On the Anisotropy of the Arrival Directions of Galactic Cosmic Rays

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TeVPA 2017, August 11, 2017
• **Standard paradigm:**
Galactic CRs accelerated in supernova remnants

✓ sufficient power: \( \sim 10^{-3} \times M_\odot \) with a rate of \( \sim 3 \) SNe per century

[Baade & Zwicky’34]

• galactic CRs via diffusive shock acceleration?

\[ n_{\text{CR}} \propto E^{-\gamma} \] (at source)

• energy-dependent **diffusion** through Galaxy

\[ n_{\text{CR}} \propto E^{-\gamma-\delta} \] (observed)

• arrival direction **mostly isotropic**
Cosmic ray anisotropies up to the level of one-per-mille at various energies
(Super-Kamiokande; Milagro; ARGO-YBJ; EAS-TOP, Tibet AS-$\gamma$; IceCube; HAWC)

HAWC & IceCube @ 10TeV

[→ talk by Dan Fiorino; IceCube & HAWC’17]
Dipole Anisotropy

- spherical harmonic expansion of relative CR intensity:

\[ I(\alpha, \delta) \simeq 1 + \delta \cdot n(\alpha, \delta) + \mathcal{O}(\{a_{\ell m}\}_{\ell \geq 2}) \]

- expected dipole anisotropy:

\[ \delta = 3K \cdot \nabla \ln n_{\text{CR}} + (2 + \Gamma_{\text{CR}}) \beta \]

- **Data-driven methods** of anisotropy reconstructions used by ground-based observatories are **only sensitive to dipole along the equatorial plane (EP)** (or, more generally, to all \( m \neq 0 \) multipoles).

\[ \Delta |\delta_{\text{EP}}| \sim \frac{f_{\text{sky}}}{\sqrt{N_{\text{tot}}}} \]

- **Monte-Carlo-based methods** are sensitive to the full dipole, but are **limited by systematic uncertainties**.
TeV-PeV CR Dipole Anisotropy

![Graph showing anisotropy of the arrival directions of Galactic CRs](image)

Markus Ahlers (NBI, Copenhagen)  Anisotropy of the Arrival Directions of Galactic CRs  August 11, 2017  slide 5
Local Magnetic Field

- reconstructed diffuse dipole:

\[ \delta^* = \delta - (2 + \Gamma_{\text{CR}})\beta = 3K \cdot \nabla \ln n^* \]

- projection onto equatorial plane:

\[ \delta_{\text{EP}}^* = (\delta_{0h}^*, \delta_{6h}^*) \]

- strong ordered magnetic fields in the local environment

\[ \rightarrow \text{diffusion tensor reduces to projector:} \]

\[ K_{ij} \rightarrow \kappa || \hat{B}_i \hat{B}_j \]

- TeV–PeV dipole data consistent with magnetic field direction inferred by IBEX data

\[ \rightarrow \text{talk by Eric Zirnstein} \]

\[ \text{[McComas et al.'09]} \]

\[ \text{[e.g. Mertsch & Funk’14; Schwadron et al.’14]} \]
Known Local Supernova Remnants

- projection maps source gradient onto $\hat{B}$ or $-\hat{B}$

→ **dipole phase** $\alpha_1$ depends on orientation of magnetic hemispheres

- intersection of magnetic equator with Galactic plane defines two source groups:

  $120^\circ \lesssim l \lesssim 300^\circ \rightarrow \alpha_1 \simeq 49^\circ$

  $-60^\circ \lesssim l \lesssim 120^\circ \rightarrow \alpha_1 \simeq 229^\circ$
Local Magnetic Field

- 1–100 TeV phase indicates dominance of a local source within longitudes:
  \[ 120^\circ \lesssim l \lesssim 300^\circ \]

- plausible scenario: Vela SNR \([\text{MA}'16]\)

  - **age**: \(\simeq 11,000\) yrs
  - **distance**: \(\simeq 1,000\) lyrs
  - **SNR rate**: \(R_{\text{SNR}} = 1/30\) yr\(^{-1}\)
  - **(effective) isotropic diffusion**: \(K_{\text{iso}} \simeq 4 \times 10^{28} (E/3\text{GeV})^{1/3} \text{cm}^2/\text{s}\)
  - **Galactic half height**: \(H \simeq 3\) kpc
  - **instantaneous CR emission** \((Q_\star)\)

\[ n/Q_\star \text{[kpc}^{-3}] \]
\[ K_{\text{iso}} |\nabla n|/Q_\star \text{[kpc}^{-3}] \]
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Small-Scale Anisotropy

Significant TeV small-scale anisotropies down to angular scales of $O(10)$ degrees.

$E_{\text{CR}} \approx 1 \text{ TeV}, N_{\text{CR}} \sim 4.9 \times 10^{10}$ [HAWC’14 (HAWC-111)]
Suggested Origin of Small-Scale Anisotropy

- magnetic reconnections in the heliotail \[\text{[Lazarian & Desiati'10]}\]
- non-isotropic particle transport in the heliosheath \[\text{[Desiati & Lazarian'11]}\]
- heliospheric electric field structure \[\text{[Drury'13]}\]
- non-uniform pitch-angle diffusion \[\text{[Malkov, Diamond, Drury & Sagdeev'10; Giacinti & Kirk'17]}\]
  \[\rightarrow \text{talk by Gwenael Giacinti}\]
- non-diffusive CR transport \[\text{[Salvati & Sacco'08; Drury & Aharonian'08]}\]
  \[\text{[Battaner, Castellano & Masip'14; Harding, Fryer & Mendel'16]}\]
- magnetized outflow from old SNRs \[\text{[Biermann, Becker, Seo & Mandelartz'12]}\]
  \[\rightarrow \text{talk by Julia Tjus}\]
- strangelet production in molecular clouds or neutron stars \[\text{[Kotera, Perez-Garcia & Silk '13]}\]
- small-scale anisotropies from local magnetic field mapping of a global dipole \[\text{[Giacinti & Sigl'12; MA'14; MA & Mertsch'15]}\]
  \[\text{[Pohl & Rettig'16; López-Barquero, Farber, Xu, Desiati & Lazarian'16]}\]
Angular Power Spectrum

- smooth function $g(\theta, \phi)$ on a sphere can be decomposed in terms of spherical harmonics $Y_m^\ell(\theta, \phi)$:

$$g(\theta, \phi) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} a_{\ell m} Y_m^\ell(\theta, \phi)$$

- angular power spectrum:

$$C_\ell = \frac{1}{2\ell + 1} \sum_{m=-\ell}^{\ell} |a_{\ell m}|^2$$

- approximate relation between angular scale and multipole $\ell$

$$\Delta \alpha \approx \frac{180^\circ}{\ell}$$

[IceCube’16 (top) & HAWC’14 (bottom)]
Analogy to Gravitational Lensing

CMB temperature fluctuations

Cosmic Ray Gradient

Local Magnetic Turbulence

Large Scale Structure

small scale temperature fluctuations

small scale anisotropies

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Simulation via CR Backtracking

- (quasi-)stationary solution of the diffusion approximation:

\[ 4\pi \langle f \rangle \simeq n + r \nabla n - 3 \hat{p} K \nabla n \]

1st order correction

- Liouville’s theorem:

\[ f(t, r(t), p(t)) = f(t', r(t'), p(t')) \]

- CR backtracking \((T \gg \tau_{\text{diff}})\):

\[ f(0) \simeq \delta f(-T) + \langle f \rangle (-T) \]

→ ensemble-averaged power spectrum \((\ell \geq 1)\):

\[
\frac{\langle C_\ell \rangle}{4\pi} \simeq \int \frac{d\hat{p}_1}{4\pi} \int \frac{d\hat{p}_2}{4\pi} P_\ell (\hat{p}_1 \hat{p}_2) \lim_{T \to \infty} \langle r_{1i}(-T) r_{2j}(-T) \rangle \frac{\partial_i n \partial_j n}{n^2}
\]

\[ \sigma^2 = 1, \frac{r_L}{L_c} = 0.1, \lambda_{\min} / L_c = 0.01, \lambda_{\max} / L_c = 100, \Omega T = 100 \]
Summary

- Observation of CR anisotropies at the level of one-per-mille is challenging.
- Reconstruction methods introduce bias.
- **Dipole anisotropy** can be understood in the context of standard diffusion theory:
  - TeV-PeV dipole phase aligns with local ordered magnetic field.
  - **New method** of measuring local magnetic fields
  - Amplitude variations as a result of local sources
  - Plausible & natural candidate: the Vela supernova remnant
- Observed CR data shows evidence of **small-scale anisotropy**.
  - Effect of heliosphere? [e.g. review by MA & Mertsch’16]
  - Result of local magnetic turbulence? [Giacinti & Sigl’12; MA’14; MA & Mertsch’15]
  - Induces cross-talk with dipole anisotropy in limited field of view.
Angular Power Spectrum

- Every smooth function $g(\theta, \phi)$ on a sphere can be decomposed in terms of spherical harmonics $Y_m^\ell$:

$$g(\theta, \phi) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} a_{\ell m} Y_m^\ell(\theta, \phi) \quad \leftrightarrow \quad a_{\ell m} = \int d\Omega(Y_m^\ell)^*(\theta, \phi)g(\theta, \phi)$$

- angular power spectrum:

$$C_\ell = \frac{1}{2\ell + 1} \sum_{m=-\ell}^{\ell} |a_{\ell m}|^2$$

- related to the two-point auto-correlation function: (n_{1/2} : unit vectors, $n_1 \cdot n_2 = \cos \eta$)

$$\xi(\eta) = \frac{1}{8\pi^2} \int d\mathbf{n}_1 \int d\mathbf{n}_2 \delta(n_1 n_2 - \cos \eta)g(n_1)g(n_2) = \frac{1}{4\pi} \sum_\ell (2\ell + 1) C_\ell P_\ell(\cos \eta)$$

Note that individual $C_\ell$’s are independent of coordinate system (assuming full sky coverage).
Multipole Cross-Talk

- relative CR intensity (including small-scale structure):

\[ I(\alpha, \delta) = 1 + \sum_{\ell \geq 1} \sum_{m \neq 0} a_{\ell m} Y_{\ell m}(\alpha, \pi/2 - \delta) \]

- dipole: \( a_{1-1} = (\delta_0h + i\delta_6h) \sqrt{2\pi/3} \) and \( a_{11} = -a_{1-1}^* \)

- traditional dipole analyses extract amplitude “\( A_1 \)” and phase “\( \alpha_1 \)” from data projected into right ascension \( (s_{1/2} \equiv \sin \delta_{1/2}) \)

\[
A_1 e^{i\alpha_1} = \frac{1}{\pi} \int_0^{2\pi} d\alpha e^{i\alpha} \frac{1}{s_2 - s_1} \int_{s_1}^{s_2} d\sin \delta \ I(\alpha, \delta) \]

- the presence of high-\( \ell \) multipole moments introduces cross-talk

Can now estimate the systematic uncertainties of dipole measures from dipole-induced small-scale power spectrum.
Systematic Uncertainty of CR Dipole

IceTop

$\left(\Delta \delta^{\ast}_{\delta}/\delta^{\ast}\right)_{\text{IceTop}} = 0.41, \delta_1 = -90^\circ, \delta_2 = -35^\circ$

IceCube

$\left(\Delta \delta^{\ast}_{\delta}/\delta^{\ast}\right)_{\text{IceCube}} = 0.39, \delta_1 = -90^\circ, \delta_2 = -25^\circ$

EAS-TOP

$\left(\Delta \delta^{\ast}_{\delta}/\delta^{\ast}\right)_{\text{EAS-TOP}} = 0.31, \delta_1 = 10^\circ, \delta_2 = 58^\circ$

ARGO-YBJ

$\left(\Delta \delta^{\ast}_{\delta}/\delta^{\ast}\right)_{\text{ARGO-YBJ}} = 0.20, \delta_1 = -10^\circ, \delta_2 = 70^\circ$

Tibet-ASγ

$\left(\Delta \delta^{\ast}_{\delta}/\delta^{\ast}\right)_{\text{Tibet-ASγ}} = 0.12, \delta_1 = -30^\circ, \delta_2 = 90^\circ$

HAWC (without IC)

$\left(\Delta \delta^{\ast}_{\delta}/\delta^{\ast}\right)_{\text{HAWC (without IC)}} = 0.07, \delta_1 = -41^\circ, \delta_2 = 79^\circ$
Systematic Uncertainty of CR Dipole
Gedankenexperiment

• **Idea:** local realization of magnetic turbulence introduces small-scale structure [Giacinti & Sigl’11]

• Particle transport in (static) magnetic fields is governed by Liouville’s equation of the CR’s phase-space distribution $f$:

$$\frac{df}{dt}(t, r, p) = 0$$

• “trivial” solution:

$$f(0, 0, p) = f(-T, r(-T), p(-T))$$

• **Gedankenexperiment:**
  Assume that at look-back time $-T$ initial condition is **homogenous, but not isotropic**:

$$f(0, 0, p) = \tilde{f}(p(-T))$$
Gedankenexperiment

- Initial configuration has power spectrum $\tilde{C}_\ell$.
- For small correlation angles $\eta$ flow remains correlated even beyond scattering sphere.
- Correlation function for $\eta = 0$:
  \[
  \xi(0) = \frac{1}{4\pi} \int d\hat{p}_1 \tilde{f}^2(p_1(-T))
  \]
- On average, the rotation in an isotropic random rotation in the turbulent magnetic field leaves an isotropic distribution on a sphere invariant:
  \[
  \langle \xi(0) \rangle = \frac{1}{4\pi} \int d\hat{p}_1 \tilde{f}^2(p_1)
  \]
  \[
  \rightarrow \text{The weighted sum of } \langle C_\ell \rangle \text{'s remains constant:}
  \]
  \[
  \frac{1}{4\pi} \sum_{\ell \geq 0} (2\ell + 1) \tilde{C}_\ell = \frac{1}{4\pi} \sum_{\ell \geq 0} (2\ell + 1) \langle C_\ell(T) \rangle
  \]
Evolution Model

- Diffusion theory motivates that each $\langle C_\ell \rangle$ decays exponentially with an effective relaxation rate

$$\nu_\ell \propto L^2 \propto \ell(\ell + 1)$$

- A linear $\langle C_\ell \rangle$ evolution equation with generation rates $\nu_{\ell \rightarrow \ell'}$ requires:

$$\partial_t \langle C_\ell \rangle = -\nu_\ell \langle C_\ell \rangle + \sum_{\ell' \geq 0} \nu_{\ell' \rightarrow \ell} \frac{2\ell' + 1}{2\ell + 1} \langle C_{\ell'} \rangle \quad \text{with} \quad \nu_\ell = \sum_{\ell' \geq 0} \nu_{\ell' \rightarrow \ell}$$

- For $\nu_\ell \simeq \nu_{\ell \rightarrow \ell+1}$ and $\tilde{C}_\ell = 0$ for $\ell \geq 2$ this has the analytic solution:

$$\langle C_\ell \rangle(T) \simeq \frac{3\tilde{C}_1}{2\ell + 1} \prod_{m=1}^{\ell-1} \nu_m \sum_n \prod_{p=1(\neq n)}^{\ell} \frac{e^{-T\nu_n}}{\nu_p - \nu_n}$$

- For $\nu_\ell \simeq \ell(\ell + 1)\nu$ we arrive at a finite asymptotic ratio:

$$\lim_{T \to \infty} \frac{\langle C_\ell \rangle(T)}{\langle C_1 \rangle(T)} \sim \frac{18}{(2\ell + 1)(\ell + 2)(\ell + 1)}$$
Comparison with CR Data

\[
\lim_{T \to \infty} \frac{\langle C_\ell \rangle(T)}{\langle C_1 \rangle(T)} \approx \frac{18}{(2\ell + 1)(\ell + 2)(\ell + 1)}
\]

[MA’14; updated with HAWC data]
Local Description: Relative Scattering

- evolution of $C_\ell$’s: 

$$ \partial_t \langle C_\ell \rangle = -\frac{1}{2\pi} \int d\hat{p}_1 \int d\hat{p}_2 P_\ell(\hat{p}_1 \hat{p}_2) \langle (p_1 \nabla f_1 + i\omega L f_1) f_2 \rangle $$

- large-scale dipole anisotropy gives an effective “source term”:

$$ -\frac{1}{2\pi} \int d\hat{p}_1 \int d\hat{p}_2 P_\ell(\hat{p}_1 \hat{p}_2) \langle (p_1 \nabla f_1) f_2 \rangle \rightarrow Q_1 \delta_\ell_1 $$

- BGK-like Ansatz for scattering term ($\langle i\omega L f \rangle \rightarrow -\frac{\nu}{2} L^2 \langle f \rangle$) [Bhatnagara, Gross & Krook’54]

$$ -\frac{1}{2\pi} \int d\hat{p}_1 \int d\hat{p}_2 P_\ell(\hat{p}_1 \hat{p}_2) \langle i\omega L f_1 \rangle f_2 \rightarrow \frac{1}{2\pi} \int d\hat{p}_1 \int d\hat{p}_2 P_\ell(\hat{p}_1 \hat{p}_2) \tilde{\nu}(\hat{p}_1 \hat{p}_2) L^2 \langle f_1 f_2 \rangle $$

- Note that $\tilde{\nu}(1) = 0$ for vanishing regular magnetic field.

$$ \tilde{\nu}(x) \simeq \nu_0 (1 - x)^p $$
Cosmic Ray Dipole Anisotropy

- cosmic-ray (CR) arrival directions described by \textbf{phase-space distribution}

\[ f(t, r, p) = \phi(t, r, p)/(4\pi) + 3\hat{p}\Phi(t, r, p)/(4\pi) + \ldots \]

- monopole

- dipole

- local CR spectral density [GeV\(^{-1}\)cm\(^{-3}\)]

\[ n(p) = p^2\phi(t, r_{\oplus}, p) \propto p^{-\Gamma_{CR}} \]

\[ \propto p^{-(\Gamma_{CR}+2)} \]

- in the absence of sources, follows Liouville’s equation (\(\dot{f} = 0\))

\[ \partial_t \phi \simeq \nabla_r (K \nabla_r \phi) \] \textit{diffusion equation}

\[ \Phi \simeq -K \nabla_r \phi \] \textit{Fick’s law}

- diffusion tensor \(K\):

\[ K_{ij} = \kappa_{||}\hat{B}_i\hat{B}_j + \kappa_{\perp}(\delta_{ij} - \hat{B}_i\hat{B}_j) + \kappa_A \epsilon_{ijk}\hat{B}_k \]

- \textbf{dipole anisotropy:} \(\delta = 3K \cdot \nabla_r \ln n\)
Compton-Getting Effect

- phase-space distribution is **Lorentz-invariant**
  
  \[ f^*(p^*) = f(p) \]

- consider **relative motion of observer** (\( \beta = \frac{v}{c} \)) in plasma rest frame (\( \star \)):
  
  \[ p^* = p + p\beta + \mathcal{O}(\beta^2) \]

- Taylor expansion:
  
  \[ f(p) \simeq f^*(p) + (p^* - p)\nabla_{p^*}f^*(p) + \mathcal{O}(\beta^2) \simeq f^*(p) + p\beta\nabla_{p^*}f^*(p) + \mathcal{O}(\beta^2) \]

→ splitting in \( \phi \) and \( \Phi \) is **not invariant**:

\[ \phi = \phi^* \quad \text{and} \quad \Phi = \Phi^* + \frac{1}{3}\beta \frac{\partial \phi^*}{\partial \ln p} \]

- remember: \( \phi \sim p^{-2n_{\text{CR}}} \propto p^{-2-\Gamma_{\text{CR}}} \)

\[ \delta = \delta^* + (2 + \Gamma_{\text{CR}})\beta \]

Compton-Getting effect