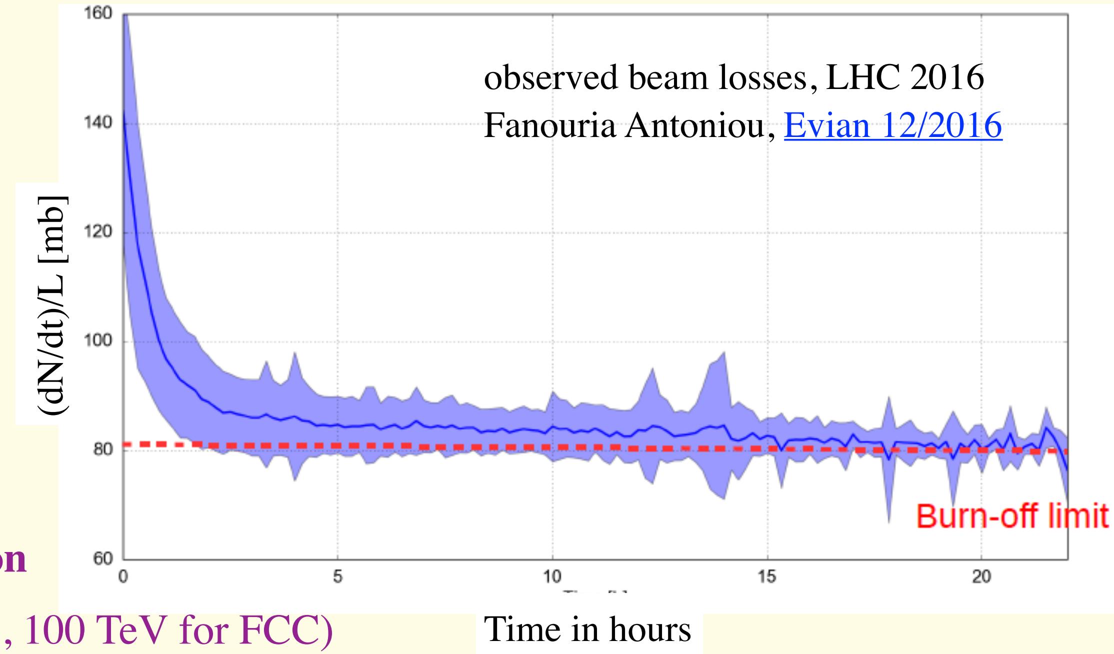
pp cross-section relevant for burn-off

Realistic prediction for pile-up and beam+luminosity lifetime needed to plan and optimize running schemes LHC, HL-LHC, HE-LHC and FCC

Predictions

pile-up, $\sigma_{inelastic}$ 81 ± 3 mb **reference**

burn-off, σ_{tot} >100 mb **too pessimistic**



Ideally :

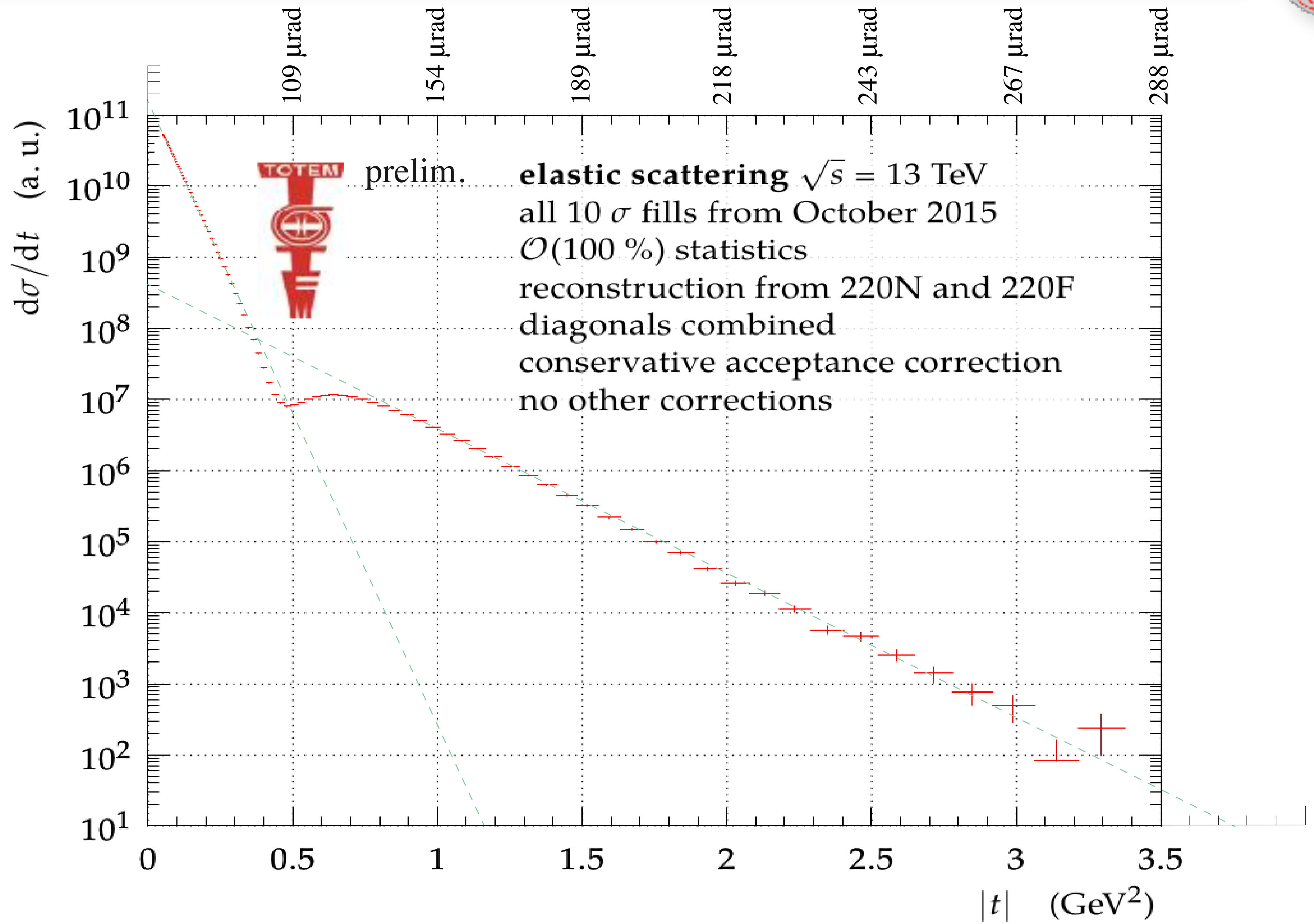
- **simple ~ analytic model with dependence on**
  $\sqrt{s}$ (LHC 0.9 - 14 TeV .. 25 TeV HE-LHC, 100 TeV for FCC)
  and beam parameters (emittance, β*)
- **event generator which can easily be interfaced with detailed machine tracking**
- **tuned to data**
- **estimate of uncertainty**

Contacted TOTEM, ALFA, Theory (Michelangelo Mangano)

----> advice and direct collaboration with TOTEM, Jan Kaspar CERN-THESIS-2011-214

BTW: e+e- burn-off σ ≈ 200 mb, R. Kleiss + H.B., ee-Fact 1999, EPAC 1994, BBBREM generator; Beam-size effect Kotkin et al.

13 TeV, β* = 90 m, $d\sigma_{el}/dt$ + θ axis
and straight (exp) dashed lines



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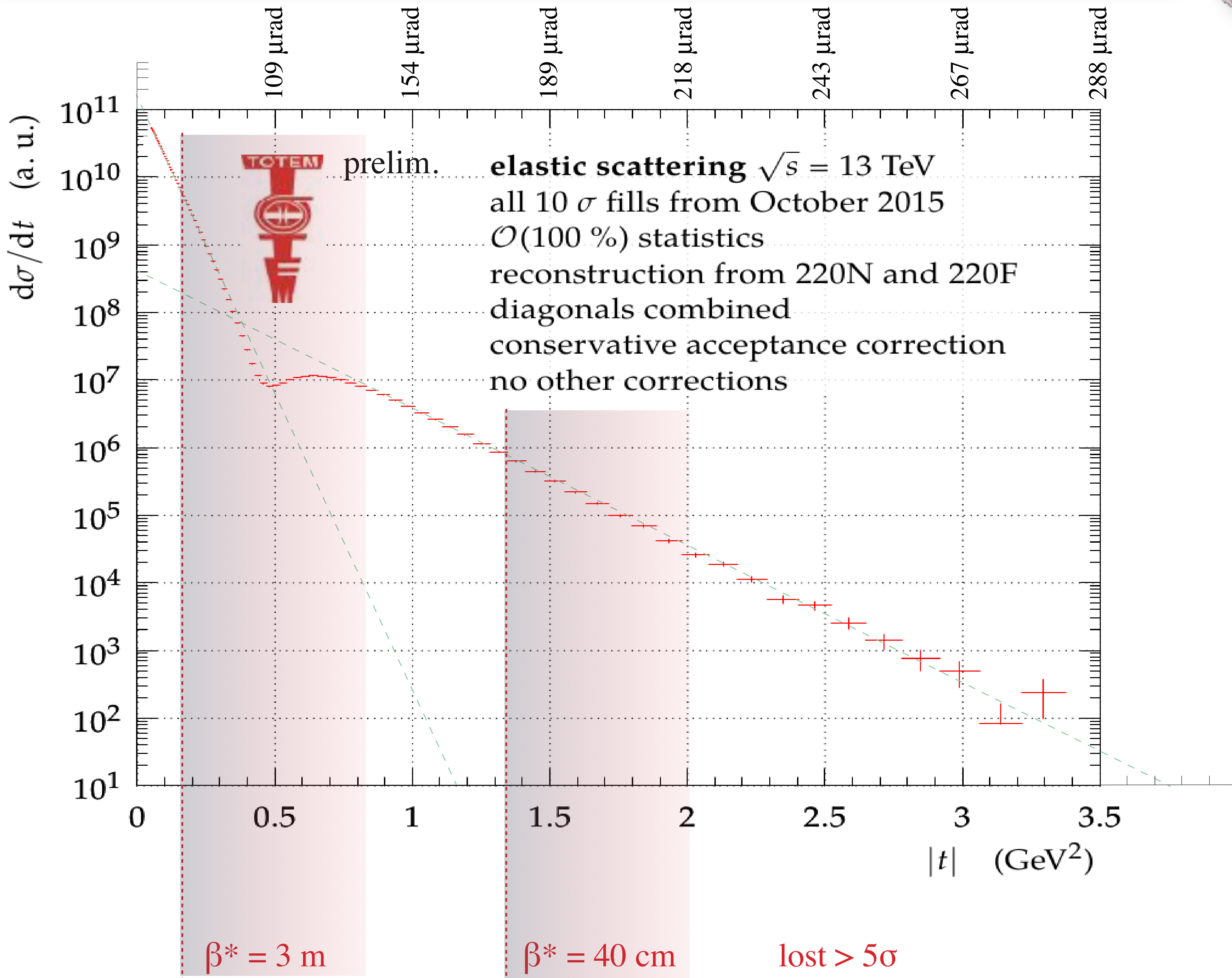


Kinematics and machine acceptance

Mandelstam variable $t = (p1+p2)^2 = (p3+p4)^2$ at high energy $\mathbf{t \approx 2\, p^2 (1 - \cos \theta)}$

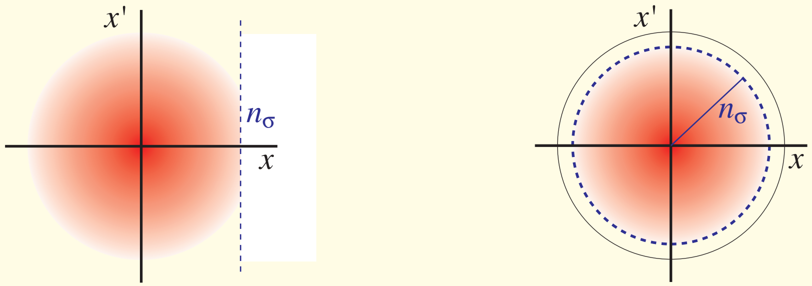
beam momentum p

for small angles $\cos \theta \approx 1 - \theta^2 / 2$ such that $\mathbf{t \approx - p^2 \theta^2}$ $\mathbf{\theta \approx \frac{\sqrt{|t|}}{p}}$

Primary collimator at 5.7 σ, calculated for a nominal normalized emittance of $\varepsilon_N = \gamma\varepsilon = 3.5$ µrad

After multiple turns, removes all particles above 5.7 σ **both in position and angle**

see   Intensity and Luminosity after Beam Scraping, H.B. + R. Schmidt, CERN-AB-2004-032-ABP



Beam divergence at the IP $(\alpha = \beta' = 0)$: $$\sigma' = \sqrt{\frac{\varepsilon}{\beta^*}}$$

2016 values, 13 TeV

ε = 505 µm

β^* = 40 cm

σ' = 35.5 µrad

θ_{acc} = 178 µrad

t_{lim} = 1.33 GeV^2

Particles on average at ~1 σ, core contributes more to collisions

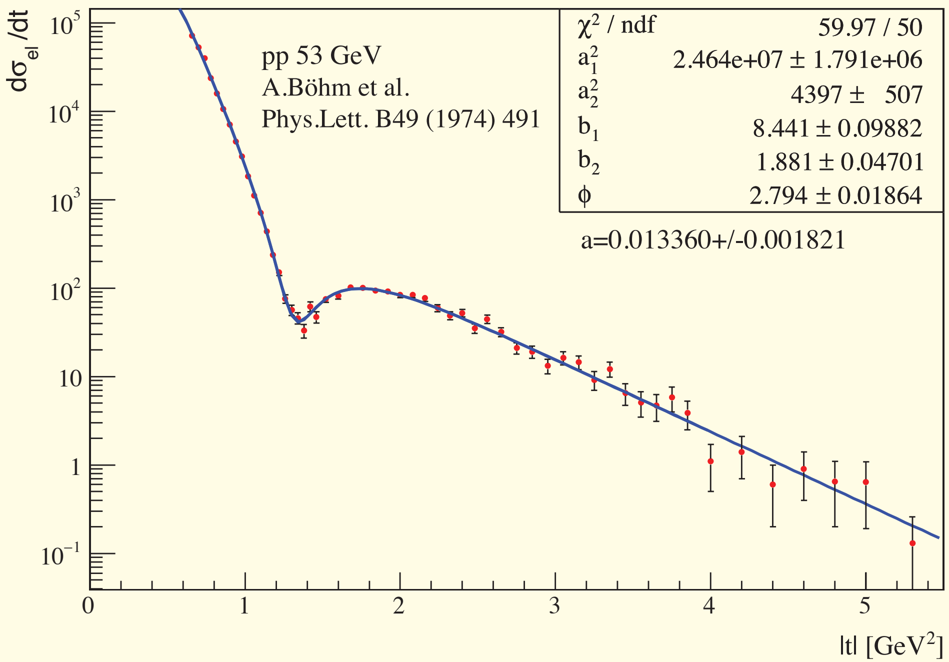
Lost when scattered by $\mathbf{\theta_{acc} = 5\,\sigma'}$ $\mathbf{t_{lim} = - p^2 \theta_{acc}^2}$

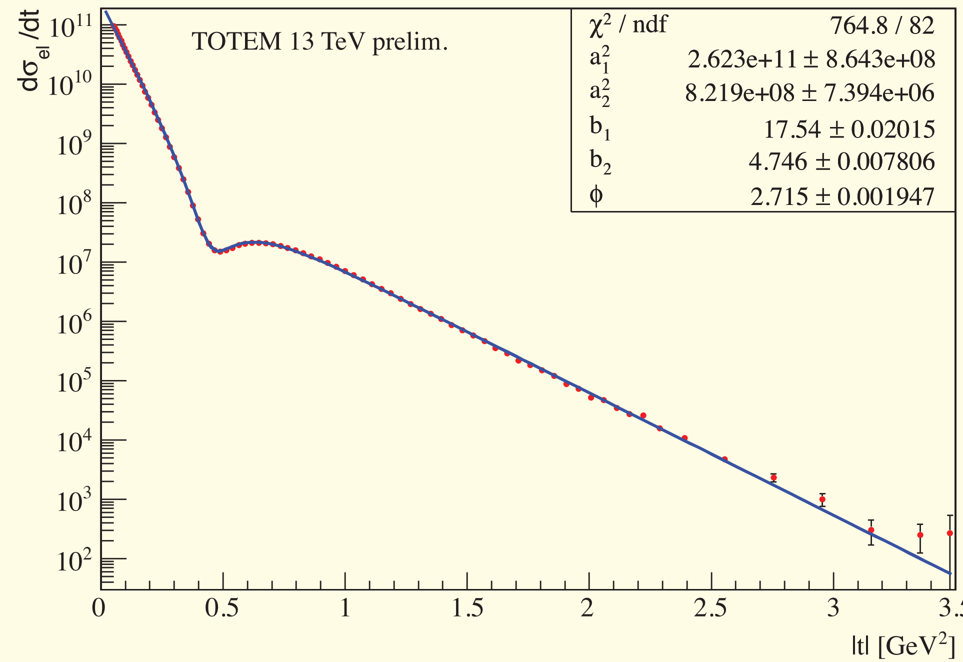
Energy acceptance :   rf-bucket height, $\Delta E/E < 3\times10^{-4}$ -- relevant for DS

Simple parametrization, suitable for generator

Complex amplitude $\mathrm{amp}(t) = a_1 e^{-b_1 t/2} + a_2 e^{-b_2 t/2} e^{i\phi}$ just two exp, with phase diff.

Diffr. cross-section $\frac{d\sigma}{dt} = |\mathrm{amp}(t)|^2 = a_1^2 e^{-b_1 t} + 2 a_1 a_2 e^{-(b_1+b_2)\frac{t}{2}} \cos\phi + a_2^2 e^{-b_2 t}$ all variables real > 0 (|t| written as t)





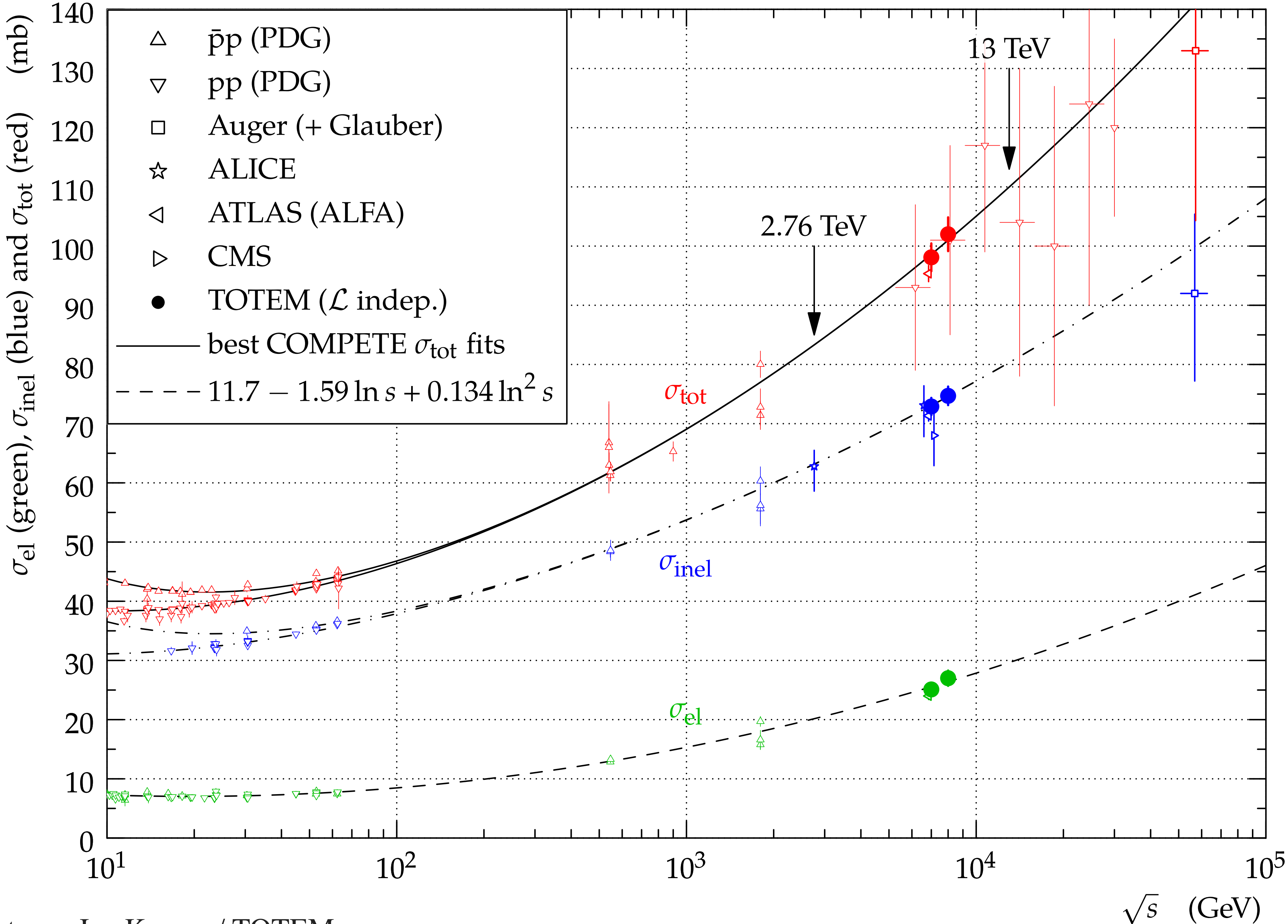
Straightforward to generate by inverse transform log(random) + hit&miss and include if required deviations from exponential.

3 - parameters, log(s) dependent, use linear extrapolation to higher energies

CoulombInterference : only relevant for very high β', where burn off no issue; can safely be ignored for standard β*

in addition : emittance increase by Coulomb scattering -- when needed included as intrabeam scattering

Total cross section, Energy dependence



courtesy Jan Kaspar / TOTEM

Results, conclusion

Main conclusion : $\sigma_{inelastic}$ also relevant as cross section for burn-off at low β*

conservative estimate -- not all diffractive lost

Details : simulation, generator works, implementation in detailed tracking started (SIXTRACK, by Kyrre Sjobaek)

Preliminary values with current parameters, to be checked and tuned (much of the forward LHC data not yet published) and simulated in detail

sqrt(s) GeV	sigma_el mb
53	7.5236
8 000	26.413
13 000	29.673
14 000	30.193
25 000	34.463
100 000	46.134

emitN= 3.5 mum emit=469.13 pm

14 TeV, cut at nsigma = 5 | cut at nsigma = 2

betastar [m]	divergence murad	theta_acc murad	tlim GeV^2	fraction lost	cut at nsigma = 2: fraction lost, ~ relevant for Luminosity
0.15	55.92	279.6	3.831	3.401e-12	0.0004628
0.3	39.54	197.7	1.916	2.518e-07	0.001346
0.4	34.25	171.2	1.437	4.153e-06	0.003504
0.55	29.21	146	1.045	4.099e-05	0.01522
3	12.51	62.53	0.1916	0.008351	0.4057
10	6.849	34.25	0.05747	0.2182	0.6614
90	2.283	11.42	0.006385	0.7048	0.7956
1000	0.6849	3.425	0.0005747	0.8036	0.8124
2500	0.4332	2.166	0.0002299	0.8099	0.8134

Backup

Code (1/2)

Standard (ISO) C++11, no external libraries, multithread safe (seed = thread number)

Can specify tmin, by default 0 to generate full spectrum

```
// ppGen.h    Helmut Burkhardt, 26/01/2017

#ifndef ppGen_h
#define ppGen_h 1

class ppgen_elastic
{
public:
  ppgen_elastic(double a,double b1,double b2,double phi,double tmin=0,unsigned int seed=0);
  ~ppgen_elastic() {} // (empty) destructor
  double t_gen(); // generate t    updates statistics, cannot be const
protected:
  double a,b1,b2,phi,tmin;
  unsigned int seed;
  std::mt19937_64 mt; // Mersenne Twister PRNG engine
  std::uniform_real_distribution<double> RanD; // flat double in default range 0 to 1
  double b_inv[2],slope2prob; // derived, used in generation
};

#endif
```

Code (2/2)

```
// ppGen.C   Helmut Burkhardt, 26/01/2017
#include <cmath>
#include <random>

#include "ppGen.h"

ppgen_elastic::ppgen_elastic(double a,double b1,double b2,double phi,double tmin,unsigned int seed)
     // constructor
: a(a),b1(b1),b2(b2),phi(phi),tmin(tmin),seed(seed) // init parameters
{
  mt.seed(seed); // random generator seed
  double G1=     exp(-b1*tmin)/b1;
  double G3=a*a*exp(-b2*tmin)/b2;
  slope2prob=G3/(G1+G3);
  b_inv[0]=1./b1; // store inverse for generation
  b_inv[1]=1./b2;
};

double ppgen_elastic::t_gen() // generate t
{
  double t,g1,g2,g3,rat;
  unsigned int i=0;
  do
  {
    if(RanD(mt)>slope2prob) i=0; else i=1;
    t=tmin-b_inv[i]*log(RanD(mt));
    g1=     exp(-b1*t);
    g2=2*a*exp(-0.5*(b1+b2)*t)*cos(phi);
    g3=a*a*exp(-b2*t);
    rat=(g1+g2+g3)/(g1+g3);
  }
  while(RanD(mt)>rat);
  return t;
}
```

generator, terms, integration

dσ/dt, divided by a1, just use amplitude ratio a = a2 / a1; normalize later anyway

probability distribution to generate $$g(t) = \frac{1}{a_1^2}\frac{d\sigma}{dt} = e^{-b_1 t} + 2a e^{-(b_1+b_2)\frac{t}{2}}\cos\phi + a^2 e^{-b_2 t}$$

interference negative $\cos\phi \leq 0,\ \pi/2 \leq \phi \leq \pi$ **dip when first and last term equal** $t_{\text{dip}} = \frac{\log a^{-2}}{b_1 - b_2} = -\frac{2\log a}{b_1 - b_2}$

Split up in 3 terms

$$g_1(t) = e^{-b_1 t}$$ soft scatter, steep exponential

$$g_2(t) = 2a e^{-(b_1+b_2)\frac{t}{2}}\cos\phi$$ interference

$$g_3(t) = a^2 e^{-b_2 t}$$ hard scatter, flat exponential

Integrate (from 0 to ∞)

$$G_1(t) = \frac{e^{-b_1 t}}{b_1}$$

$$G_2(t) = e^{-\frac{1}{2}t(b_1+b_2)}\frac{4a\cos(\varphi)}{b_1+b_2}$$

$$G_3(t) = \frac{a^2 e^{-b_2 t}}{b_2}$$

Generate $$g(t)_{\text{appr}} = \frac{1}{a_1^2}\frac{d\sigma}{dt} = e^{-b_1 t} + a^2 e^{-b_2 t}$$

by randomly switching between terms 1,3

probability for term 3 = $\frac{G_1}{G_1 + G_3}$

Get exact g(t) by hit & miss on ratio exact/appr $= \frac{g_1 + g_2 + g_3}{g_1 g_3}$

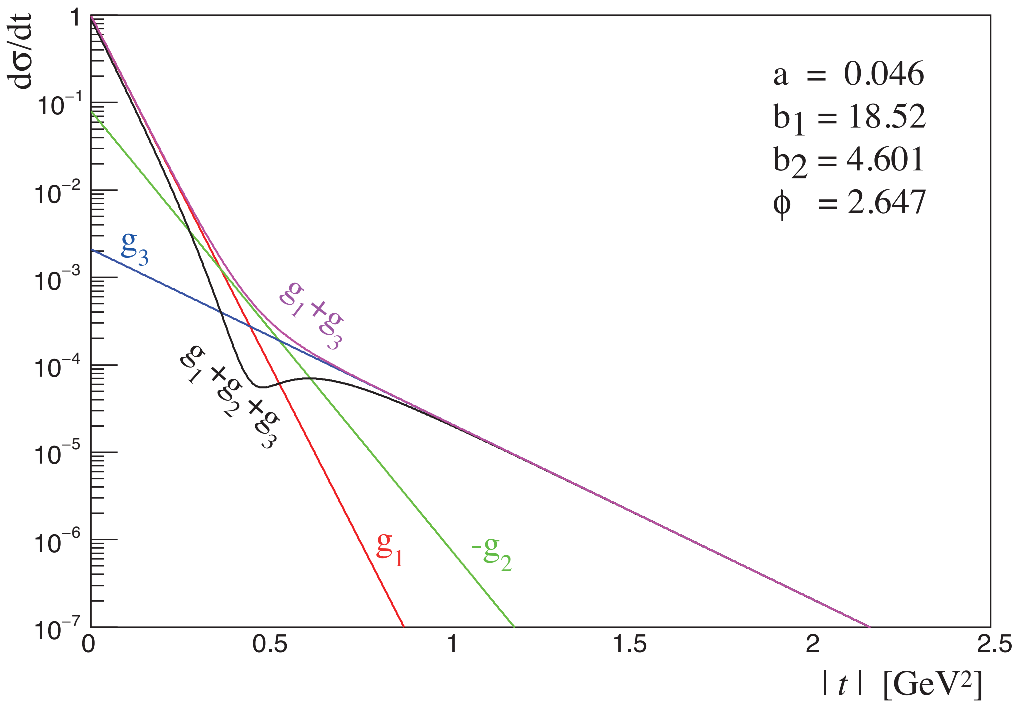
$d_1 = 11.7,\ d_2 = 1.59,\ d_3 = 0.134$

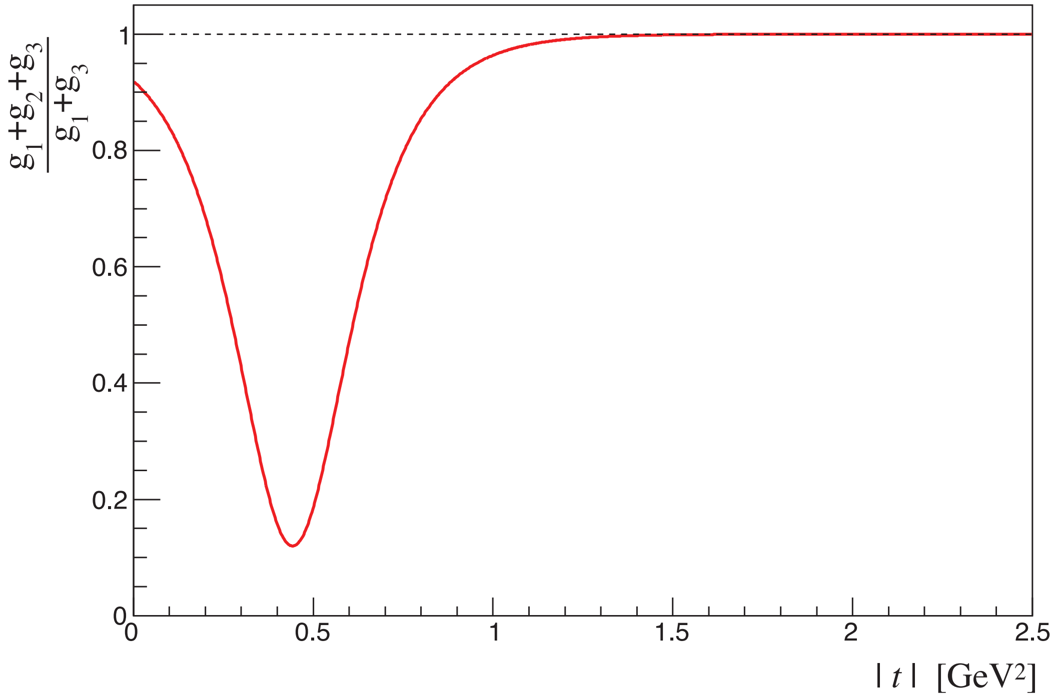
Parametrization for normalization $\sigma_{\text{el}}(s) = d_1 - d_2\log s + d_3\log^2 s$ TOTEM PRL 111 (012001) 2013 s in GeV^2

terms g_1, g_2, g_3

sqrt(s) = 13 TeV parameters

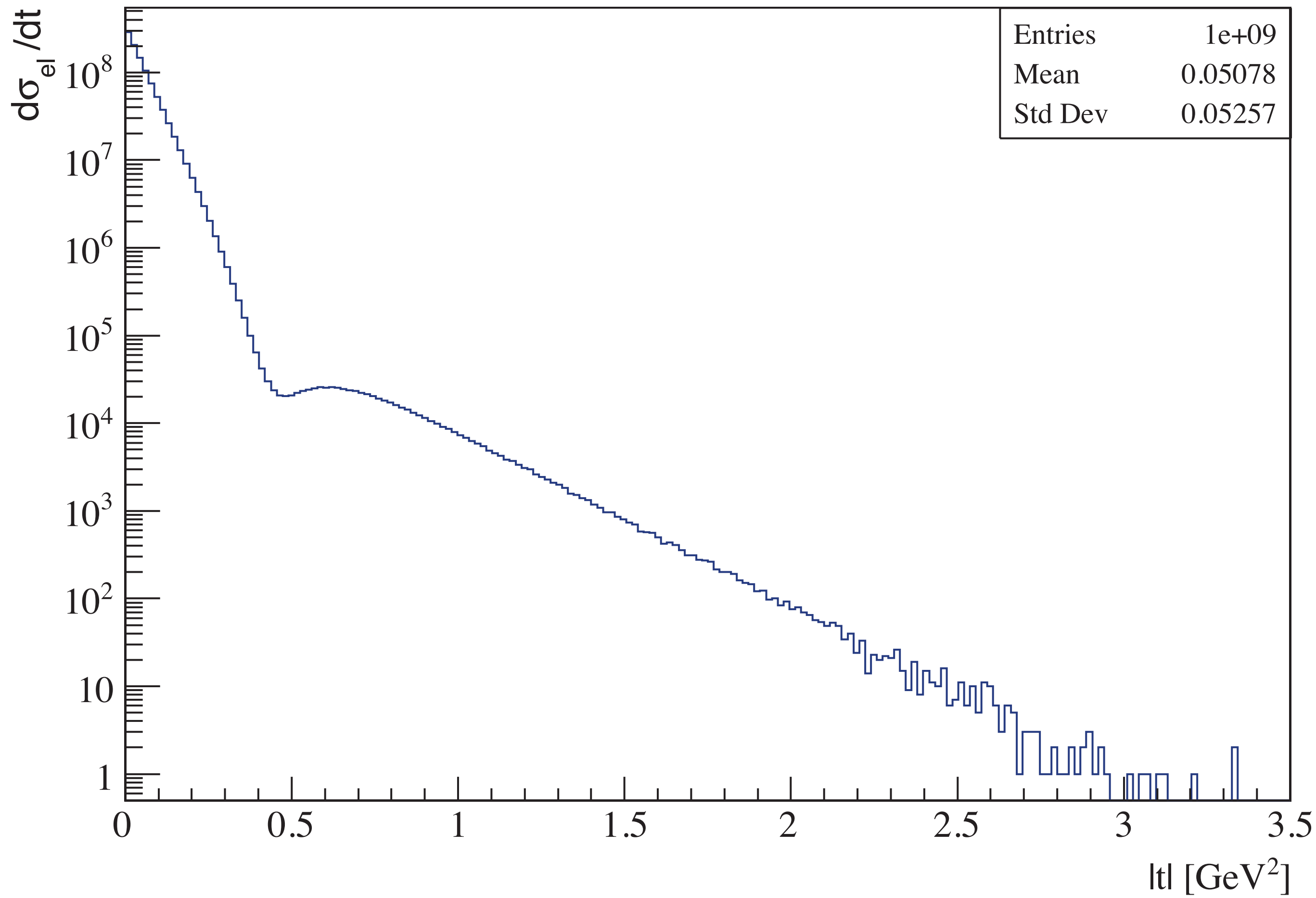
$$\frac{G_1}{G_1 + G_3} = 0.008426$$






```
Statistics, for generation of 1.e7 accepted events
ngen[0]=  11378513 nacc[0]=   9933329 efficiency=0.87299
ngen[1]=     97050 nacc[1]=     66671 efficiency=0.686976
rat_min[0]=  0.119838 rat_max[0]=  0.935963
rat_min[1]=  0.119838 rat_max[1]=  0.999999
```

Generated spectrum



sqrt(s) = 13 TeV parameters

1.e9 events generated + histogram filling in 57 sec on iMaci7, using 5 threads