Gauge independent renormalization of the 2HDM

R. Santos ISEL & CFTC-UL & LIP-Braga RISE - U. Toyama 2017

# A gauge independent renormalization procedure for the 2HDM

KRAUSE, LORENZ, MUHLLEITNER, RS, ZIESCHE (2016) KRAUSE, MUHLLEITNER, RS, ZIESCHE (1609.04185)

**BARROSO, RS (1997)** 

KANEMURA, OKADA, SENAHA, YUAN, YAMADA, LOPEZ-VAL SOLA, PILAFTSIS, FREITAS, STÖCKINGER BOUDJEMA, BARO, DENNER, JENNICHES, LANG, STURM, ...

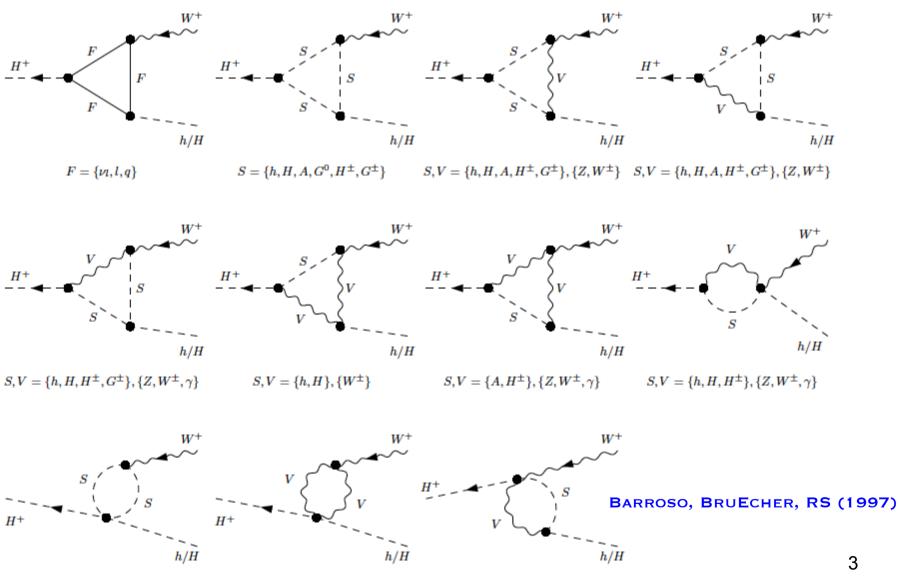
#### **Two Steps**

1. Use a scheme for the SM where the renormalization constants are all gauge independent except for the wave function renormalization constants.

2. If the renormalization constants for the remaining parameters are gauge independent no new gauge dependences are introduced.

Input parameters:  $m_h, m_H, m_A, m_{H^{\pm}}, T_1, T_2, \alpha, \tan\beta, m_{12}^2, M_W^2, M_Z^2, e, m_f$ 

For historical reasons we have started with the corrections to a charged Higgs decaying to a neutral Higgs and a W boson.



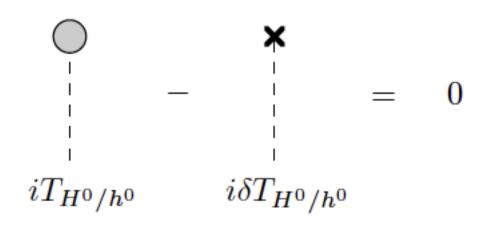
 $S = \{h, H, G^0, H^{\pm}, G^{\pm}\} \qquad \qquad V = \{Z, W^{\pm}, \gamma\} \qquad \qquad S, V = \{A, H^{\pm}\}, \{Z, W^{\pm}\}$ 

#### Quantities are redefined according to

 $\rho_{i,0} = \rho_i + \delta \rho_i$  for the parameters.

$$\phi_{j,0} = \sqrt{Z_{\phi_j}} \phi_j \approx \left(1 + \frac{\delta Z_{\phi_j}}{2}\right) \phi_j$$
 for the fields.

#### Renormalization condition for the tadpoles



#### Step 1 - moving constants around

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#### Radiative corrections to Higgs-boson decays in the Weinberg-Salam model

J. Fleischer and F. Jegerlehner

Fakultät für Physik, Universität Bielefeld, D-4800 Bielefeld I, Federal Republic of Germany (Received 26 August 1980)

One-loop corrections to the Higgs-boson decays  $H\tau^+\tau^-$ ,  $HW^+W^-$ , and HZZ are calculated up to Higgs-boson masses of about 1 TeV. The corrections are of the order of 10% for 200 GeV  $\leq m_H \leq 1$  TeV within the renormalization scheme adopted. Renormalization problems are discussed in detail. A complete set of one-loop counterterms in the 't Hooft gauge is presented.

$$-i\Delta m_{H}^{2} = -i6\lambda v_{0}\delta v_{t} =$$

$$\delta v_{t} = \circ - - - \left( \times \right) = \frac{i}{-m_{Ho}^{2}} \left\{ - - - - \left( \times \right) \right\}$$

$$= \frac{\delta t}{m_{Ho}^{2}} \qquad (A13)$$

In our choice of the renormalization procedure, we make use of the fact that apart from the gaugedependent wave-function renormalizations, all other counterterms may be chosen to be gauge invariant. Starting from the bare Lagrangian we generate counterterms by the shifts

<u>Difference</u> - the condition for the tadpole is the same. However, the parameter is v and not T! The difference in what we call "the tadpole scheme" is the inclusion of the tadpole graph in the calculation of the self-energies

<u>Easier do understand in a three-point function</u> - the other difference relative to the usual on-shell scheme is to include diagrams with tadpoles whenever there is a vacuum expectation value in the vertex

$$\begin{split} ig_{H^{0}Z^{0}Z^{0}} &\to ig_{H^{0}Z^{0}Z^{0}} + \frac{ig^{2}}{2c_{W}^{2}} \left( c_{\alpha}\delta v_{1} + s_{\alpha}\delta v_{2} \right) \\ &= ig_{H^{0}Z^{0}Z^{0}} + \frac{ig^{2}}{2c_{W}^{2}} \left[ \left( c_{\alpha}^{2} + s_{\alpha}^{2} \right) \frac{\delta T_{H^{0}}}{m_{H^{0}}^{2}} + \left( s_{\alpha}c_{\alpha} - s_{\alpha}c_{\alpha} \right) \frac{\delta T_{h^{0}}}{m_{h^{0}}^{2}} \right] \\ &= ig_{H^{0}Z^{0}Z^{0}} + \frac{ig^{2}}{2c_{W}^{2}} \frac{-i}{m_{H^{0}}^{2}} i\delta T_{H^{0}} \\ &= ig_{H^{0}Z^{0}Z^{0}} + \left( \underbrace{\begin{array}{c} \frac{1}{M^{0}} & \frac{1}{2} \\ \frac{1}{M^{0}} & \frac{1}{M^{0}} \\ \frac{1}{M^{0}} & \frac{1}{M^$$

- **Self-energies:** The self-energies appearing in the definitions of the wave function renormalization constants and counterterms are changed such that they contain additional tadpole contributions:  $\Sigma(p^2) \rightarrow \Sigma^{\text{tad}}(p^2)$ .
- **Tadpole counterterms:** The tadpole counterterms  $\delta T_{\phi_i \phi_j}$  (i, j = 1, 2) in the scalar sector vanish:  $\delta T_{\phi_i \phi_j} \rightarrow 0$ .
- **Vertex corrections:** The virtual vertex corrections change to contain additional tadpole contributions if the resulting coupling exists within the 2HDM.

### Results for the alternative tadpole scheme

$$\mathcal{M}_{H^{\pm} \to W^{\pm}h} \Big|_{\mathrm{ct}, \xi, \, \delta c_{\beta-\alpha} \, \mathrm{only}}^{\mathrm{standard}} = \frac{g\Lambda_5 c_{\beta-\alpha} s_{\beta-\alpha}^2 \, p_1 \cdot \epsilon^*(p_3)}{32\pi^2 (m_H^2 - m_h^2)} \left[ 2M_W^2 (1 - \xi_W) \alpha_W + M_Z^2 (1 - \xi_Z) \alpha_Z \right] \,.$$

$$\mathcal{M}_{H^{\pm} \to W^{\pm} h} |_{\text{NLO}, \xi, \delta c_{\beta-\alpha}=0}^{\text{tad}} = 0 .$$

The virtue of the alternative tadpole scheme is to lead to gauge independent amplitudes when the angular countertems are set to zero.

Now we just need a gauge independent way to define the angles and soft breaking counterterms.

### Step 2 – gauge independent renormalization for angles and soft breaking parameter

**Process dependent** - On-shell plus two particular processes to renormalize the angles and one more (with a triple Higgs coupling) for the soft breaking parameter

**Process independent** - On-shell plus conditions for the angles based on the mixing matrix properties plus MS for the soft breaking parameter

$$\delta O_{ij} = \frac{1}{4} \left( \delta Z_{il} - \delta Z_{li} \right) O_{lj}$$

PILAFTSIS (1997)

KANEMURA, OKADA, SENAHA, YUAN (2004)

# But the wave function renormalization constants are gauge dependent. So what to do?

Input parameters:  $m_h, m_H, m_A, m_{H^{\pm}}, T_1, T_2, \alpha, \tan\beta, m_{12}^2, M_W^2, M_Z^2, e, m_f$ 

9

Since the angle counterterms are defined with the help of wave function renormalization constants if these are gauge independent the angle counterterms will also be gauge independent.

There is an unambiguous way to remove the gauge dependent part of the selfenergies

$$\Sigma^{\text{pinch}}_{\phi_1\phi_2}(p^2) = \left[\Sigma^{\text{tad}}_{\phi_1\phi_2}(p^2)\right]_{\xi=1} + \Sigma^{\text{add}}_{\phi_1\phi_2}(p^2)$$

$$\begin{split} \Sigma_{H^0h^0}^{\text{add}}(p^2) &= \frac{g^2 \mathbf{s}_{\beta-\alpha} \mathbf{c}_{\beta-\alpha}}{32\pi^2 c_W^2} \left( p^2 - \frac{m_{H^0}^2 + m_{h^0}^2}{2} \right) \left\{ \left[ B_0(p^2; m_Z^2, m_{A^0}^2) - B_0(p^2; m_Z^2, m_Z^2) \right. \\ &+ 2 \mathbf{c}_W^2 \left[ B_0(p^2; m_W^2, m_{H^\pm}^2) - B_0(p^2; m_W^2, m_W^2) \right] \right\} , \\ \Sigma_{G^0A^0}^{\text{add}}(p^2) &= \frac{g^2 \mathbf{s}_{\beta-\alpha} \mathbf{c}_{\beta-\alpha}}{32\pi^2 c_W^2} \left( p^2 - \frac{m_{A^0}^2}{2} \right) \left[ B_0(p^2; m_Z^2, m_{H^0}^2) - B_0(p^2; m_Z^2, m_{h^0}^2) \right] , \\ \Sigma_{G^\pm H^\pm}^{\text{add}}(p^2) &= \frac{g^2 \mathbf{s}_{\beta-\alpha} \mathbf{c}_{\beta-\alpha}}{16\pi^2} \left( p^2 - \frac{m_{H^\pm}^2}{2} \right) \left[ B_0(p^2; m_W^2, m_{H^0}^2) - B_0(p^2; m_W^2, m_{h^0}^2) \right] \end{split}$$

The Pinch Technique and its Applications to Non-Abelian

> JOHN M. CORNWALL Joannis papavassiliou And Daniele Binosi

CAMBRIDGE MONOGRAPHS ON PARTICLE PHYSICS, NUCLEAR PHYSICS AND COSMOLOGY

**Gauge Theories** 

### Scale - masses of the scalars - OS pinched

Scale – 
$$p_*^2=rac{m_{\phi_1}^2+m_{\phi_2}^2}{2}$$
 p\* pinched

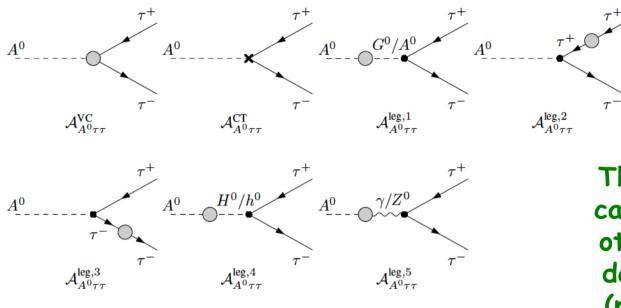
Alternative tadpole scheme, OS-pinched

$$\begin{split} \delta \alpha &= \frac{\text{Re}\Big[\left[\Sigma_{H^0h^0}^{\text{tad}}(m_{H^0}^2) + \Sigma_{H^0h^0}^{\text{tad}}(m_{h^0}^2)\right]_{\xi=1} + \Sigma_{H^0h^0}^{\text{add}}(m_{H^0}^2) + \Sigma_{H^0h^0}^{\text{add}}(m_{h^0}^2)\Big]}{2\left(m_{H^0}^2 - m_{h^0}^2\right)} \ ,\\ \delta \beta^{(1)} &= -\frac{\text{Re}\Big[\left[\Sigma_{G^0A^0}^{\text{tad}}(m_{A^0}^2) + \Sigma_{G^0A^0}^{\text{tad}}(0)\right]_{\xi=1} + \Sigma_{G^0A^0}^{\text{add}}(m_{A^0}^2) + \Sigma_{G^0A^0}^{\text{add}}(0)\Big]}{2m_{A^0}^2} \ ,\\ \delta \beta^{(2)} &= -\frac{\text{Re}\Big[\left[\Sigma_{G^{\pm}H^{\pm}}^{\text{tad}}(m_{H^{\pm}}^2) + \Sigma_{G^{\pm}H^{\pm}}^{\text{tad}}(0)\right]_{\xi=1} + \Sigma_{G^{\pm}H^{\pm}}^{\text{add}}(m_{H^{\pm}}^2) + \Sigma_{G^{\pm}H^{\pm}}^{\text{add}}(0)\Big]}{2m_{H^{\pm}}^2} \end{split}$$

# Alternative tadpole scheme, $p_*$ -pinched

$$\begin{split} \delta \alpha &= \frac{1}{m_{H^0}^2 - m_{h^0}^2} \mathrm{Re} \left[ \Sigma_{H^0 h^0}^{\mathrm{tad}} \left( \frac{m_{H^0}^2 + m_{h^0}^2}{2} \right) \right]_{\xi=1} \;, \\ \delta \beta^{(1)} &= -\frac{1}{m_{A^0}^2} \mathrm{Re} \left[ \Sigma_{G^0 A^0}^{\mathrm{tad}} \left( \frac{m_{A^0}^2}{2} \right) \right]_{\xi=1} \;, \\ \delta \beta^{(2)} &= -\frac{1}{m_{H^\pm}^2} \mathrm{Re} \left[ \Sigma_{G^\pm H^\pm}^{\mathrm{tad}} \left( \frac{m_{H^\pm}^2}{2} \right) \right]_{\xi=1} \;. \end{split}$$

# Process dependent renormalization



This process takes care of  $\beta$ . And the other process is H decaying into taus (not shown). That takes care of  $\alpha$ .

 $\Gamma_{A^0\tau\tau}^{\text{LO}} \stackrel{!}{=} \Gamma_{A^0\tau\tau}^{\text{NLO,weak}}$ 

$$\begin{split} \delta\beta &= \frac{-Y_3}{1+Y_3^2} \bigg[ \mathcal{F}_{A^0\tau\tau}^{\rm VC} + \frac{\delta g}{g} + \frac{\delta m_\tau}{m_\tau} - \frac{\delta m_W^2}{2m_W^2} + \frac{\delta Z_{A^0A^0}}{2} - \frac{1}{Y_3} \frac{\delta Z_{G^0A^0}}{2} + \frac{\delta Z_{\tau\tau}^{\rm L}}{2} + \frac{\delta Z_{\tau\tau}^{\rm R}}{2} \bigg] ,\\ \delta\alpha &= \frac{-Y_2}{Y_1} \bigg[ \mathcal{F}_{H^0\tau\tau}^{\rm VC} + \frac{\delta g}{g} + \frac{\delta m_\tau}{m_\tau} - \frac{\delta m_W^2}{2m_W^2} + Y_3\delta\beta + \frac{\delta Z_{H^0H^0}}{2} + \frac{Y_1}{Y_2} \frac{\delta Z_{h^0H^0}}{2} + \frac{\delta Z_{\tau\tau}^{\rm L}}{2} + \frac{\delta Z_{\tau\tau}^{\rm R}}{2} \bigg] , \end{split}$$

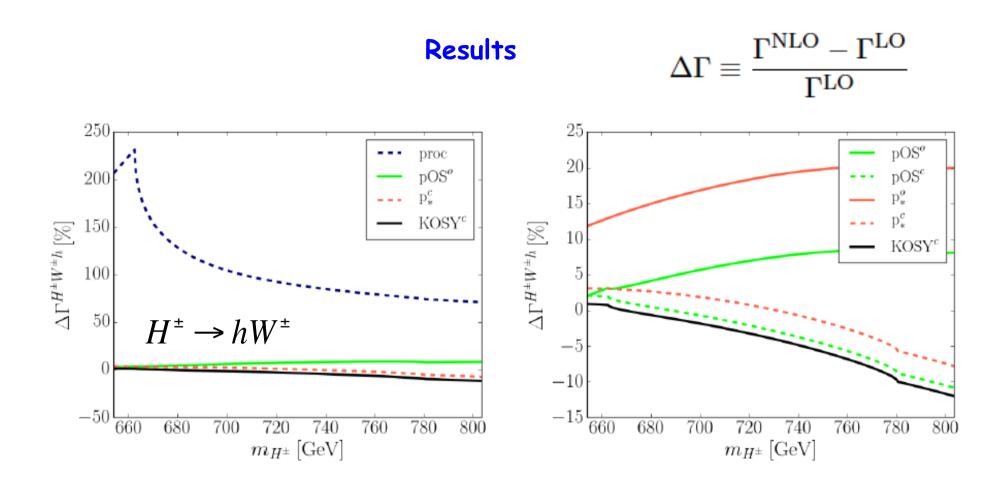
## Process dependent renormalization

$$\delta\alpha^{\text{sta}} = \left.\delta\alpha^{\text{sta}}\right|_{\xi=1} - (1-\xi_W) \frac{\Lambda_5 m_W^2 c_{\beta-\alpha} s_{\beta-\alpha}}{16\pi^2 (m_{H^0}^2 - m_{h^0}^2)} \alpha_W - (1-\xi_Z) \frac{\Lambda_5 m_Z^2 c_{\beta-\alpha} s_{\beta-\alpha}}{32\pi^2 (m_{H^0}^2 - m_{h^0}^2)} \alpha_Z$$

 $\delta \alpha^{\rm alt} = \left. \delta \alpha^{\rm alt} \right|_{\xi=1}$ 

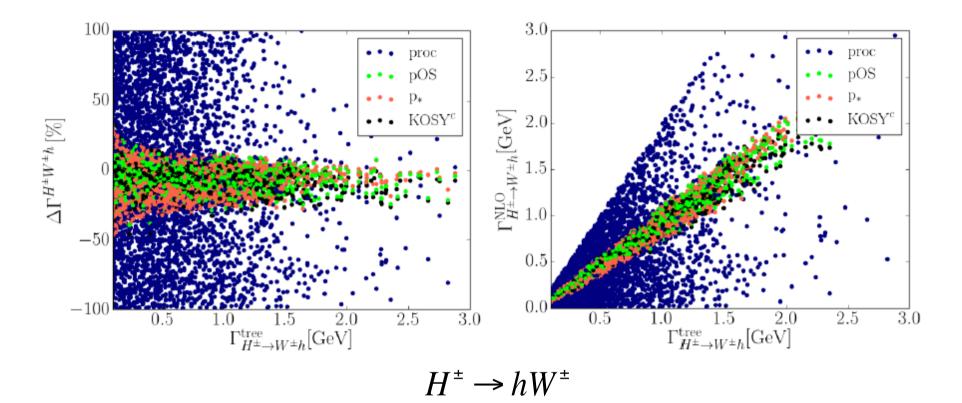
For the two angles the conclusion is that by using the alternative tadople scheme the counterterms for both  $\alpha$  and  $\beta$  become gauge independent making it much easier to control the overall gauge dependence

Moreover, by analysing the amplitudes it is clear that the 3 schemes proposed lead to gauge independent amplitudes: process-dependent, p\*pinched and O-pinched



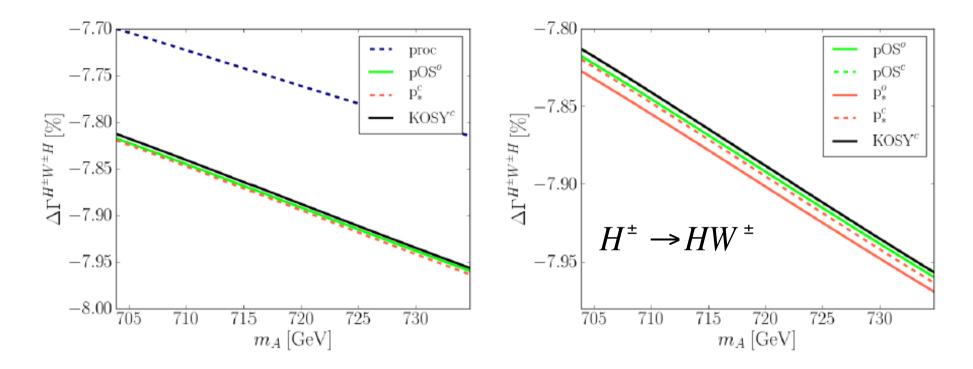
<u>Scen2</u>:  $m_{H^{\pm}} = (654...804) \text{ GeV}, \quad m_H = 742.84 \text{ GeV}, \quad m_A = 700.13 \text{ GeV}, \\ \tan \beta = 1.46, \quad \alpha = -0.57, \quad m_{12}^2 = 2.076 \cdot 10^5 \text{ GeV}^2$ 

Set of parameters consistent with main theoretical and experimental constraints.



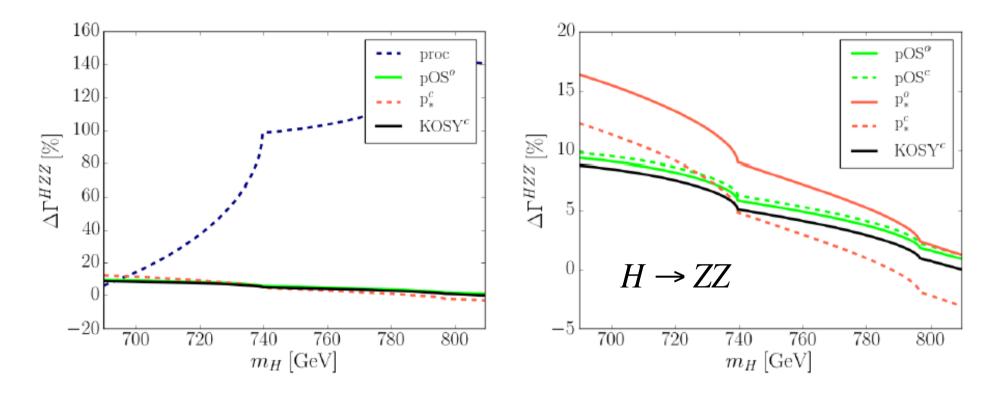
Values blow up for very small tree-level width due to different behaviour with  $\cos(\beta - \alpha)$ . In the right plot it is clear that NLO behaviour is good relative to the LO one.

For  $H \rightarrow ZZ$  that when the tree-level width is zero the NLO correction is of the order of  $A \rightarrow ZZ$ .



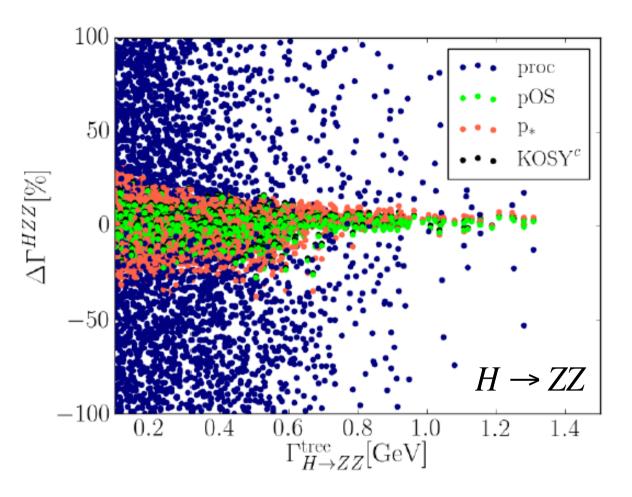
<u>Scen3:</u>  $m_{H^{\pm}} = 745.54 \text{ GeV}, \quad m_{H} = 594.55 \text{ GeV}, \quad m_{A} = (704...735) \text{ GeV}, \\ \tan \beta = 1.944, \quad \alpha = -0.458, \quad m_{12}^{2} = 1.941 \cdot 10^{5} \text{ GeV}^{2}.$ 

$$\Delta\Gamma \equiv \frac{\Gamma^{\rm NLO} - \Gamma^{\rm LO}}{\Gamma^{\rm LO}}$$
Set of parameters consistent with  
main theoretical and experimental  
constraints.



# kinks are due to the following thresholds

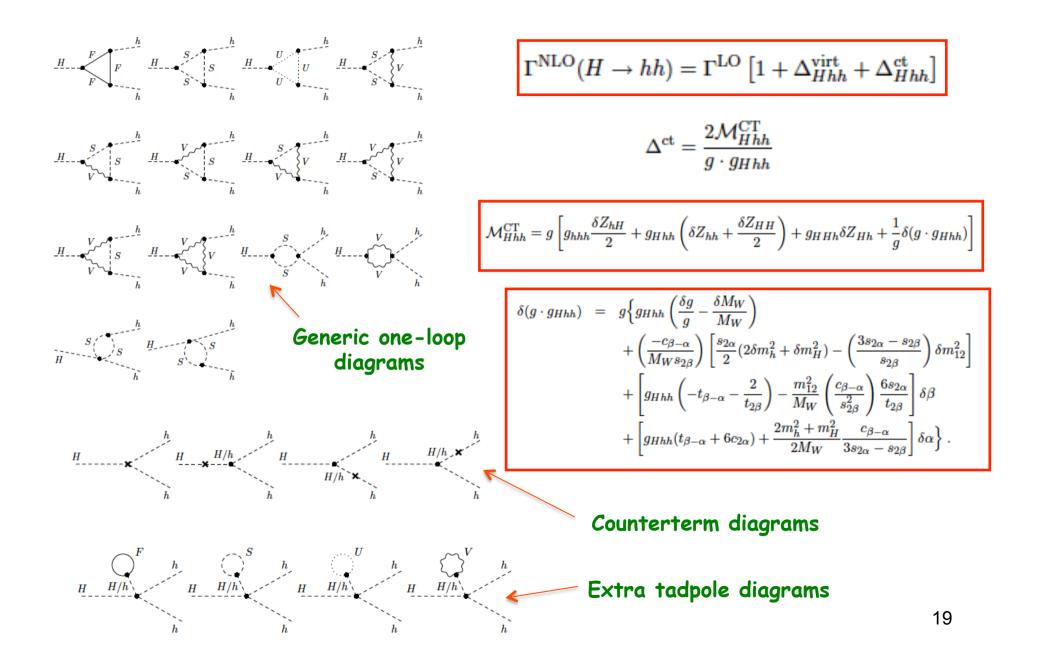
Kink	Kinematic point	Origin
1	$m_H(739.55 \text{ GeV}) = m_{H^{\pm}}(659.16 \text{ GeV}) + M_W$	$\delta Z_{HH}, \delta Z_{hH}$
2	$m_H(796.63 \text{ GeV}) = m_A(705.44 \text{ GeV}) + M_Z$	$\delta Z_{HH}, \delta Z_{hH}$



Using scans makes it clear that the renormalization schemes are stable at least for the parameter points allowed after Run 1.

<u>Conclusion</u>: among these schemes the OS tadpole-pinched scheme turns out to be more stable when changing the renormalization scale than the p\* scheme for our investigated scenarios. It is clear that the process dependent scheme is the more unstable of them all.

#### Renormalization of the soft breaking term



## Renormalization of the soft breaking term

a) MSbar scheme counterterm for  $(m_{12})^2$  is chosen such that it cancels all residual terms in the amplitude proportional to

$$\Delta = \frac{1}{\epsilon} - \gamma_E + \ln(4\pi)$$

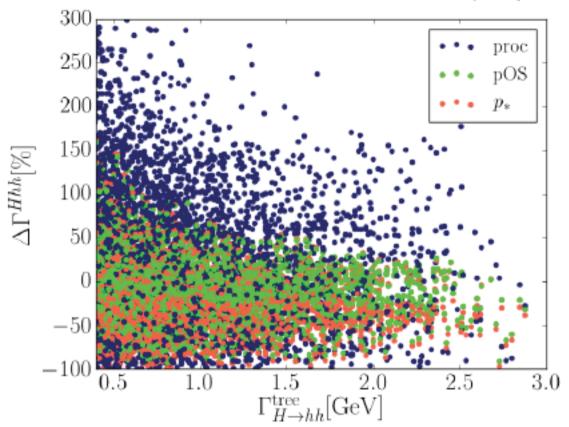
The condition is written as

$$\delta m_{12}^2 = \delta m_{12}^2(\Delta)|_{\overline{\rm MS}}$$

b) We have also used a process dependent scheme using H-> AA

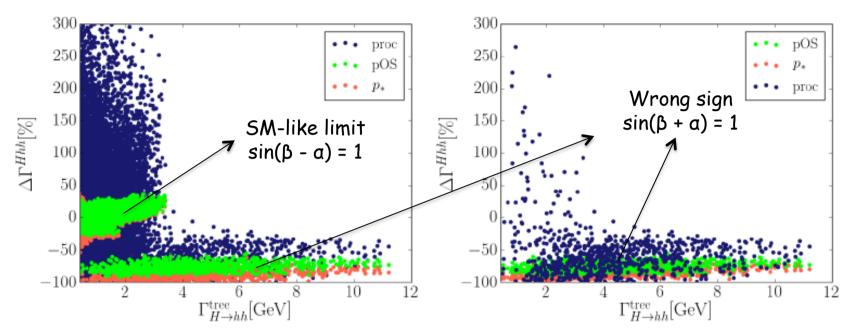
$$\Gamma^{\rm LO}(H \to AA) \stackrel{!}{=} \Gamma^{\rm NLO}(H \to AA)$$
$$\delta m_{12}^2 = -\frac{v s_{2\beta}^2}{4s_{\alpha+\beta}} \operatorname{Re} \left[ \mathcal{M}_{HAA}^{\rm VC} + (\mathcal{M}_{HAA}^{\rm CT})_{\delta m_{12}^2 = 0} \right]$$

The counterterm for  $(m_{12})^2$  is gauge independent irrespective of the scheme and can be MS or process-dependent renormalized.



 $m_{12}^2$  has been  $\overline{\text{MS}}$  renormalized with  $\mu_R = 2m_h$ .

Scatter plot for the relative NLO corrections to H-> hh for all points passing main experimental and theoretical constraints, as a function of the LO width.



#### Scatter plot same data, but with following restrictions:

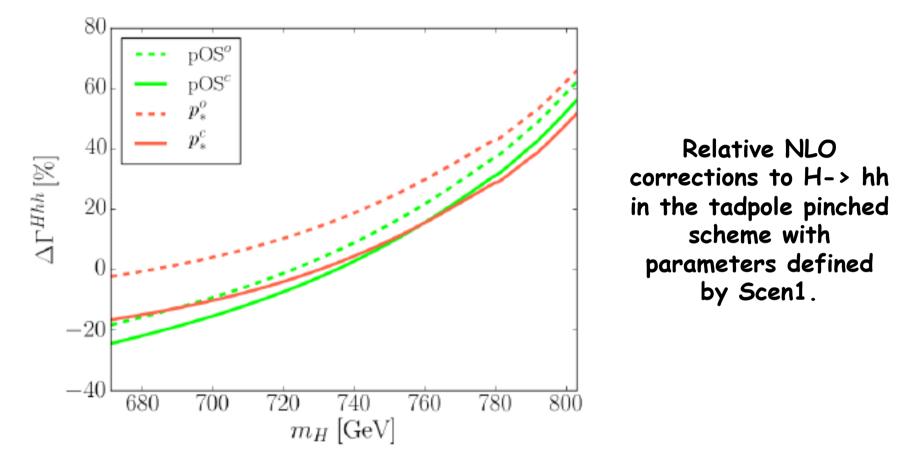
(i) The parameter sets are chosen such that the decay  $H \rightarrow hh$  is kinematically possible,

Condition (i): 
$$M_H \stackrel{!}{\geq} 2M_h$$

(ii) The parameter sets are chosen such that the decay  $H \to hh$  is kinematically possible. Additionally, we require the heavy Higgs boson masses to maximally deviate by  $\pm 5\%$  from M, with  $M^2 \equiv m_{12}^2/(s_\beta c_\beta)$ . We hence have

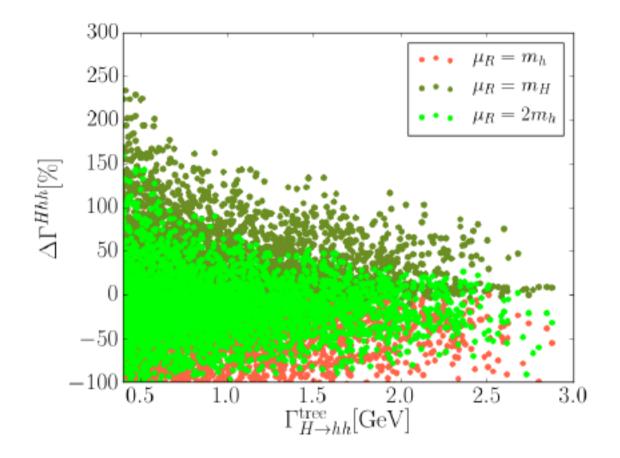
Condition (ii): 
$$M_H \stackrel{!}{\geq} 2M_h$$
 and  
 $m_{\phi_{\text{heavy}}} \stackrel{!}{=} M \pm 5\%$ , with  $m_{\phi_{\text{heavy}}} \in \{m_H, m_A, m_{H^{\pm}}\}$ .

In these scenarios the non-SM Higgs bosons are approximately mass degenerate and of the order of the  $\mathbb{Z}_2$  breaking scale.

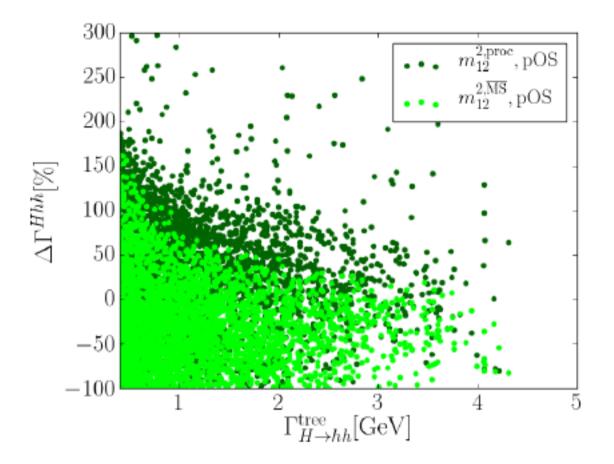


 $\begin{array}{ll} \underline{Scen1:} & m_{H} = (671.05...803.12) \; {\rm GeV}\,, & m_{A} = 700.13 \; {\rm GeV}\,, & m_{H^{\pm}} = 700.35 \; {\rm GeV}\,, \\ & \tan\beta = 1.45851\,, & \alpha = -0.570376\,, & m_{12}^2 = 2.0761 \cdot 10^5 \; {\rm GeV}^2 \end{array}$ 

Thank you



Scatter plot for the relative NLO corrections to H->hh (same data set) as a function of the LO width for three different scales. Angles are pOS tadpole pinched renormalized and soft breaking parameter is Msbar renormalized.



Comparison between minimal subtraction and process dependent for the soft breaking term.

In this appendix we consider the problems related to the shift (2.4) of the Higgs field. The quantities considered in this appendix are the bare ones if not indicated otherwise. If the shift parameter v has the correct value, the physical Higgs field satisfies the gauge-invariant condition

$$\langle H \rangle = \circ - - - - - \star + \circ - - - - - \bigcirc = \circ (A1)$$

when the trivial tadpole

$$t = v m_0^2 = v (\lambda v^2 - \mu^2)$$
 (A2)

and the Higgs-boson mass

$$m_H^2 = 3\lambda v^2 - \mu^2 \tag{A3}$$

are given the ground-state values

$$m_0^2 = 0 \text{ and } m_H^2 = 2\lambda v^2$$
 (A4)

The proper value of v, however, is not known a priori and it must be determined order by order in the perturbation expansion. We denote by  $v_0$  and v the proper values of v to the *n*th and the (n + 1)th order, respectively. Thus we write

$$v = v_0 + \delta v_t . \tag{A5}$$

 $-i\Delta m_{H}^{2} = -i6\lambda v_{0}\delta v_{t} =$ 

$$\delta v_t = \circ - - - (x) = \frac{i}{-m_{Ho}^2} \{- - - (x)\}$$

$$= \frac{\delta t}{m_{Ho}^2}$$
(A13)

J. FLEISCHER AND F. JEGERLEHNER (1981).

27

The gauge-dependences are similar in structure, no matter their origin. In the language of Feynman diagrams, it can be shown that all gauge dependences of a certain process have structures like e.g. self-energies. The pinch technique allows to isolate and extract these gauge-dependences in a unique way. It is then possible to construct e.g. self-energies and mass counterterms which are manifestly gauge-independent by themselves.

- the extension of the PT from one-loop to higher orders is neither unique nor trivial,
- the process-independence of the PT is not generally proven, but only shown for specific examples,
- the PT is not generally applicable to all possible one-loop Green's functions,

# Choose a process where by collapsing the vertices you obtain the self energy you want to calculate.

The process-independence of the PT is indeed not proven in general, which has to be accepted as one of the major weak points when using the technique. While the gauge-independence of the pinched self-energies is ensured by the BRST symmetry [93], a process-dependence of the PT might result in different forms of the additional gauge-independent terms presented in Eqs. (4.159) - (4.161). However, the PT was applied to a variety of different toy processes, so far yielding a universal result [109]. As a consequence, the process-independence of the PT is postulated until falsified with a suitable counter-example. The gauge-dependences are similar in structure, no matter their origin. In the language of Feynman diagrams, it can be shown that all gauge dependences of a certain process have structures like e.g. self-energies. The pinch technique allows to isolate and extract these gauge-dependences in a unique way. It is then possible to construct e.g. self-energies and mass counterterms which are manifestly gauge-independent by themselves.

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