Non-Zero Velocity Effects in Dielectric Haloscopes

Alex Millar on behalf of the MADMAX Working Group

SFB 1258 | Neutrinos Dark Matter Messengers

Max-Planck-Institut für Physik (Werner-Heisenberg-Institut)

Dark Matter Axions

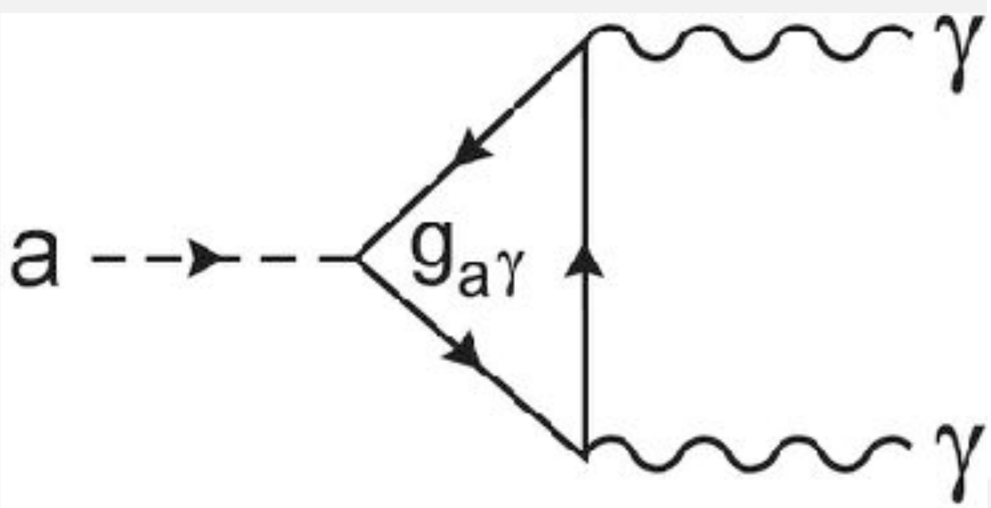
- The axion is highly non-relativistic, but not stationary.
- As the axion acts as a classical field, the axion's velocity gives a spacial variation of the axion's phase - physically large detectors may be sensitive to this change.
- The goal of MADMAX is to build a device that can be physically much larger than a traditional cavity… do we need to worry about the axions velocity?

Axion-electrodynamics

- Axions and ALPs interact with photons through an anomaly term

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - J^{\mu}A_{\mu} + \frac{1}{2}\partial_{\mu}a\partial^{\mu}a - \frac{1}{2}m_a^2a^2 - \frac{g_{a\gamma}}{4}F_{\mu\nu}\widetilde{F}^{\mu\nu}a,$$

- This coupling is tiny, but still important



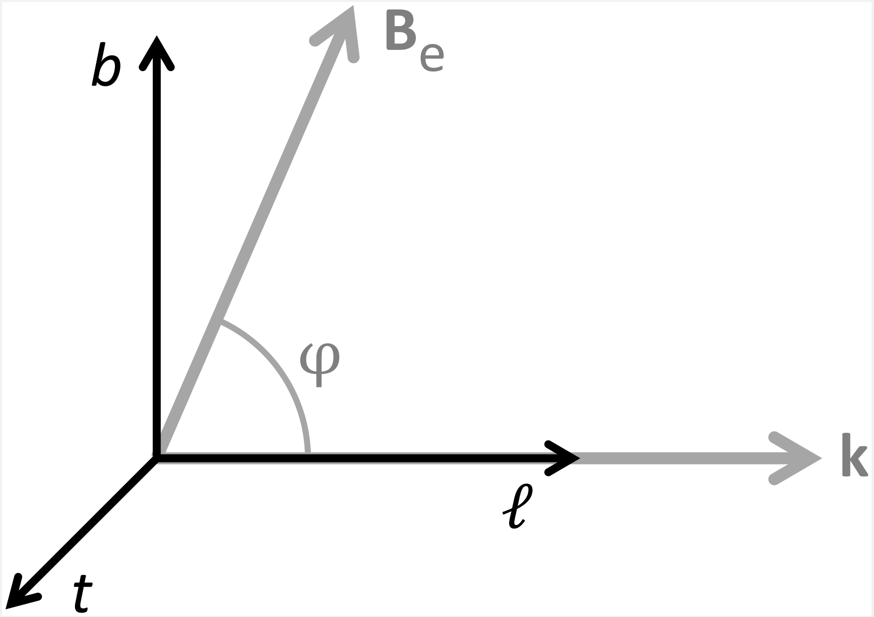
$$m_a = 5.70(7)\,\mu\text{eV}\,\frac{10^{12}\text{GeV}}{f_a}\,;$$

$$g_{a\gamma} = \frac{\alpha}{2\pi f_a}C_{a\gamma} = 2.04(3)\times 10^{-16}\ \text{GeV}^{-1}\,\frac{m_a}{\mu\text{eV}}C_{a\gamma}\,;$$

$$C_{a\gamma} = \frac{E}{N} - 1.92(4)\,,$$

Axion-Photon Mixing

- By providing an external magnetic field, we can induce a mixing between axions and photons.
- Alignment of the axion with the magnetic field matters



$$\begin{pmatrix} \omega^2 - k^2 - m_a^2 & \Delta_b \omega & \Delta_\ell \omega & 0 \\ \Delta_b \omega & \epsilon\omega^2 - k^2/\mu & 0 & 0 \\ \Delta_\ell \omega & 0 & \epsilon\omega^2 & 0 \\ 0 & 0 & 0 & \epsilon\omega^2 - k^2/\mu \end{pmatrix} \begin{pmatrix} a \\ iA_b \\ iA_\ell \\ iA_t \end{pmatrix} = 0$$

$$\Delta_b = g_{a\gamma} B_{\mathrm{e}} \sin\varphi \text{ and } \Delta_\ell = g_{a\gamma} B_{\mathrm{e}} \cos\varphi.$$

Axion-Photon Mixing

- Ignoring any dynamical longitudinal photon modes (i.e., longitudinal plasmons), this mixing gives rise to a "photon-like" state and an "axion-like" state,

$$\text{Axion like:} \qquad \begin{pmatrix} a \\ iA_b \\ iA_\ell \end{pmatrix} = \begin{pmatrix} 1 \\ -\chi_b \mu \sin\varphi \\ -\chi_\ell \mu \cos\varphi \end{pmatrix} + \mathcal{O}(\chi^2),$$

$$\text{Photon like:} \qquad \begin{pmatrix} a \\ iA_b \\ iA_\ell \end{pmatrix} = \begin{pmatrix} \chi_b \sin\varphi \\ 1 \\ 0 \end{pmatrix} + \mathcal{O}(\chi^2),$$

$$\chi_b = \frac{g_{a\gamma} B_e \, \omega}{n^2\omega^2 - \omega^2 + m_a^2},$$

$$\chi_\ell = \frac{g_{a\gamma} B_e \, \omega}{n^2\omega^2}.$$

Axion-Photon Mixing

- The “axion-like” mass eigenstate means that the axion gives rise to a small E field aligned with the magnetic field

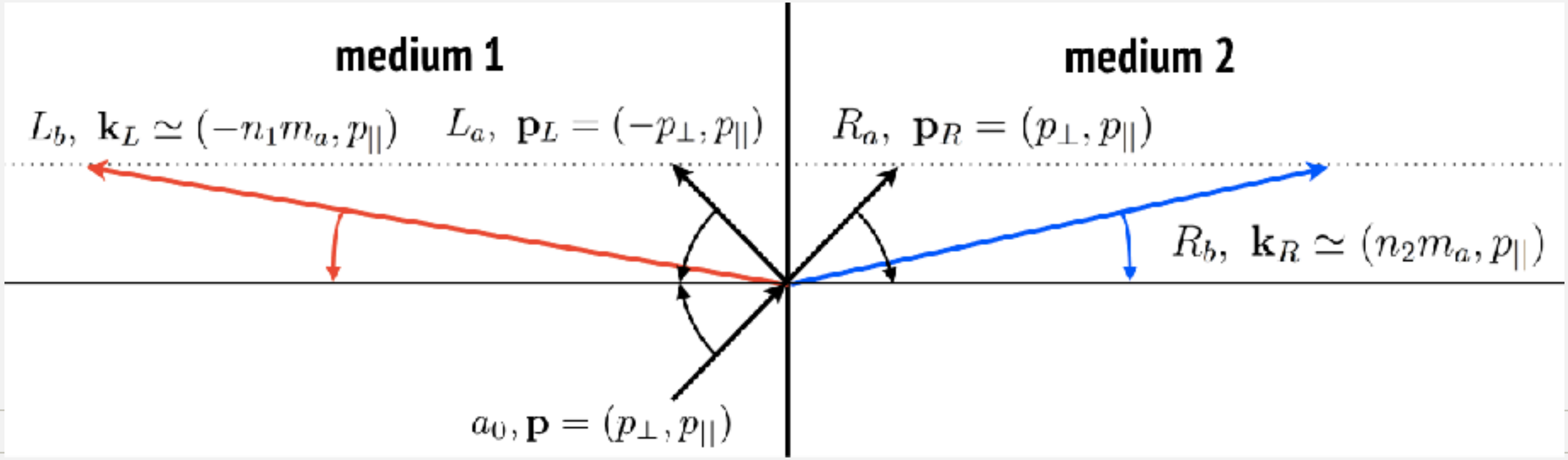
$$\mathbf{E}_a(t,\mathbf{x}) = -\boldsymbol{\nabla} A_0 - \dot{\mathbf{A}} = -\chi\mu\omega\, a(t,\mathbf{x})\hat{\mathbf{B}}_e = -\frac{g_{a\gamma}\mathbf{B}_e}{\epsilon}\, a(t,\mathbf{x}) \,,$$

- The axion now has a very transverse small H field, suppressed by the velocity.

$$\mathbf{H}_a(t,\mathbf{x}) = \frac{1}{\mu}\nabla \times \mathbf{A} = -v\chi\omega a(t,\mathbf{x})\hat{\mathbf{t}}_a = -\frac{v g_{a\gamma} B_e}{\mu\epsilon} a(t,\mathbf{x})\hat{\mathbf{t}}_a .$$

Single Interface

- As in the zero velocity case, the axion induced E and H fields experience a discontinuity when encountering a change in dielectric media.
- Maxwell's equations imply that the total parallel E and H field must be continuous — regular EM waves must be emitted. See arXiv:1307.7181 for more on dish antennas.



Single Interface

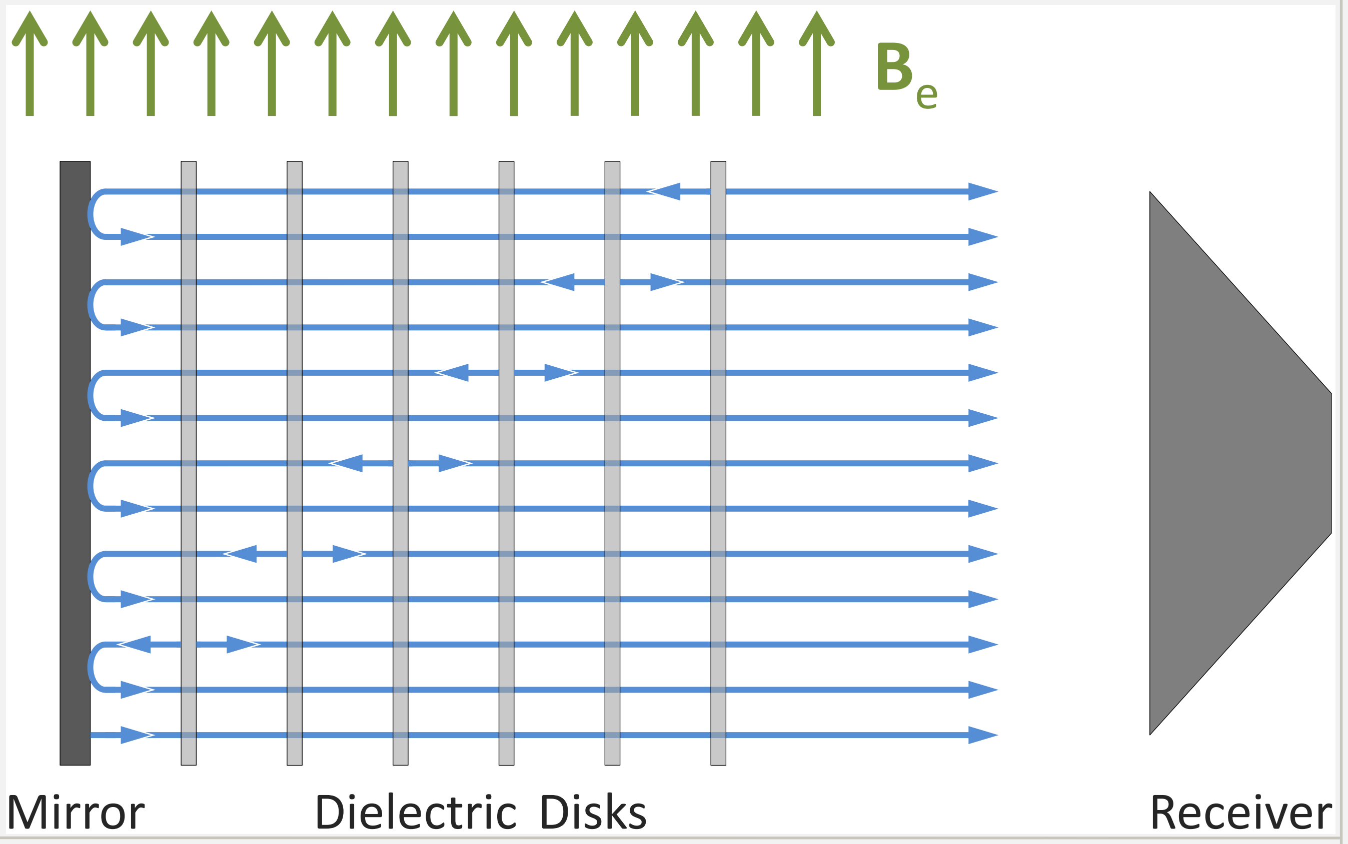
- Assuming that the magnetic field and interface are parallel, then the emitted waves are given by

$$L = (E_2^a - E_1^a)\frac{n_2/\mu_2}{(n_1/\mu_1 + n_2/\mu_2)} - \frac{v_x}{v}(H_2^a - H_1^a)\frac{1}{(n_1/\mu_1 + n_2/\mu_2)},$$
$$R = -(E_2^a - E_1^a)\frac{n_1/\mu_1}{(n_1/\mu_1 + n_2/\mu_2)} - \frac{v_x}{v}(H_2^a - H_1^a)\frac{1}{(n_1/\mu_1 + n_2/\mu_2)},$$

- The first term is the same as for the zero-velocity case, but now the H-field of the axion also contributes.

Dielectric Haloscopes

- The idea of MADMAX is to enhance the conversion of axions to photons by using many dielectric disks — a dielectric haloscope.
- This means that we will need to worry about the change of phase of the axion over the haloscope.



Transfer matrix formalism

- Encode every interface and distance as a matrix
- Add in a new source term at each interface to account for the axions, but with a change in phase from the axion velocity along the haloscope

$$\begin{pmatrix} R \\ L \end{pmatrix}_m = \mathsf{T} \begin{pmatrix} R \\ L \end{pmatrix}_0 + a_0 \chi \mathsf{M} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\mathsf{G}_r = \frac{1}{2n_{r+1}} \begin{pmatrix} n_{r+1} + n_r & n_{r+1} - n_r \\ n_{r+1} - n_r & n_{r+1} + n_r \end{pmatrix},$$

$$\mathsf{P}_r = \begin{pmatrix} e^{i\delta_r} & 0 \\ 0 & e^{-i\delta_r} \end{pmatrix},$$

$$\mathsf{S}_r = e^{i v_x \omega x_{r+1}} \frac{\chi_{r+1} - \chi_r}{2\chi} \begin{pmatrix} 1 + v_x/n_{r+1} & 0 \\ 0 & 1 - v_x/n_{r+1} \end{pmatrix}.$$

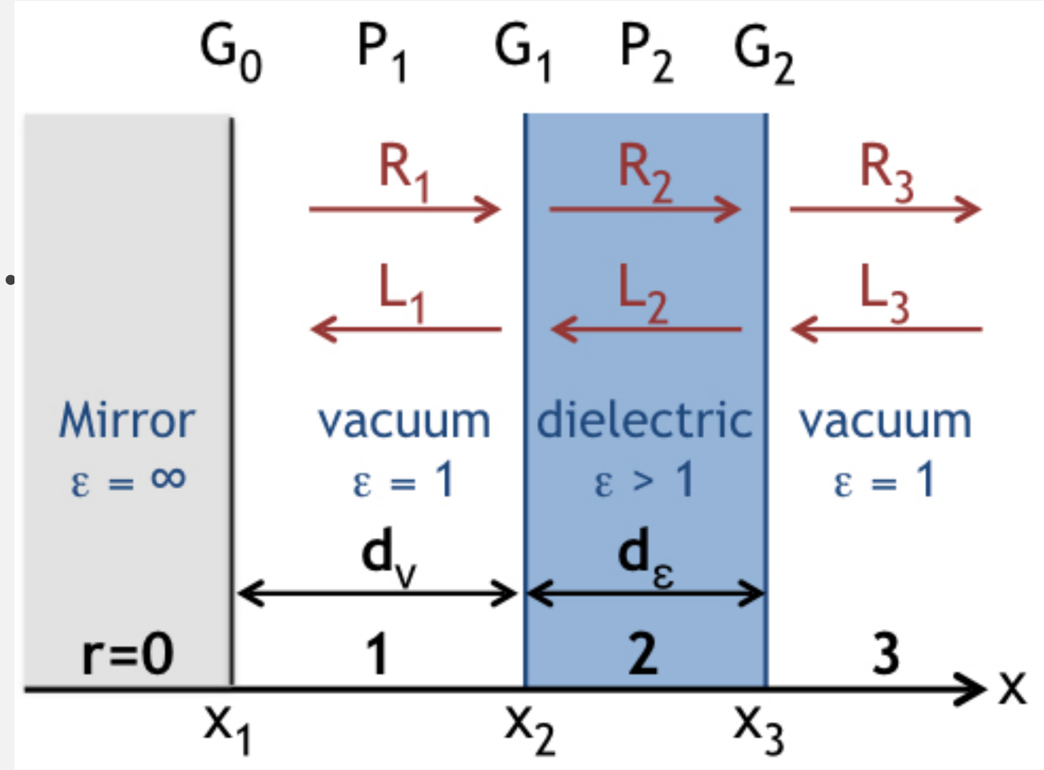
$$\mathsf{M} = \sum_{s=1}^{m} \mathsf{T}_s^m \mathsf{S}_{s-1}$$

$$\chi = \frac{g_{a\gamma} B_e}{n^2 m_a}$$

$$\mathsf{T}_b^a = \mathsf{G}_{a-1} \mathsf{P}_{a-1} \mathsf{G}_{a-2} \mathsf{P}_{a-2} \ldots \mathsf{G}_{b+1} \mathsf{P}_{b+1} \mathsf{G}_b \mathsf{P}_b$$

Simple Resonant Cavity

- Simple analytic case: mirror and single disk arranged for a resonance.
- If the disk spacing is small, no significant velocity effects regardless of resonant conditions.



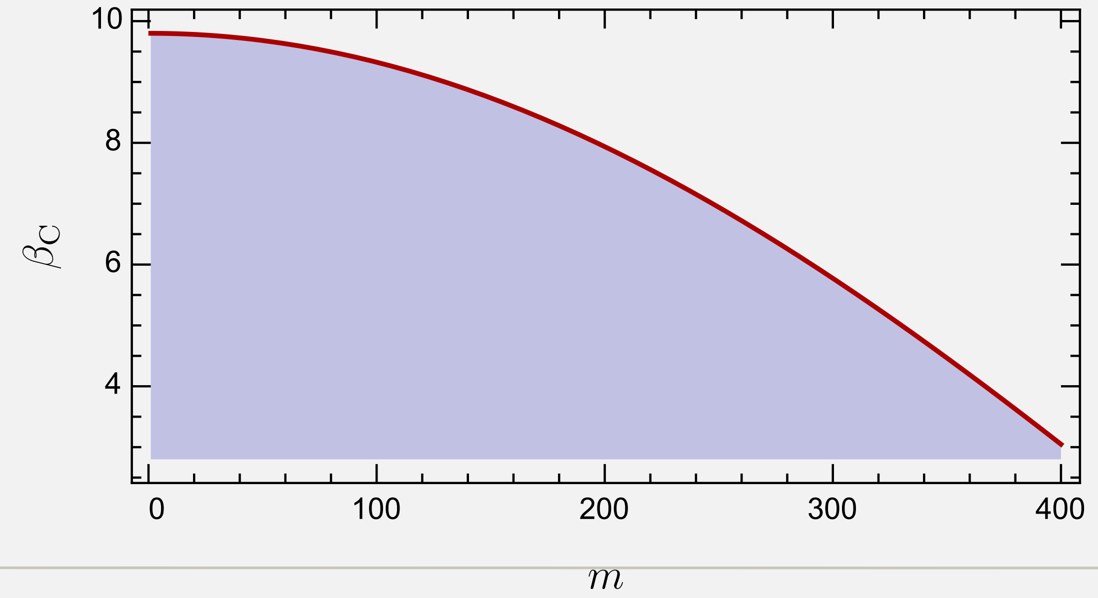
$$\beta_C \sim n\left(2 - \frac{\pi^2 v^2}{4}\right)$$

Simple Resonant Cavity

- What happens if we increase the disk spacing?

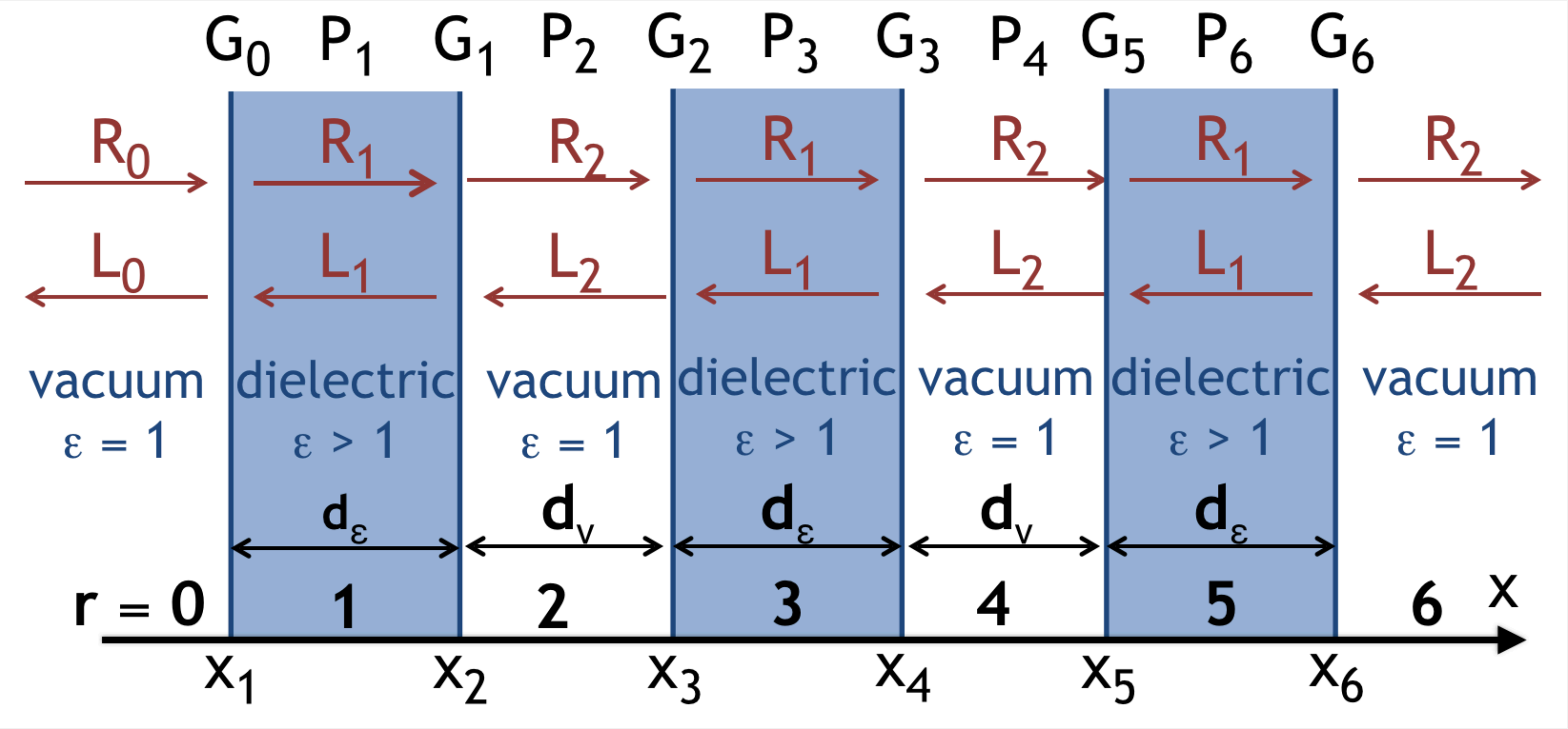
$$\mathcal{B}_C = n\left(e^{\frac{i\pi v}{2n}} - e^{\frac{i\pi(v+2n(2m-1)(1+v))}{2n}}\right) - \frac{ie^{\frac{i\pi v}{2n}}}{n} - v\left(1 - \frac{1}{n^2}\right)$$

- Velocity effects start becoming important around 15-20% of the axion's de Broglie wavelength
- For v~10^{-3} the Compton wavelength is smaller by a factor of 1000



Transparent Mode

- Nice analytic test bed for studying effects in dielectric haloscopes
- Equally spaced disks, each with a half wavelength thickness and spacing gives a transparent device, so the waves generated at each interface simply add.



Transparent Mode

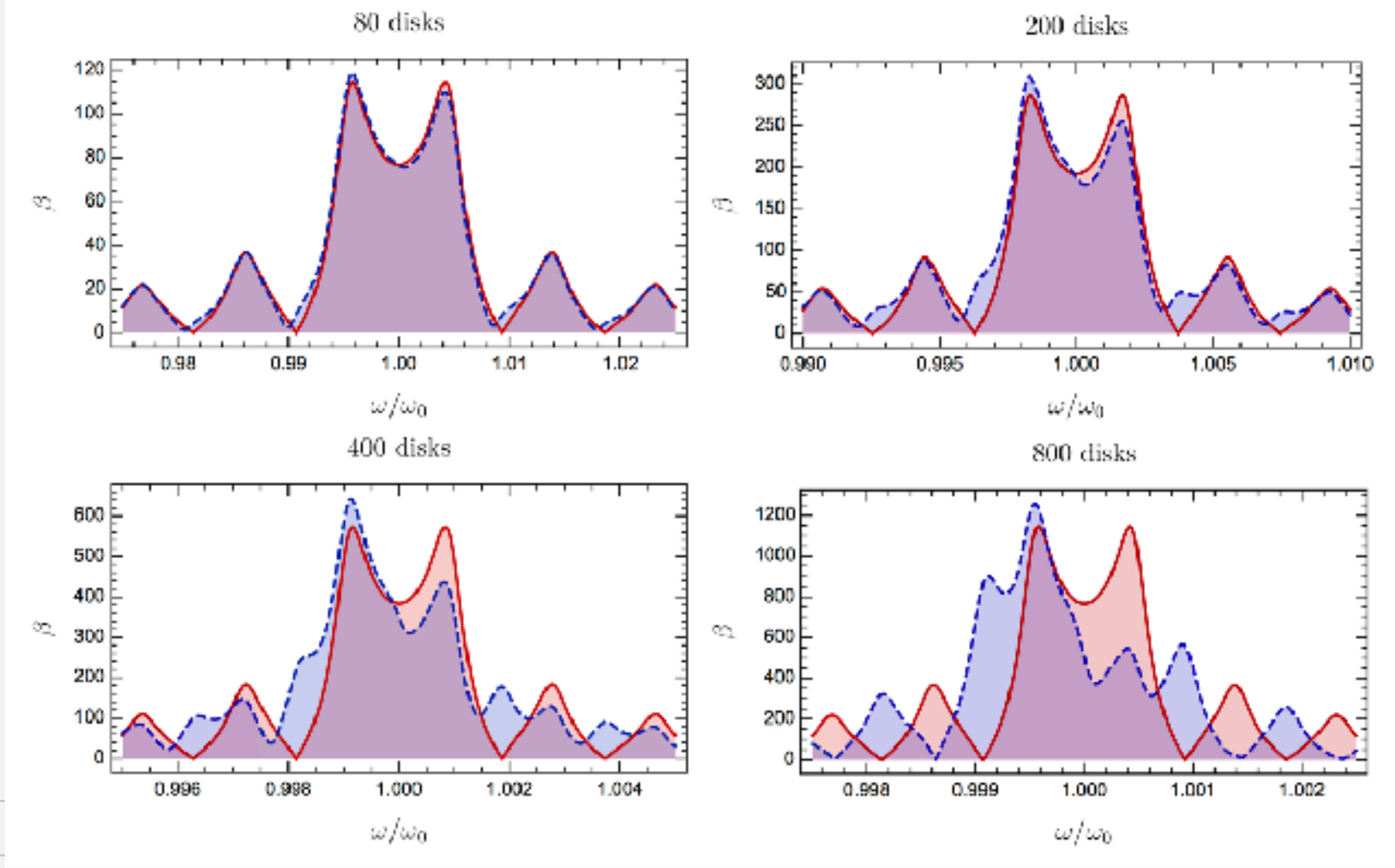
- At the transparent frequency, we can find a closed form for the boost amplitude for N disks.

$$\mathcal{B}_{\rm N} = N\frac{1-v}{2}\left(1-\frac{1}{n^2}\right)\frac{\left(1+e^{\frac{i\pi v}{n}}\right)\left(-1+e^{\frac{i(1+n)N\pi v}{n}}\right)}{-1+e^{\frac{i(1+n)N\pi v}{n}}}.$$

- As the phases come in with factors of Nv, even though the axion velocity is small these can become very significant effects for large N.
- Note that N is simply a proxy for the linear distance of the dielectric L~Nƛ/2.

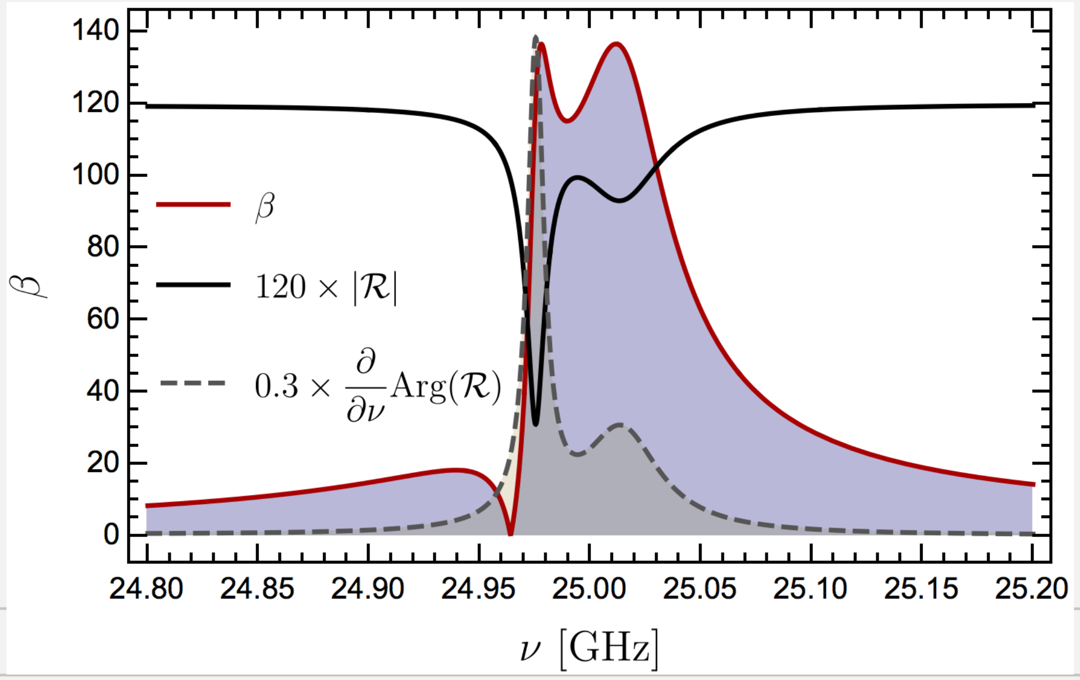
Transparent Mode

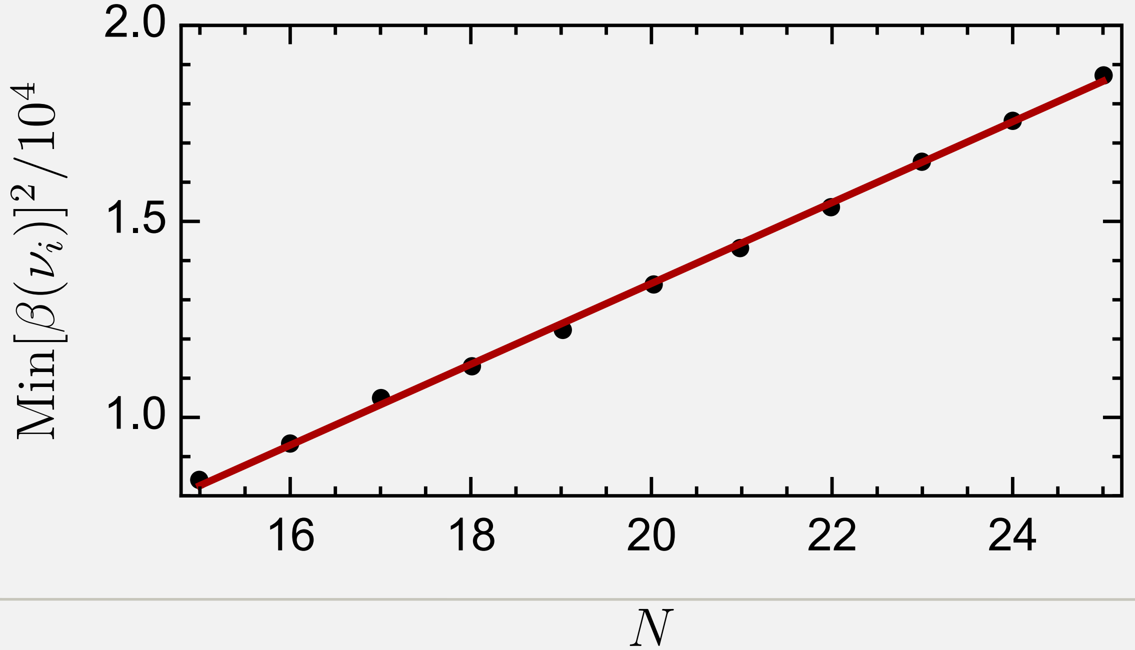
- Velocity effects start to be become important when the haloscope is around 20% of the axion's de Broglie wavelength.



Realistic example: 80 Disks

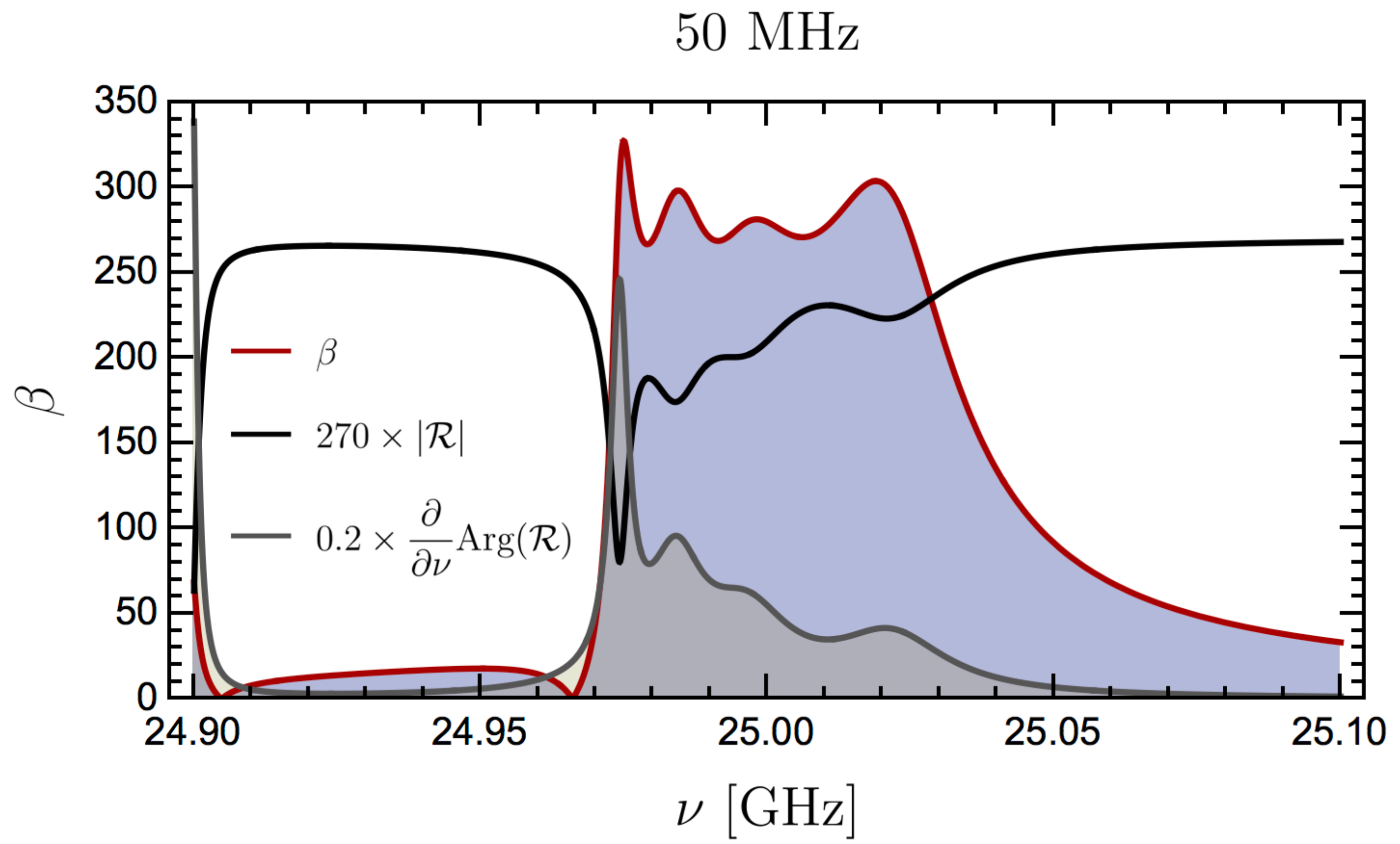
- The transparent mode suggests that the axion's velocity can be neglected for MADMAX, we should check this explicitly.
- Recall: MADMAX is to have ~80 disks and will be optimised for broadband solutions.
- We used the area law to extrapolate from 20 disks to 80 disks: expect β~270.





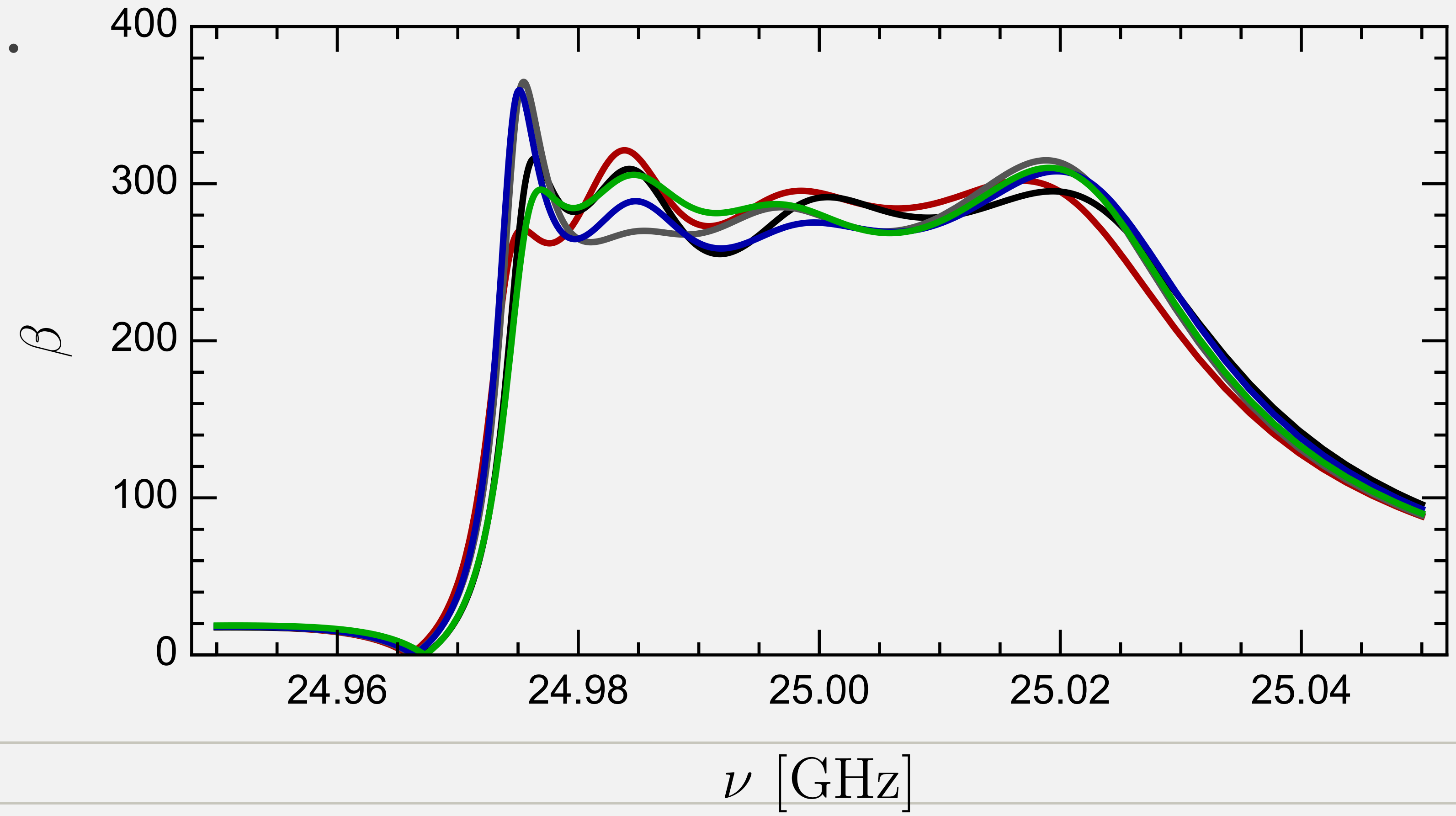
Realistic example: 80 Disks

- This 80 disk solution achieves a power boost within ~5% of expected (~70,000).
- Still a good correlation between the boost factor and reflectivity.



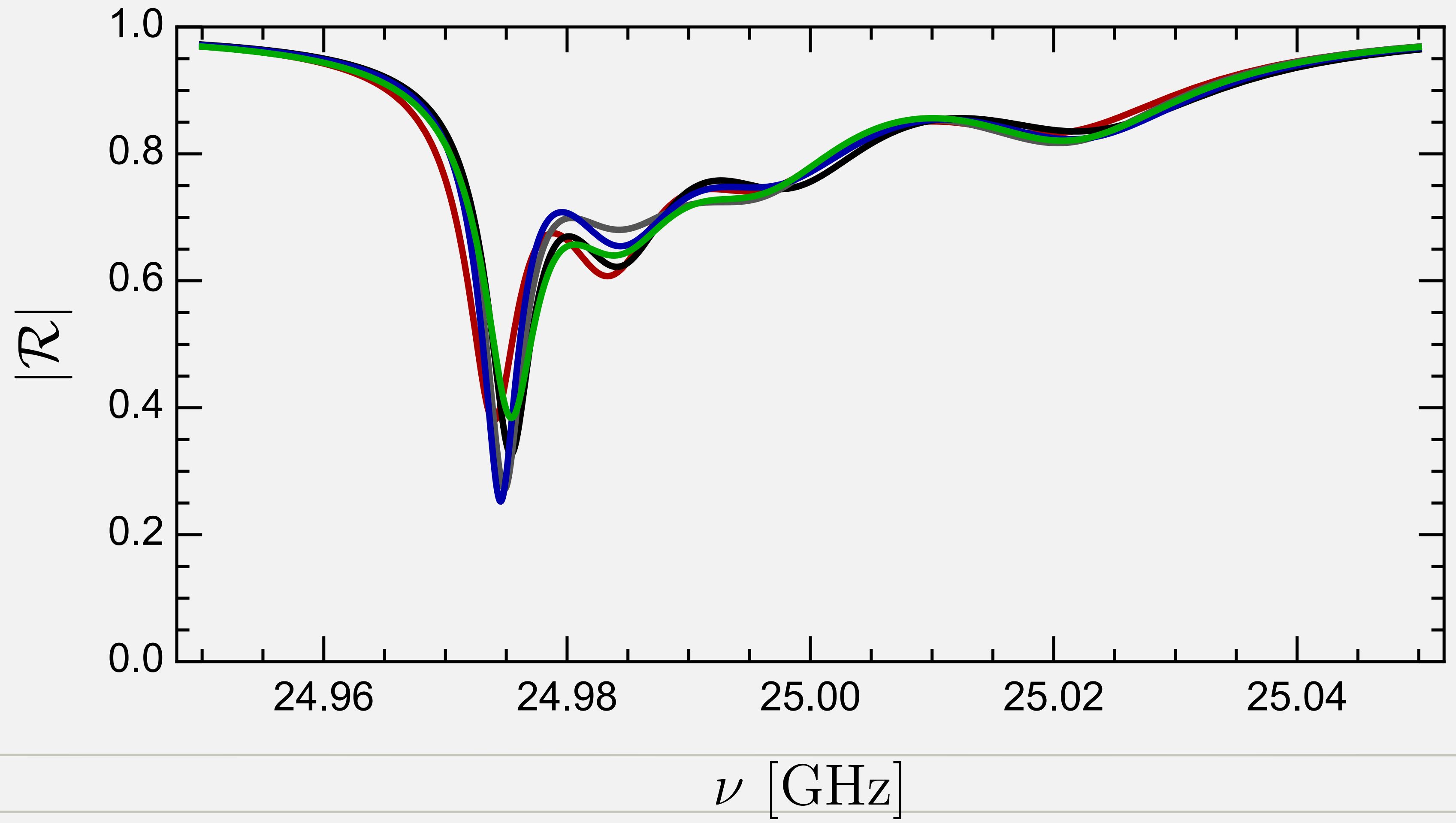
Sensitivity to Error

- With this solution we can make a rough check of the sensitivity to mispositioning errors that we expect.
- Errors of a few μm seem to be tolerable without error correction.



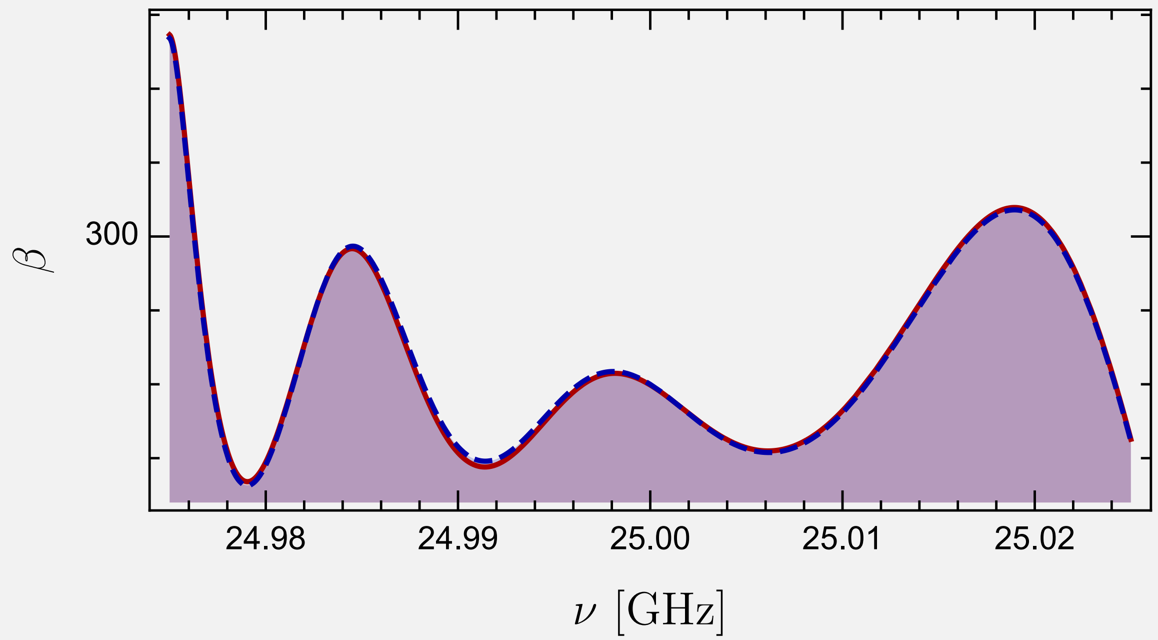
Sensitivity to Error

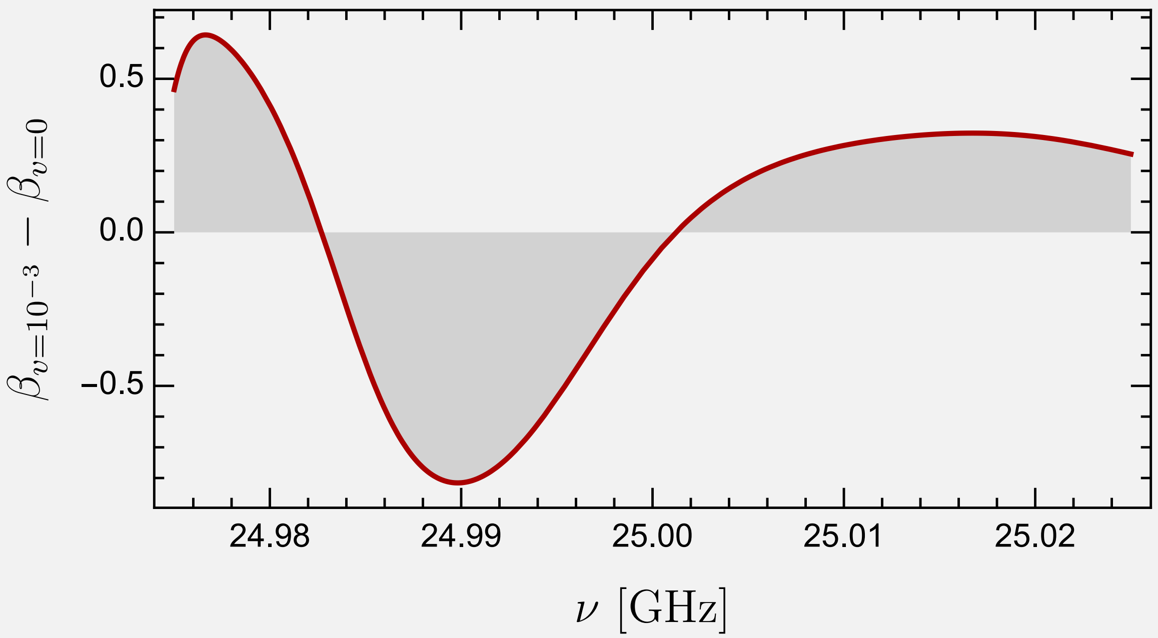
- But mispositioning also effects the reflectivity!
- Should be possible to use this to reject bad positions, and to reconstruct the actual boost factor (see Stefan's talk).



Velocity effects

- The axion velocity is entirely negligible!
- Actually less sensitive than the transparent mode due to a lack of symmetry…



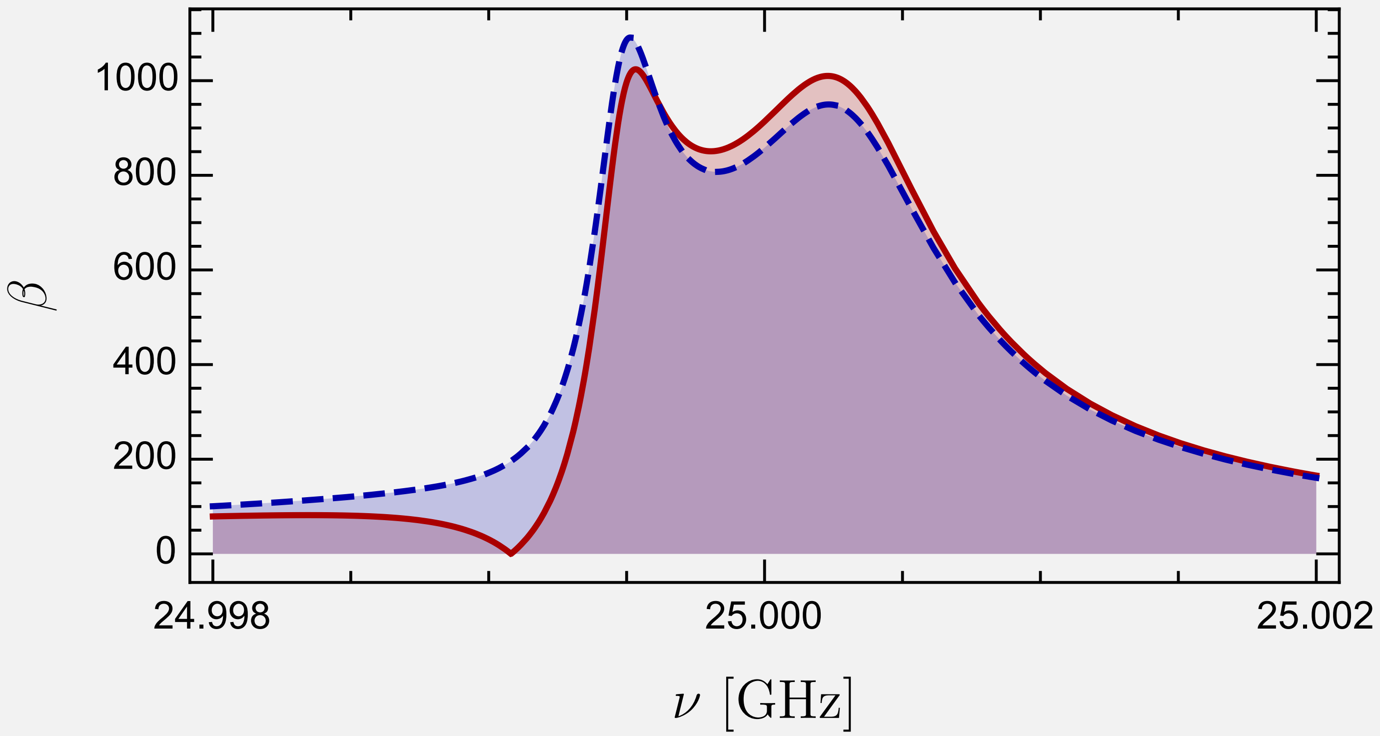


Directional Sensitivity

- Velocity dependence of dielectric haloscope depends primarily on v_x.
- The transverse velocities only enter the axion's frequency, so effect the boost factor at Lv^2 at most (where L is the haloscopes length).
- However, transverse velocities effect the emitted angle of photons, and the phase of the axion across the disk.
- This means that in principal a dielectric haloscope can be made to have directional sensitivity.

Directional Sensitivity

- In the high mass range of MADMAX, much of the magnet's volume would be unused.
- Potentially add disks, or in the event of discovery space the disks out further.



Conclusions

- Existing formalisms can be extended to include the axion’s velocity.
- 1D velocity effects are negligible for devices less than ~20% of the axion’s de Broglie wavelength, such as an 80 disk MADMAX.
- Projections based on ~20 disk solutions seem to be accurate.
- Second generation experiments, or possible high-mass extensions of MADMAX could be in principal sensitive to the axion’s velocity in a directional way, giving a sizeable diurnal modulation.

Non-trivial Velocity Dispersions

- Unless dark matter is discovered, there is significant uncertainty to the velocity dispersion of the axion.
- Power generated is a convolution of the boost factor and axion density,

$$P_{m_a,v_1,v_2} \propto \int_{v_1}^{v_2} v|\mathcal{B}(\omega, v_x)a(m_a, v)|^2 dv$$

$$\beta_{\rm int}(m_a) = \left(\frac{\int_0^\infty v|\mathcal{B}(\omega, v_x)a(m_a, v)|^2 dv}{\int_0^\infty v a(m_a, v)^2 dv}\right)^{1/2}$$