The High scale flavor-aligned 2HDM

Howard E. Haber
RISE Collaboration
Meeting:
NonminimalHiggs
Toyama, Japan
6 March 2017
Outline

• Introduction—Higgs-mediated flavor changing neutral currents (FCNCs) in the two Higgs doublet model (2HDM)

• The flavor-aligned 2HDM [A2HDM]

• Imposing flavor alignment at $\Lambda = M_P$

• Higgs-mediated FCNCs in the one-loop leading logarithmic approximation

• Phenomenological consequences
  - $t \rightarrow ch$
  - $B$ and $K$ meson mixing
  - $B_s \rightarrow \mu^+ \mu^-$
  - $B \rightarrow \tau \nu$
  - $H \rightarrow b\bar{s}, \bar{b}s$

• Conclusions

This talk is based on work in collaboration with Stefania Gori and Edward Santos, arXiv:1703.xxxxx.
Higgs-mediated FCNCs and the 2HDM

The 2HDM Higgs-quark Yukawa Lagrangian (in terms of quark mass-eigenstates), summed over $i = 1, 2$, is:

$$- \mathcal{L}_Y = \overline{U}_L \Phi_i^0 h_i^U U_R - \overline{D}_L K^\dagger \Phi_i h_i^U U_R + \overline{U}_L K \Phi_i^+ h_i^D \dagger D_R + \overline{D}_L \Phi_i^0 h_i^D \dagger D_R + \text{h.c.},$$

where there is an implicit sum over $i$, $K$ is the CKM mixing matrix, and the $h_i^{U,D}$ are $3 \times 3$ Yukawa coupling matrices.

We can re-express the Yukawa couplings in terms of the Higgs basis, $\{H_1, H_2\}$, where $\langle H_1^0 \rangle = v/\sqrt{2}$ and $\langle H_2 \rangle = 0$ and $v \simeq 246$ GeV. If $\langle \Phi_i^0 \rangle = v_i/\sqrt{2}$ where $\hat{v}_i \equiv v_i/v$ is a unit vector in Higgs “flavor” space, then the Higgs basis fields are

$$H_1 = (H_1^+, H_1^0) = \hat{v}_i^* \Phi_i, \quad H_2 = (H_2^+, H_2^0) = \hat{w}_i^* \Phi_i,$$

where $\hat{w}_i = v_j \epsilon_{ji}$. We can then expand

$$h_i^D = \kappa^F \hat{v}_i + \rho^F \hat{w}_i,$$

with $F = U, D$ and $\kappa^F \equiv \sqrt{2}M_F/v$ where $M_F$ is a diagonal mass matrix.
The resulting Higgs basis Yukawa Lagrangian is

$$\mathcal{L}_Y = \overline{U}_L \left( \frac{\sqrt{2} M_U}{v} H_1^{0\dagger} + \rho^U H_2^{0\dagger} \right) U_R - \overline{D}_L K^\dagger \left( \frac{\sqrt{2} M_U}{v} H_1^- + \rho^U H_2^- \right) U_R$$

$$+ \overline{U}_L K \left( \frac{\sqrt{2} M_D}{v} H_1^+ + \rho^D H_2^+ \right) D_R + \overline{D}_L \left( \frac{\sqrt{2} M_D}{v} H_1^0 + \rho^D H_2^0 \right) D_R$$

In the most general 2HDM, $\rho^U$ and $\rho^D$ are arbitrary complex $3 \times 3$ matrices, which yield neutral Higgs-mediated CP-violating and flavor-changing interactions.

For simplicity, assume that the Higgs scalar potential and vacuum are CP-invariant. Then, the neutral Higgs interactions are

$$\mathcal{L}_Y = \frac{1}{v} \sum_{F=U,D,E} \overline{F} \left\{ s_{\beta-\alpha} M_F - c_{\beta-\alpha} M_F^{1/2} \left[ \rho_F^R + i \epsilon_F \gamma_5 \rho_F^I \right] M_F^{1/2} \right\} F h$$

$$+ \frac{1}{v} \sum_{F=U,D,E} \overline{F} \left\{ c_{\beta-\alpha} M_F + s_{\beta-\alpha} M_F^{1/2} \left[ \rho_F^R + i \epsilon_F \gamma_5 \rho_F^I \right] M_F^{1/2} \right\} F H$$

$$- \frac{1}{v} \sum_{F=U,D,E} \overline{F} \left\{ M_F^{1/2} \left( \rho_I^F - i \epsilon_F \gamma_5 \rho_R^F \right) M_F^{1/2} \right\} F A$$

where $\epsilon_F = +1 \ [-1]$ for $F = U \ [F = D, E]$; the $\rho_{R,I}^F$ are Hermitian matrices.
Note: In the $\Phi_1-\Phi_2$ real basis, $\tan \beta \equiv v_2/v_1$ (which is a real number of either sign) and $\alpha$ is the angle that diagonalizes the CP-even Higgs squared-mass matrix. The CP-conserving Higgs potential in the Higgs basis is

$$V \ni \frac{1}{2} Z_1 (H_1^\dagger H_1)^2 + \left\{ \frac{1}{2} Z_5 (H_1^\dagger H_2)^2 + Z_6 (H_1^\dagger H_1)(H_1^\dagger H_2) + \text{h.c.} \right\}.$$

where $Z_5$ and $Z_6$ can be chosen real. The CP-odd Higgs boson, $A = \sqrt{2} \text{Im} H_2^0$, has squared-mass $m_A^2$. The CP-even Higgs squared-masses are obtained by diagonalizing the $2 \times 2$ squared-mass matrix, $M_H^2$, with respect to Higgs basis states, $\{\sqrt{2} \text{Re} \, H_1^0 - v, \sqrt{2} \text{Re} \, H_2^0\}$,

$$M_H^2 = \begin{pmatrix} Z_1 v_2^2 & Z_6 v_2^2 \\ Z_6 v_2^2 & m_A^2 + Z_5 v_2^2 \end{pmatrix}.$$

The CP-even Higgs bosons are $h$ and $H$ with $m_h \leq m_H$. In particular, the CP-even mass eigenstates are:

$$\begin{pmatrix} H \\ h \end{pmatrix} = \begin{pmatrix} c_{\beta-\alpha} & -s_{\beta-\alpha} \\ s_{\beta-\alpha} & c_{\beta-\alpha} \end{pmatrix} \begin{pmatrix} \sqrt{2} \text{Re} \, H_1^0 - v \\ \sqrt{2} \text{Re} \, H_2^0 \end{pmatrix},$$

with $s_{\beta-\alpha} \equiv \sin(\beta - \alpha)$ and $c_{\beta-\alpha} \equiv \cos(\beta - \alpha)$, where $0 \leq \beta - \alpha \leq \frac{1}{2} \pi$. 
Indeed, tree-level FCNCs mediated by neutral Higgs bosons are present (as well as new sources of CP-violation), governed by the $3 \times 3$ Hermitian matrices $\rho_{R,I}^F$, which are generically non-diagonal.

**Definitions of $\rho_{R,I}^F$**

\[
M_F^{1/2} \rho_R^F M_F^{1/2} = \frac{v}{2\sqrt{2}} (\rho^F + [\rho^F]^\dagger), \quad iM_F^{1/2} \rho_I^F M_F^{1/2} = \frac{v}{2\sqrt{2}} (\rho^F - [\rho^F]^\dagger).
\]

In the CP-conserving Type-I and Type-II 2HDM, $\rho_{I,D}^{U,D} = 0$ and

**Type I:** \[
\rho_R^D = \rho_U^U = -1 \cot \beta,
\]

**Type II:** \[
\rho_R^D = 1 \tan \beta, \quad \rho_U^U = -1 \cot \beta,
\]

where $1$ is the $3 \times 3$ identity matrix. Thus, the neutral Higgs-fermion couplings are flavor diagonal!

In Type-I and Type-II models, the couplings to leptons follows the pattern of the down-type quark couplings. In the so-called Types Y and X models, the Types I and II quark Yukawa couplings are associated with Types II and I lepton Yukawa couplings, respectively.
The flavor-aligned two-Higgs doublet model (A2HDM)

We can by fiat declare that \( \rho^F = a^F \kappa^F \) for \( F = U, D, E \), were \( a^F \) is called the alignment parameter.* It follows that

\[
\rho^F_R = (\text{Re } a^F) \mathbb{1} , \quad \rho^F_I = (\text{Im } a^F) \mathbb{1} .
\]

The corresponding neutral Higgs–fermion Yukawa couplings are flavor-diagonal and are given by

\[
-L_Y = \frac{1}{v} \sum_{F=U,D,E} \overline{F} M_F \left\{ s_{\beta-\alpha} - c_{\beta-\alpha} [\text{Re } a^F + i \epsilon^F \text{Im } a^F \gamma_5] \right\} F h
\]

\[
+ \frac{1}{v} \sum_{F=U,D,E} \overline{F} M_F \left\{ c_{\beta-\alpha} + s_{\beta-\alpha} [\text{Re } a^F + i \epsilon^F \text{Im } a^F \gamma_5] \right\} F H
\]

\[
- \frac{1}{v} \sum_{F=U,D,E} \overline{F} M_F \left\{ [\text{Im } a^F - i \epsilon^F \text{Re } a^F \gamma_5] \right\} F A .
\]

Is the flavor aligned 2HDM radiatively stable?

The flavor-alignment conditions of the A2HDM are not radiatively stable, except in the case of the Types I, II X and Y 2HDMs. Indeed, flavor alignment is preserved by the renormalization-group (RG) running of the Yukawa coupling matrices only in the cases of the standard type-I, II, X, and Y models.†

Suppose that the flavor alignment condition is imposed by new physics at the Planck scale.‡ due to new physics that is presently unknown. One can then use an RG analysis to determine the structure of the Yukawa couplings at the electroweak scale. This in turn will lead to small flavor-violation in the neutral Higgs-quark interactions that can be constrained by experiment.

‡This ansatz was first considered by C.B. Braeuninger, A. Ibarra and C. Simonetto, arXiv:1005.5706 [hep-ph].
Defining $D \equiv 16\pi^2 \mu (d/d\mu)$, here are two examples of the RGEs that govern the running of the Yukawa coupling matrices,

$$
D \kappa^U = -(8g_s^2 + \frac{9}{4}g^2 + \frac{17}{12} g'\mu^2) \kappa^U + \left\{ 3\text{Tr}[\kappa^U \kappa^U] + \kappa^D \kappa^D + \text{Tr}[\kappa^E \kappa^E] \right\} \kappa^U
$$

$$
+ \left\{ 3\text{Tr}[\kappa^U \kappa^U + \kappa^D \kappa^D] + \text{Tr}[\kappa^E \kappa^E] \right\} \rho^U - 2K(\kappa^D \kappa^D K^\dagger \kappa^U + \rho^D \kappa^D K^\dagger \rho^U)
$$

$$
+ \kappa^U (\kappa^U + \rho^U) + \frac{1}{2} K(\kappa^D + \rho^D \kappa^D) K^\dagger \kappa^U + \frac{1}{2}(\kappa^U + \rho^U \rho^U) \kappa^U,
$$

$$
D \rho^U = -(8g_s^2 + \frac{9}{4}g^2 + \frac{17}{12} g'\mu^2) \rho^U + \left\{ 3\text{Tr}[\rho^U \kappa^U] + \rho^D \kappa^D \right\} \kappa^U
$$

$$
+ \left\{ 3\text{Tr}[\rho^U \rho^U + \rho^D \rho^D] + \text{Tr}[\rho^E \rho^E] \right\} \rho^U - 2K(\rho^D \kappa^D K^\dagger \kappa^U + \rho^D \kappa^D K^\dagger \rho^U)
$$

$$
+ \rho^U (\kappa^U + \rho^U) + \frac{1}{2} K(\kappa^D + \rho^D \kappa^D) K^\dagger \rho^U + \frac{1}{2}(\kappa^U + \rho^U \rho^U) \rho^U.
$$

The diagonalization of the fermion mass matrices is carried out at the electroweak scale. Only Yukawa couplings evolve under RG running. The end result is that the RGEs for the $\kappa^F$ and $\rho^F$ explicitly contain factors of the CKM matrix $K$. Thus, if $\kappa^F$ and $\rho^F$ are proportional at one energy scale, they will no longer be proportional at another scale.
We shall assume flavor-alignment at the Planck scale, $\Lambda = \mathcal{M}_P$,

$$\rho^Q(\Lambda) = a^Q \kappa^Q(\Lambda).$$

We further assume the existence of a low-energy scale $\Lambda_H$ that characterizes the mass scale of the second Higgs doublet. We take $\Lambda_H = 400 \text{ GeV}$, in order that the observed Higgs boson possess SM-like properties (within about 20%). To be consistent with the observed diagonal quark mass matrix $M_Q$, we impose

$$\kappa^Q(\Lambda_H) = \sqrt{2} M_Q(\Lambda_H)/v.$$

We therefore have two boundary conditions, one at the high scale and one at the low scale.
We begin by assuming flavor-alignment at $\Lambda_H$ via a low-scale alignment parameter $a'Q$ in the first approximation of an iterative process, $\rho^Q(\Lambda_H) = a'Q \kappa^Q(\Lambda_H)$. We then decompose $\rho^Q(\Lambda)$ into parts that are aligned and misaligned with $\kappa^Q(\Lambda)$, respectively,

$$
\rho^Q(\Lambda) = a^Q \kappa^Q(\Lambda) + \delta \rho^Q,
$$

where $a^Q$ represents the aligned part (in general, different from $a'Q$), and $\delta \rho^Q$ the corresponding degree of misalignment at the high scale.

To minimize the misaligned part of $\rho^Q(\Lambda)$, we implement the cost function,

$$
\Delta^Q \equiv \sum_{i,j=1}^{3} |\delta \rho^Q_{ij}|^2 = \sum_{i,j=1}^{3} |\rho^Q_{ij}(\Lambda) - a^Q \kappa^Q_{ij}(\Lambda)|^2,
$$

which once minimized, provides the optimal value of the complex parameter $a^Q$ for flavor-alignment at the high scale.
The iteration procedure

- Impose flavor-alignment at the high scale using the optimized alignment parameters $a^Q$.
- Evolve the one-loop RGEs back down to $\Lambda_H$.
- At $\Lambda_H$, match the boundary conditions for the 2HDM and SM.
- Re-diagonalize $\kappa^U$ and $\kappa^D$ at the scale $\Lambda_H$ (which are no longer diagonal), while respectively transforming $\rho^U$ and $\rho^D$.
- Evolve $\kappa^U$ and $\kappa^D$ down to the electroweak scale using the one-loop SM RGEs.
- If any of the quark masses differ from their known values by more than 3%, re-establish the correct quark masses, run back up to $\Lambda_P$, and then rerun this procedure repeatedly until the two boundary conditions are satisfied.
The one-loop leading logarithmic approximation

\[ \rho^U (\Lambda_H) \sim a^U \kappa^U (\Lambda_H) + \frac{1}{16\pi^2} \log \left( \frac{\Lambda_H}{\Lambda} \right) (\mathcal{D} \rho^U - a^U \mathcal{D} \kappa^U), \]

\[ \rho^D (\Lambda_H) \sim a^D \kappa^D (\Lambda_H) + \frac{1}{16\pi^2} \log \left( \frac{\Lambda_H}{\Lambda} \right) (\mathcal{D} \rho^D - a^D \mathcal{D} \kappa^D). \]

where \( \kappa^U (\Lambda_H) \) and \( \kappa^D (\Lambda_H) \) are proportional to the diagonal quark mass matrices, \( M^U \) and \( M^D \) respectively, at the scale \( \Lambda_H \). Working to one loop order and neglecting higher order terms,

\[ \rho^U (\Lambda_H)_{ij} \approx a^U \delta_{ij} \frac{\sqrt{2}(M^U)_{jj}}{v} + \frac{(M^U)_{jj}}{4\sqrt{2}\pi^2 v^3} \log \left( \frac{\Lambda_H}{\Lambda} \right) \left\{ (a^E - a^U) [1 + a^U (a^E)^*] \delta_{ij} \sum_k (M^2_E)_{kk} \right. \]

\[ \left. \quad + (a^D - a^U) [1 + a^U (a^D)^*] \sum_k \left[ 3\delta_{ij} (M^2_D)_{kk} - 2(M^2_D)_{kk} K_{ik} K^*_{kj} \right] \right\}, \]

\[ \rho^D (\Lambda_H)_{ij} \approx a^D \delta_{ij} \frac{\sqrt{2}(M^D)_{ii}}{v} + \frac{(M^D)_{ii}}{4\sqrt{2}\pi^2 v^3} \log \left( \frac{\Lambda_H}{\Lambda} \right) \left\{ (a^E - a^D) [1 + a^D (a^E)^*] \delta_{ij} \sum_k (M^2_E)_{kk} \right. \]

\[ \left. \quad + (a^U - a^D) [1 + a^D (a^U)^*] \sum_k \left[ 3\delta_{ij} (M^2_U)_{kk} - 2(M^2_U)_{kk} K_{ki} K^*_{kj} \right] \right\}. \]
The validity of the one-loop leading log approximation breaks down for large values of the alignment parameters.

Blue: region of the A2HDM parameter space where the prediction for all the off-diagonal terms of the $\rho^Q$ matrices lies within a factor of 3 from the results obtained with the full running. Red: region where the one-loop leading log approximation differs significantly from the results obtained by numerically solving the RGEs.

**Remark:** In our numerical analysis, we require that no Landau pole singularities appear below $\Lambda = M_P$. This constraint is reflected in the upper boundary of the red curve shown above.
The significance of the parameter $\tan \beta$ in the A2HDM

Since $\tan \beta$ is a basis-dependent quantity, it has no significance in the A2HDM. In the CP-conserving case, only $\beta - \alpha$ (which is basis independent) has significance. Indeed, $\tan \beta$ does not appear in the Yukawa couplings of the A2HDM.

In our analysis, we have neglected neutrino masses, so that alignment in the leptonic sector is preserved by RG running. Thus, it is convenient to define $\tan \beta$ via

$$ a^E \equiv \tan \beta, $$

which is a real number of either sign. The significance of $\tan \beta$ is that in the $\Phi_1 - \Phi_2$ basis, we have $h_2^E = 0$, although this is not enforced by a discrete symmetry. The Yukawa couplings to leptons then resemble those of a Type II or Type X 2HDM.
Phenomenological consequences

Flavor-changing top decays

$$\text{BR}(t \rightarrow u_i h) = c_{\beta-\alpha}^2(|\rho^U_{i3}|^2 + |\rho^U_{3i}|^2) \times \frac{v^2}{4m_t^2} \frac{(1 - m_h^2/m_t^2)^2}{(1 - m_W^2/m_t^2)^2(1 + 2m_W^2/m_t^2)} \eta_{QCD},$$

where $\eta_{QCD} = 1 + 0.97\alpha_s \sim 1.10$ is the NLO QCD correction to the branching ratio. Note the dependence on $c_{\beta-\alpha}$.

Remark: In the SM, $\text{BR}(t \rightarrow ch) \sim 3 \times 10^{-15}$. Projections for the HL-LHC show that the bounds on the branching ratios of flavor violating top decays will likely be at the $10^{-4}$ level. At a future 100 TeV proton-proton machine with a large luminosity, recent estimates suggest that branching ratios as small as $\sim 10^{-7}$ could be probed with $10 \text{ ab}^{-1}$ luminosity.
Tree-level contributions to top flavor changing branching ratios as a function of the alignment parameters $a^U$ and $a^D$. Yellow, red, green and blue colors correspond to branching ratios $< 10^{-11}$, $[10^{-11} - 10^{-10}]$, $[10^{-10} - 10^{-8}]$, $> 10^{-8}$. We have fixed $\beta - \alpha = \pi/2 - 0.2$ and $\Lambda_H = 400$ GeV.

**Remark**: Flavor changing top decays are also generated at one-loop via charged Higgs exchange. These contributions (not included above) can dominate for light charged Higgs masses (e.g., $m_{H^\pm} \lesssim 200$ GeV). These contributions decouple when $m_{H^\pm} \gg m_h$. 
Bounds from Meson Mixing

Higgs mediated contributions to neutral meson mixing \((B_{d,s}\rightarrow \overline{B}_{d,s}, K\rightarrow \overline{K}\) and \(D\rightarrow \overline{D}\) mixing) arise in our model. Integrating out the Higgs bosons, we obtain the following dimension six effective Lagrangian describing \(B_s\) meson mixing

\[
\mathcal{L}_{\text{eff}} = C_2(\bar{b}_R s_L)^2 + \tilde{C}_2(\bar{b}_L s_R)^2 + C_4(\bar{b}_R s_L)(\bar{b}_L s_R) + \text{h.c.},
\]

with Wilson coefficients

\[
C_2 = \frac{(\rho^D_{32})^2}{4} \left( \frac{\sin^2(\beta - \alpha)}{m_H^2} + \frac{\cos^2(\beta - \alpha)}{m_h^2} - \frac{1}{m_A^2} \right),
\]
\[
\tilde{C}_2 = \frac{(\rho^D_{23}^*)^2}{4} \left( \frac{\sin^2(\beta - \alpha)}{m_H^2} + \frac{\cos^2(\beta - \alpha)}{m_h^2} - \frac{1}{m_A^2} \right),
\]
\[
C_4 = \frac{(\rho^D_{32})(\rho^D_{23}^*)}{2} \left( \frac{\sin^2(\beta - \alpha)}{m_H^2} + \frac{\cos^2(\beta - \alpha)}{m_h^2} + \frac{1}{m_A^2} \right),
\]

and corresponding ones for \(B_d\), \(K\), and \(D\) mixing.
Bounds from meson mixing observables. Left panel: experimentally preferred regions, as computed in our model at the leading log. The dark purple region is favored by the measurement of $B_s$ mixing, the purple region by $B_d$ mixing, and the dark pink (pink) region by the phase (mass difference) of the Kaon mixing system. $D$ meson mixing does not give any interesting bound on the parameter space and it is not shown. Right panel: corresponding $B_s$ results as obtained scanning the parameter space and using the full RG running. The yellow, red, and green points corresponds to a Wilson coefficient whose magnitude relative to the present bound from $B_s$ mixing is $< 1/3, [1/3, 1], > 1$. 
For our calculations, we use\(^\S\)

\[
\frac{\text{BR}(B_{s,d} \to \mu^+ \mu^-)}{\text{BR}(B_{s,d} \to \mu^+ \mu^-)_{\text{SM}}} \simeq \left( |S_{s,d}|^2 + |P_{s,d}|^2 \right) \\
\times \left( 1 + y_{s,d} \frac{\text{Re}(P_{s,d}^2) - \text{Re}(S_{s,d}^2)}{|S_{s,d}|^2 + |P_{s,d}|^2} \right) \left( \frac{1}{1 + y_{s,d}} \right).
\]

Above, \(\text{BR}(B_{s,d} \to \mu^+ \mu^-)_{\text{SM}}\) is the prediction in the SM for the branching ratio extracted from an untagged rate, \(y_s = (8.8 \pm 1.4)\%\) and \(y_d \sim 0\), and

\[
S_{s,d} \equiv \frac{m_{B_{s,d}}}{2m_\mu} \frac{(C_{s,d}^S - C_{s,d}'^S)}{C_{10 s,d}^{SM}} \sqrt{1 - \frac{4m_\mu^2}{m_{B_{s,d}}^2}}, \\
P_{s,d} \equiv \frac{m_{B_{s,d}}}{2m_\mu} \frac{(C_{s,d}^P - C_{s,d}'^P)}{C_{10 s,d}^{SM}} + \frac{(C_{s,d}^{10} - C_{10 s,d}'^{10})}{C_{10 s,d}^{SM}}.
\]

The $C_i$ are the Wilson coefficients corresponding to the Lagrangian

$$ \mathcal{L}_s = \sum_i (C_i O_i + C'_i O'_i) + \text{h.c.} $$

The relevant operators for the $B_s$ decay are

$$ O^{(i)S}_s = \frac{m_b}{m_{B_s}} (\bar{s}P_{R(L)}b)(\bar{\ell}\ell), \quad O^{(i)P}_s = \frac{m_b}{m_{B_s}} (\bar{s}P_{R(L)}b)(\bar{\ell}\gamma^5\ell), $$

$$ O^{(i)}_{10s} = (\bar{s}\gamma_{\mu}P_{L(R)}b)(\bar{\ell}\gamma^\mu\gamma^5\ell), $$

The heavy Higgs $s$-channel tree-level diagrams contributing to $B_s$ decay yield

$$ C_s^P = -\frac{m_{B_s}}{m_b} \frac{\rho_{32}^{D*} m_\mu}{\sqrt{2} v} \tan \beta \frac{1}{m_A^2}, $$

$$ C'_s^P = \frac{m_{B_s}}{m_b} \frac{\rho_{23}^D m_\mu}{\sqrt{2} v} \tan \beta \frac{1}{m_A^2} \ll C_s^P, $$

$$ C_s^S = -\frac{m_{B_s}}{m_b} \frac{\rho_{32}^{D*} m_\mu}{\sqrt{2} v} \tan \beta \frac{1}{m_H^2}, $$

$$ C'_s^S = -\frac{m_{B_s}}{m_b} \frac{\rho_{23}^D m_\mu}{\sqrt{2} v} \tan \beta \frac{1}{m_H^2} \ll C_s^S, $$

in the approximation that $\cos(\beta - \alpha) \simeq 0$. Similar expressions are obtained for $B_d$ decay.
For the SM prediction, we take

\[ C_{10s,d}^{\text{SM}} = -4.1 \frac{e^2}{16\pi^2} \frac{4G_F}{\sqrt{2}} K_{tb} K^*_{t(s,d)}, \]

and

\[
\begin{align*}
\text{BR}(B_s \to \mu^+\mu^-)_{\text{SM}} &= (3.65 \pm 0.23) \times 10^{-9}, \\
\text{BR}(B_d \to \mu^+\mu^-)_{\text{SM}} &= (1.06 \pm 0.09) \times 10^{-10}.
\end{align*}
\]

These values are in good agreement with the combination of the LHCb and the CMS measurements at Run I for the $B_s$ decay, which yields

\[
\begin{align*}
\text{BR}(B_s \to \mu^+\mu^-)_{\text{exp}} &= (2.8_{-0.6}^{+0.7}) \times 10^{-9}, \\
\text{BR}(B_d \to \mu^+\mu^-)_{\text{exp}} &= (3.9_{-1.4}^{+1.6}) \times 10^{-10}.
\end{align*}
\]

Leading log prediction for the branching ratios for $B_s \rightarrow \mu^+ \mu^-$ (left panel) and $B_d \rightarrow \mu^+ \mu^-$ (right panel) relative the the SM, as a function of $a^U$ and $a^D$, with fixed $\tan \beta = 10$, $\cos(\beta - \alpha) = 0$, and $m_A = m_H = 400$ GeV. The regions in pink are allowed at the $2\sigma$ level by the present measurements. The regions in purple denote the regions favored by the more precise HL-LHC measurements, assuming a measured central value equal to the SM prediction. The gray shaded regions produce Landau poles in the Yukawa couplings below $M_P$. 
The branching ratio for $B_s \to \mu^+\mu^-$ (left panel) and for $B_d \to \mu^+\mu^-$ (right panel) relative to the SM, obtained via scanning the parameter space and using the full RG running, with fixed $\tan\beta = 10$, $\cos(\beta - \alpha) = 0$, and $m_A = m_H = 400$ GeV. The yellow, red, green and blue points corresponds to branching ratios normalized to the SM prediction $< 0.4, [0.4, 1.1], [1.1, 10], > 10$. In boldface we denote the range preferred by the LHCb and ATLAS measurements of $B_s \to \mu^+\mu^-$. 

The red points shown in the left plot above correspond roughly to the regions allowed by the experimental measurements at the $2\sigma$ level.
The charged Higgs couplings to fermions are given by

\[ \mathcal{L} = -\frac{\sqrt{2}}{v} \left\{ \bar{U} \left[ K M_D^{1/2} (\rho_R^D - i \rho_I^D) M_D^{1/2} P_R - M_U^{1/2} (\rho_R^U - i \rho_I^U) M_U^{1/2} K P_L \right] D H^+ ight. \]

\[ \left. \overline{N} M_E^{1/2} (\rho_R^E - i \rho_I^E) M_E^{1/2} P_R E H^+ + \text{h.c.} \right\}, \]

where \( K \) is the CKM mixing matrix.

We apply this to the decay \( B \to \tau \nu \), which is mediated at tree-level by charged \( W^\pm \) and \( H^\pm \) exchange. The present data yields

\[ \text{BR}(B \to \tau \nu)_{\text{exp}} = (1.06 \pm 0.19) \times 10^{-4}, \]

which is in a relatively good agreement with the SM prediction,

\[ \text{BR}(B \to \tau \nu)_{\text{SM}} = (0.848^{+0.036}_{-0.055}) \times 10^{-4}. \]
The branching ratio in the 2HDM relative to that of the SM is given by

$$\frac{BR(B \to \tau \nu)}{BR(B \to \tau \nu)_{\text{SM}}} = \left| 1 - \frac{m_B^2}{m_b} \frac{v \tan \beta}{\sqrt{2} K_{ub} m_{H^\pm}^2} \sum_i \left[ K_{ui} D_{3i}^* + K_{ib} U_{1i}^* \right] \right|^2.$$

The ratio $BR(B \to \tau \nu)/BR(B \to \tau \nu)_{\text{SM}}$ as a function of the two alignment parameters $a^U$ and $a^D$, at fixed $\tan \beta = 10$ and $m_{H^\pm} = 400$ GeV. Left panel: predictions at the leading log, where the pink region is favored by the measurement of $B \to \tau \nu$. The purple region are favored by future measurement at Belle II, under the assumption that the central value of the measurement is given by the SM prediction for this branching ratio. Right panel: result of the parameter space scan, using the full RG running. Yellow, red, green and blue points correspond to the ratios $< 0.2, [0.79, 1.71], [1.71, 3], > 3$, respectively. In boldface we denote the range preferred by the present world average for $BR(B \to \tau \nu)$. 
Result of the scan of parameter space, using the full RGEs, and having fixed $\cos(\beta - \alpha) = 0$, $m_A = m_H = m_{H^\pm} = 400$ GeV, and $\tan \beta = 10$. Blue points correspond to points allowed by the measurement of $B \to \tau \nu$, but not by the measurement of $B_s$ mixing or $B_s \to \mu^+ \mu^-$. Green points are allowed by the measurements of $B \to \tau \nu$ and of meson mixing but not by $B_s \to \mu^+ \mu^-$. Red points are allowed by all constraints. The left and right panels represent the bounds as they are now and as projected for the coming years.
Future consequences: $H$ decay BRs

If a heavy CP-even Higgs boson $H$ is discovered, then its branching ratios provide critical tests of the A2HDM approach.

- Possible flavor non-diagonal decays, e.g. $H \rightarrow b\bar{s}, \bar{b}s$
- Non-standard ratios of BRs, e.g.

$$\frac{\text{BR}(H \rightarrow \bar{b}b)}{\text{BR}(H \rightarrow \tau^+\tau^-)} \neq \frac{3m_b^2}{m_\tau^2}.$$  

$$\Gamma(H \rightarrow f_i f_j) = \frac{3G_Fv^2}{16\sqrt{2}\pi}m_Hs^2_{\alpha-\beta}(|\rho_{ij}^F|^2 + |\rho_{ji}^F|^2)$$

$$\times \left[1 - \left(\frac{m_{f_i} - m_{f_j}}{m_H}\right)^2\right] \left[\left(1 - \frac{m_{f_i}^2 + m_{f_j}^2}{m_H^2}\right)^2 - \frac{4m_{f_i}^2m_{f_j}^2}{m_H^4}\right]^{1/2} (i \neq j).$$
Assuming $\tan \beta = 10$, $\cos(\beta - \alpha) = 0$, and $m_H = 400$ GeV. **Left panel:** Leading log prediction for BR($H \to \bar{b}s$, $b\bar{s}$). The blue shaded regions have been probed by the LHC searches for $H, A \to \tau^+\tau^-, b\bar{b}$. The gray shaded regions produce Landau poles below the Planck scale $M_P$. **Right panel:** BR($H \to \bar{b}s$, $b\bar{s}$) obtained by scanning the parameter space and using the full RG running. Yellow, red, green and blue colors correspond to BR < 0.0005, [0.0005, 0.01], [0.01, 0.1], and > 0.1 based on a full numerical scan.
Assuming $\tan \beta = 10$, $\cos(\beta - \alpha) = 0$, and $m_H = 400$ GeV. **Left panel:** Leading log prediction for the branching ratios of the heavy Higgs boson, $H$. The blue contours represent the prediction of a Type II 2HDM. The gray shaded regions produce Landau poles below the Planck scale $M_P$. The blue shaded regions have been probed by the LHC searches for heavy scalars. **Right panel:** Branching ratios obtained by scanning the parameter space and using the full RG running. The yellow, red, green and blue points correspond to: upper left panel, $\text{BR}(H \rightarrow \bar{b}b) m_{\tau}^2 / \text{BR}(H \rightarrow \tau^+ \tau^-) 3m_b^2 < 1$, $[1, 10]$, $[10, 100]$, $> 100$. 
Conclusions

• In the search for new Higgs bosons, one should try to make the minimal set of assumptions that are consistent with the observed Higgs data.

• Current electroweak and Higgs data suggest a SM-like Higgs boson and highly suppressed FCNCs mediated by tree-level neutral Higgs exchange.

• Although special forms of the Higgs-fermion Yukawa couplings can naturally suppress FCNCs, one can imagine a more general set of assumptions that yield sufficiently suppressed Higgs-mediated FCNCs.

• In this talk, a framework was considered in which there is flavor alignment at a very high energy scale, which induces small Higgs-mediated FCNCs at the electroweak scale that can be consistent with current data.

• Some phenomenological consequences were examined, with an emphasis on processes that can distinguish among different models for the flavor structure of Higgs-fermion Yukawa interactions.