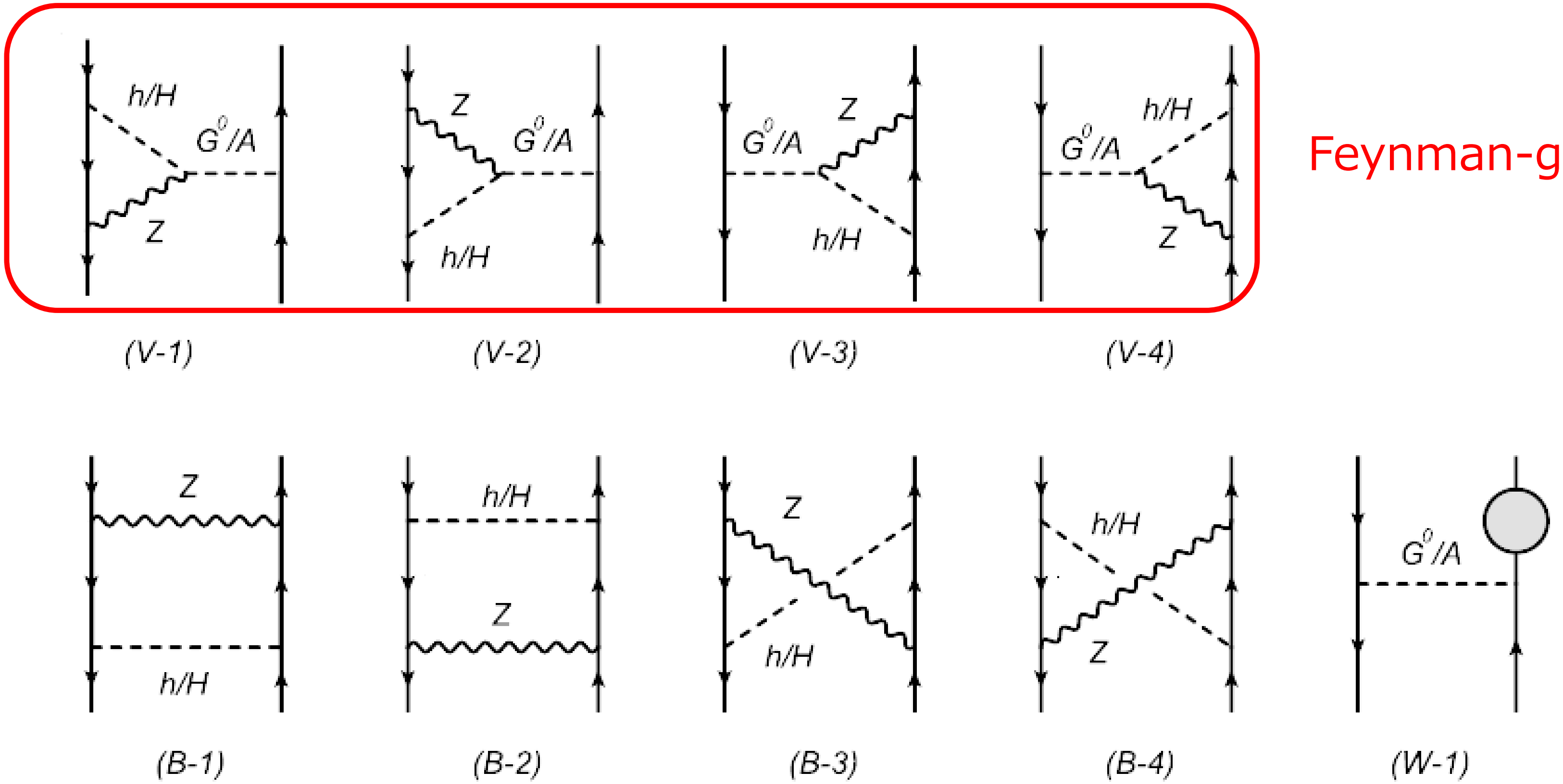


$$\Delta_\xi \Pi_{AG^0}(p^2) = -\frac{g_Z^2}{64\pi^2} s_{\beta-\alpha} c_{\beta-\alpha} (1-\xi_Z)$$

$$\times [g_{AG^0}(p^2, m_h^2) C_0(p^2; Z, G^0, h) - g_{AG^0}(p^2, m_H^2) C_0(p^2; Z, G^0, H)]$$

There is no ξ_W dependence in the A-G mixing

Pinch technique for A-G mixing

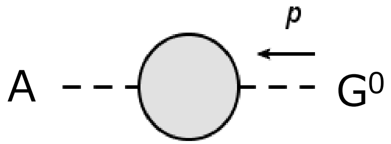


Feynman-gauge

Krause, Muhlleitner, Santos, Ziesche, JHEP1609 (2016)

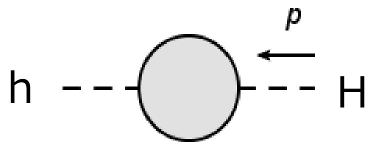
$$\Pi^{\rm PT}_{AG^0}(q^2) = \frac{g_Z^2}{64\pi^2}(2q^2 - m_A^2)s_{\beta-\alpha}c_{\beta-\alpha}[B_0(q^2;Z,H) - B_0(q^2;Z,h)]$$

Gauge invariant mixing angles



$= 0$ → $\delta\beta$, δC_A

@ $p^2 = m_A{}^2$ and $p^2 = 0$



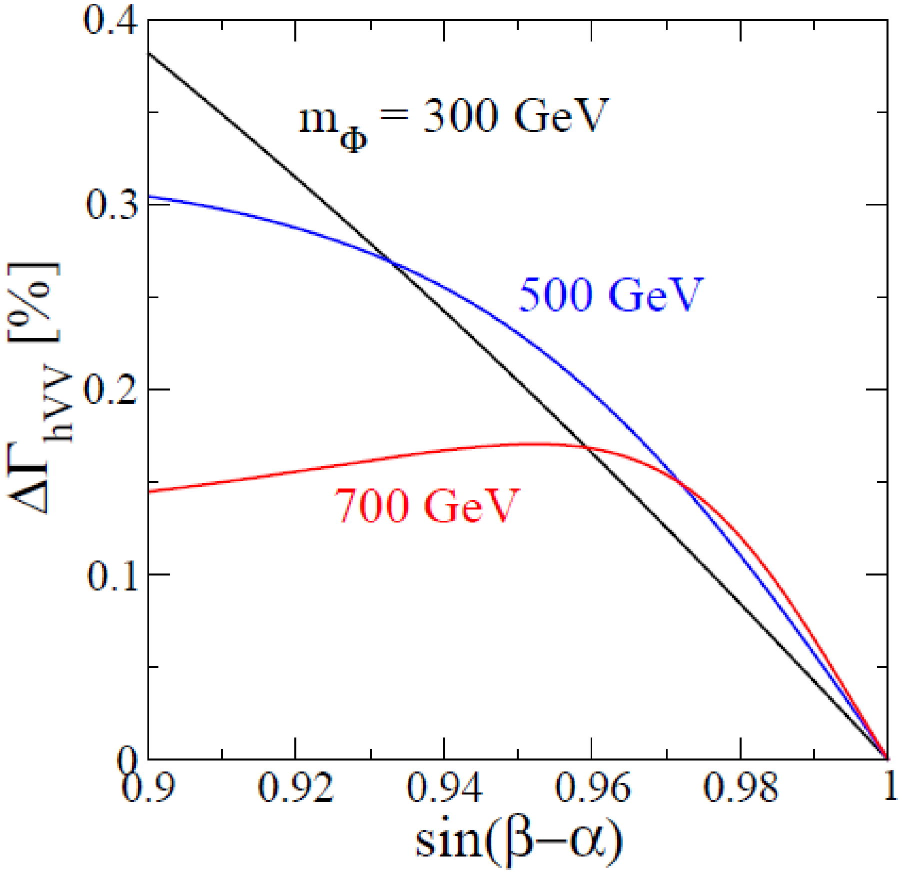
$= 0$ → $\delta\alpha$, δC_h

@ $p^2 = m_h{}^2 = m_H{}^2$

$$\delta\beta = -\frac{1}{2m_A^2}\left[\Pi^{\rm 1PI}_{AG}(m_A^2)_{\xi=1} + \Pi^{\rm 1PI}_{AG}(0)_{\xi=1} + \Pi^{\rm PT}_{AG}(m_A^2) + \Pi^{\rm PT}_{AG}(0)\right]$$

$$\delta\alpha = \frac{1}{2(m_H^2 - m_h^2)}\left[\Pi^{\rm 1PI}_{Hh}(m_H^2)_{\xi=1} + \Pi^{\rm 1PI}_{Hh}(m_h^2)_{\xi=1} + \Pi^{\rm PT}_{Hh}(m_H^2) + \Pi^{\rm PT}_{Hh}(m_h^2)\right]$$

Issue of Gauge Dependence



$$-\frac{m_V^2}{m_A^2 v}c_{\beta-\alpha}[\Pi_{AG}^{\rm PT}(m_A^2)+\Pi_{AG}^{\rm PT}(0)]$$

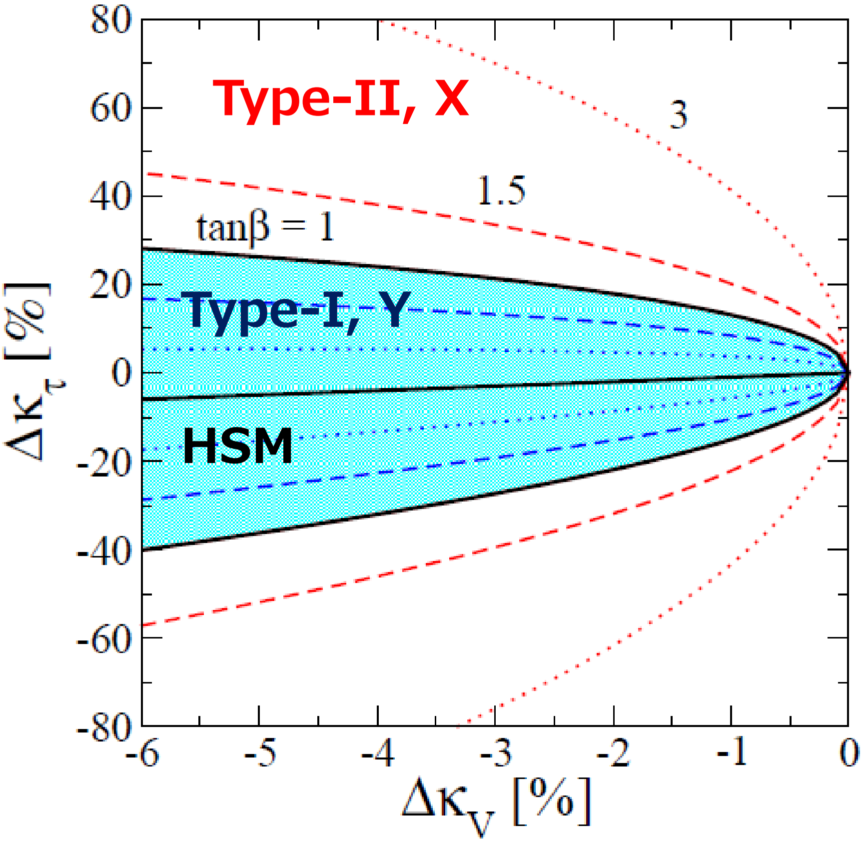
$$\Delta\hat{\Gamma}_{hVV}\equiv\frac{\hat{\Gamma}_{hVV}^{\overline{\rm OS}}-\hat{\Gamma}_{hVV}^{\rm OS}}{\hat{\Gamma}_{hVV}^{\overline{\rm OS}}}$$

Typically, the difference

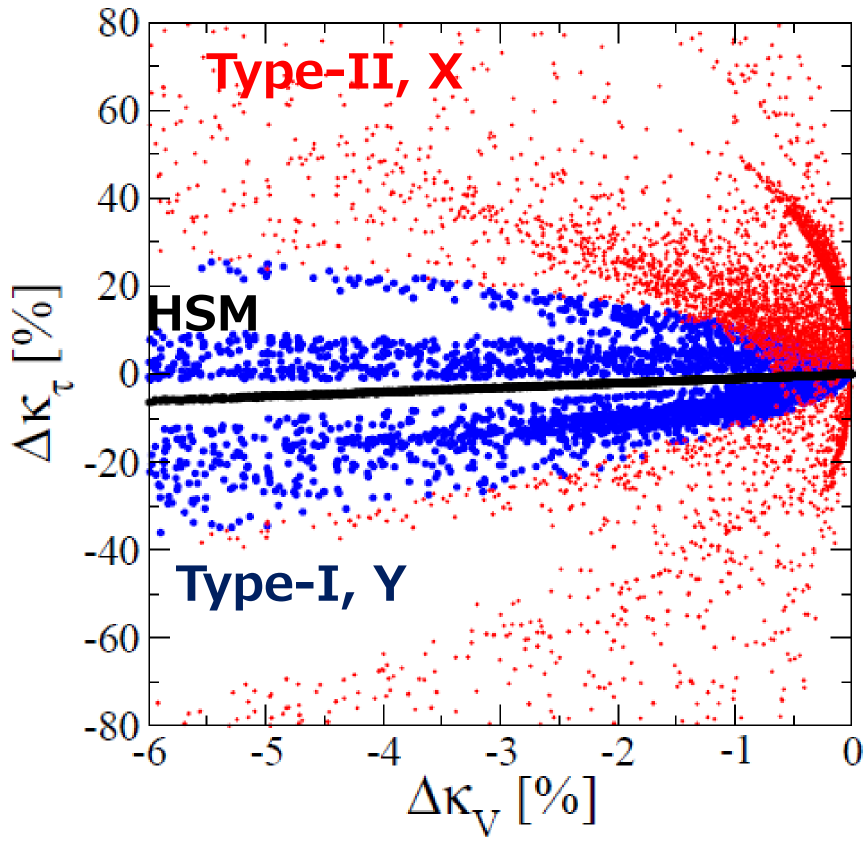
is **O(0.1)%** level.

$\Delta\kappa_V$-$\Delta\kappa_\tau$ at 1-loop level

Tree Level



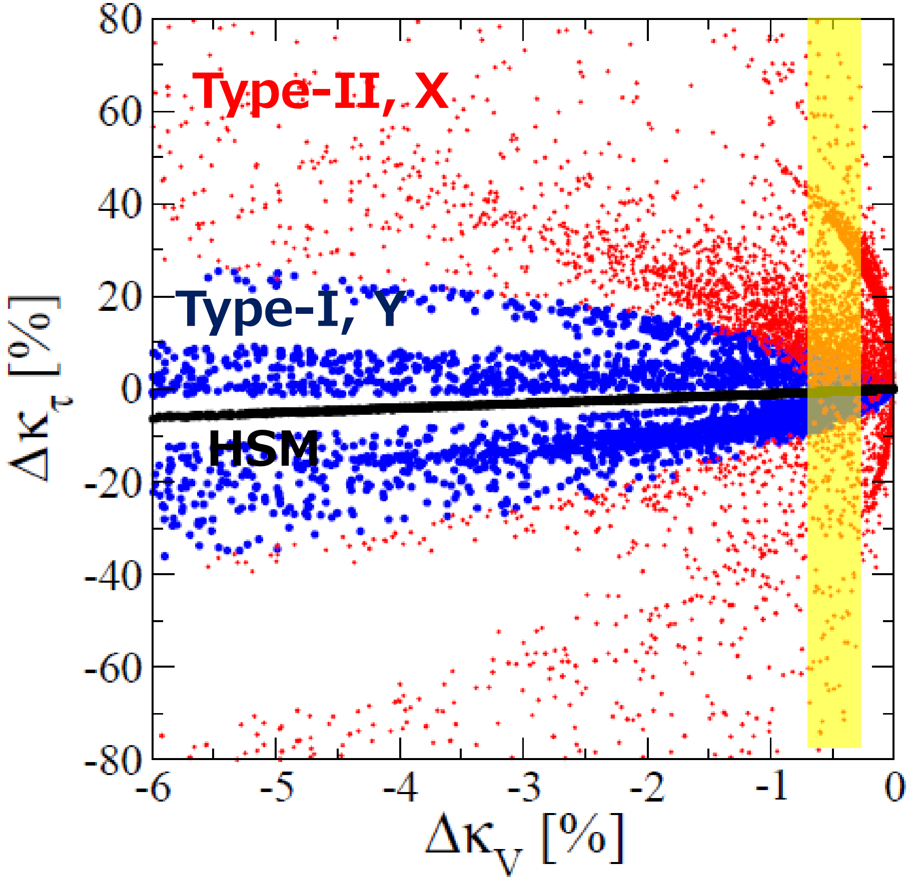
1-loop Level (m_Φ >300 GeV)



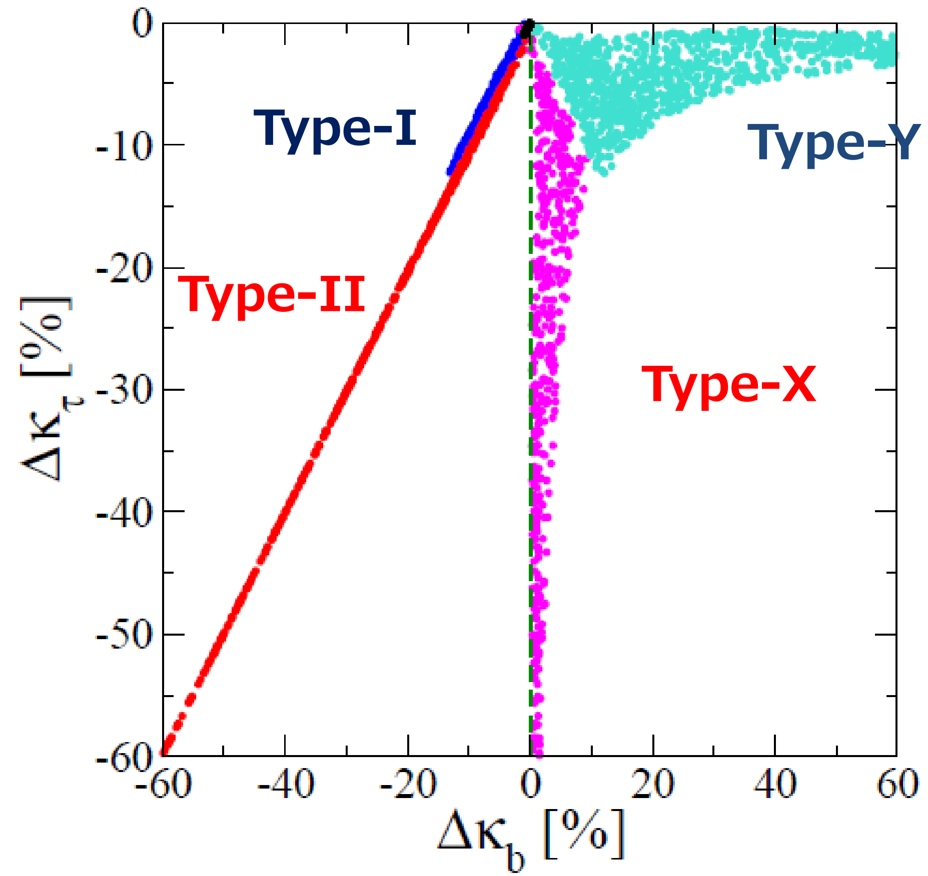
$\tan\beta \geq 1$, $\Lambda_{cutoff} \geq 3$ TeV

$\Delta\kappa_b$-$\Delta\kappa_\tau$ at 1-loop level

$\Delta\kappa_V = (-\mathbf{0.5} \pm 0.4)\%$



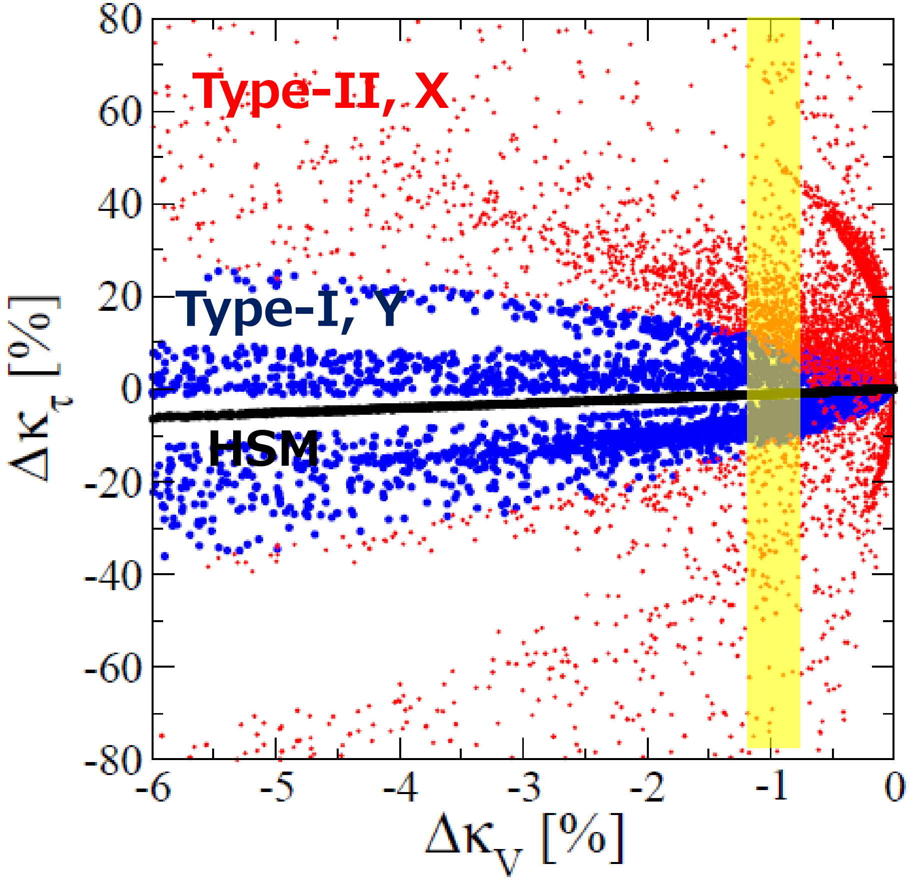
HSM



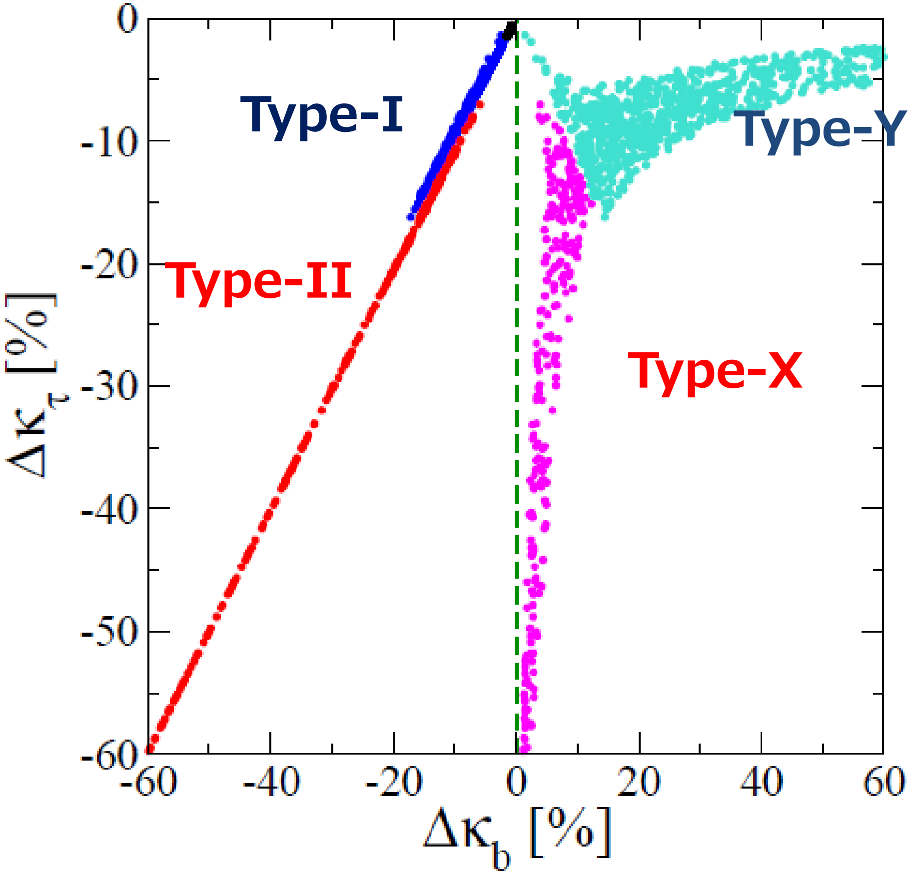
$\tan\beta \geq 1$, $\Lambda_{cutoff} \geq 3$ TeV

$\Delta\kappa_b$-$\Delta\kappa_\tau$ at 1-loop level

$\Delta\kappa_V = (-\mathbf{1.0} \pm 0.4)\%$



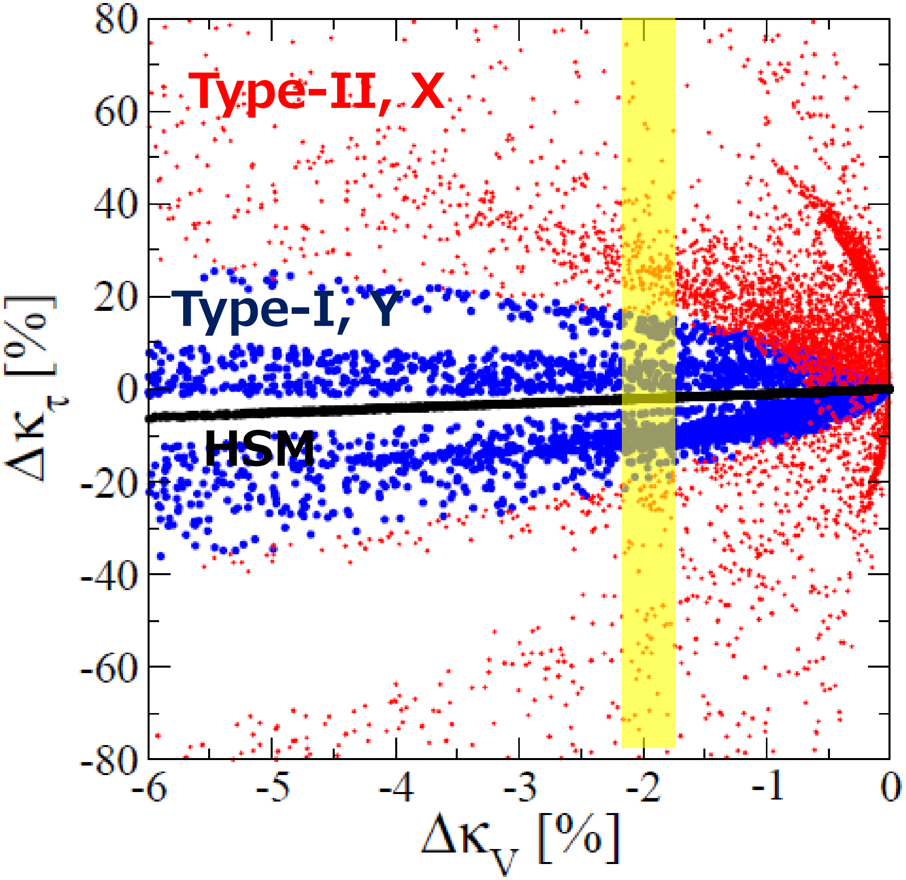
HSM



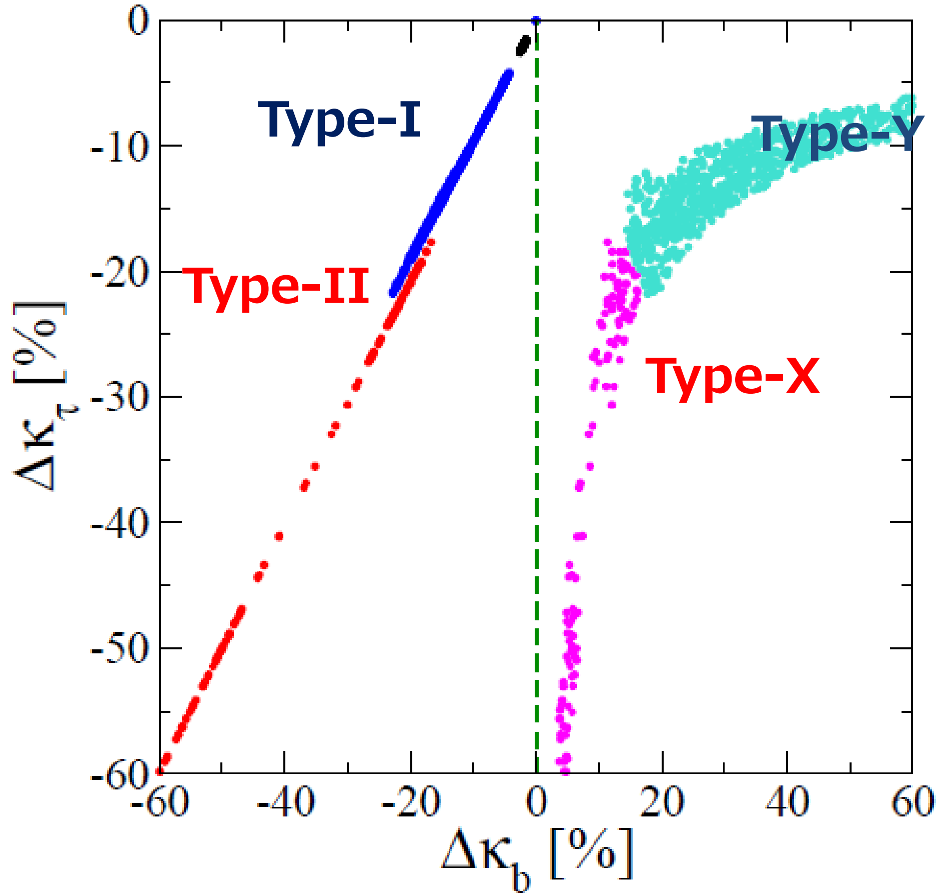
$\tan\beta \geq 1$, $\Lambda_{cutoff} \geq 3$ TeV

$\Delta\kappa_b$-$\Delta\kappa_\tau$ at 1-loop level

$\Delta\kappa_V = (-\mathbf{2.0} \pm 0.4)\%$



HSM



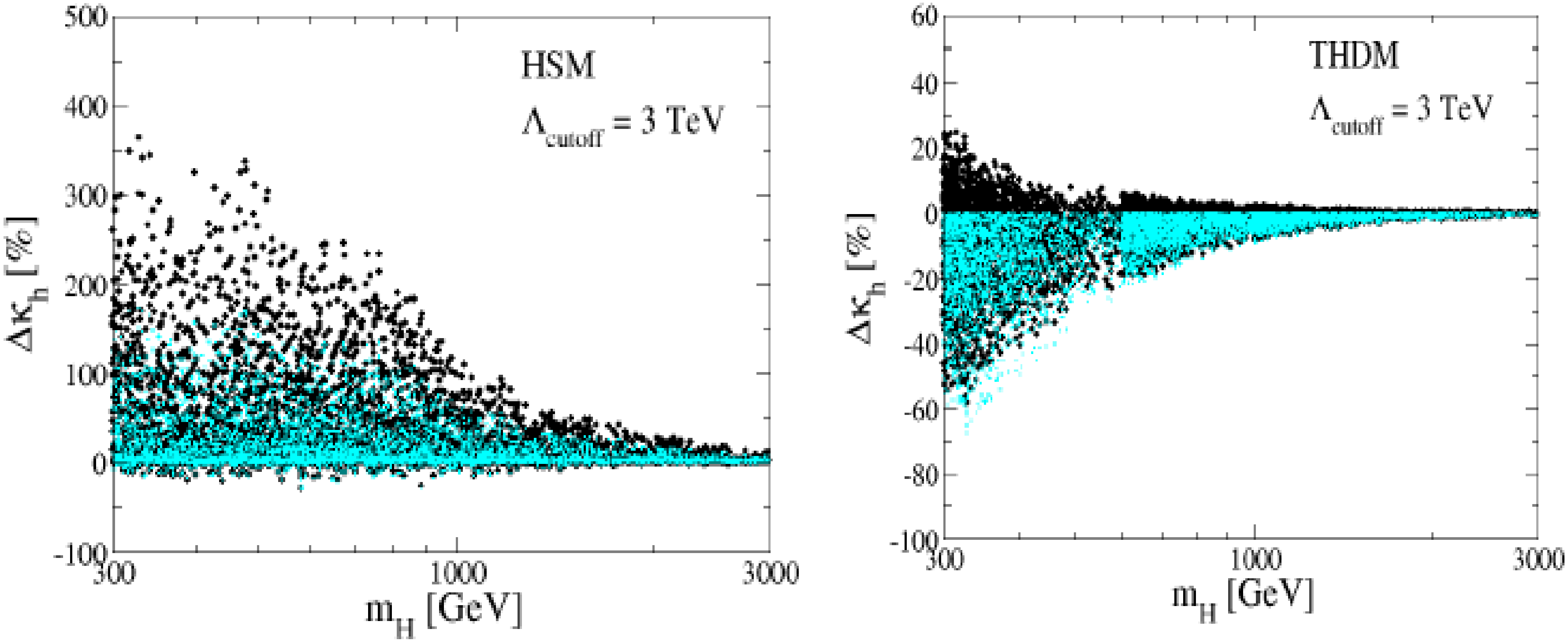
$\tan\beta \geq 1$, $\Lambda_{cutoff} \geq 3$ TeV

hhh coupling

Kanemura, Kikuchi, KY, NPB917 (2017)

Tree Level,

1-loop Level



Summary

- **H-COUP** is the tool to calculate the Higgs couplings at 1-loop level.
- Gauge dependence remains in the **mixing angles** in the OS scheme.
- Gauge dependence can be removed by using the **pinch-technique.**
- Numerical impact of the gauge dependence is negligibly small **~ O(0.1)%**
- Using H-COUP, we can do the **fingerprinting** of the Higgs sector in a quite precise way.

Thank You!

Current and Future Measurements

$\kappa_X = g_{hXX}(\mathrm{Exp})/g_{hXX}(\mathrm{SM})$

Present (LHC Run-I: ATLAS + CMS)

arXiv: 1606.02266 [hep-ex]

~10%	κ_Z	-0.98 $[-1.08, -0.88] \cup [0.94, 1.13]$
	κ_W	0.87 $[0.78, 1.00]$
~20%	κ_t	$1.40^{+0.24}_{-0.21}$
~15%	$\lvert\kappa_\tau\rvert$	$0.84^{+0.15}_{-0.11}$
~20%	$\lvert\kappa_b\rvert$	$0.49^{+0.27}_{-0.15}$
	$\lvert\kappa_g\rvert$	$0.78^{+0.13}_{-0.10}$
~10%	$\lvert\kappa_\gamma\rvert$	$0.87^{+0.14}_{-0.09}$

Future

arXiv: 2210.8361 [hep-ex]

Facility	LHC	HL-LHC	ILC500
$\sqrt{s}$ (GeV)	14,000	14,000	250/500
$\int \mathcal{L} dt$ (fb^{-1})	300/expt	3000/expt	250+500
κ_γ	$5-7\%$	$2-5\%$	8.3%
κ_g	$6-8\%$	$3-5\%$	2.0%
κ_W	$4-6\%$	$2-5\%$	0.39%
κ_Z	$4-6\%$	$2-4\%$	0.49%
κ_ℓ	$6-8\%$	$2-5\%$	1.9%
$\kappa_d = \kappa_b$	$10-13\%$	$4-7\%$	0.93%
$\kappa_u = \kappa_t$	$14-15\%$	$7-10\%$	2.5%

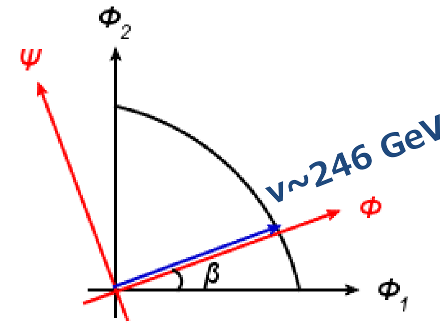
To compare future precise measurements, precise calculations are necessary!

Ex. 1 2HDM

- The Higgs basis *Davidson, Haber PRD71 (2005)*

$$\begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix} = \begin{pmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} \Phi \\ \Psi \end{pmatrix}$$

$\tan\beta = v_2/v_1$



NG boson

Charged Higgs

$$\Phi = \begin{bmatrix} G^+ \\ \frac{1}{\sqrt{2}}(h_1' + v + iG^0) \end{bmatrix}$$

$$\Psi = \begin{bmatrix} H^+ \\ \frac{1}{\sqrt{2}}(h_2' + iA) \end{bmatrix}$$

CP-even Higgs

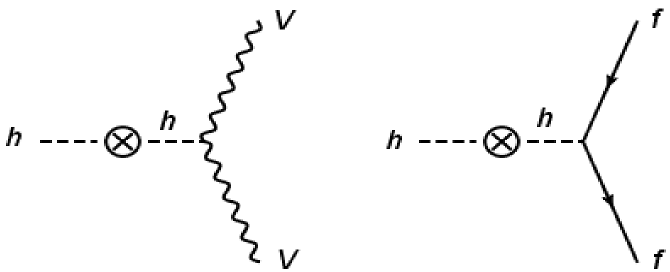
CP-odd Higgs

$$\begin{pmatrix} h_1' \\ h_2' \end{pmatrix} \begin{pmatrix} \cos(\beta-\alpha) & \sin(\beta-\alpha) \\ -\sin(\beta-\alpha) & \cos(\beta-\alpha) \end{pmatrix} \begin{pmatrix} H \\ h \end{pmatrix}$$

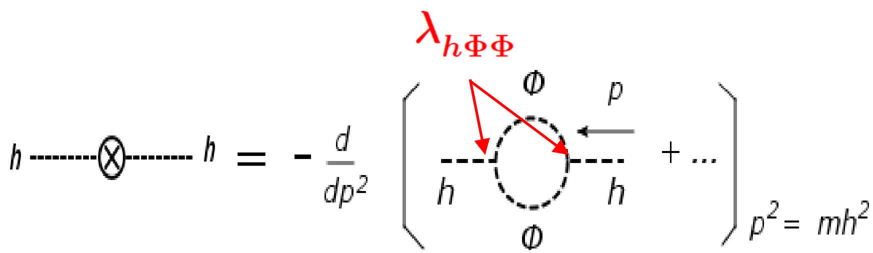
SM-like Higgs with 125 GeV

Important diagrams

$\kappa_X = g_{hXX}(\mathrm{MHM})/g_{hXX}(\mathrm{SM}), \quad \Delta\kappa_X = \kappa_X - 1$

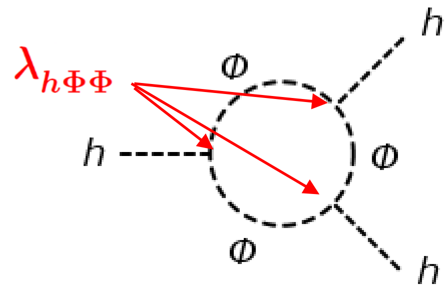


$$\simeq (\mathrm{tree}) \times \left[-\frac{1}{16\pi^2}\frac{1}{6}\sum_\Phi \lambda^2_{h\Phi\Phi}\frac{1}{m_\Phi^2} \right]$$



$\Delta\kappa_V$ ($\Delta\kappa_F$) ~ - 0.6% for m_Φ = 300 GeV,

$\lambda_{h\Phi\Phi}$ = 1.5v (Φ=H,A,H$^\pm$)



$$\simeq -\frac{1}{16\pi^2} 4 \sum_\Phi \lambda^3_{h\Phi\Phi}\frac{1}{m_\Phi^2}$$

$$= +(\mathrm{tree}) \times \frac{1}{16\pi^2}\frac{4}{3}\sum_\Phi \frac{v\lambda^3_{h\Phi\Phi}}{m_\Phi^2 m_h^2}$$

$\Delta\kappa_h$ ~ +30% for m_Φ = 300 GeV,

$\lambda_{h\Phi\Phi}$ = 1.5v (Φ=H,A,H$^\pm$)

Ex. 1 2HDM with NFC

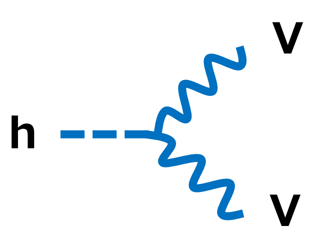
- Kinetic term

$$\mathcal{L}_{\rm kin} = |D_\mu\Phi_1|^2 + |D_\mu\Phi_2|^2 = |D_\mu\Phi|^2 + |D_\mu\Psi|^2$$

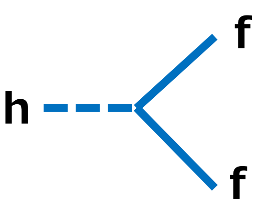
- Yukawa couplings

$$\mathcal{L}_Y = \bar{Q}_L Y_d \Phi_d d_R + \cdots$$
$$= \frac{\sqrt{2}}{v}\bar{Q}_L M_d(\Phi + \xi_d\Psi) d_R + \cdots$$

	ξ_u	ξ_d	ξ_e
Type I	**cotβ**	**cotβ**	**cotβ**
Type II	**cotβ**	**-tanβ**	**-tanβ**
Type X	**cotβ**	**cotβ**	**-tanβ**
Type Y	**cotβ**	**-tanβ**	**cotβ**



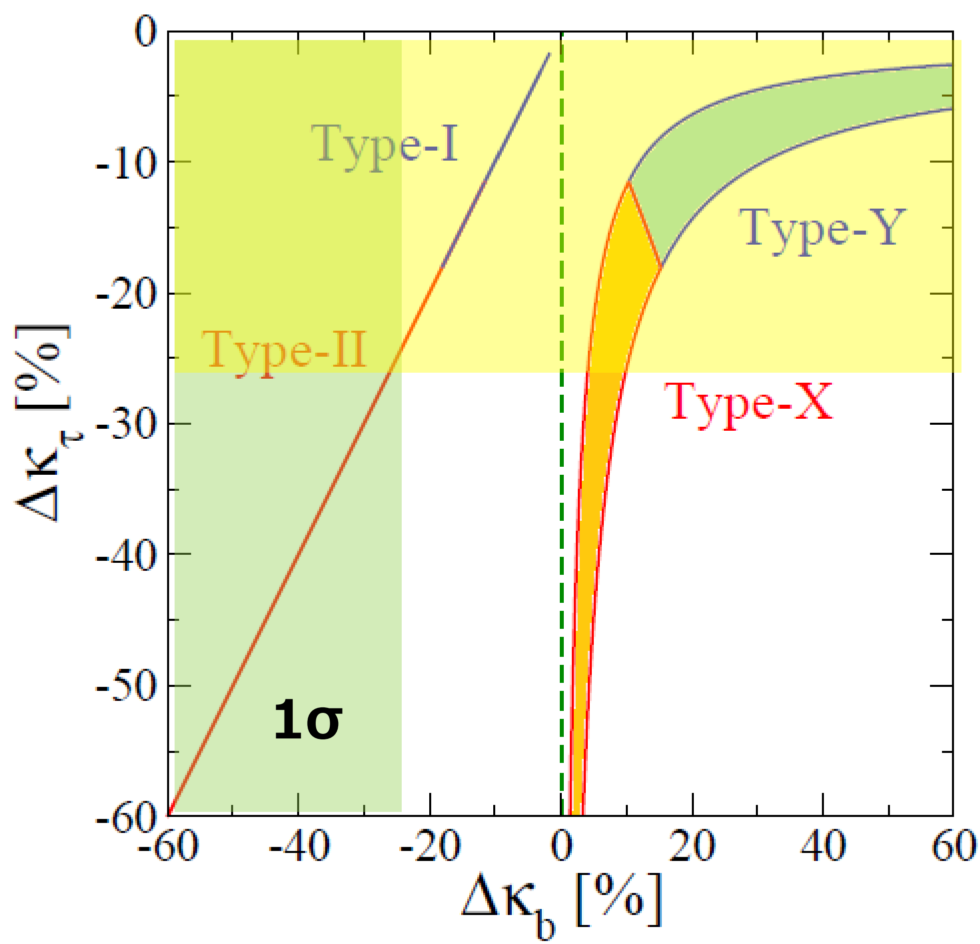
= (SM) × sin(β-α)



= (SM) × [sin(β-α) +ξ_f cos(β-α)]

Coupling deviations at the tree level

$\Delta\kappa_V = (-1 \pm 0.4)\%$, $\tan\beta \geq 1$



Type-I and Y (Type-II and X) can be distinguished by the sign of $\Delta\kappa_b$!!

Type-II seems to be favored.

But, we need more data to really say excluded or determined.

Buck up

Enomoto and Watanabe, JHEP 1605, 002 (2016)

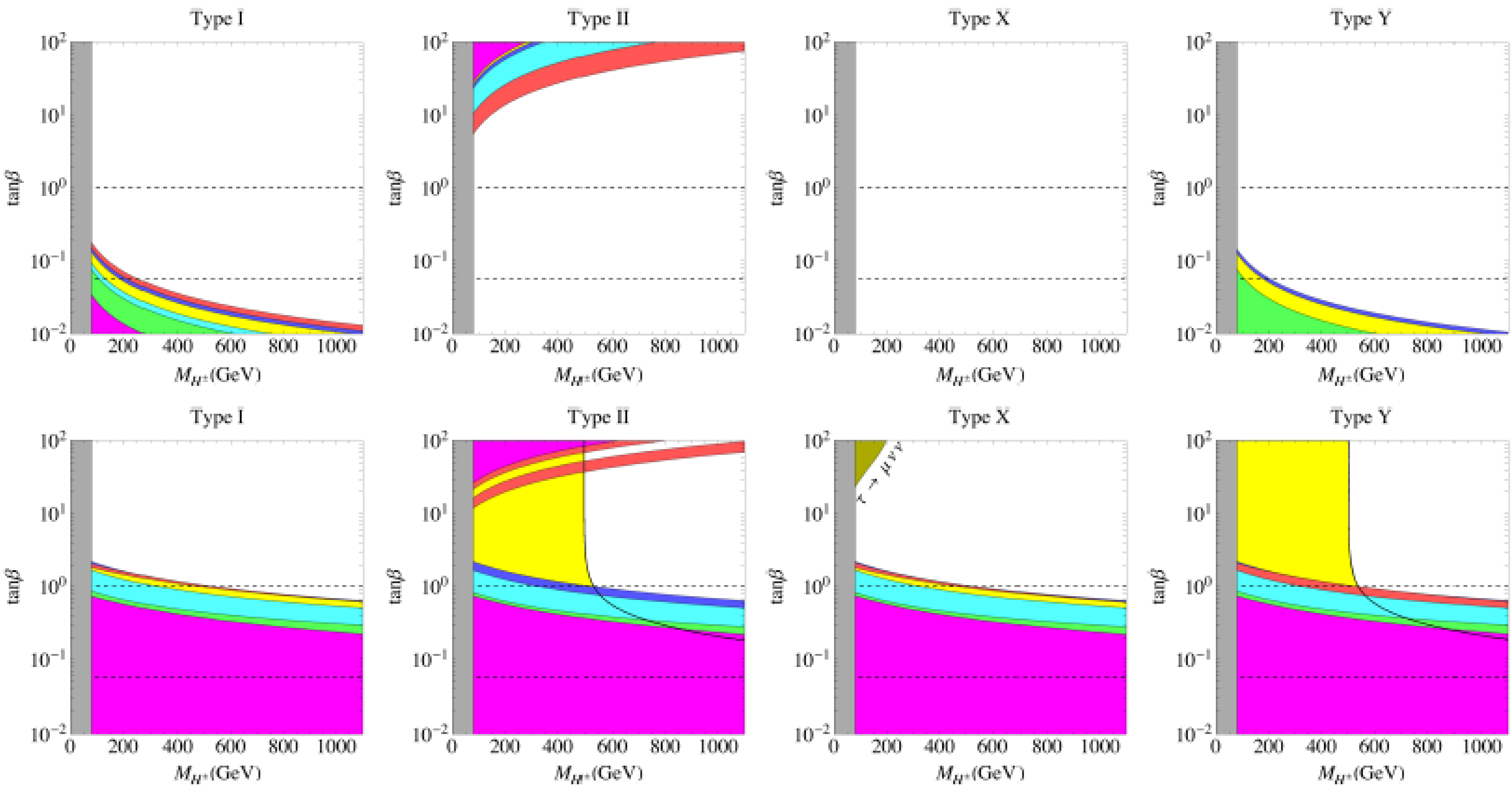
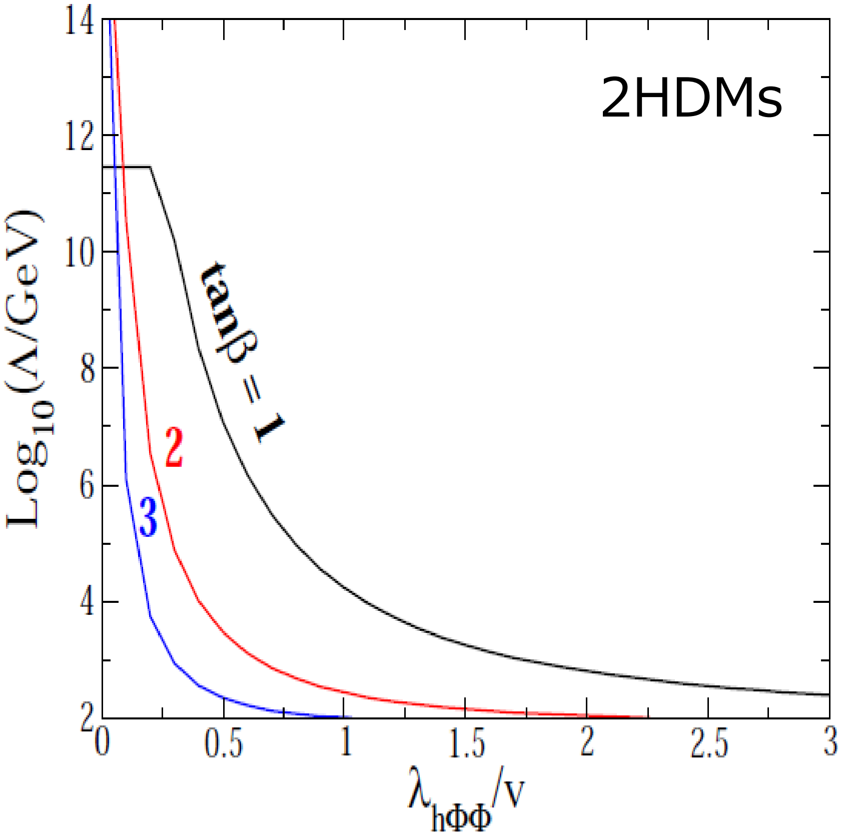
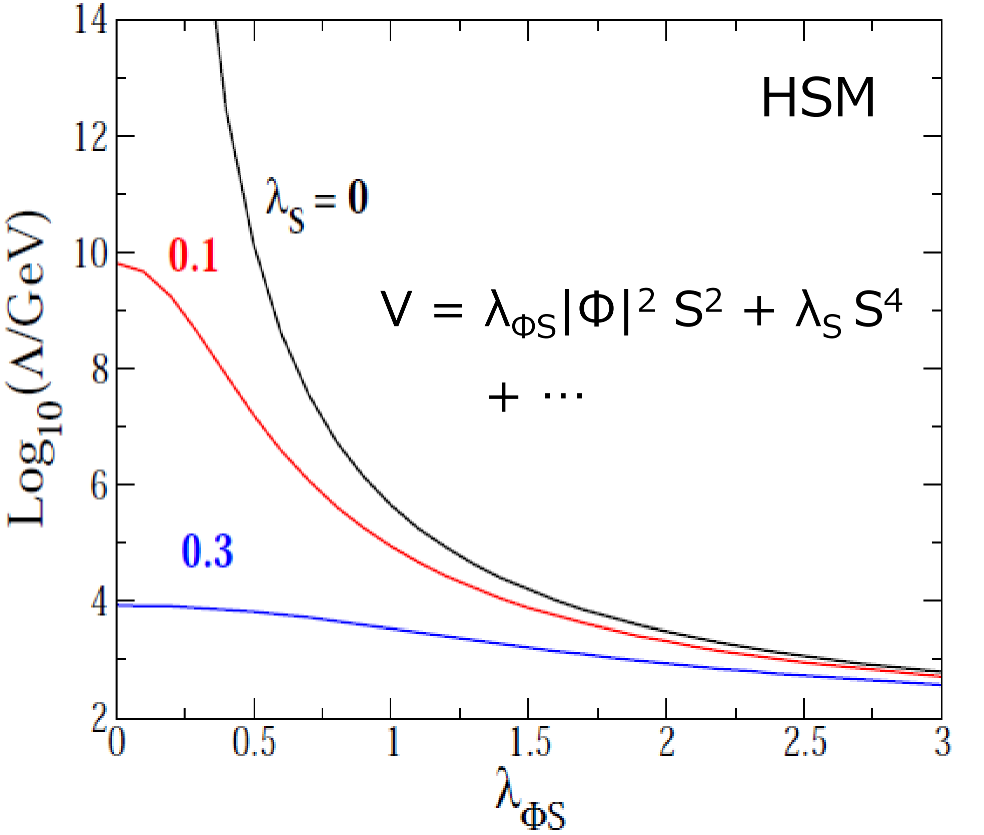


Figure 3. Excluded regions in the Z_2 symmetric models on the $(m_{H^+}, \tan\beta)$ plane at 95% CL individually from the tree level processes $B \to \tau\nu$ (red), $D \to \mu\nu$ (green), $D_s \to \tau\nu$ (blue), $D_s \to \mu\nu$ (yellow), $K \to \mu\nu/\pi \to \mu\nu$ (cyan), $\tau \to K\nu/\tau \to \pi\nu$ (magenta) in the upper panels, and the loop induced processes $B_s^0 \to \mu^+\mu^-$ (red), $B_d^0 \to \mu^+\mu^-$ (magenta), $\bar{B} \to X_s\gamma$ (yellow), ΔM_s (blue), ΔM_d (cyan), $|\epsilon_K|$ (green) in the lower panels. The black line contour in the type II and Y is the

Upper limit on $\lambda_{h\Phi\Phi}$ from triviality





Higgs potential of HSM

- The most general potential

$$V(\Phi, S) = m_\Phi^2|\Phi|^2 + \lambda|\Phi|^4 + \mu_{\Phi S}|\Phi|^2 S + \lambda_{\Phi S}|\Phi|^2 S^2 + t_S S + m_S^2 S^2 + \mu_S S^3 + \lambda_S S^4$$

- 7 parameters

v (=246 GeV), m_h (=125 GeV), m_H, sin(α), λ_S, $\lambda_{\Phi S}$, and μ_S

- Scalar Masses

$$V_{\text{mass}} = \frac{1}{2}(s,\ \phi)\begin{pmatrix} M_{11}^2 & M_{12}^2 \\ M_{12}^2 & M_{22}^2 \end{pmatrix}\begin{pmatrix} s \\ \phi \end{pmatrix}$$

$$M_{11}^2 = 2m_S^2 + v^2\lambda_{\Phi S}, \quad M_{22}^2 = 2\lambda v^2, \quad M_{12}^2 = v\mu_{\Phi S}.$$

Uncertainty for QCD corrections

Lepage, Mackenzie and Peskin, 1404.0319 [hep-ph]

$$\delta_A = \frac{1}{2}\frac{\Delta\Gamma(h \to A\overline{A})}{\Gamma(h \to A\overline{A})}.$$

	$\delta m_b(10)$	$\delta\alpha_s(m_Z)$	$\delta m_c(3)$	δ_b	δ_c	δ_g
current errors [10]	0.70	0.63	0.61	0.77	0.89	0.78
+ PT	0.69	0.40	0.34	0.74	0.57	0.49
+ LS	0.30	0.53	0.53	0.38	0.74	0.65
+ LS^2	0.14	0.35	0.53	0.20	0.65	0.43
+ PT + LS	0.28	0.17	0.21	0.30	0.27	0.21
+ PT + LS^2	0.12	0.14	0.20	0.13	0.24	0.17
+ PT + LS^2 + ST	0.09	0.08	0.20	0.10	0.22	0.09
ILC goal				0.30	0.70	0.60

Table 1: Projected fractional errors, in percent, for the $\overline{\text{MS}}$ QCD coupling and heavy quark masses under different scenarios for improved analyses. The improvements considered are: PT - addition of 4$^{\text{th}}$ order QCD perturbation theory, LS, LS^2 - reduction of the lattice spacing to 0.03 fm and to 0.023 fm; ST - increasing the statistics of the simulation by a factor of 100. The last three columns convert the errors in input parameters into errors on Higgs couplings, taking account of correlations. The bottom line gives the target values of these errors suggested by the projections for the ILC measurement accuracies.