Searching for high density effects in photon induced reactions

Martin Hentschinski
martin.hentschinski@udlap.mx

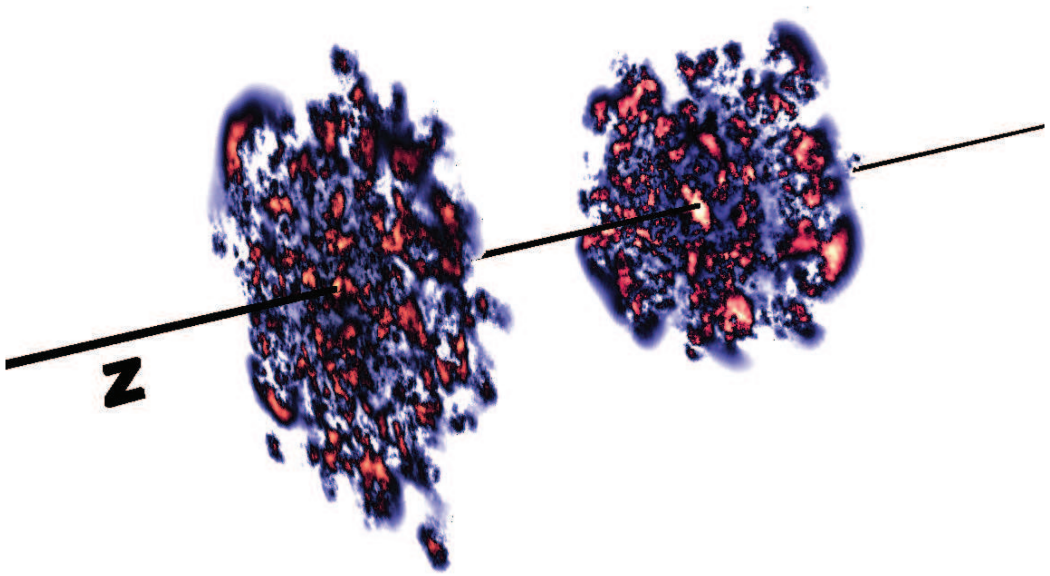
IN COLLABORATION WITH

A. Ayala, J. Jalilian-Marian, M.E. Tejeda Yeomans
and I. Bautista, A. Fernandez Tellez

arXiv:1701.07143/Nucl. Phys. B 920, 232 (2017)
arXiv:1607.05203/Phys.Rev. D94 (2016) no.5, 054002
arXiv:1604.08526/Phys. Lett. B 761, 229 (2016)

2nd International Workshop on QCD Challenges from pp to AA,
October 31 - November 3 2017, Puebla, Mexico

Initial collisions ....



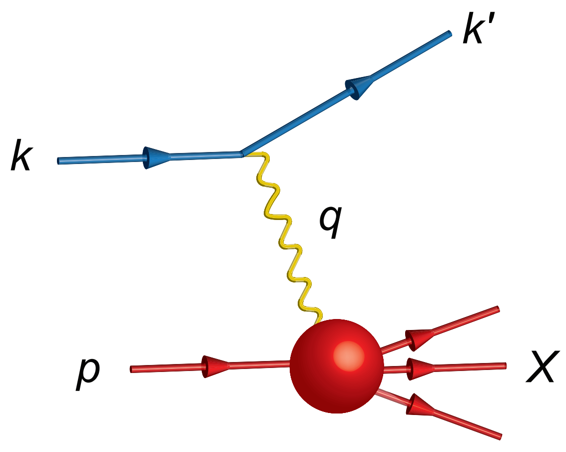
- common believe: AA collisions = collisions of two Color Glass Condensates ....

in general: there is quite some activity, a lot of models, (impressive)(lattice) calculations and much more

question: what do we really know about the validity of this formalism, its applicability etc + how can we improve?

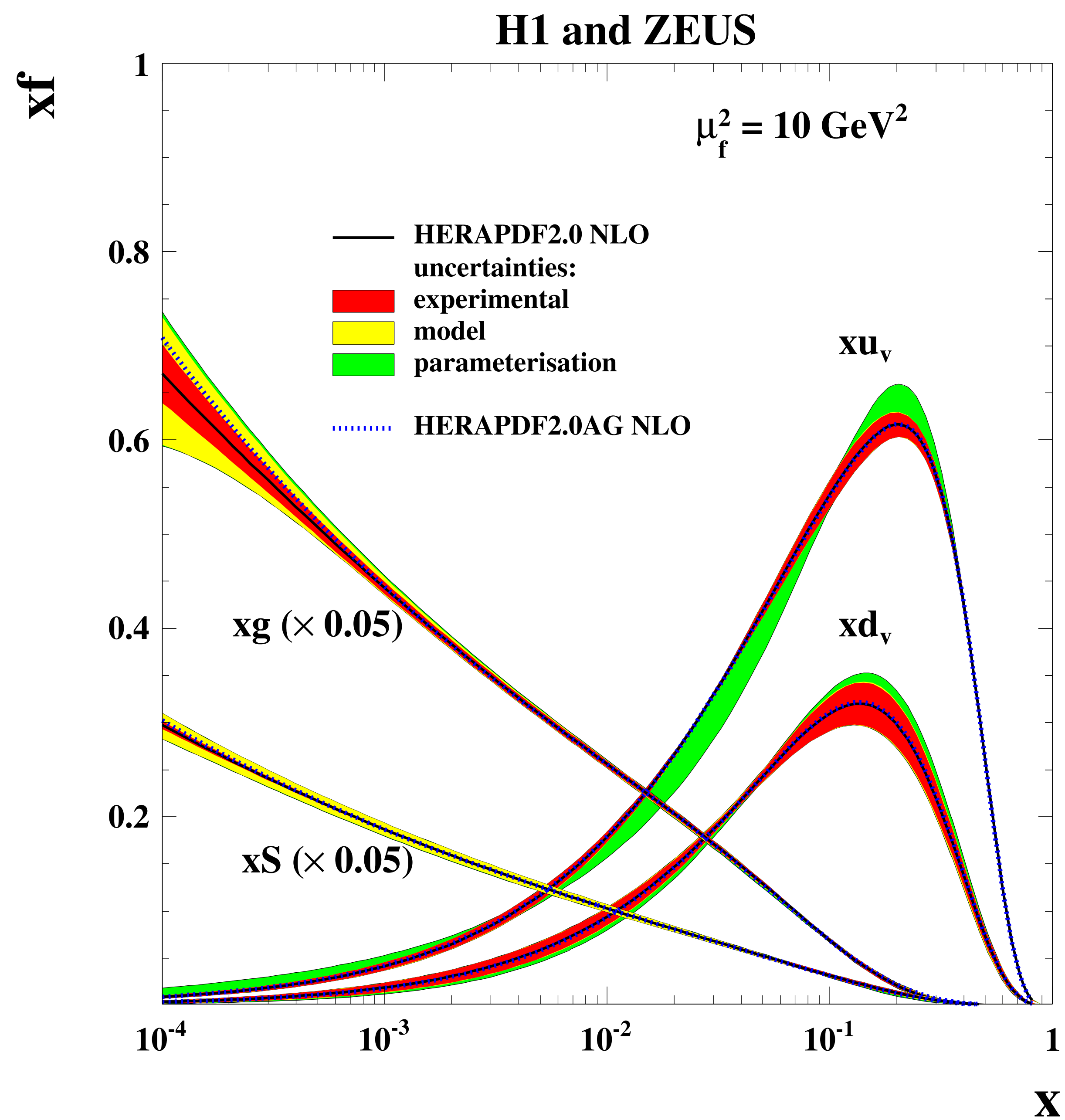
The origins — DIS at HERA: parton distribution functions

HERA collider (92-07): Deep Inelastic Scattering (DIS) of
of electrons on protons



Photon virtuality
$$Q^2 = -q^2$$

Bjorken $$x = \frac{Q^2}{2p \cdot q}$$



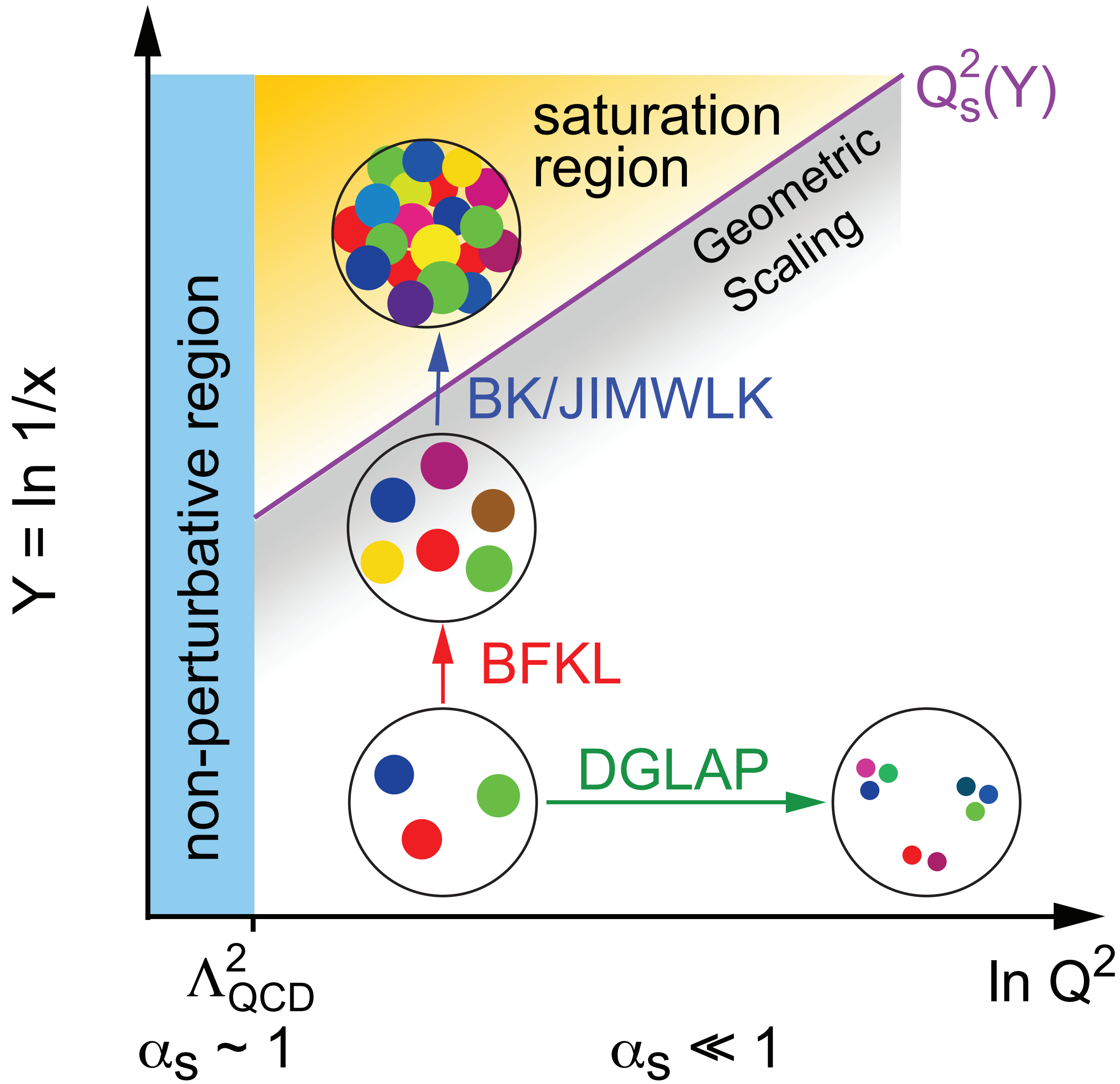
gluon g(x) and sea-quark S(x) distribution like powers ~ $x^{-\lambda}$ for $x \rightarrow 0$

→ invalidates probability interpretation if continued forever (integral over x diverges)

→ at some x, new QCD dynamics must set in

The proton at high energies: saturation

theory considerations:



- effective finite size $1/Q$ of partons at finite Q^2
- at some $x \ll 1$, partons 'overlap' $=$ recominbation effects
- turning it around: system is characterized by saturation scale Q_s
- grows with energy $Q_s \sim x^{-\Delta}$, $\Delta > 0$ & can reach in principle perturbative values $Q_s > 1$GeV

One strong hint: BK evolution and geometrical scaling

$$\frac{\partial}{\partial Y}N(x_{01},Y) = \frac{\overline{\alpha}_s}{2\pi}\int d^2\vec{x}_2\,\frac{x_{01}^2}{x_{02}^2x_{12}^2}\left[N(x_{02},Y)+N(x_{12},Y)-N(x_{01},Y)\right.$$
$$\left. - N(x_{02},Y)N(x_{12},Y)\right]$$

Y.V. Kovchegov, Phys. Rev. D 61 (2000) 074018

- non-linear evolution equation in Y = ln(1/x) for dipole amplitude N;
- low density N ≪1, high density N~1

One strong hint: BK evolution and geometrical scaling

$$f(x,k^2) = \mathcal{F}\left(\frac{k^2}{Q_s^2(x)}\right)$$

$$Q_s(x) = Q_0\left(\frac{x_0}{x}\right)^{\lambda/2}$$

observable a function of

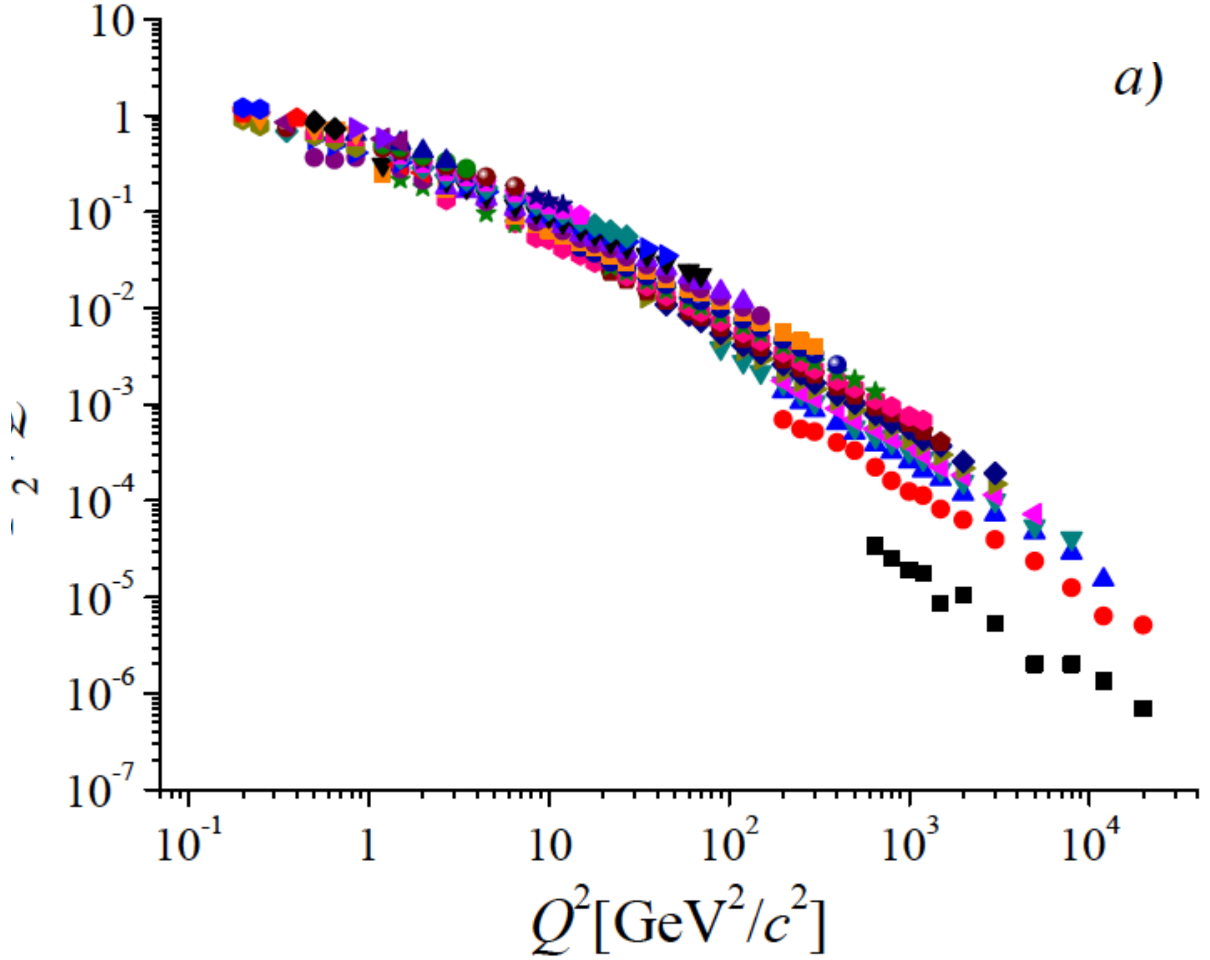
τ = $k^2/Qs^2(x)$

- BK evolution “generates” saturation scale
- “travelling waves” solution to BK evolution equation
- implies: observable which depend only on one scale →geometric scaling

Example process: DIS

Saturation scale: energy and x dependence

$$Q^2_{\mathrm{sat}}(x) = Q_0^2 \left(\frac{x}{x_0}\right)^{-\lambda}$$



A.M. Stasto, K. J. Golec-Biernat,
J. Kwiecinski
PRL 86 (2001) 596-599

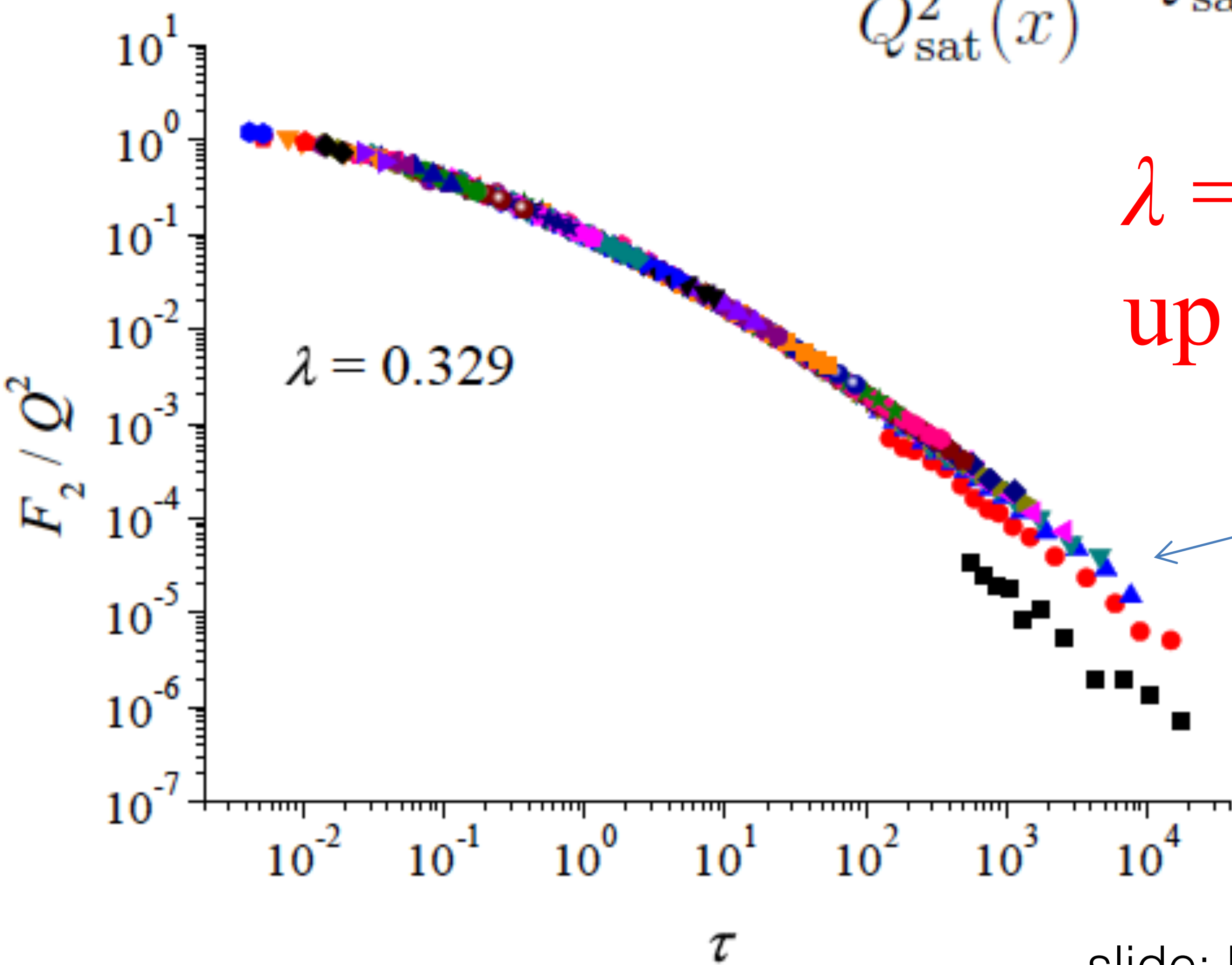
M.Praszalowicz and T.Stebel
JHEP 1303, 090 (2013)
arXiv:1211.5305 [hep-ph]
and
JHEP 1304, 169 (2013)
arXiv:1302.4227 [hep-ph]

slide: Michal Praszalowicz; QCD challenges 2016

Example process: DIS

Saturation scale: energy and x dependence

$$\tau = \frac{Q^2}{Q^2_{\rm sat}(x)} \quad Q^2_{\rm sat}(x) = Q_0^2\left(\frac{x}{x_0}\right)^{-\lambda}$$



$\lambda = 0.329 \pm 0.005$
up to $x = 0.08$ (!)

large x

more "sophisticated" scaling variables do not work well

slide: Michal Praszalowicz; QCD challenges 2016

Phenomenological evidence

- geometric scaling: a property of the BK evolution equation → seen in data ✅

- problem: same data well described by intrinsically dilute framework (= collinear factorization)
  geometric scaling in BK evolution requires non-linear term N~1 → gluon densities are high (we see that), but are they sufficiently high?

- common argument: collinear fits "abuse" their freedom to fix initial conditions at low Q^2 and all x; likely to be true, but need to demonstrate failure of dilute approach

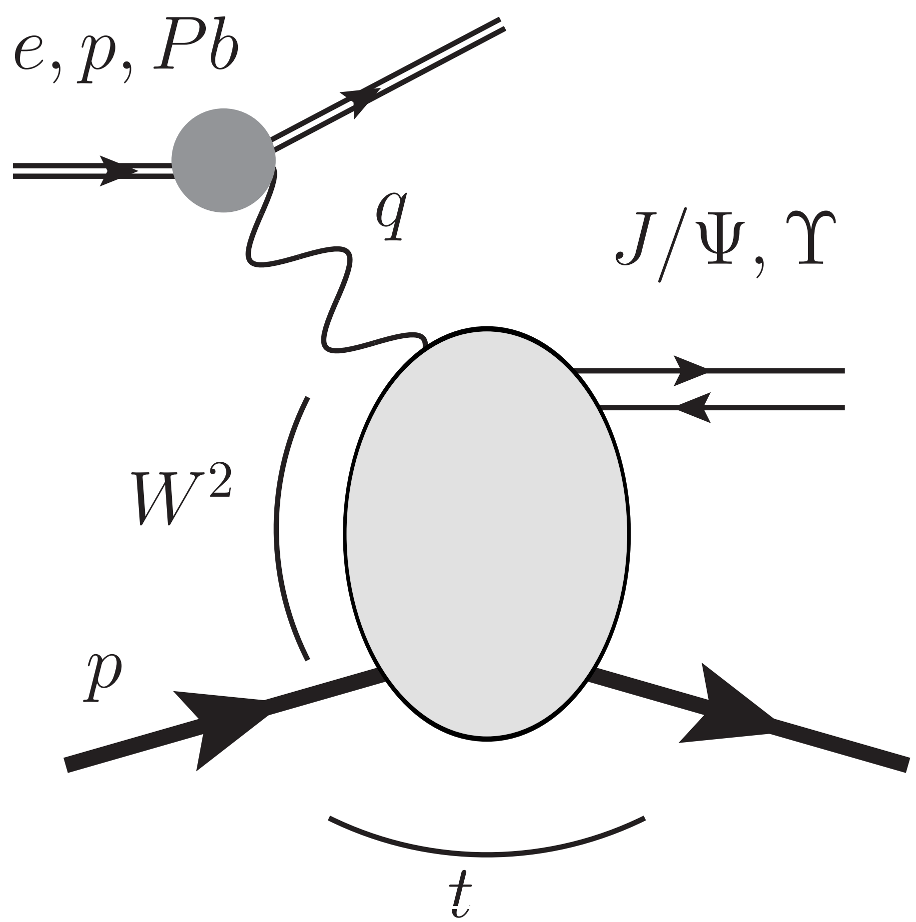
- in general: strong (& convincing) hints, not yet substantial evidence will discuss 2 processes/questions:

- J/Psi and Upsilon production in ultra-peripheral collisions → pA collisions where the nucleus acts as a photon source (+ HERA data)

- Are there processes that can tell us whether we are in a dilute or in a dense regime → tentative yes

exclusive VM production in UPC@LHC

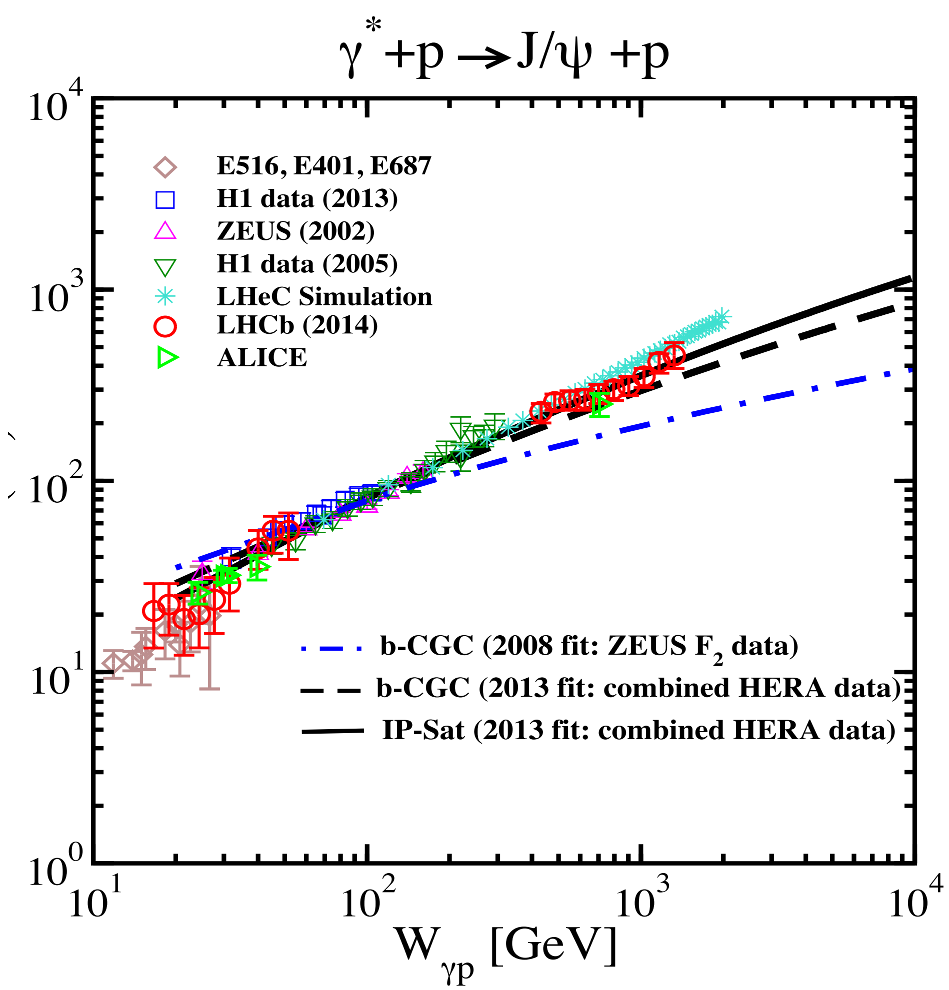
[Bautista, Ferandez-Tellez, MH; 1607.05203]



- measured at HERA (ep) and LHC (pp, ultra-peripheral pPb)
- charm and bottom mass provide hard scale ➡ pQCD
- exclusive process, but allows to relate to inclusive gluon

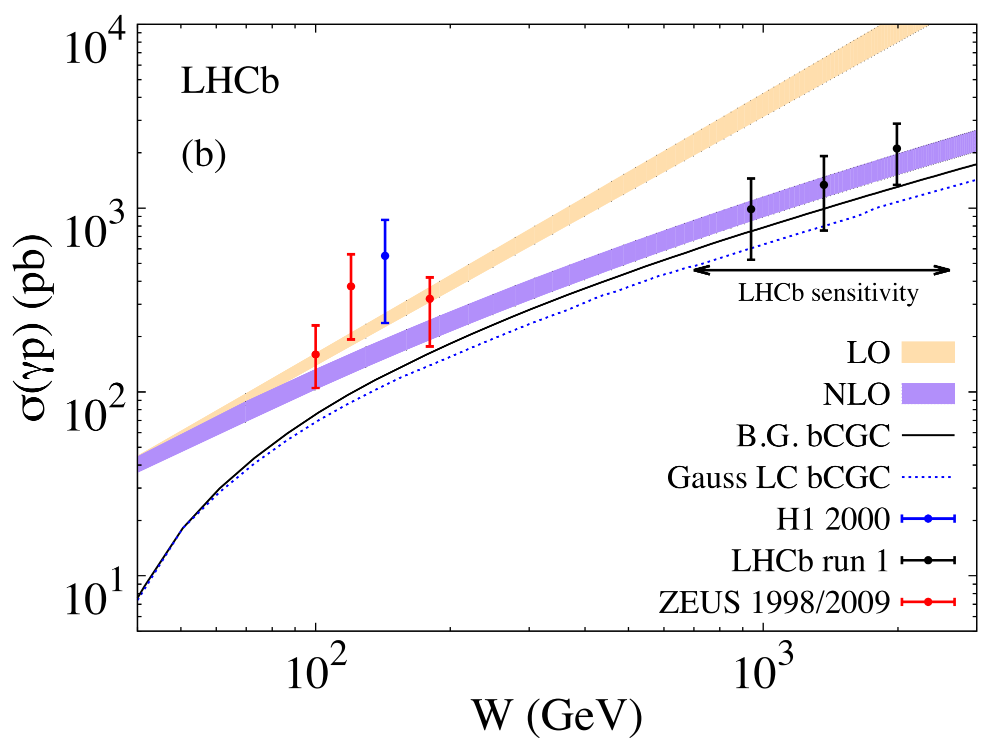
reach values down to $x = 4 \times 10^{-6} \rightarrow$ (unique ?) opportunity to explore the low x gluon

data well described by saturation models

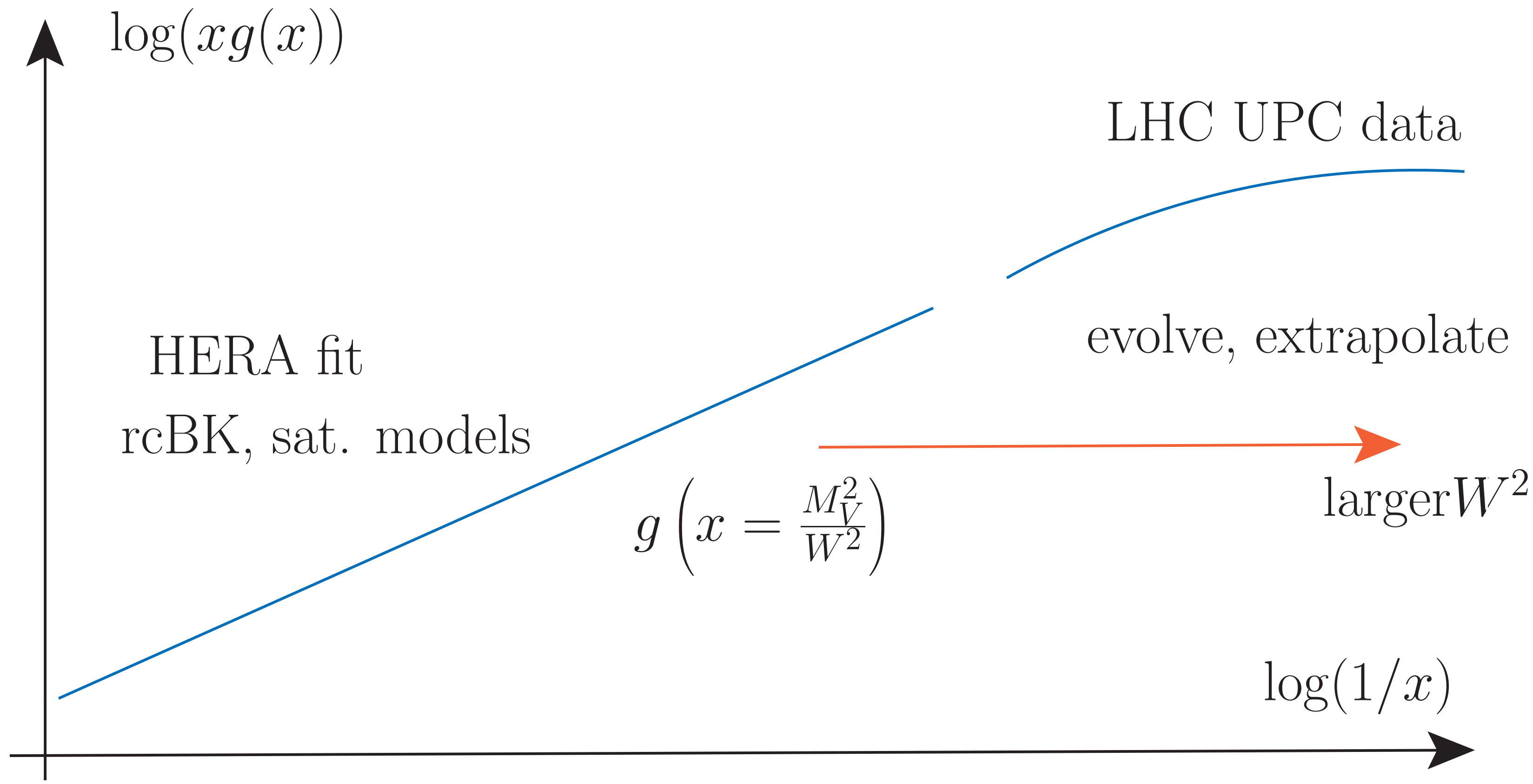


[Armesto, Rezaeian; 1402.4831], [Goncalves, Moreira, Navarra; 1405.6977]

- there exists description based on saturation models (reproduce essential feature of BK + phenomenological corrections)
- And there are DGLAP fits → also work pretty well **[Jones, Martin, Ryskin, Teubner, 1507.06942, 1312.6795]**



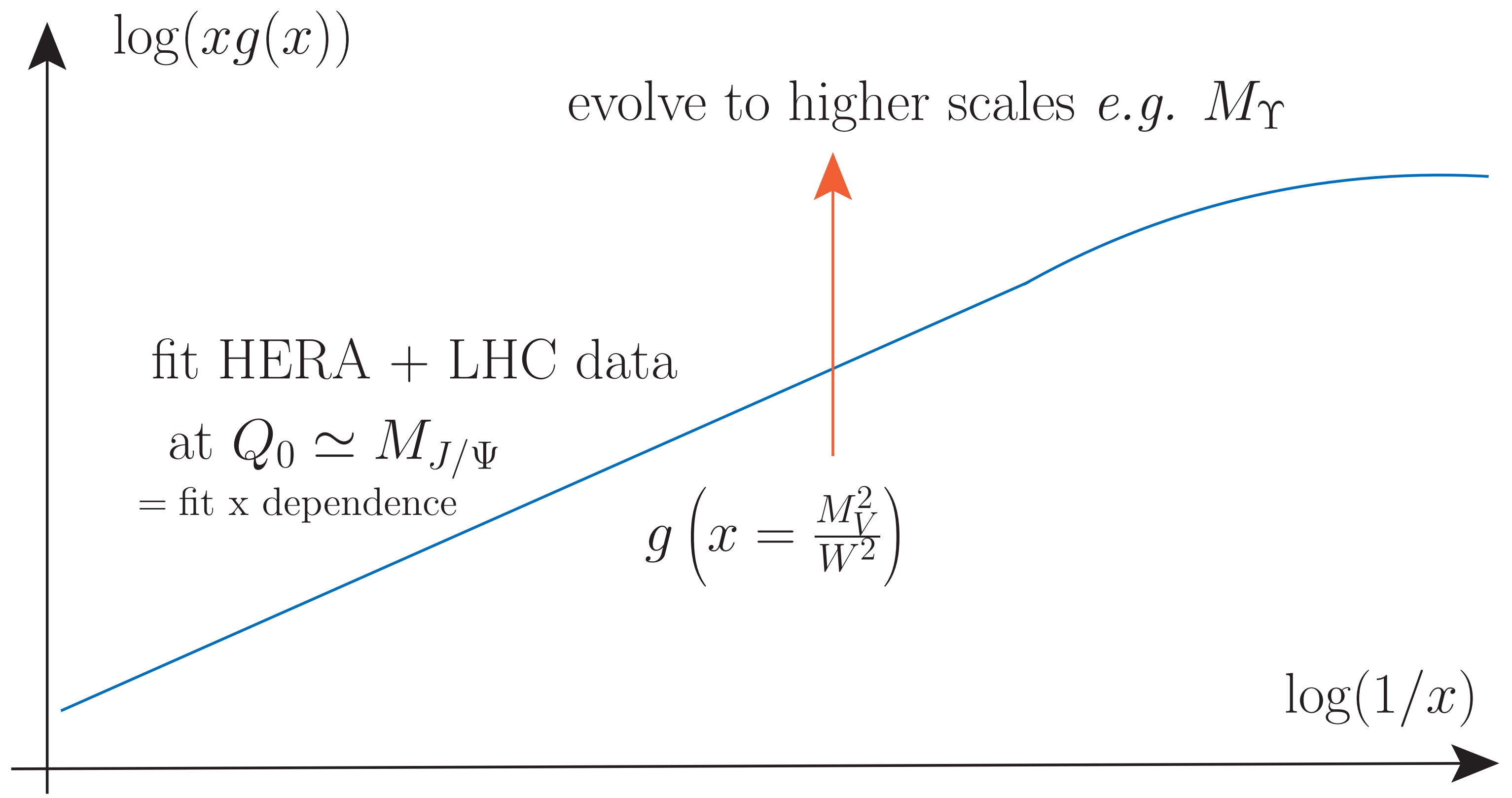
DGLAP vs. saturation (I)



- describes data or not $\rightarrow$ re-fit
- if yes: do we really see saturation effects?

i.e. BK type evolution $$\frac{d}{d\ln 1/x}G(x) = K \otimes G(x) - \underbrace{G \otimes G}_{\text{present, relevant?}}$$

DGLAP vs. saturation (II)



- $J/\Psi \to \Upsilon \simeq$ evolution 2.4 GeV^2 $\to$ 22.4 GeV^2
- high density effects die away in collinear limit
- DGLAP unstable at ultra-small x and small scales ...
- convinced: pdf studies highly valuable $\to$ constrain pdf's at ultra-small x
- useful benchmark for saturation searches (?)

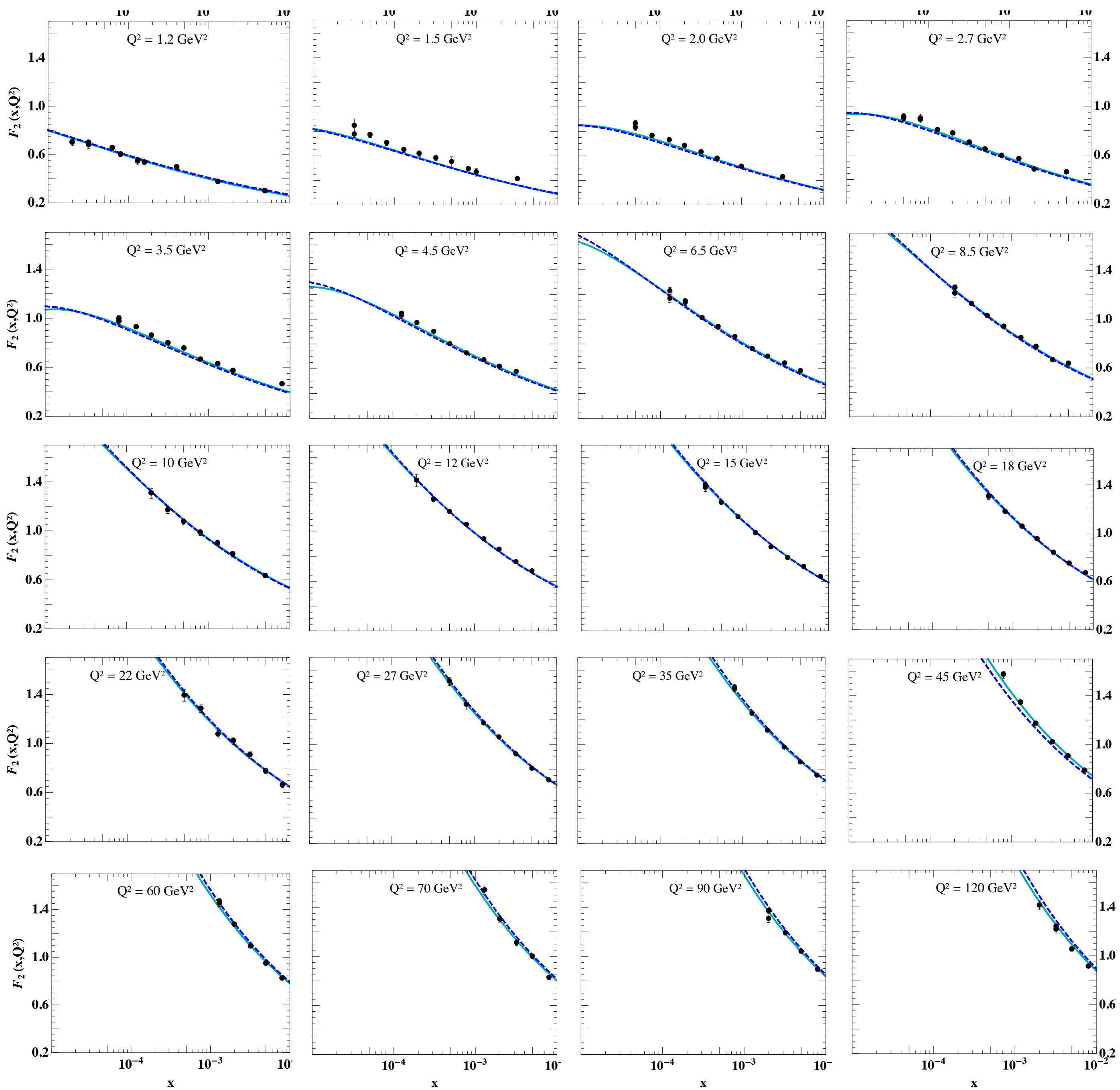
- a far better dilute benchmark might be given by BFKL evolution

- BFKL evolution = low x evolution **without** saturation/non-linear effects

- available up to NLO **[Fadin, Lipatov; PLB 429 (1998) 127]; [Ciafaloni, Camici; PLB 430 (1998) 349]** + resummation schemes for addressing collinear log's etc are relatively well studied by now **[Salam; hep-ph/9806482] etc.**

NLO BFKL fit to HERA data

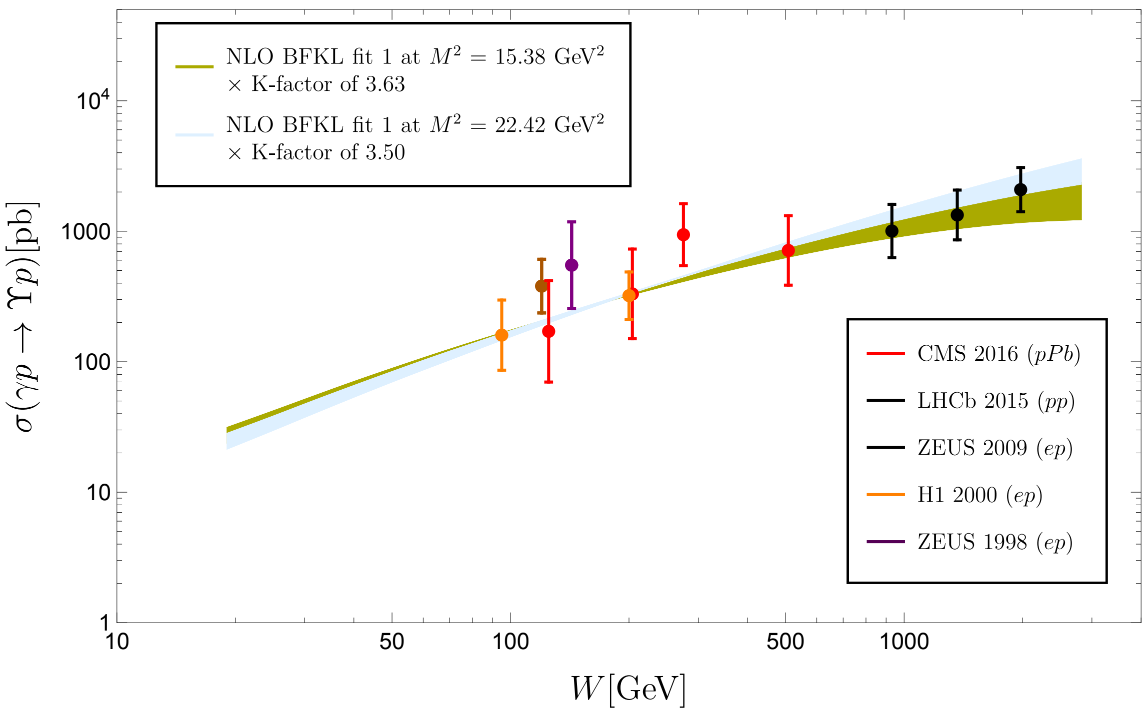
[MH, Salas, Sabio Vera; 1209.1353; 1301.5283]

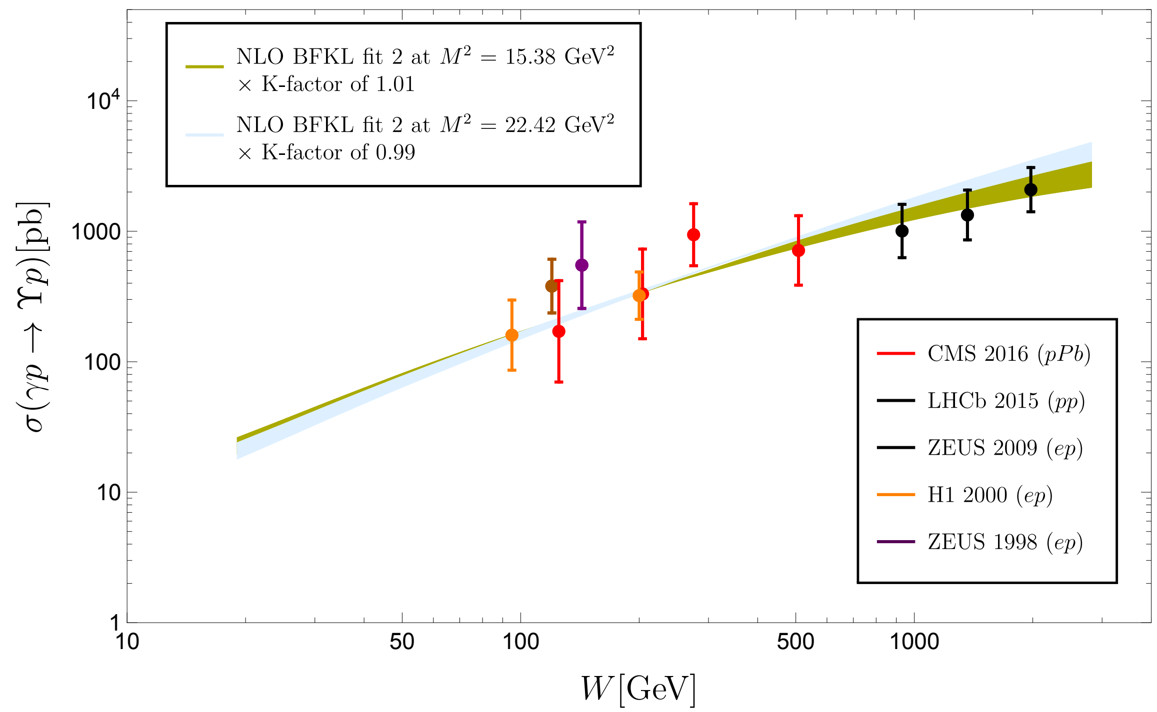


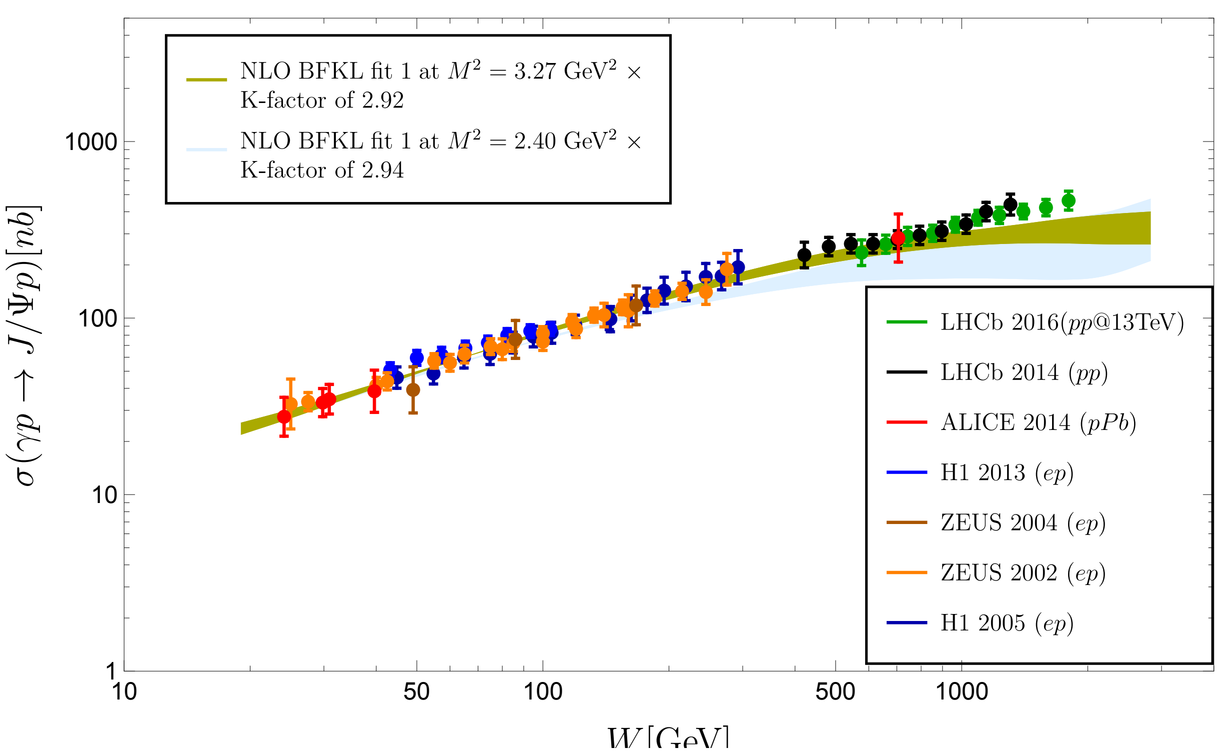
- very good description of combined HERA data
  [H1 & ZEUS collab. 0911.0884]
- allows extraction of unintegrated gluon density → apply fit to other processes

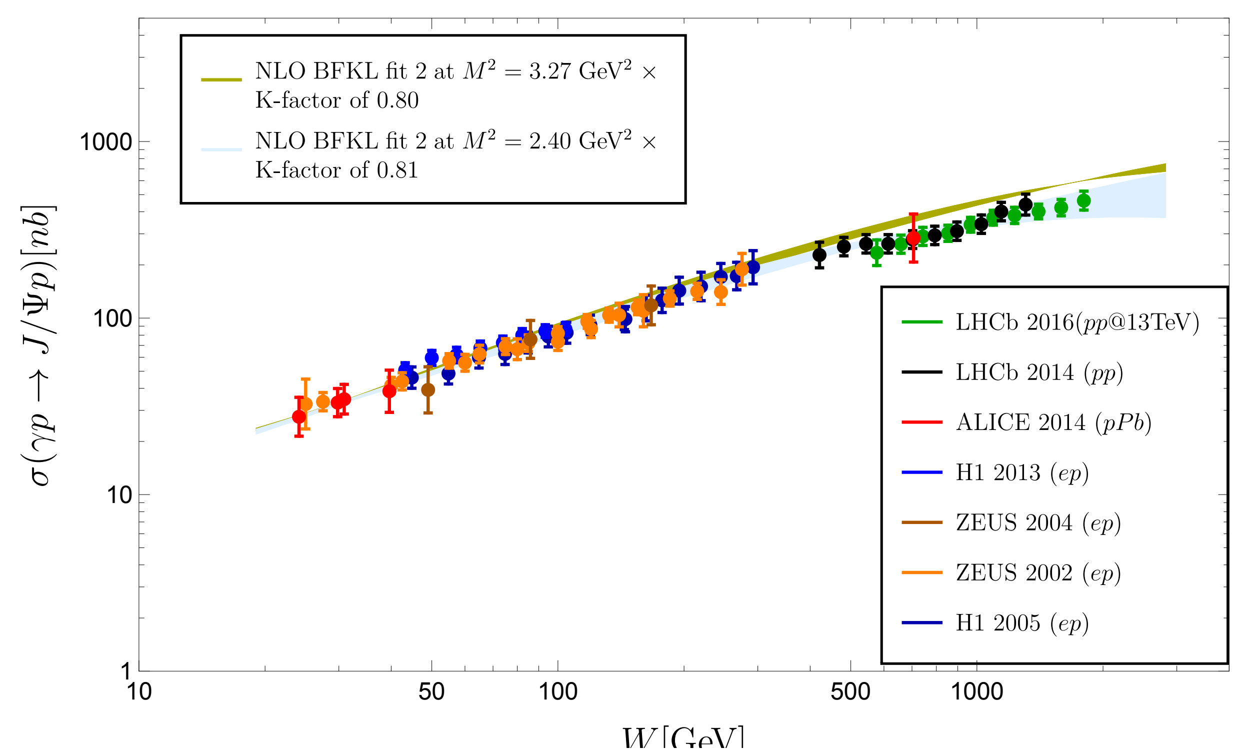
Very good description of data

[Bautista, Fernando Tellez, MH; 1607.05203]









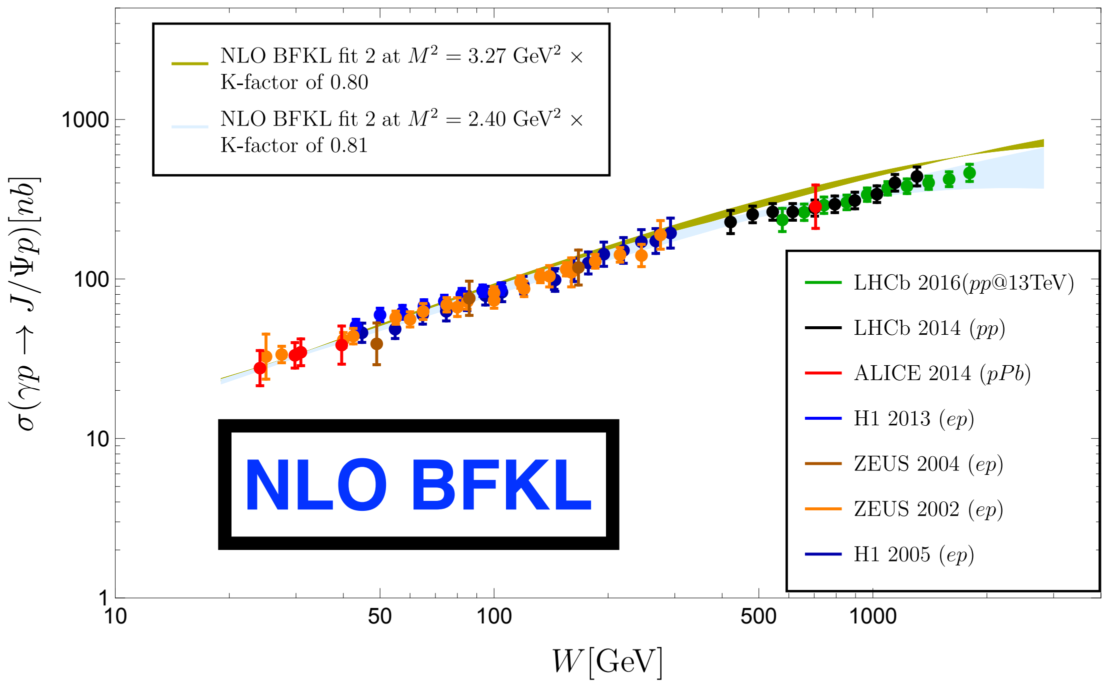
NLO BFKL works pretty well

but there are some issues

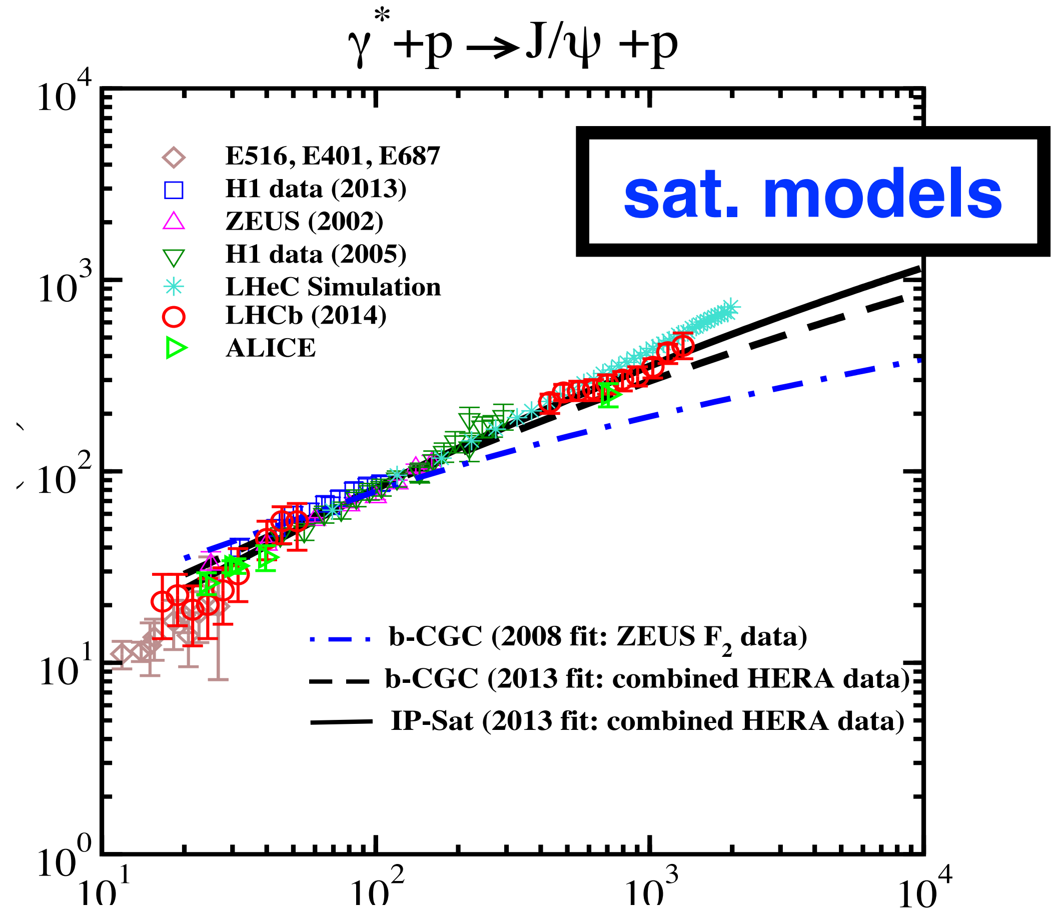
- solutions uses partial perturbative treatment of NLO corrections; at very small x/large W this leads to an instability → need to wait for the next collider to really see this

- normalization needs to be adjusted by hand (in general this is a non-trivial corrections, also for pdf/saturation study → would need extra calculation)

- still pretty good for taking a HERA only fit + LO coefficient



[Bautista, Fernandez-Tellez, MH; 1607.05203]



[Armesto, Rezaeian; 1402.4831],
[Goncalves, Moreira, Navarra; 1405.6977]

does mean there's no saturation/high density effects
2 potential explanations:

a) saturation still far away
b) BFKL can mimic effects in "transition region"→both connected!

$$2\int d^2\boldsymbol{b}\,\mathcal{N}(x,r,b) = \frac{4\pi}{N_c}\int \frac{d^2\boldsymbol{k}}{\boldsymbol{k}^2}\left(1 - e^{i\boldsymbol{k}\cdot\boldsymbol{r}}\right)\alpha_s G(x,\boldsymbol{k}^2)\,.$$

dipole amplitude/includes saturation

BFKL unintegrated gluon

evolution differs (presence or absence of nonlinear terms),

.... but essentially same object

- to manifest non-linear effects, need to evolve over (relatively large) regions of phase space

  - BFKL:
    $\partial_{\ln 1/x} G(x, k) = K \otimes G$

  - BK:
    $\partial_{\ln 1/x} G(x, k) = K \otimes G - G \otimes G$

not clear how fast the non-linear term becomes relevant

- an alternative: observables which reveal non-linear effects without evolution

Observable ~ $G + \# G^2 + \# G^4 + ...$

a possibility: observables which depend on the quadrupole

$$\mathcal{N}^{(4)}(\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{x}_3, \boldsymbol{x}_4) = \frac{1}{N_c}\mathrm{Tr}\left(1 - V(\boldsymbol{x}_1)V^\dagger(\boldsymbol{x}_2)V(\boldsymbol{x}_3)V^\dagger(\boldsymbol{x}_4)\right)$$

$$\sim \quad G + \#G^2 + \#G^4 + \ldots$$

(= 4 gluon exchange doesn’t reduce to effective 2 gluon exchange on Xsec. level)

$$\mathcal{N}(\boldsymbol{r}, \boldsymbol{b}) = \frac{1}{N_c}\mathrm{Tr}\left(1 - V(\boldsymbol{x})V^\dagger(\boldsymbol{y})\right)$$

contains also 4 gluon exchange, but gathered in 2 Wilson lines

$$V(\boldsymbol{z}) \equiv V_{ij}(\boldsymbol{z}) \equiv \mathrm{P}\exp ig \int_{-\infty}^{\infty} dx^+ A^{-,c}(x^+, \boldsymbol{z}) t^c$$

$$U(\boldsymbol{z}) \equiv U^{ab}(\boldsymbol{z}) \equiv \mathrm{P}\exp ig \int_{-\infty}^{\infty} dx^+ A^{-,c}(x^+, \boldsymbol{z}) T^c$$

well known example where this happens:

production of 2 partons in DIS

[Dominguez, Marquet, Xiao,Yuan; 1101.0715]

believe: worthwhile to go a step beyond and consider 3 cartons

→more constrains on the quadrupole (= the object where we expect effects)

→ technically a part of the NLO corrections to 2 cartons in DIS

→ related calculation for diffraction (includes already virtual)
[Boussarie, Grabovsky, Szymanowski, Wallon; 1405.7676, 1606.00419]

calculation non-trivial

- # of diagrams grows fast with number of final states
- complex Dirac & Lorentz structure
- turns out: momentum space calculation and use of spinor helicity techniques help a lot

→ wont talk on this here, details: arXiv:1701.07143

the large N_C result

$$\frac{d\sigma^{T,L}}{d^2\boldsymbol{p}\, d^2\boldsymbol{k}\, d^2\boldsymbol{q}\, dz_1 dz_2} = \frac{\alpha_s \alpha_{em} e_f^2 N_c^2}{z_1 z_2 z_3 (2\pi)^2} \prod_{i=1}^{3} \prod_{j=1}^{3} \int \frac{d^2\boldsymbol{x}_i}{(2\pi)^2} \int \frac{d^2\boldsymbol{x}'_j}{(2\pi)^2} e^{i\boldsymbol{p}(\boldsymbol{x}_1 - \boldsymbol{x}'_1) + i\boldsymbol{q}(\boldsymbol{x}_2 - \boldsymbol{x}'_2) + i\boldsymbol{k}(\boldsymbol{x}_3 - \boldsymbol{x}'_3)}$$

$$\Bigg\langle (2\pi)^4 \Bigg[\Bigg(\delta^{(2)}(\boldsymbol{x}_{13}) \delta^{(2)}(\boldsymbol{x}_{1'3'}) \sum_{h,g} \psi^{T,L}_{1;h,g}(\boldsymbol{x}_{12}) \psi^{T,L,*}_{1';h,g}(\boldsymbol{x}_{1'2'}) + \{1,1'\} \leftrightarrow \{2,2'\} \Bigg) N^{(4)}(\boldsymbol{x}_1, \boldsymbol{x}'_1, \boldsymbol{x}'_2, \boldsymbol{x}_2)$$

$$+ \Bigg(\delta^{(2)}(\boldsymbol{x}_{23}) \delta^{(2)}(\boldsymbol{x}_{1'3'}) \sum_{h,g} \psi^{T,L}_{2;h,g}(\boldsymbol{x}_{12}) \psi^{T,L,*}_{1';h,g}(\boldsymbol{x}_{1'2'}) + \{1,1'\} \leftrightarrow \{2,2'\} \Bigg) N^{(22)}(\boldsymbol{x}_1, \boldsymbol{x}'_1 | \boldsymbol{x}'_2, \boldsymbol{x}_2) \Bigg]$$

$$+ (2\pi)^2 \Bigg[\delta^{(2)}(\boldsymbol{x}_{13}) \sum_{h,g} \psi^{T,L}_{1;h,g}(\boldsymbol{x}_{12}) \psi^{T,L,*}_{3';h,g}(\boldsymbol{x}_{1'3'}, \boldsymbol{x}_{2'3'}) N^{(24)}(\boldsymbol{x}_{3'}, \boldsymbol{x}_{1'} | \boldsymbol{x}_{2'}, -_2, \boldsymbol{x}_1, \boldsymbol{x}_{3'}) + \{1\} \leftrightarrow \{2\}$$

$$+ \delta^{(2)}(\boldsymbol{x}_{1'3'}) \sum_{h,g} \psi^{T,L}_{3;h,g}(\boldsymbol{x}_{13}, \boldsymbol{x}_{23}) \psi^{T,L,*}_{1';h,g}(\boldsymbol{x}_{1'2'}) N^{(24)}(\boldsymbol{x}_1, \boldsymbol{x}_3 | \boldsymbol{x}_{2'}, \boldsymbol{x}_2, \boldsymbol{x}_3, \boldsymbol{x}_{1'}) + \{1'\} \leftrightarrow \{2'\} \Bigg]$$

$$+ \sum_{h,g} \psi^{T,L}_{3;h,g}(\boldsymbol{x}_{13}, \boldsymbol{x}_{23}) \psi^{T,L,*}_{3';h,g}(\boldsymbol{x}_{1'3'}, \boldsymbol{x}_{2'3'}) N^{(44)}(\boldsymbol{x}_1, \boldsymbol{x}_{1'}, \boldsymbol{x}_{3'}, \boldsymbol{x}_3 | \boldsymbol{x}_3, \boldsymbol{x}_{3'}, \boldsymbol{x}_{2'}, \boldsymbol{x}_2) \Bigg\rangle_{A\,-},$$

in terms of correlators of Wilson lines
& wave functions

the details: correlators of Wilson lines

- contain the target information

- written in terms of dipoles and quadrupoles

$$\boxed{S^{(2)}_{(\boldsymbol{x}_1\boldsymbol{x}_2)} \equiv \frac{1}{N_c}\mathrm{tr}\left[V(\boldsymbol{x}_1)V^\dagger(\boldsymbol{x}_2)\right]}$$

$$\boxed{S^{(4)}_{(\boldsymbol{x}_1\boldsymbol{x}_2\boldsymbol{x}_3\boldsymbol{x}_4)} \equiv \frac{1}{N_c}\mathrm{tr}\left[V(\boldsymbol{x}_1)V^\dagger(\boldsymbol{x}_2)V(\boldsymbol{x}_3)V^\dagger(\boldsymbol{x}_4)\right]}$$

$$\begin{aligned}
&N^{(4)}(\boldsymbol{x}_1,\boldsymbol{x}_2,\boldsymbol{x}_3,\boldsymbol{x}_4) \equiv \\
&\qquad \equiv 1 + S^{(4)}_{(\boldsymbol{x}_1\boldsymbol{x}_2\boldsymbol{x}_3\boldsymbol{x}_4)} - S^{(2)}_{(\boldsymbol{x}_1\boldsymbol{x}_2)} - S^{(2)}_{(\boldsymbol{x}_3\boldsymbol{x}_4)}\,, \\
&N^{(22)}(\boldsymbol{x}_1,\boldsymbol{x}_2|\boldsymbol{x}_3,\boldsymbol{x}_4) \equiv \\
&\qquad \equiv \left[S^{(2)}_{(\boldsymbol{x}_1\boldsymbol{x}_2)} - 1\right]\left[S^{(2)}_{(\boldsymbol{x}_3\boldsymbol{x}_4)} - 1\right] \\
&N^{(24)}(\boldsymbol{x}_1,\boldsymbol{x}_2|\boldsymbol{x}_3,\boldsymbol{x}_4,\boldsymbol{x}_5,\boldsymbol{x}_6) \equiv \\
&\qquad 1 + S^{(2)}_{(\boldsymbol{x}_1\boldsymbol{x}_2)}S^{(4)}_{(\boldsymbol{x}_3\boldsymbol{x}_4\boldsymbol{x}_5\boldsymbol{x}_6)} \\
&\qquad\qquad - S^{(2)}_{(\boldsymbol{x}_1\boldsymbol{x}_2)}S^{(2)}_{(\boldsymbol{x}_3\boldsymbol{x}_6)} - S^{(2)}_{(\boldsymbol{x}_4\boldsymbol{x}_5)}\,, \\
&N^{(44)}(\boldsymbol{x}_1,\boldsymbol{x}_2,\boldsymbol{x}_3,\boldsymbol{x}_4|\boldsymbol{x}_5,\boldsymbol{x}_6,\boldsymbol{x}_7,\boldsymbol{x}_8) \equiv \\
&\qquad \equiv 1 + S^{(4)}_{(\boldsymbol{x}_1\boldsymbol{x}_2\boldsymbol{x}_3\boldsymbol{x}_4)}S^{(4)}_{(\boldsymbol{x}_5\boldsymbol{x}_6\boldsymbol{x}_7\boldsymbol{x}_8)} \\
&\qquad\qquad - S^{(2)}_{(\boldsymbol{x}_1\boldsymbol{x}_4)}S^{(2)}_{(\boldsymbol{x}_5\boldsymbol{x}_8)} - S^{(2)}_{(\boldsymbol{x}_2\boldsymbol{x}_3)}S^{(2)}_{(\boldsymbol{x}_6\boldsymbol{x}_7)}
\end{aligned}$$

- quadrupole $S^{(4)}$ linear & quadratic
  → extra handle to explore it wrt. 2 partons

(quadrupole only linear)

the details: wave functions & amplitudes

$$\psi^L_{j,hg} = -2\sqrt{2} Q K_0\left(QX_j\right) \cdot a^{(L)}_{j,hg}, \qquad j=1,2$$

$$\psi^T_{j,hg} = 2ie^{\mp i\phi_{\boldsymbol{x}_{12}}}\sqrt{(1-z_3-z_j)(z_j+z_3)} Q K_1\left(QX_j\right) \cdot a^{\pm}_{j,hg} \qquad j=1,2$$

$$\psi^L_{3,hg} = 4\pi i Q\sqrt{2z_1z_2} K_0\left(QX_3\right)(a^{(L)}_{3,hg} + a^{(L)}_{4,hg}),$$

$$\psi^T_{3,hg} = -4\pi Q\sqrt{z_1z_2}\frac{K_1\left(QX_3\right)}{X_3}(a^{\pm}_{3,hg} + a^{\pm}_{4,hg}) .$$

symmetry relation between amplitudes

$$a^{T,L}_{k+1,hg} = -a^{T,L}_{k,-hg}(\{p,\boldsymbol{x}_1\} \leftrightarrow \{q,\boldsymbol{x}_2\}), \qquad k = \text{1,3}$$

$$a^{T,L}_{j,hg} = a^{(-T,L)*}_{j,-h-g}, \quad j=1,\ldots,4.$$

longitudinal photon

$$a_{1,++}^{(L)} = -\frac{(z_1 z_2)^{3/2}\,(z_1+z_3)}{z_3 e^{-i\theta_p}|\boldsymbol{p}| - z_1 e^{-i\theta_k}|\boldsymbol{k}|},$$

$$a_{1,-+}^{(L)} = -\frac{\sqrt{z_1}\,z_2^{3/2}\,(z_1+z_3)^{\,2}}{z_3 e^{-i\theta_p}|\boldsymbol{p}| - z_1 e^{-i\theta_k}|\boldsymbol{k}|},$$

$$a_{3,++}^{(L)} = \frac{z_1 z_2}{|\boldsymbol{x}_{13}|e^{-i\phi_{\boldsymbol{x}_{13}}}},$$

$$a_{3,-+}^{(L)} = \frac{z_2(1-z_2)}{|\boldsymbol{x}_{13}|e^{-i\phi_{\boldsymbol{x}_{13}}}},$$

transverse photon

for precise def. see paper

take away message:

very compact expressions

$$a^{(+)}_{1,++} = -\frac{(z_1 z_2)^{3/2}}{z_3 e^{-i\theta_p}|\boldsymbol{p}| - z_1 e^{-i\theta_k}|\boldsymbol{k}|},$$

$$a^{(+)}_{1,+-} = \frac{\sqrt{z_1}(z_2)^{\frac{3}{2}}(z_1 + z_3)}{z_1 e^{i\theta_k}|\boldsymbol{k}| - z_3 e^{i\theta_p}|\boldsymbol{p}|},$$

$$a^{(+)}_{1,-+} = \frac{\sqrt{z_1 z_2}(z_1 + z_3)^2}{z_3 e^{-i\theta_p}|\boldsymbol{p}| - z_1 e^{-i\theta_k}|\boldsymbol{k}|},$$

$$a^{(+)}_{1,--} = \frac{z_1^{3/2}\sqrt{z_2}(z_1 + z_3)}{z_3 e^{i\theta_p}|\boldsymbol{p}| - z_1 e^{i\theta_k}|\boldsymbol{k}|},$$

$$a^{(+)}_{3,++} = \frac{z_1 z_2 (z_2 z_3 |\boldsymbol{x}_{23}| e^{-i\phi_{\boldsymbol{x}_{23}}} + z_3 |\boldsymbol{x}_{13}| e^{-i\phi_{\boldsymbol{x}_{13}}} - z_1 z_2 |\boldsymbol{x}_{12}| e^{-i\phi_{\boldsymbol{x}_{12}}})}{(z_1 + z_3)|\boldsymbol{x}_{13}| e^{-i\phi_{\boldsymbol{x}_{13}}}},$$

$$a^{(+)}_{3,+-} = \frac{z_2^2 (z_3 |\boldsymbol{x}_{23}| e^{-i\phi_{\boldsymbol{x}_{23}}} - z_1 |\boldsymbol{x}_{12}| e^{-i\phi_{\boldsymbol{x}_{12}}})}{|\boldsymbol{x}_{13}| e^{i\phi_{\boldsymbol{x}_{13}}}},$$

$$a^{(+)}_{3,-+} = -\frac{z_2 (z_1 + z_3)(z_3 |\boldsymbol{x}_{23}| e^{-i\phi_{\boldsymbol{x}_{23}}} - z_1 |\boldsymbol{x}_{12}| e^{-i\phi_{\boldsymbol{x}_{12}}})}{|\boldsymbol{x}_{13}| e^{-i\phi_{\boldsymbol{x}_{13}}}},$$

$$a^{(+)}_{3,--} = \frac{z_1 z_2 (z_1 |\boldsymbol{x}_{12}| e^{-i\phi_{\boldsymbol{x}_{12}}} - z_3 |\boldsymbol{x}_{23}| e^{-i\phi_{\boldsymbol{x}_{23}}})}{|\boldsymbol{x}_{13}| e^{i\phi_{\boldsymbol{x}_{13}}}}.$$

First attempts in phenomenology

- differential Xsec: given in terms of dipole and quadrupole operators

- need to be evaluated for a given background field configuration = represents dynamics of target

$$\langle \ldots \rangle_{A^-} = \int D[\rho] \ldots e^{-W[\rho]}$$

$\boldsymbol{\rho}$: color carge, relates to back-ground field through Yang-Mills equation

$$\boxed{-\boldsymbol{\partial}^2 A^{c,-}(z^+, \boldsymbol{x}) = g_s \rho_c(z^+, \boldsymbol{x})}$$

- in general: weight function W[$\boldsymbol{\rho}$] not known … what can be extracted from inclusive DIS data is the dipole amplitude

$$\langle S^{(2)}(\boldsymbol{x}_1, \boldsymbol{x}_2)\rangle_{A^-} = \frac{1}{N_c}\langle \mathrm{tr}\left(V(\boldsymbol{x}_1)V^\dagger(\boldsymbol{x}_2)\right)\rangle_{A^-}$$

→ higher correlators not known; way out: “Gaussian approximation” (McLerran-Venugopalan model) for weight function with width $\boldsymbol{\mu}$

$$W[\rho] = \int d^2\boldsymbol{x} \int d^2\boldsymbol{y} \int dz^+ \, \frac{\rho_c(z^+, \boldsymbol{x})\rho_c(z^+, \boldsymbol{y})}{2\mu^2(z^+)}$$

can argue: good approximation in dilute limit

- allows to calculate dipole in terms of μ^2 and 2 point correlator of fields → fix this combination from DIS inclusive fits of $S^{(2)}$

- calculate quadrupole correlator in terms of dipole correlator
[Dominguez, Marquet, Xiao,Yuan; 1101.0715]

$$S^{(4)}(\boldsymbol{x}_1, \boldsymbol{x}_{1'}, \boldsymbol{x}_{2'}, \boldsymbol{x}_2) = S^{(2)}(\boldsymbol{x}_1, \boldsymbol{x}_2) S^{(2)}(\boldsymbol{x}_{1'}, \boldsymbol{x}_{2'}) - \frac{\Gamma(\boldsymbol{x}_1, \boldsymbol{x}_{2'}; \boldsymbol{x}_2, \boldsymbol{x}_{1'})}{\Gamma(\boldsymbol{x}_1, \boldsymbol{x}_2; \boldsymbol{x}_{2'}, \boldsymbol{x}_{1'})} S^{(2)}(\boldsymbol{x}_1, \boldsymbol{x}_{1'}) S^{(2)}(\boldsymbol{x}_2, \boldsymbol{x}_{2'})$$

$$\Gamma(\boldsymbol{x}_1, \boldsymbol{x}_{2'}; \boldsymbol{x}_2, \boldsymbol{x}_{1'}) = \ln \frac{S^{(2)}(\boldsymbol{x}_1, \boldsymbol{x}_{2'}) S^{(2)}(\boldsymbol{x}_{1'}, \boldsymbol{x}_2)}{S^{(2)}(\boldsymbol{x}_{1'}, \boldsymbol{x}_{1'}) S^{(2)}(\boldsymbol{x}_2, \boldsymbol{x}_{2'})}$$

- numerical study: a good approximation to full expression
[Dumitru, Jalilian-Marian, Lappi, Schenke, Venugoplana; 1108.4764]

- in general: known for finite N_C; here: large N_C limit→ argue that expectation values of combinations of $S^{(2)}$ and $S^{(4)}$ factorise

- our treatment: use $S^{(2)}=1 - N^{(2)}$ and expand for small $N^{(2)}$ to linear and quadratic order → large quadratic corrections: sensitive to non-linear effects

- For $S^{(2)}$ use model with parameters fitted to rcBK DIS fit
[Quiroga-Arias,Albacete, Armesto, Milhano, Salgado, 1107.0625]

$$S^{(2)}(\boldsymbol{x}_1,\boldsymbol{x}_2) = \int d^2\boldsymbol{l}\, e^{-i\boldsymbol{l}\cdot\boldsymbol{x}_{12}}\ \Phi(\boldsymbol{l}^2)$$

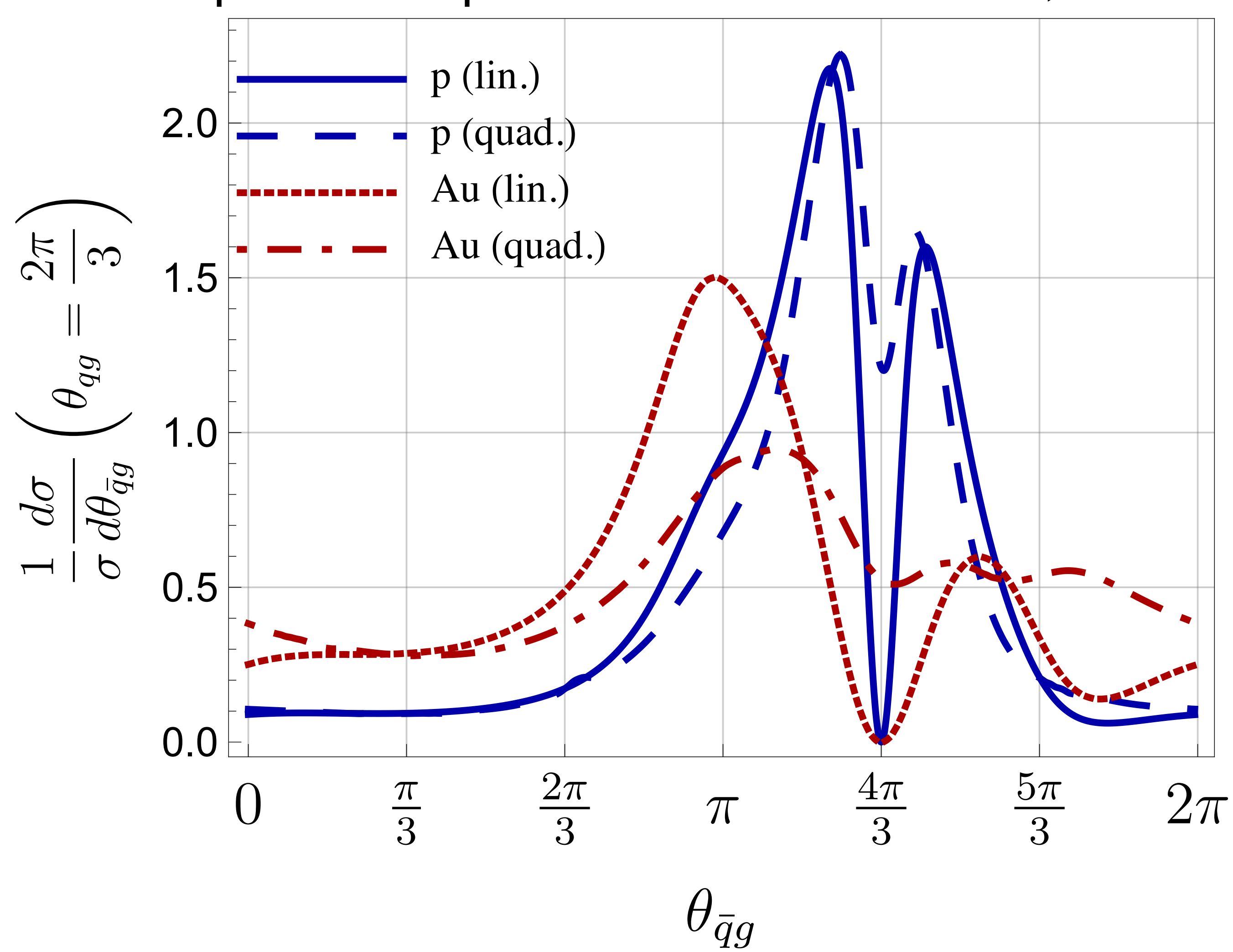
$$= 2\left(\frac{Q_0|\boldsymbol{x}_{12}|}{2}\right)^{\alpha-1}\frac{K_{\alpha-1}(Q_0|\boldsymbol{x}_{12}|)}{\Gamma(\alpha-1)},$$

$$\Phi(\boldsymbol{l}^2) = \frac{\Gamma(\alpha)}{Q_0^2\pi\Gamma(\alpha-1)}\left(\frac{Q_0^2}{Q_0^2+\boldsymbol{l}^2}\right)^{\alpha},$$

- parameters: $\boldsymbol{\alpha} = 2.3$
proton: $Q_0^{\text{prot.}} = 0.69$ GeV; corresponds to $x = 0.2 \cdot 10^{-3}$
gold: $Q_0^{\text{gold}} = A^{1/6}\, Q_0^{\text{prot.}} = 1.67$ GeV

First study at partonic level

- explore deviations from Mercedes star configuration→back-to-back for three particles
- parton p_T fixed to 2 GeV, Q=3 GeV



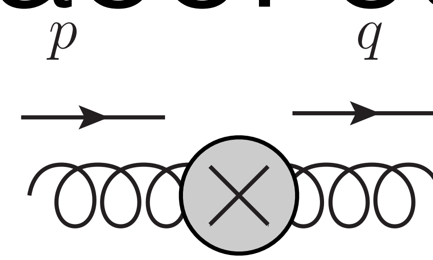
- fix one angle (quark-gluon), vary antiquark-gluon
- sizeable quadratic corrections for gold

Summary:

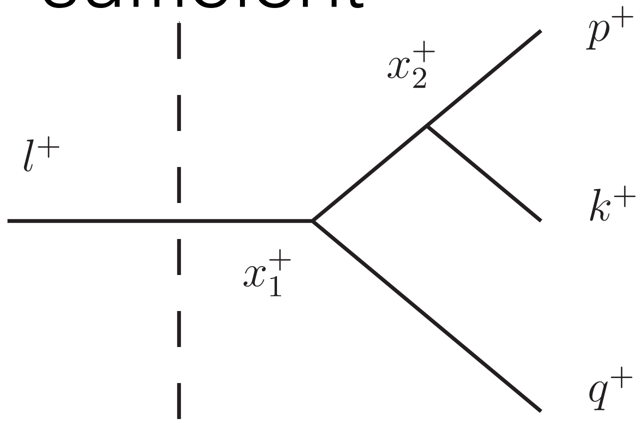
- NLO BFKL serves to evolve from HERA energies to LHC energies
- to detect high gluon density effects, observables directly sensitive to such effects should help (“evolution only” might require too much phase space)
- studied such an observables and showed that this could actually work (at partonic level so far)
- more work left be done!

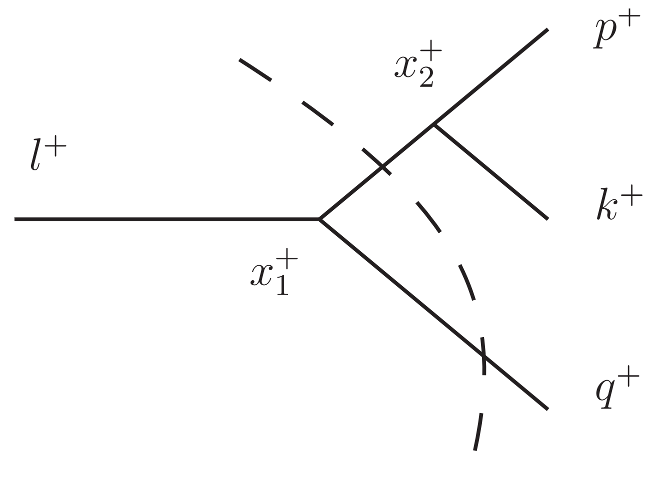
Gracias!

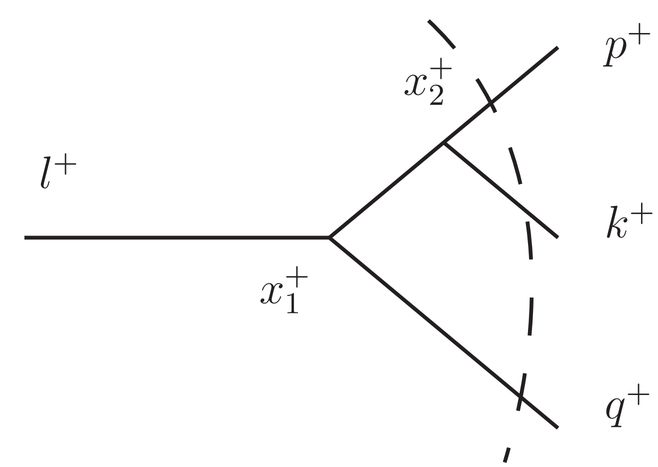
Configuration space: cuts at $x^+=0$

- start without special vertices 

- divide x_i^+ integral $\int_{-\infty}^{\infty} dx^+ \to \int_{-\infty}^{0} dx^+ + \int_{0}^{\infty} dx^+$ + theta functions in plus momenta & coordinates → each of our diagrams cut by a line separating positive & negative light-cone time (left: negative; right: positive)
- only plus coordinates & momenta → skeleton diagrams sufficient



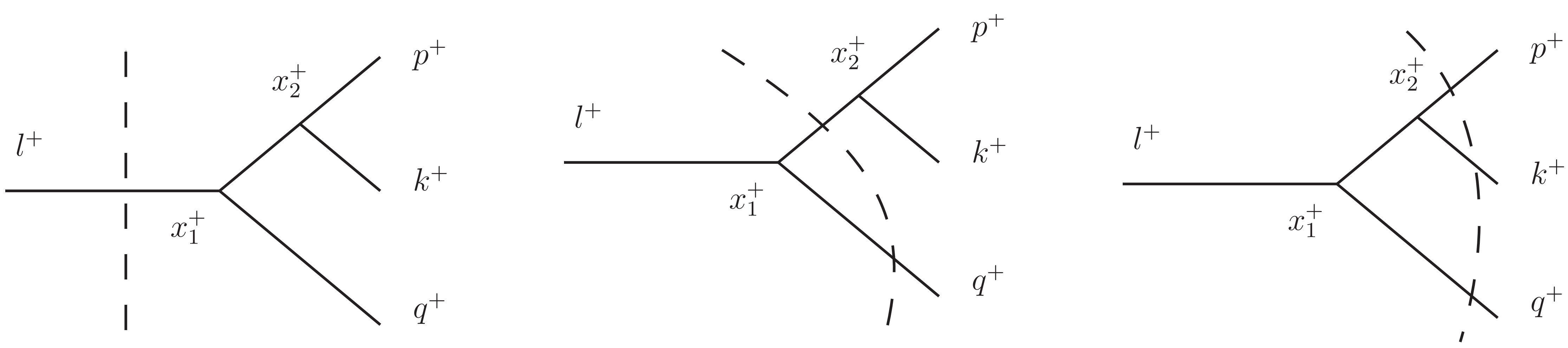




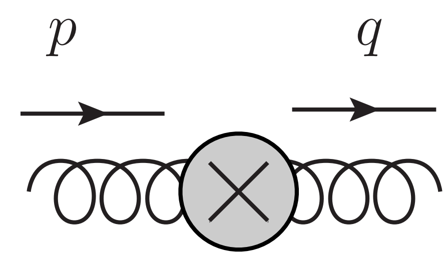
- a "cut" propagator crosses light-cone time $x^+=0$

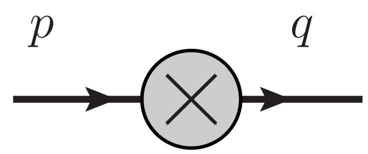
Which cuts are possible?

- in general: any line through the diagram
- fix kinematics to s-channel kinematics [l⁺=p⁺ +q⁺ +k⁺, all plus momenta positive always]
  → only s-channel type cuts possible (~vertical cuts)



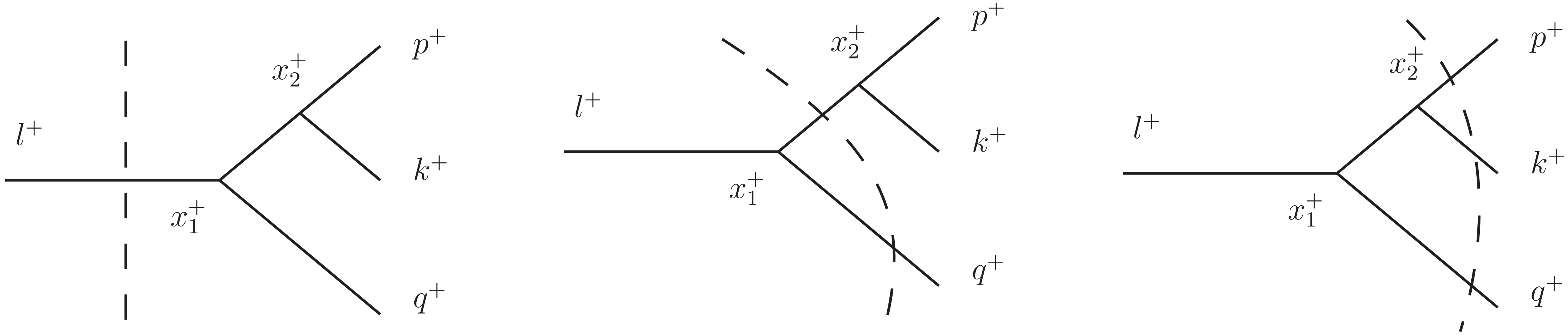
- for this topology, these are the only possible cuts

- NEXT: add special vertices 

- recall: 

$\sim \delta(p^+ - q^+)$ plus momentum flow not altered + placed at z⁺=0 ⇒ by default on the cut

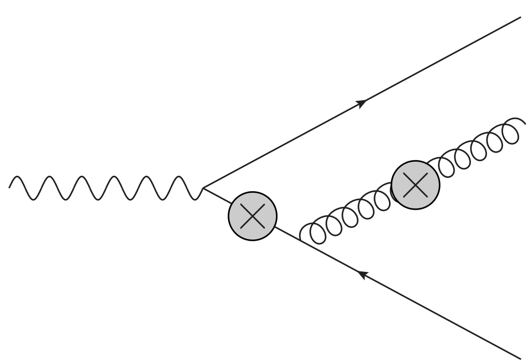
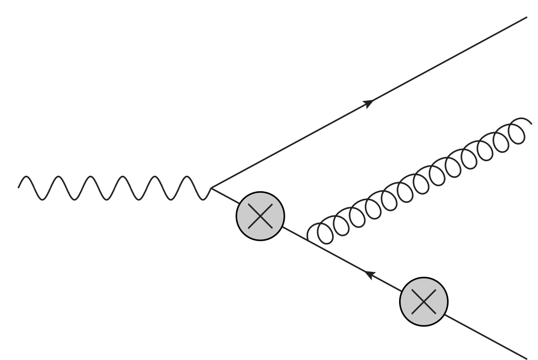
- go back to momentum space: special vertices still must be aligned along the cut



- at a cut: “propagator ⊗ special vertex ⊗ propagator” or “propagator” only; no special vertex anywhere else

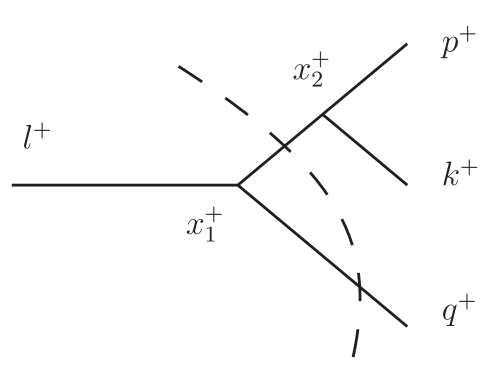
How does it help?

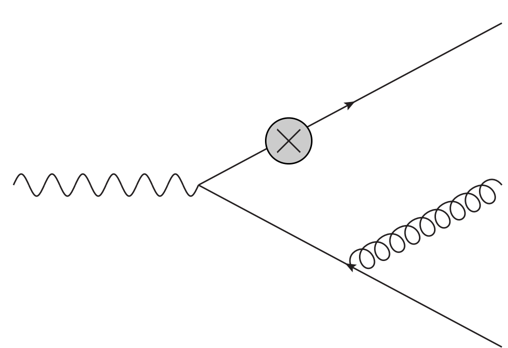
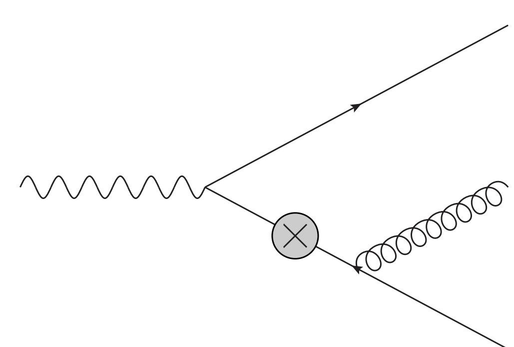
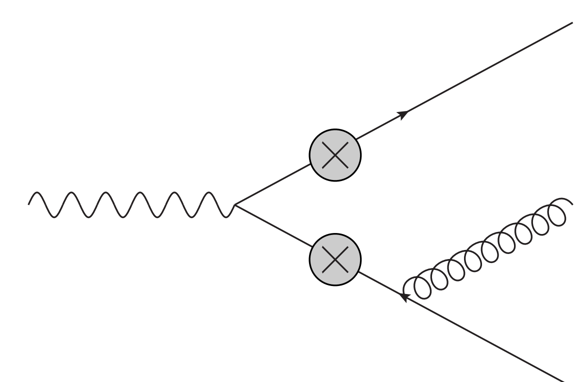
- evaluates 50% of possible momentum diagrams to zero

not possible for s-channel kinematics

- ….. but each cut contains still several diagrams



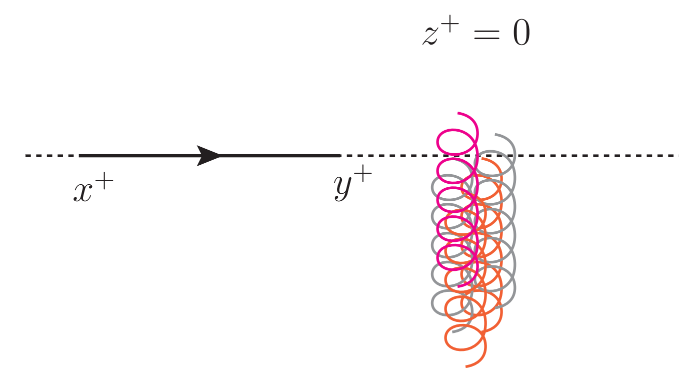




Configuration space knows more …
(partial) Fourier transform for complete propagator

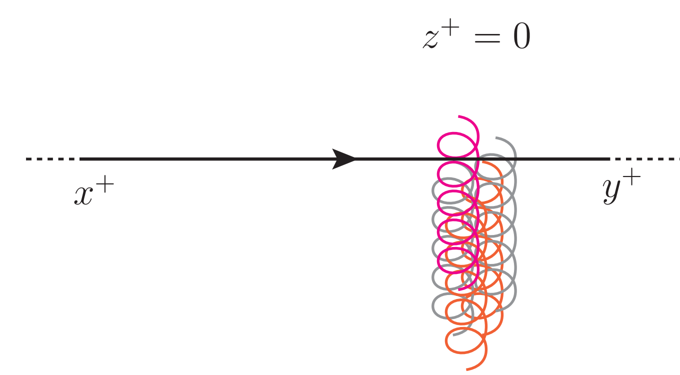
$$\int \frac{dp^-}{2\pi} \int \frac{dq^-}{2\pi} e^{-ip^-x^+} e^{iq^-y^+} \left[S^{(0)}_{F,il}(p)(2\pi)^4\delta^{(4)}(p-q) + S^{(0)}_{F,ij}(p) \cdot \tau_{F,jk}(p,q) \cdot S^{(0)}_{kl}(q) \right]$$

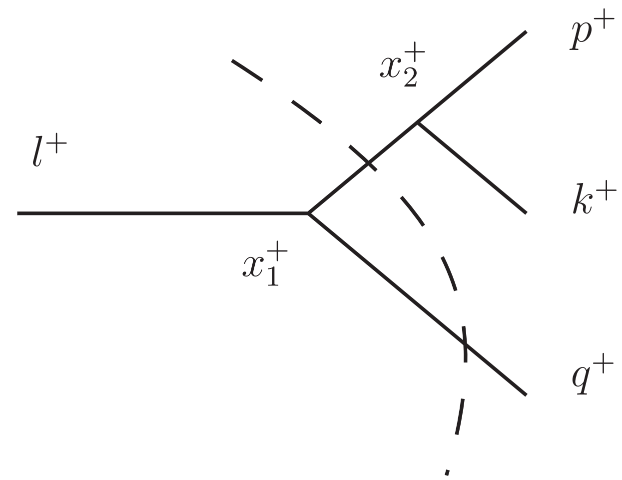
obtain free propagation for

- $x^+, y^+ < 0$ ("before interaction")
- $x^+, y^+ > 0$ ("after interaction")

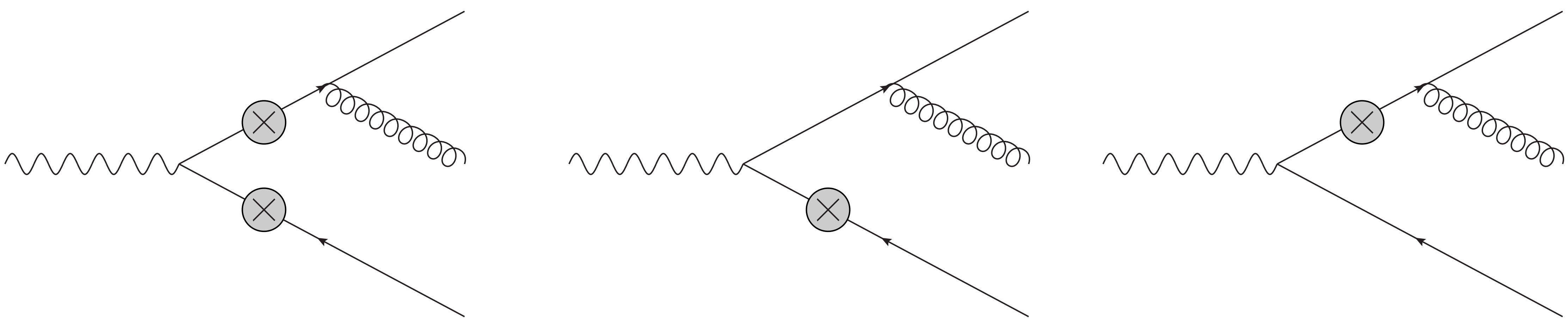


propagator proportional to complete Wilson line V (fermion) or U (gluon) if we cross light-cone time $z^+=0$
→ must pass through the cuts

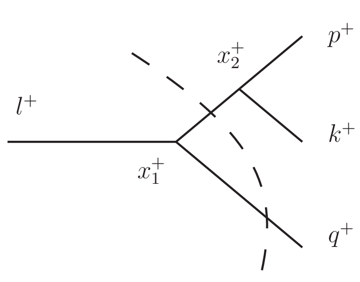


- for a single cut: 

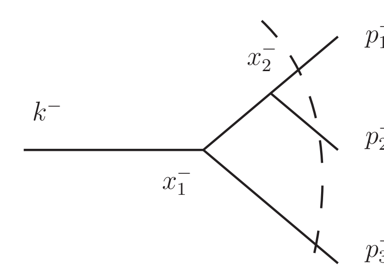
effectively adds up



- reality: more complicated due to mixing of different cuts



vs.



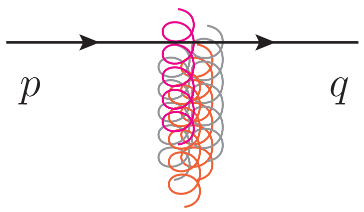
- crucial: positive plus momenta in all lines for tree diagrams

- allows to formulate a new set of effective ”Feynman rules”

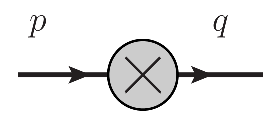
Theory: Propagators in background field

use light-cone gauge, with $k^- = n^+ \cdot k$, $(n^+)^2 = 0$, $n^+ \sim$ target momentum

$$\tilde{S}_F^{(0)}(p) = \frac{i\not{p} + m}{p^2 - m^2 + i0} \qquad \tilde{G}_{\mu\nu}^{(0)}(p) = \frac{i d_{\mu\nu}(p)}{p^2 + i0}$$

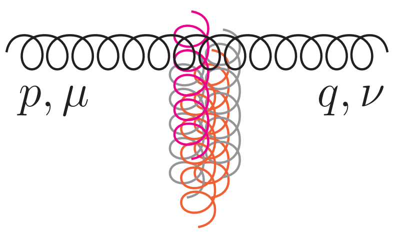


$$= (2\pi)^d \delta^{(d)}(p-q)\tilde{S}_F^{(0)}(p) + \tilde{S}_F^{(0)}(p)$$

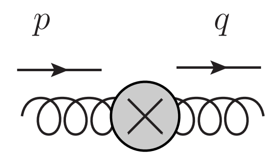


$$\tilde{S}_F^{(0)}(q)$$

$$d_{\mu\nu}(p) = -g_{\mu\nu} + \frac{n_\mu^- p_\nu + p_\mu n_\nu^-}{n^- \cdot p}$$



$$= (2\pi)^d \delta^{(d)}(p-q)\tilde{G}_{\mu\nu}^{(0)}(p) + \tilde{G}_{\mu\alpha}^{(0)}(p)$$



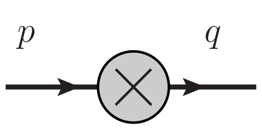
$$\tilde{G}_{\alpha\nu}^{(0)}(q)$$

[Balitsky, Belitsky; NPB 629 (2002) 290], [Ayala, Jalilian-Marian, McLerran, Venugopalan, PRD 52 (1995) 2935-2943], ...

interaction with the background field:

$$V(\boldsymbol{z}) \equiv V_{ij}(\boldsymbol{z}) \equiv \mathrm{P}\exp ig \int_{-\infty}^{\infty} dx^+ A^{-,c}(x^+, \boldsymbol{z}) t^c$$

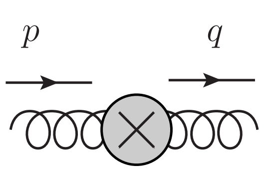
$$U(\boldsymbol{z}) \equiv U^{ab}(\boldsymbol{z}) \equiv \mathrm{P}\exp ig \int_{-\infty}^{\infty} dx^+ A^{-,c}(x^+, \boldsymbol{z}) T^c$$



$$= \tau_{F,ij}(p,q) = 2\pi\delta(p^+ - q^+)\ \not{n}$$

$$\times \int d^2\boldsymbol{z}\, e^{i\boldsymbol{z}\cdot(\boldsymbol{p}-\boldsymbol{q})} \left\{\theta(p^+)\left[V_{ij}(\boldsymbol{z}) - 1_{ij}\right] - \theta(-p^+)\left[V_{ij}^\dagger(\boldsymbol{z}) - 1_{ij}\right]\right\}$$

strong background field resummed into path ordered exponentials (Wilson lines)



$$= \tau_G^{ab}(p,q) = 2\pi\delta(p^+ - q^+)\ (-2p^+)$$

$$A^-(x^+, x_t) = \delta(x^+)\alpha(x_t)$$

$$\times \int d^2\boldsymbol{z}\, e^{i\boldsymbol{z}\cdot(\boldsymbol{p}-\boldsymbol{q})} \left\{\theta(p^+)\left[U^{ab}(\boldsymbol{z}) - 1\right] - \theta(-p^+)\left[\left(U^{ab}\right)^\dagger(\boldsymbol{z}) - 1\right]\right\}$$

momentum vs. configuration space

	conventional pQCD (use known techniques)	inclusion of finite masses (charm mass!)	intuition: interaction at t=0 with Lorentz contracted target
momentum space	well explored	complication, but doable	lose intuitive picture at first -> large # of cancelations
configuration space	poorly explored	very difficult	many diagrams automatically zero

our approach:
work in momentum space + exploit configuration space to set a large fraction of all diagrams to zero

How to do that?

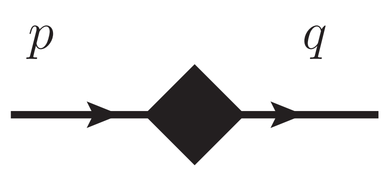
Essentially: re-install configuration space rules at the level of a single diagram

essential results: can use configuration space simplification also for momentum space calculations

Result: New effective rules for momentum space

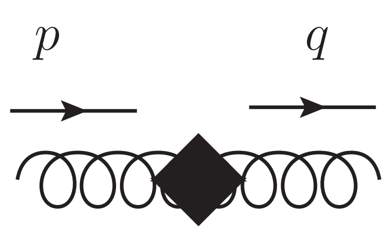
A. Determine zero light-cone time cuts of a given diagram

B. Place new vertices at these cuts



$$= \bar{\tau}_{F,ij}(p,q) = 2\pi\delta(p^+ - q^+) \cdot \not{n}$$

$$\cdot \int d^2\boldsymbol{z} e^{i\boldsymbol{z}\cdot(\boldsymbol{p}-\boldsymbol{q})} \left\{\theta(p^+)V_{ij}(\boldsymbol{z}) - \theta(-p^+)V^\dagger_{ij}(\boldsymbol{z})\right\}$$



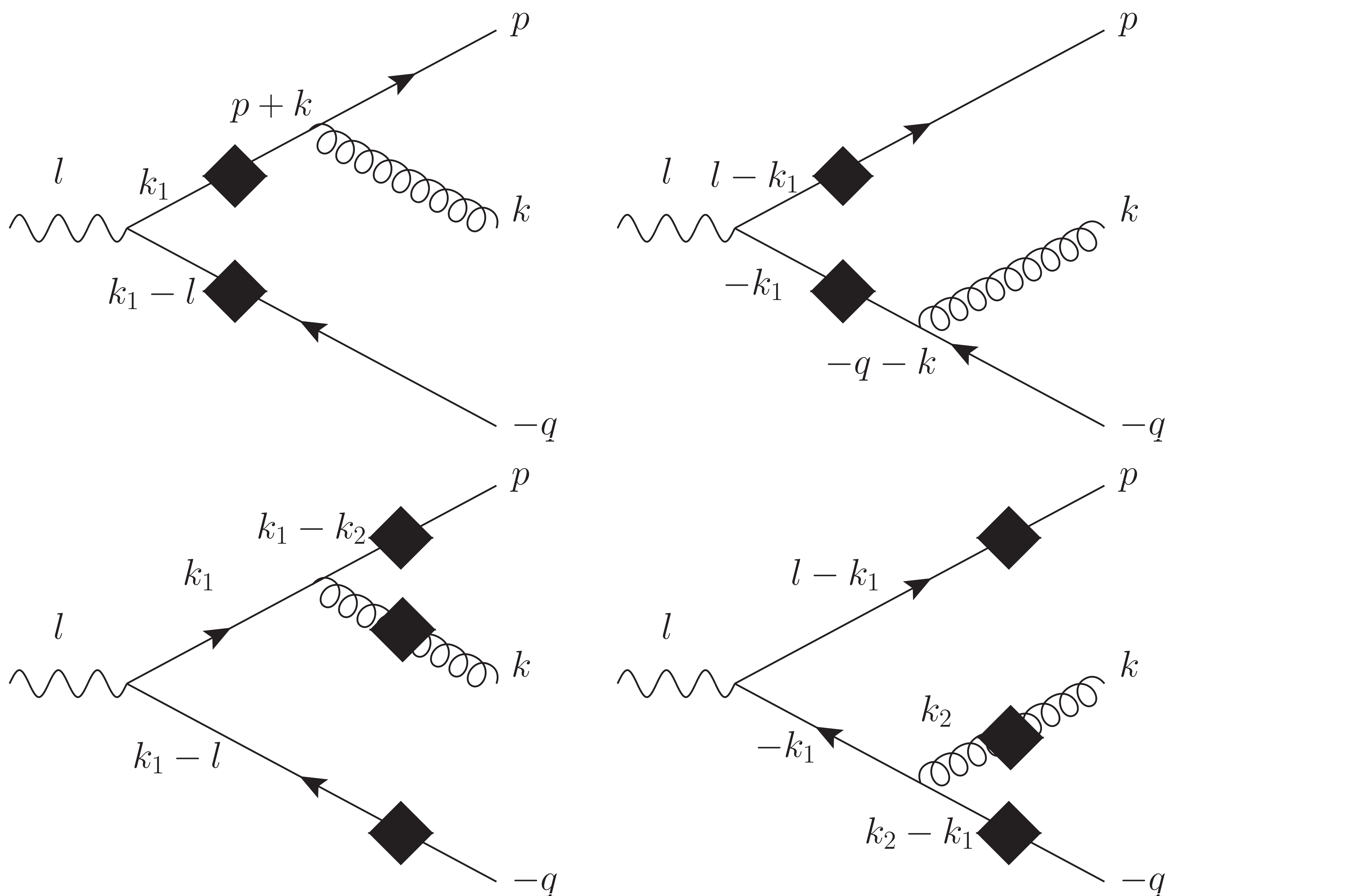
$$= \bar{\tau}^{ab}_G(p,q) = 2\pi\delta(p^+ - q^+) \cdot (-2p^+)$$

$$\cdot \int d^2\boldsymbol{z} e^{i\boldsymbol{z}\cdot(\boldsymbol{p}-\boldsymbol{q})} \left\{\theta(p^+)U^{ab}(\boldsymbol{z}) - \theta(-p^+)\left(U^{ab}\right)^\dagger(\boldsymbol{z})\right\}$$

verified by explicit calculation for tree level diagrams; in general also extendable to loop diagrams …

First result: minimal set of amplitudes

(nothing new if you're used to work in coordinate space, momentum space: reduction by factor of 4)



What do we win with new momentum space rules?

can use techniques explored in (conventional) Feynman diagram calculations

- loop integrals (d-dimensional, covariant) → won’t talk about this today …. in general: complication due to Fourier factors remain

- **spinor helicity techniques** (calculate amplitudes not Xsec. + exploit helicity conservation in mass-less QCD) → compact expressions (→ for a different application to h.e.f. see **[van Hameren, Kotko, Kutak, 1211.0961]**)

Spinor-helicity formalism

see *e.g.* [Mangano, Parke; Phys. Rept. 200, 301 (1991)] ,[Dixon; hep-ph/9601359]

central idea: express both external spinors & polarisation vectors in terms of spinors of **massless** momenta of definite helicity

$u_\pm(k) = \frac{1\pm\gamma_5}{2}u(p)$	$v_\mp(k) = \frac{1\pm\gamma_5}{2}v(p)$
$\bar{u}_\pm(k) = \bar{u}(k)\frac{1\mp\gamma_5}{2}$	$\bar{v}_\pm(k) = \bar{v}(k)\frac{1\pm\gamma_5}{2}$

$$|i^\pm\rangle \equiv |k_i^\pm\rangle \equiv u_\pm(k_i) = v_\mp(k_i)$$

$$\langle i^\pm| \equiv \langle k_i^\pm| \equiv \bar{u}_\pm(k_i) = \bar{v}_\mp(k_i)$$

$$\epsilon_\mu^{(\lambda=+)}(k,n) \equiv +\frac{\langle k^+|\gamma_\mu|n^+\rangle}{\sqrt{2}\langle n^-|k^+\rangle} = \left(\epsilon_\mu^{(\lambda=-)}(k,n)\right)^*$$

$$\epsilon_\mu^{(\lambda=-)}(k,n) \equiv -\frac{\langle k^-|\gamma_\mu|n^-\rangle}{\sqrt{2}\langle n^+|k^-\rangle} = \left(\epsilon_\mu^{(\lambda=+)}(k,n)\right)^*$$

… and make heavy use of various IDs

→many cancelations already at amplitude level

A reminder from before we realised that …

Dirac traces from Computer Algebra Codes

- possible to express elements of Dirac trace in terms of scalar, vector and rank 2 tensor integrals
- Evaluation requires use of computer algebra codes; use 2 implementations: FORM [Vermaseren, math-ph/0010025] & Mathematica packages FeynCalc and FormLink
- result (3 partons) as coefficients of "basis"-functions $f_{(a)}$ and $h_{(a,b)}$; result lengthy (~100kB), but manageable
- currently working on further simplification through integration by parts relation between basis function (work in progress)