Diffusive Shock Acceleration Model with Postshock Turbulence for Radio Relics

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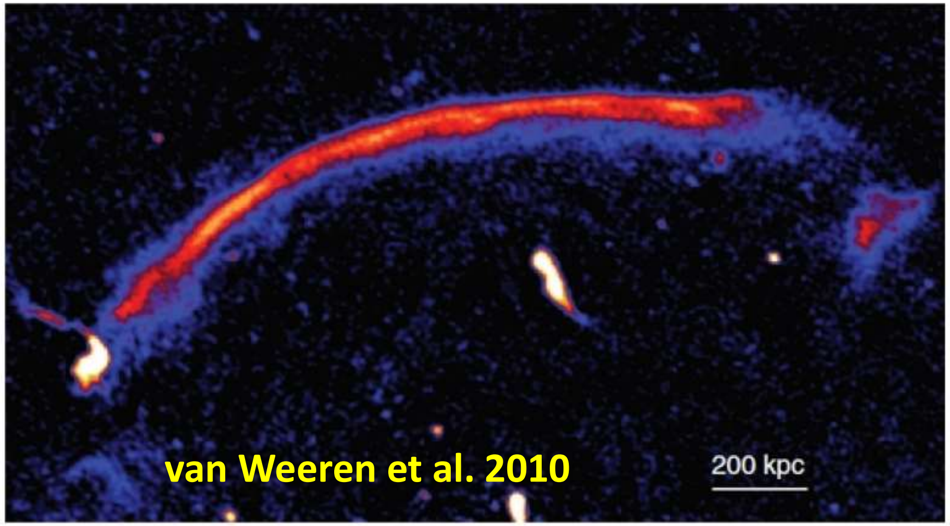
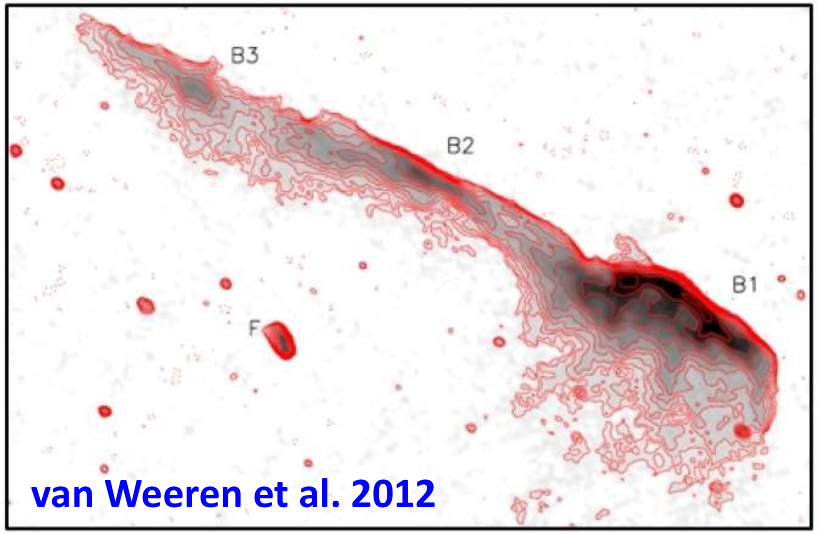
Kang, Ryu, Jones, 2017, ApJ

Sausage Relic in CIZA J2242.8+5301

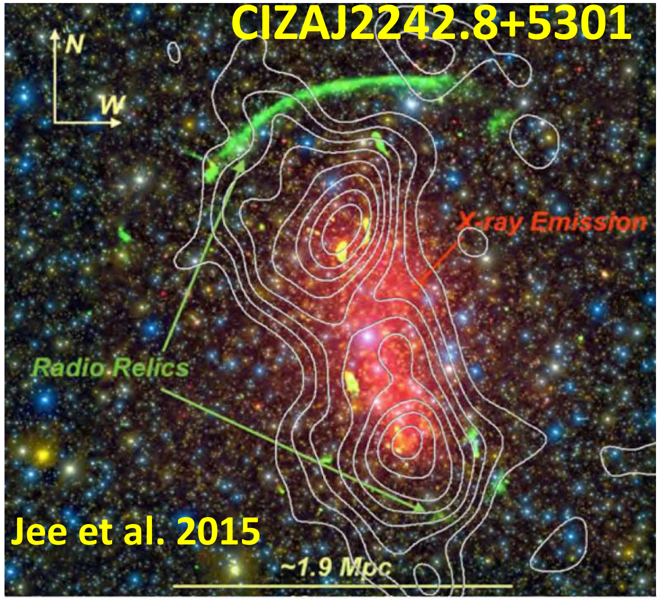


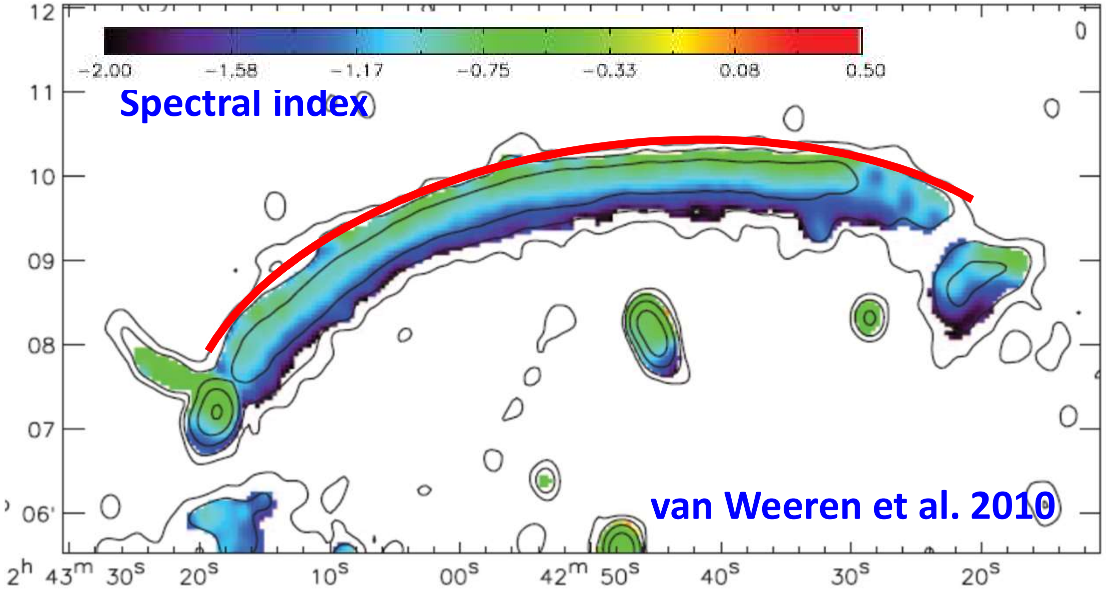
Toothbrush Relic in 1RXS J0603.3+4214

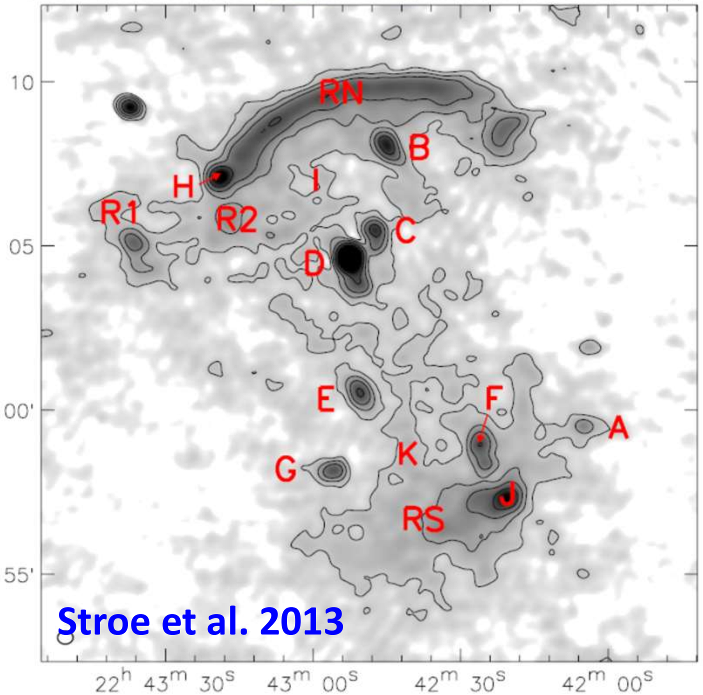


synchrotron radiation emitted by ~GeV electrons accelerated at structure formation shocks via DSA (Fermi I) process.

Sausage Relic







$$M_{radio}^2 = \frac{(3+2\alpha_{sh})}{(2\alpha_{sh}-1)}$$

$$\Rightarrow M_{radio} \approx 4.6$$

$$\frac{T_2}{T_1} = \frac{(M_X^2+3)(5M_X^2-1)}{16M_X^2}$$

$$\Rightarrow M_X \approx 2.7$$

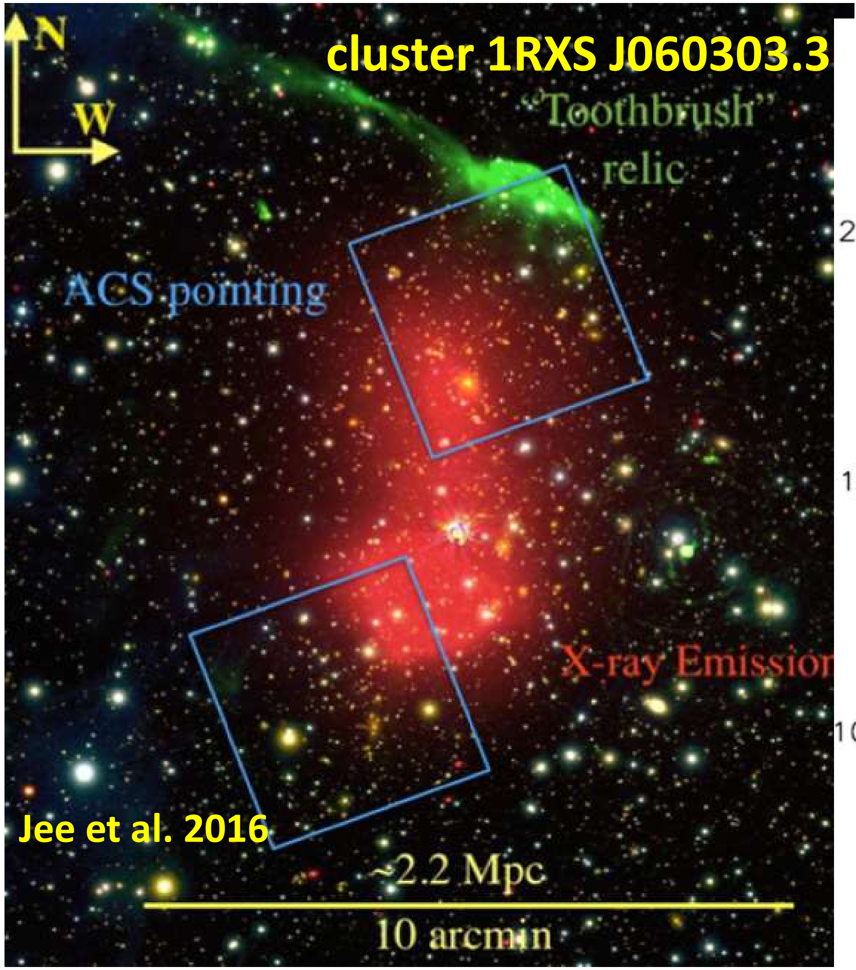
new $M_{radio} \approx 2.7$ (Hoang + 2017)

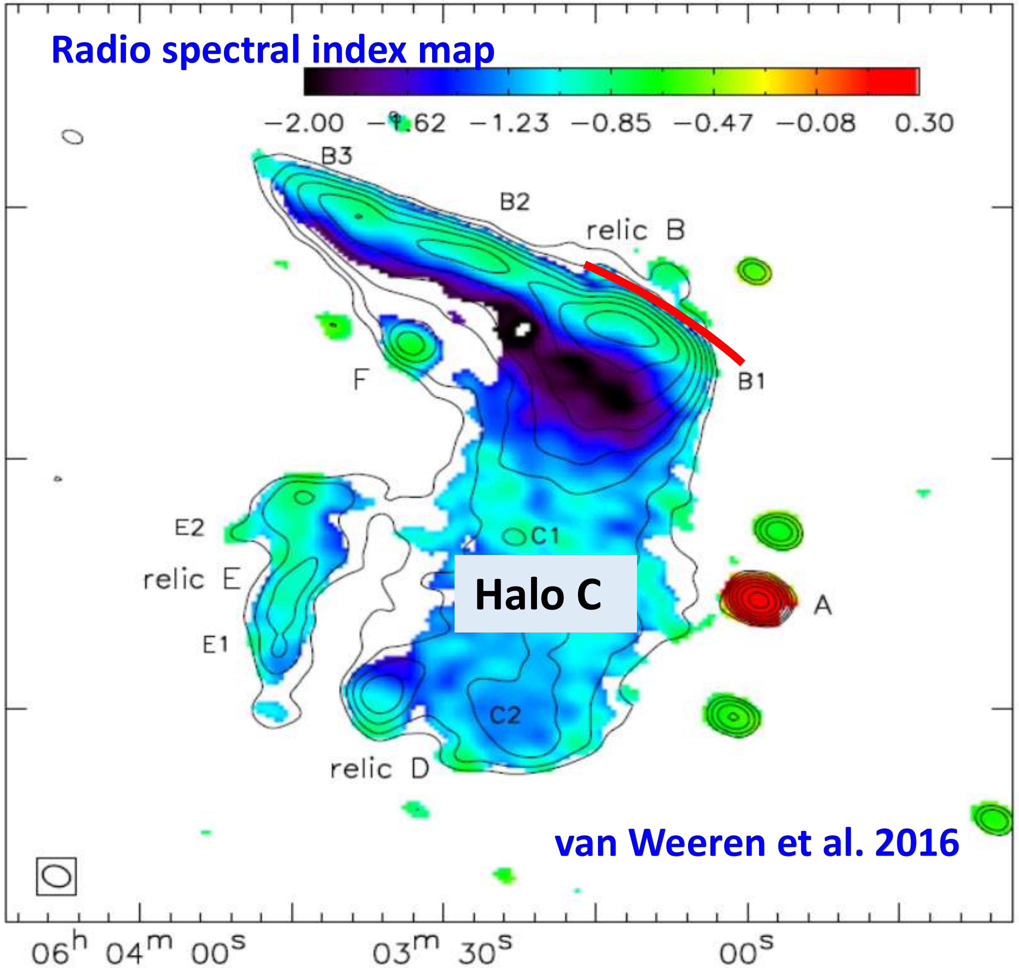
➔ **halo + radio galaxies + radio relics (RN + RS)**

Shocks run into radio tails ?

→ re-acceleration of fossil CRe

Toothbrush Relic: halo + radio galaxies + radio relics





B1 relic: $M^2_{\rm radio} = \frac{(3+2\alpha_{sh})}{(2\alpha_{sh}-1)} \Rightarrow M_{radio} \approx 2.8$ but $M_X \approx 1.5$

new $M_{\rm radio} \approx 3.3-3.8$ (Rajpurohit + 2017)

Some puzzles in DSA model with *in situ* injection only

Observations:	M_X	M_{radio}	new M_{radio}
Sausage	2.7	4.6	2.7 (Hoang + 2017)
Toothbrush	1.5	2.8	3.3 – 3.8 (Rajpurohit + 2017)

(1) For some radio relics, $M_{radio} > M_X$

(2) Only ~10 % of merging clusters host radio relics, while numerous shocks are expected to form in ICM.

(3) Some X-ray shocks do not have associated radio relics

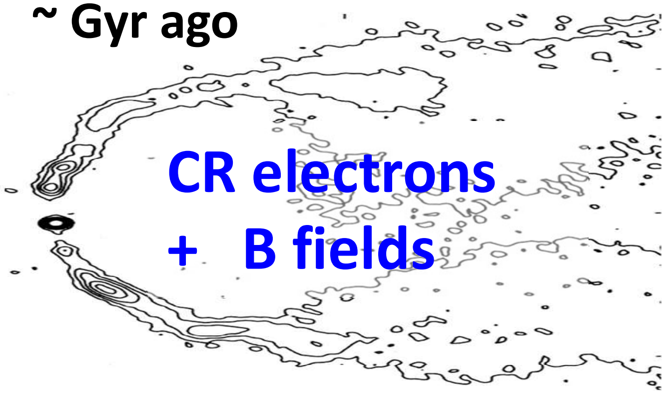
(4) Injection of thermal electrons to DSA may be inefficient.

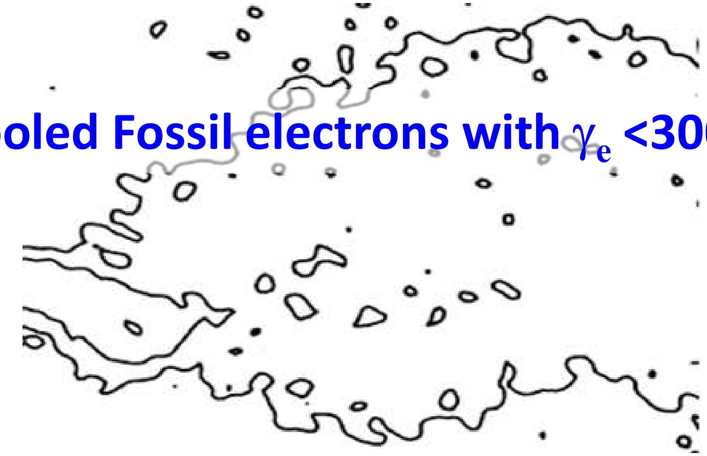
Possible solution for (2), (3), (4) is Re-acceleration model:
a radio relic forms when a weak shock encounters the ICM plasma with pre-existing live or fossil electrons.

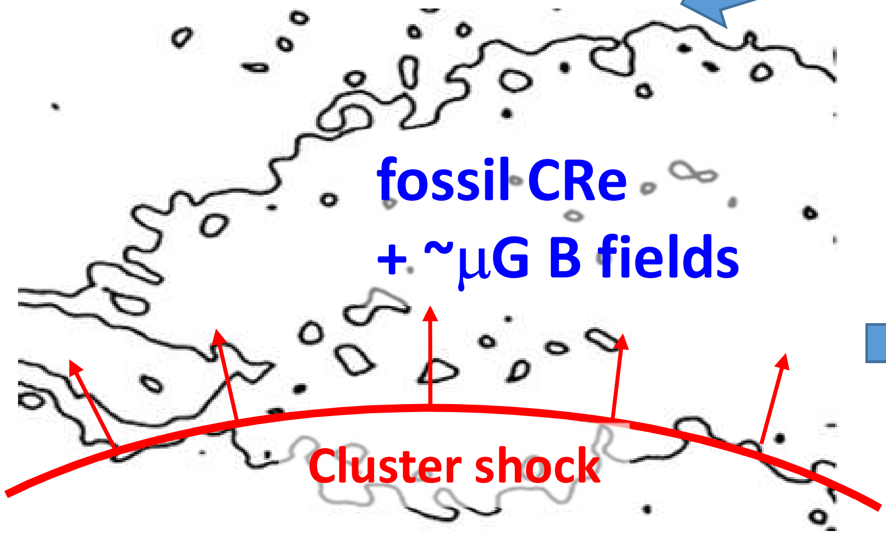
But can re-acceleration model solve (1) puzzle of $M_{radio} > M_X$ **?**

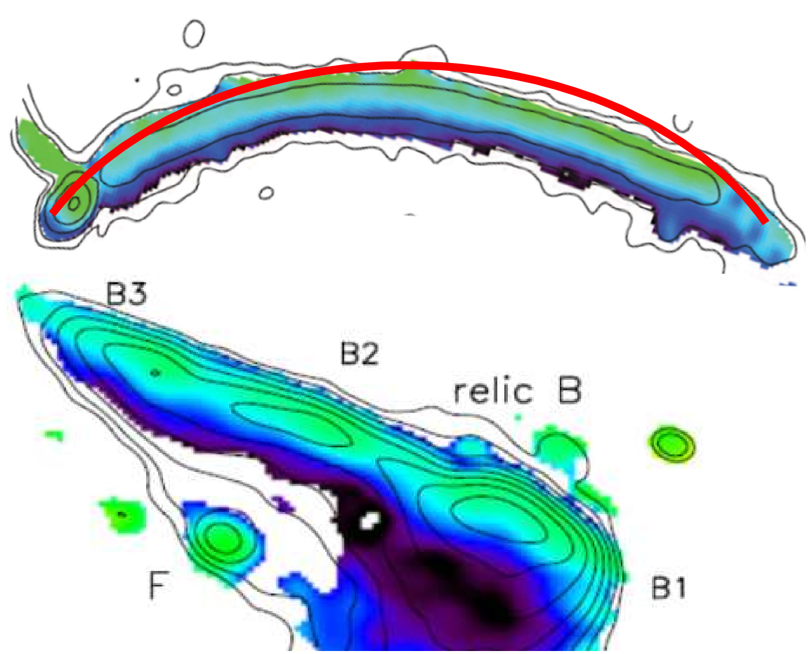
Reacceleration Model for Formation of Giant Radio Relics

~ Gyr ago









$M_s \approx 3.0$

$q_{\mathrm{DSA}} \approx 4.5 \Rightarrow \alpha_{\mathrm{sh}} \approx 0.75$

DSA model parameters for Sausage & Toothbrush

fossil CRe $f(p)$

$s \sim 4.6, \gamma_{e,c} \sim 300$

$B_1 \sim 1 \mu\text{G}$

shock properties

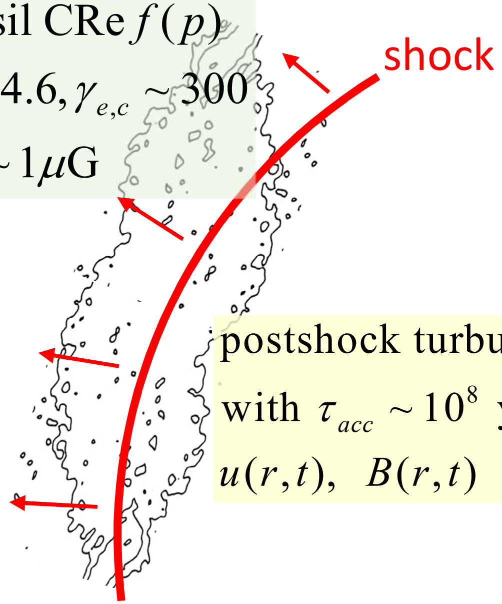
$M_s \sim 3, M_A \sim 20,$

$u_s \sim 2.5 - 3 \times 10^3$ km/s

background plasma

$n_{H,1} \sim 10^{-4} \text{cm}^{-3}$

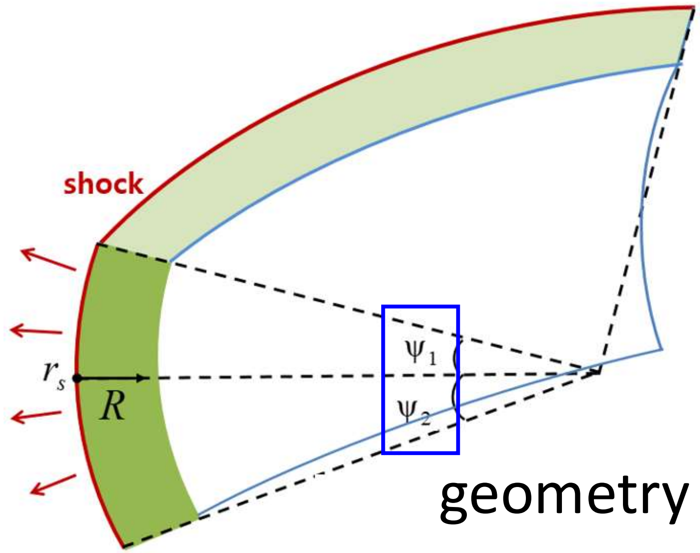
$kT_1 \sim 2 - 3$ keV



postshock turbulence

with $\tau_{acc} \sim 10^8$ yr

$u(r,t), \; B(r,t)$



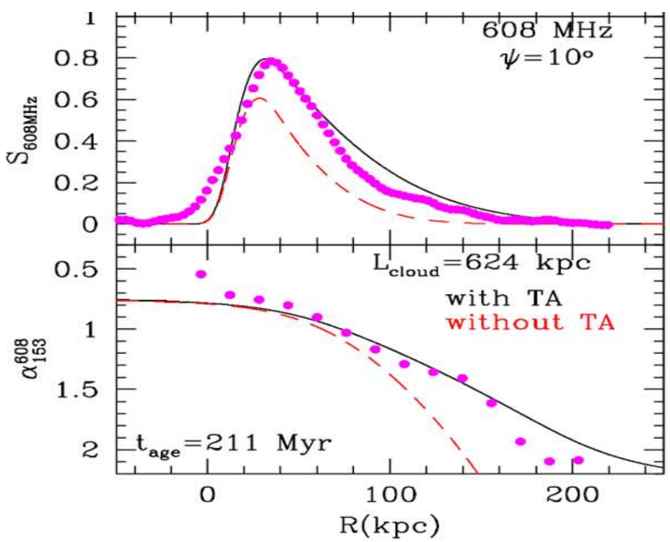
Observational Test

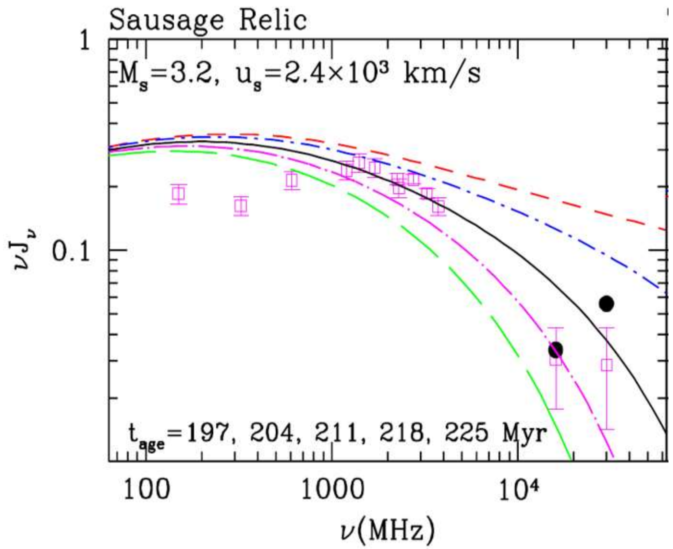
observables

$S_\nu(R)$

$\alpha_\nu(R)$

$\nu \cdot J_\nu$

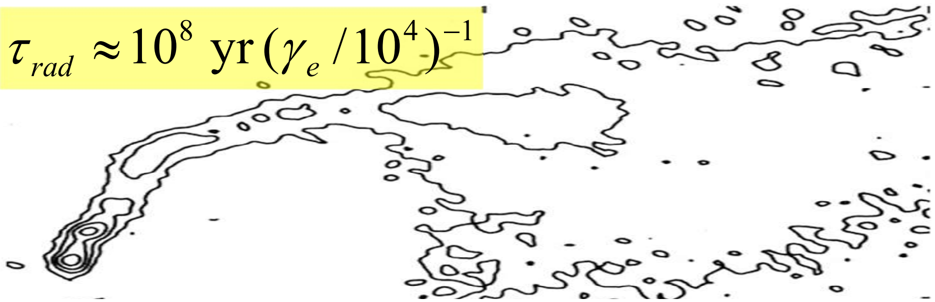


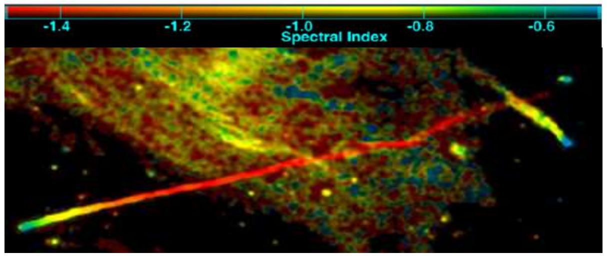


-Fitting S_ν, α_ν, & νJ_ν simultaneously is necessary.

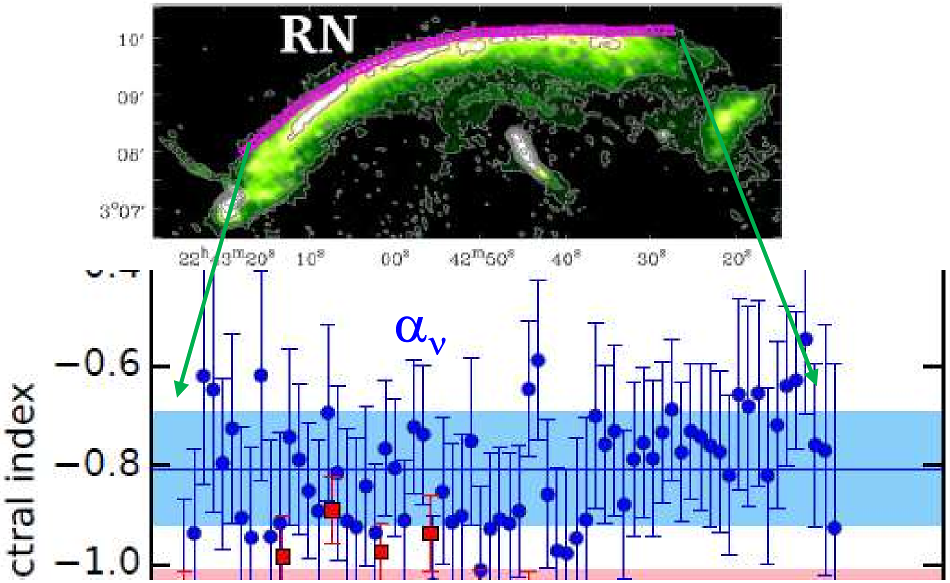
Problem with re-acceleration model with fossil CRe

$\tau_{rad} \approx 10^8\ \mathrm{yr}\,(\gamma_e/10^4)^{-1}$

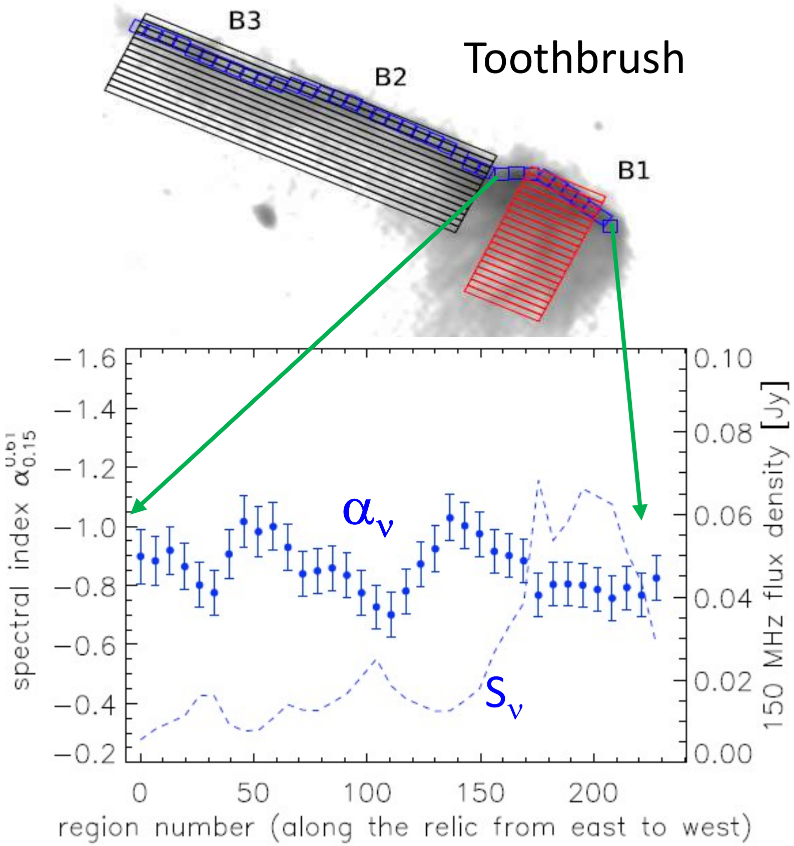




Spectral steepening due to aging electrons
➔ gradient of spectral index, α along the relic edge ?

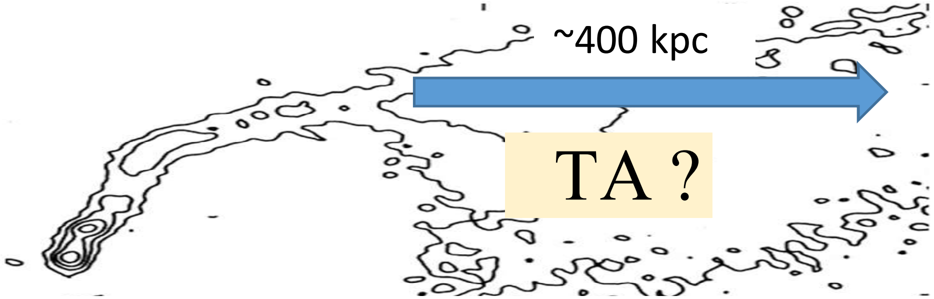


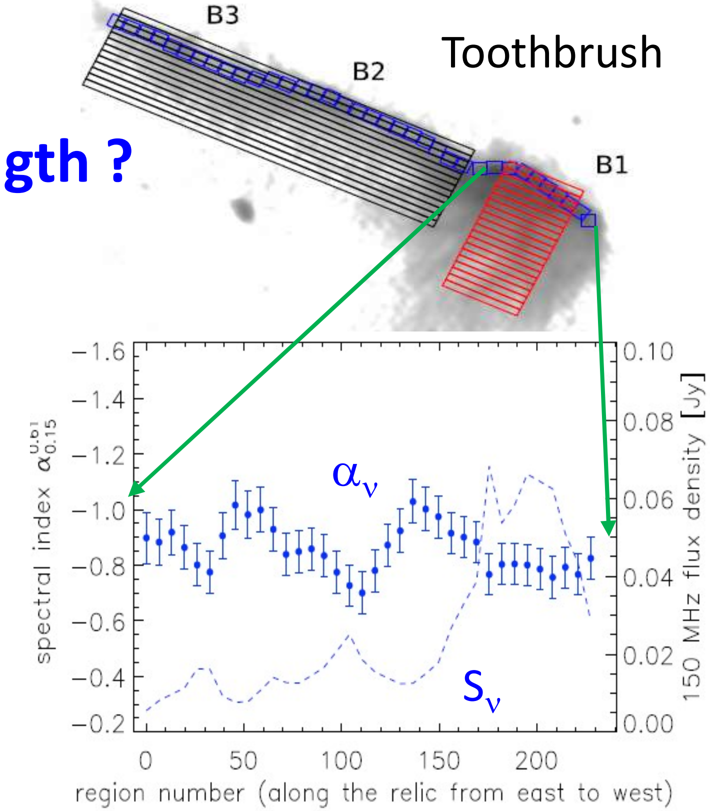
Hoang + 2017



van Weeren + 2016

Problem with re-acceleration model with fossil CRe





van Weeren et al. 2016

Q: uniform spectral index along the relic length ?

1. **strong shock model**: $M_s \approx 3$

-fossil CRe provide low E seed electrons ($\gamma_{e,c}$~300)

- $M_{\rm radio} > M_{\rm X}$: projection, multiple shocks ?

2. **weak shock model**: $M_s \approx 1.5$

additional re-energization processes (e.g. TA), so fossil CRe spectrum maintains $s \approx 4.5, \gamma_{e,c} \sim 10^5$ over ~400kpc.

$$f_{\rm pre}(p) = f_o \cdot p^{-s} \exp\left[-\left(\frac{p}{p_{e,c}}\right)^2\right]$$

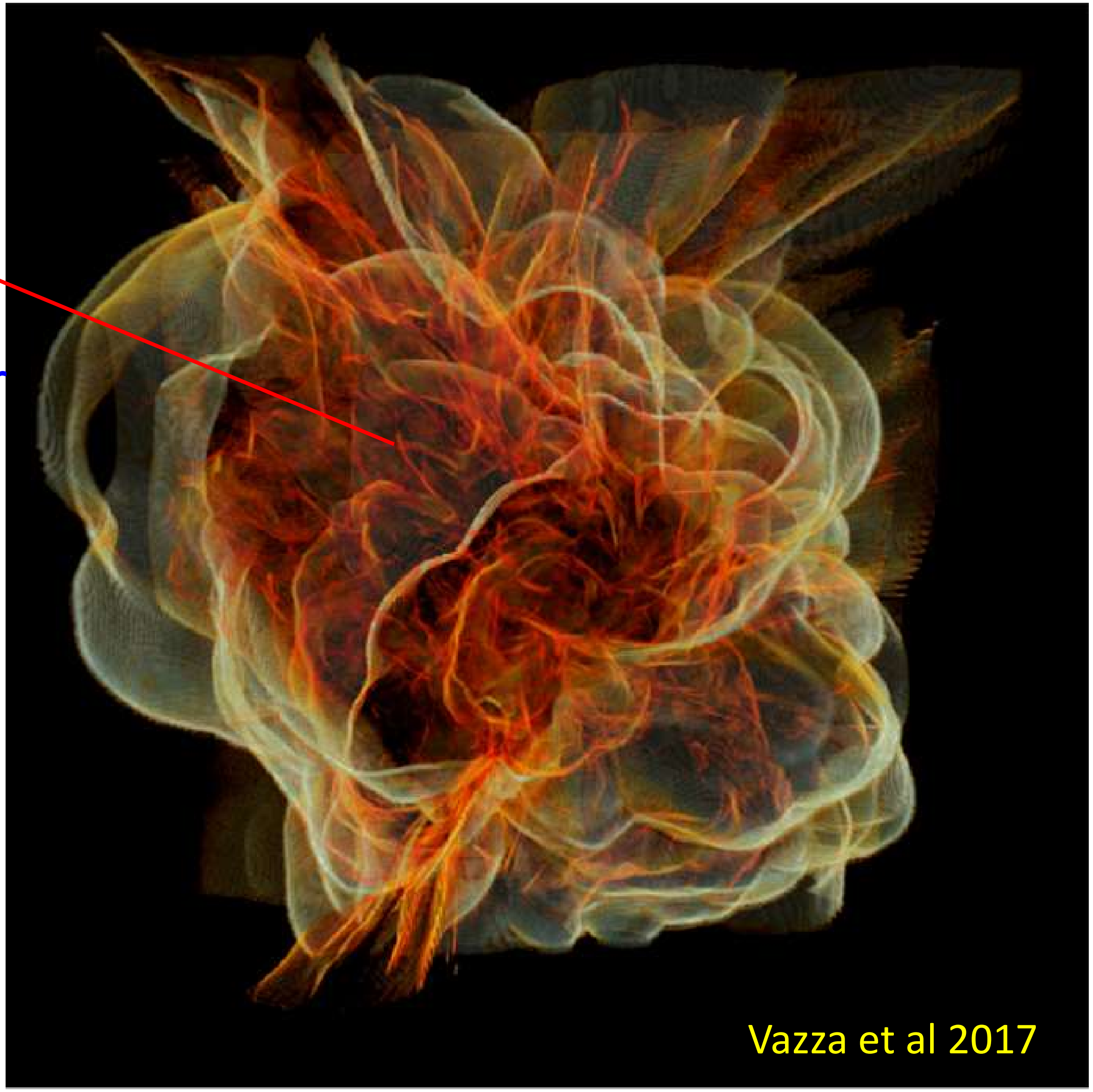
Kang, Ryu, Jones 2017

Shocks in Clusters of Galaxies in the Structure Formation Simulations

Weak shocks with M<4 **(red)**

Spherical bubbles blowing out from the cluster center during major episodes of mergers or infalls from adjacent filaments.

➔ spherically expanding shocks



DSA simulations in test-particle limit

in a co-expanding frame which expands with 1D spherical shock.

ordinary gasdynamic Eqs (high beta)

$$\frac{\partial \tilde{\rho}}{\partial t} + \frac{1}{a}\frac{\partial(\upsilon\tilde{\rho})}{\partial x} = -\frac{2}{ax}\tilde{\rho}\upsilon$$

$$\frac{\partial(\tilde{\rho}\upsilon)}{\partial t} + \frac{1}{a}\frac{\partial(\tilde{\rho}\upsilon^2 + \tilde{P}_g)}{\partial x} = -\frac{2}{ax}\tilde{\rho}\upsilon^2 - \frac{\dot{a}}{a}\tilde{\rho}\upsilon - \ddot{a}x\tilde{\rho}$$

$$\frac{\partial(\tilde{\rho}\tilde{e}_g)}{\partial t} + \frac{1}{a}\frac{\partial(\tilde{\rho}\tilde{e}_g\upsilon + \tilde{P}_g\upsilon)}{\partial x} = -\frac{2}{ax}(\tilde{\rho}\tilde{e}_g\upsilon + \tilde{P}_g\upsilon) - 2\frac{\dot{a}}{a}\tilde{\rho}\tilde{e}_g - \ddot{a}x\tilde{\rho}\upsilon - \tilde{L}(x,t)$$

$$x = r/a : \text{co-moving coordinate}, \quad a = \text{expansion factor}$$

CR transport Equation for electron distribution function

$$g_e = f_e \cdot p^4, \;\; y = \ln(p/m_e c)$$

$$\frac{\partial g_e}{\partial t} + u\frac{\partial g_e}{\partial r} = \frac{1}{3r^2}\frac{\partial(r^2 u)}{\partial r}\left(\frac{\partial g_e}{\partial y} - 4g_e\right) + \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\kappa(r,p)\frac{\partial g_e}{\partial r}\right] + p\frac{\partial}{\partial y}\left[\frac{D_{pp}}{p^3}\left(\frac{\partial g_e}{\partial y} - 4g_e\right)\right] + p\frac{\partial}{\partial y}\left(\frac{b}{p^2}g_e\right)$$

Spatial diffusion = Fermi I

Momentum diffusion = Fermi II

Coulomb/ Synchrotron/iC losses

Table 1. Parameters for Model Spherical Shocks

Model	M_X	M_{radio}	$M_{s,i}$	kT_1 (keV)	B_1 (μG)	t_{obs} (Myr)	$M_{s,obs}$	$kT_{2,obs}$ (keV)	$u_{s,obs}$ (km s^{-1})	N (10^{-4})
Sausage	2.7	4.6	4.0	2.1	1	211	3.21	8.6	2.4×10^3	1.2
Toothbrush	1.5	2.8	3.6	3.0	1	144	3.03	11.2	2.7×10^3	5.0

M_X: Mach number inferred from X-ray observations

M_{radio}: Mach number estimated from observed radio spectral index at the relic edge

$M_{s,i}$: initial shock Mach number at the onset of the simulations ($t_{age} = 0$)

kT_1: gas temperature in the preshock ICM

B_1: magnetic field strength in the preshock ICM

t_{obs}: shock age when the simulated results match the observations

$M_{s,obs}$: shock Mach number at t_{obs}

$kT_{2,obs}$: postshock temperature at t_{obs}

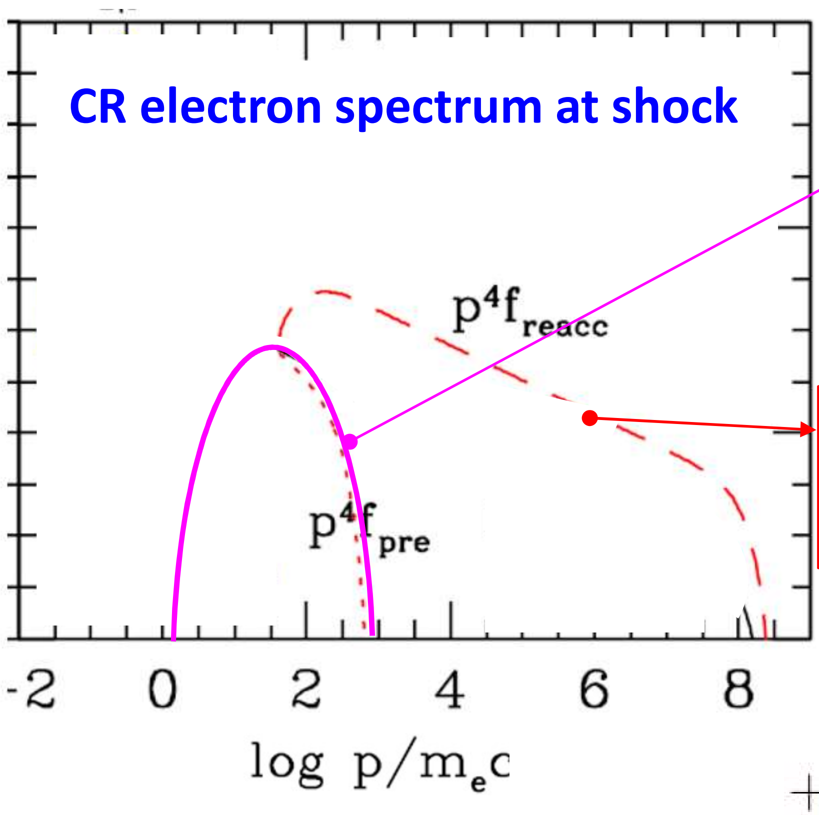
$u_{s,obs}$: shock speed at t_{obs}

$N = P_{CRe}/P_g$: the ratio of seed CR electron pressure to gas pressure in the preshock region

$$D_{pp} \approx \frac{p^2}{4\tau_{acc}}, \quad \tau_{acc} \approx 10^8 \text{ yr}$$

The spherical shock slows down and its Mach number decreases in time.

pre-existing fossil electrons: utilizing analytic solutions at the shock



Low-energy fossil electrons: pre-existing

$$f_{\rm pre}(p) = f_o \cdot p^{-s} \exp\left[-\left(\frac{p}{p_{e,c}}\right)^2\right]$$

Shock-accelerated spectrum: at the shock

$$f_{\rm reacc}(r_s, p) = q \cdot p^{-q} \int_{p_{\rm inj}}^{p} p'^{q-1} f_{\rm pre}(p') dp'$$

in the postshock region

$$+p\frac{\partial}{\partial y}\left[\frac{D_{pp}}{p^3}\left(\frac{\partial g_e}{\partial y} - 4 g_e\right)\right] + p\frac{\partial}{\partial y}\left(\frac{b}{p^2} g_e\right)$$

Momentum diffusion = Fermi II acceleration

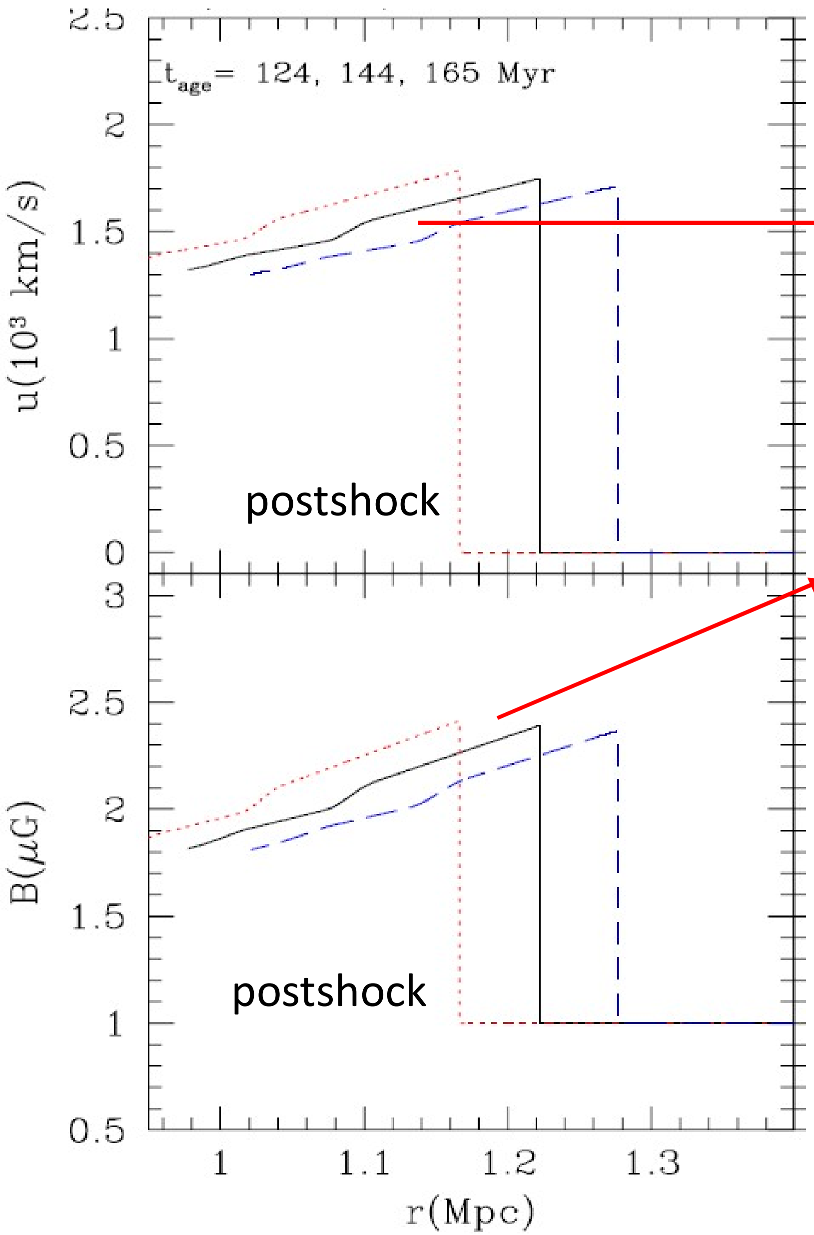
Coulomb/ Synchrotron/iC losses

Adopting the analytic solution at the shock can ease severe computing requirements to cover a wide dynamic range of length scales.

(1) strong shocks with $M_s \simeq 3$: $\gamma_{e,c} \sim 300$

(2) weak shocks with $M_s \simeq 1.5$: $\gamma_{e,c} \sim 10^5$

Additional simplification: spherical shock & postshock TA



-flow speed behind the shock

$$u_{\mathrm{dn}}(r,t)$$

-magnetic field behind the shock

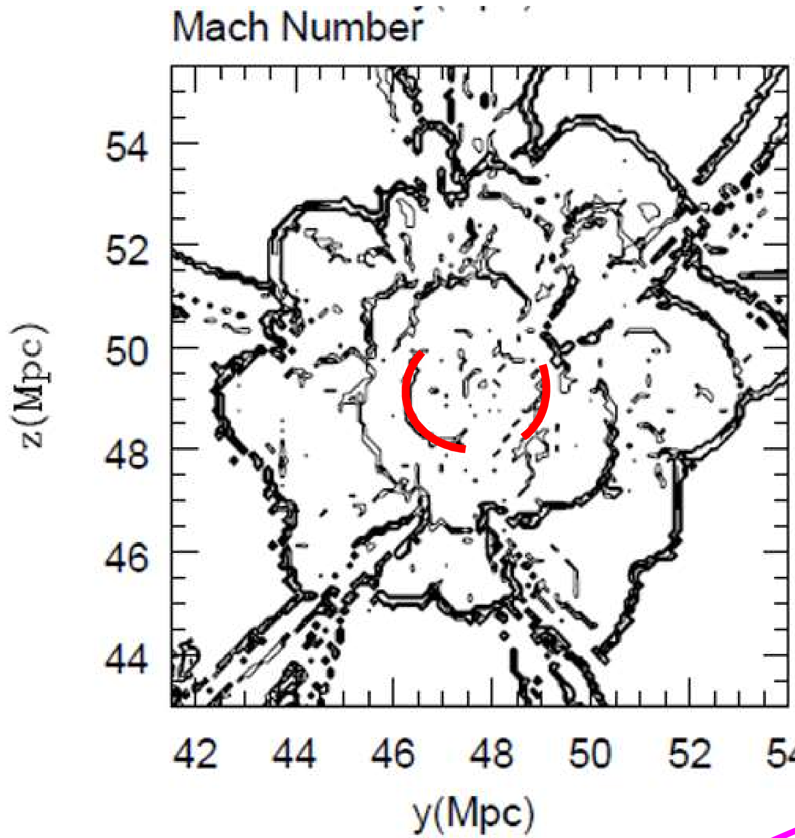
$$B_{\mathrm{dn}}(r,t) = B_2(t) \cdot [P_g(r,t)/P_{g,2}(t)]^{1/2}$$

-TTD of fast mode turbulence

$$D_{pp} \approx \frac{p^2}{4\tau_{\mathrm{acc}}}, \quad \tau_{\mathrm{acc}} \approx 10^8 \text{ yr}$$

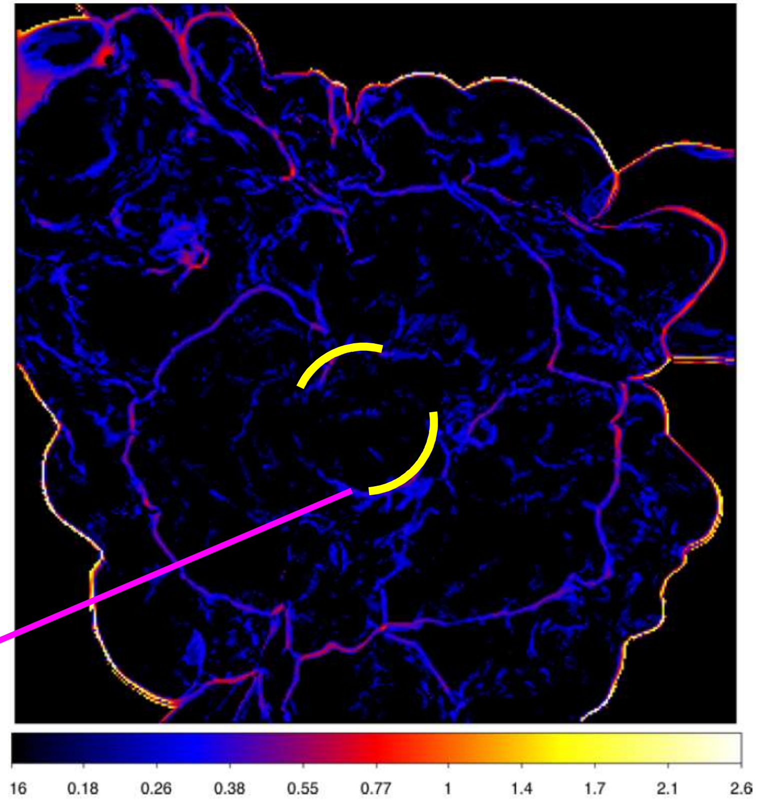
$$\tau_{\mathrm{acc}} = \tau_{\mathrm{acc},0} \cdot \exp\left[\frac{(r_s - r)}{r_{\mathrm{dec}}}\right]$$

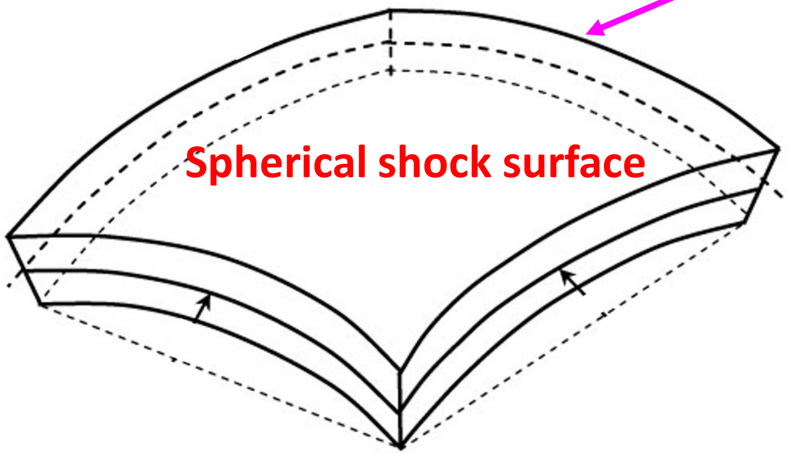
decay of turbulence behind the shock

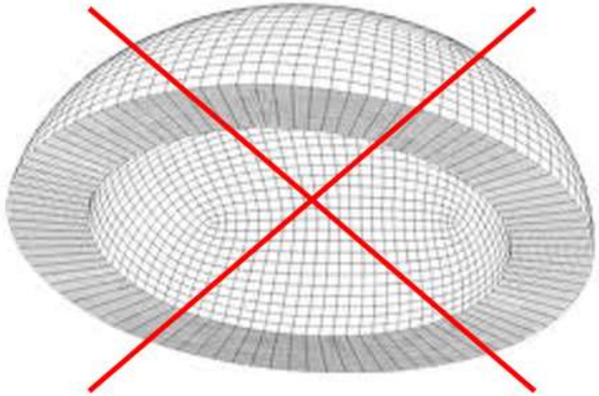


6.3Mpc Vazza et al 2017

$\log M_s$

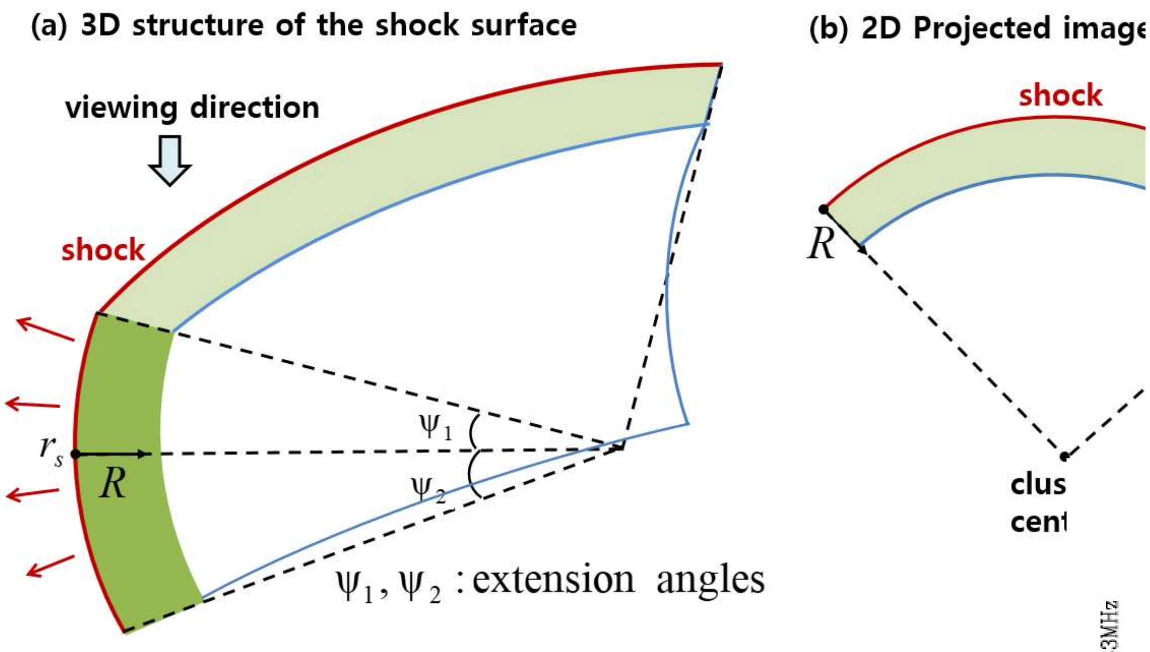


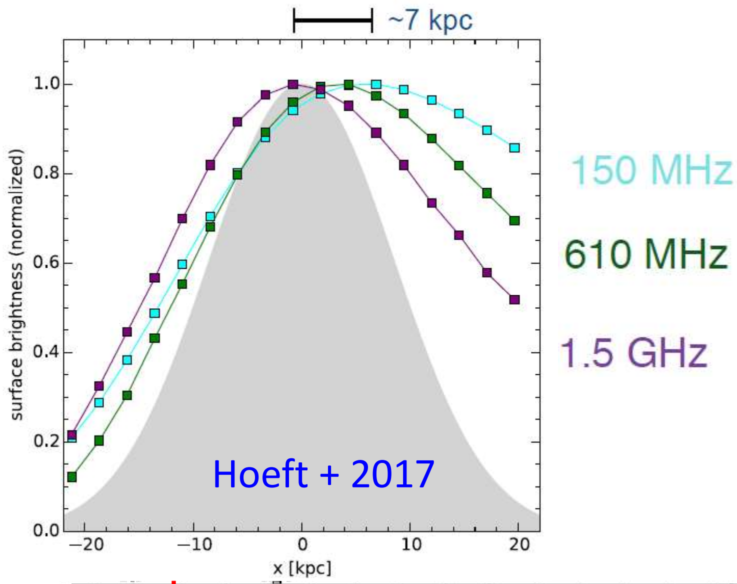


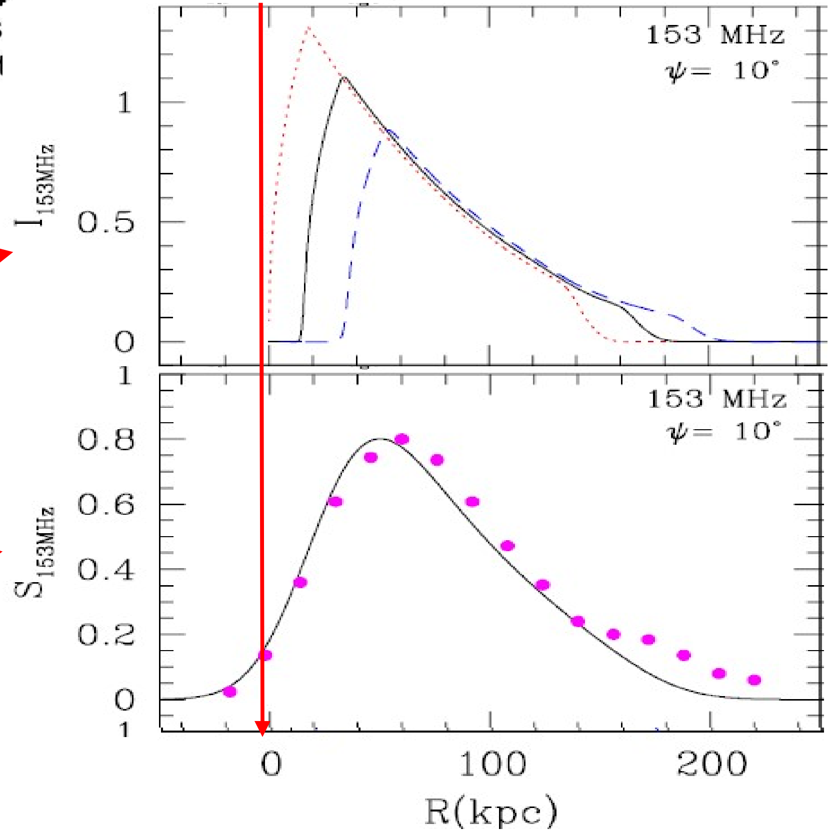


to explain uniform surface brightness (projection along line of sight)

Modeling the projection with extension ar







1. Surface brightness profile

$$I_\nu(R) = \int_0^{h_{1,\max}} j_\nu(r)dh_1 + \int_0^{h_{2,\max}} j_\nu(r)dh_2$$

2. Radio flux density profile

$$S_\nu(R) = \int_{beam} I_\nu(R)d\Omega$$

Smoothed over Gaussian telescope beam

Simple picture:

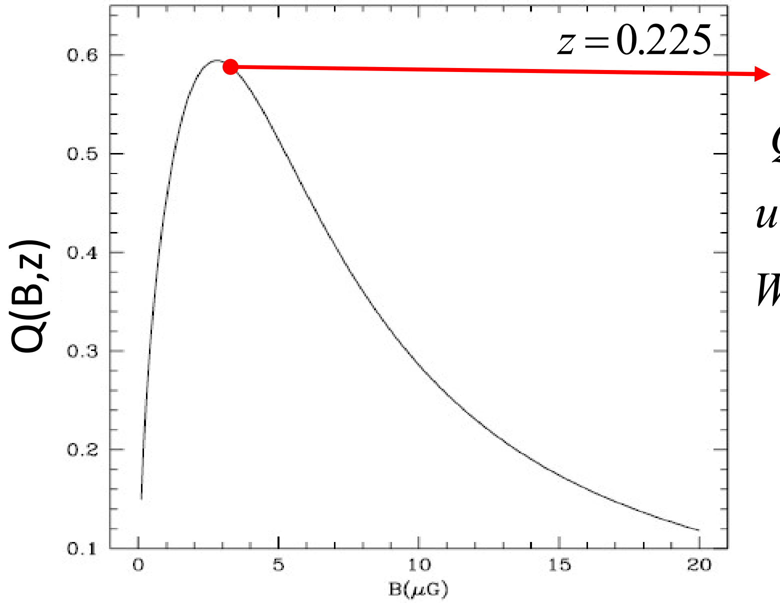
Relic width at a given frequency ~ cooling length of electrons

$$l_{cool} \approx u_2 \cdot t_{cool}(B,z) \approx 100\text{kpc} \cdot W_h \cdot u_{2,3} \cdot Q(B,z) \cdot [\frac{\nu_{obs}(1+z)}{0.63\text{GHz}}]^{-1/2}$$

depends on u_2 and B_2 for given ν_{obs}, z

$$Q(B,z) \equiv \left[\frac{(5\ \mu\text{G})^2}{B^2 + B_{\text{rad}}(z)^2}\right]\left(\frac{B}{5\ \mu\text{G}}\right)^{1/2}$$

$$l_{cool} \approx 66\text{kpc} \cdot (\frac{Q}{0.6}) \text{ at 610 MHz}$$

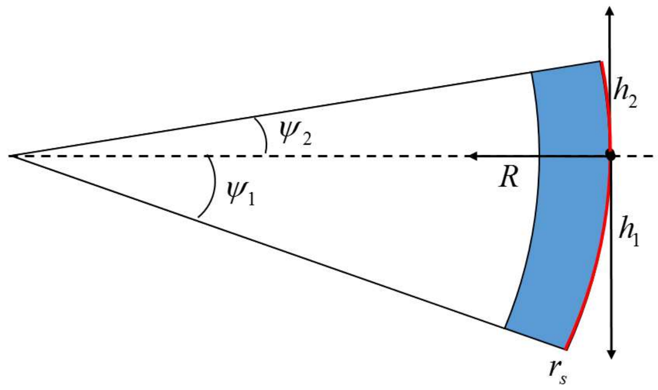


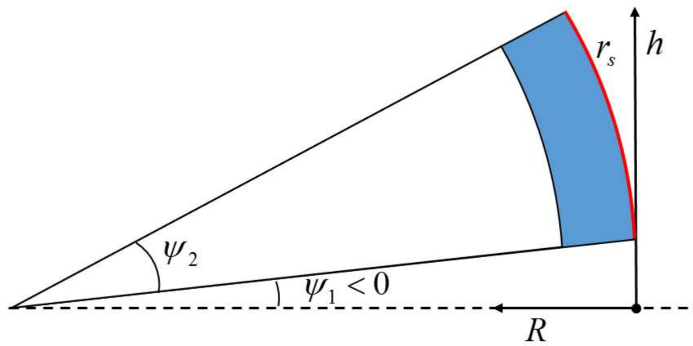
$Q_{\max}(z,B) \approx 0.6$ for $B_2 \approx 2.5$ -3.0μG,

$u_{2,3} = u_2 / 1000\text{km/s}$

$W_h \sim 1.1 - 1.2$ depends on $u(r)$ of the postshock flow

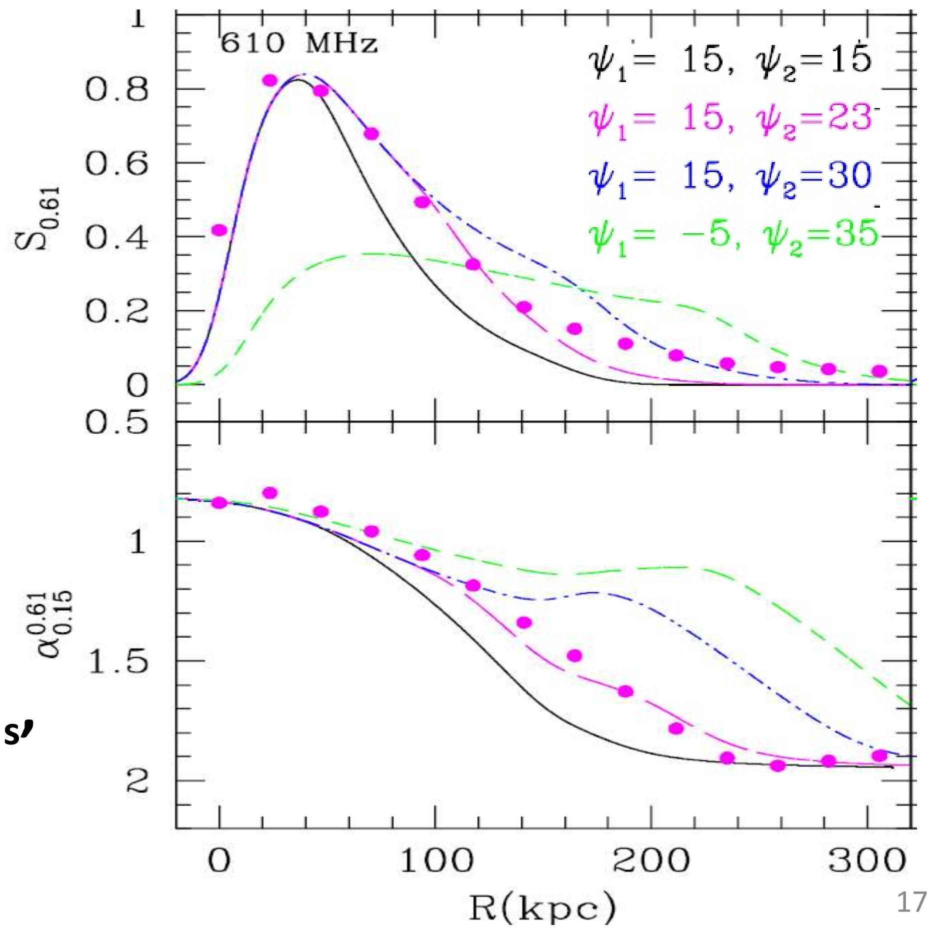
Projection of a partial shell: extension depth and viewing angle





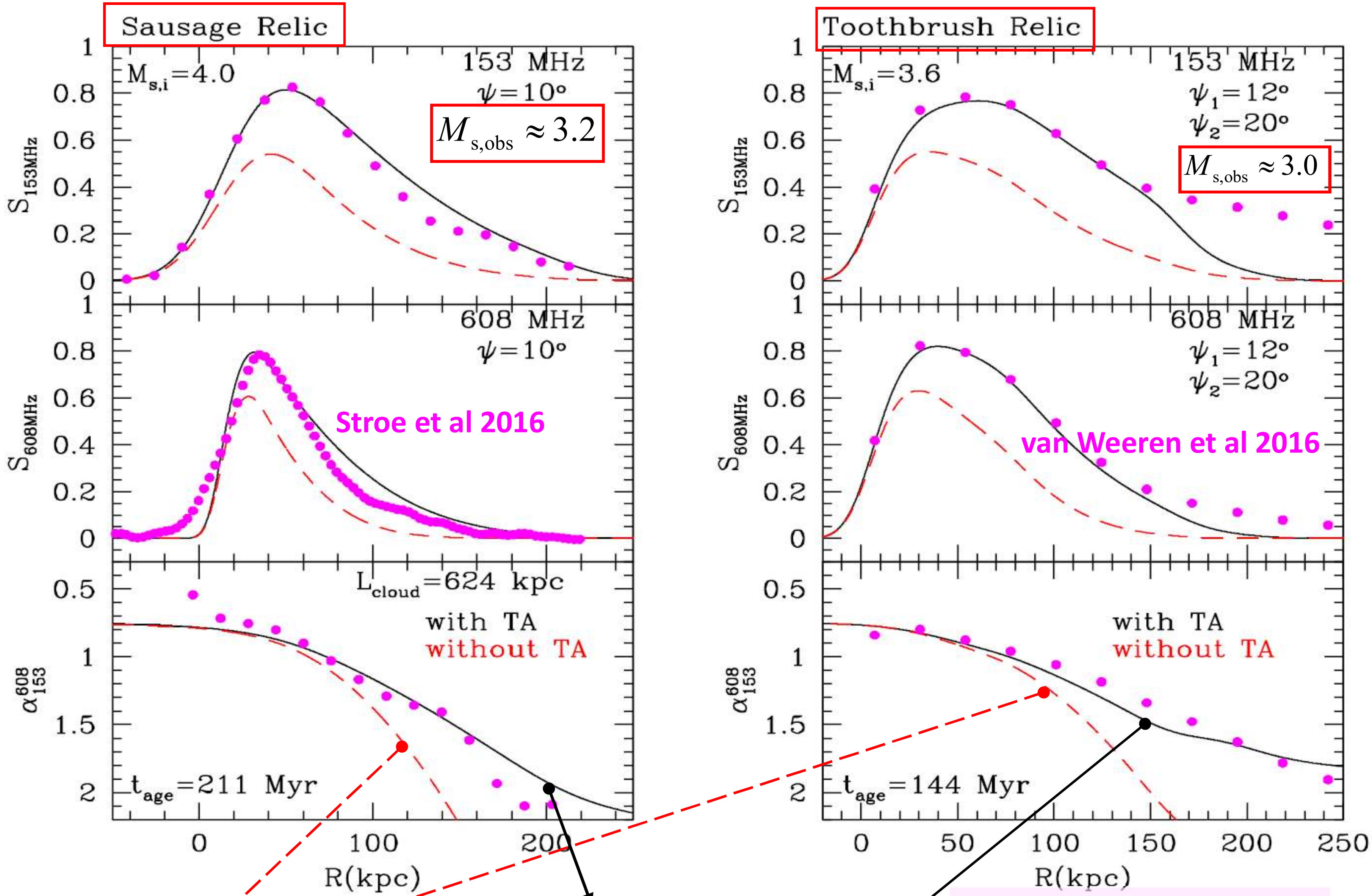
$$I_\nu(R) = \int_0^{h_{1,\max}} j_\nu(r)dh_1 + \int_0^{h_{2,\max}} j_\nu(r)dh_2$$

$$S_\nu(R) = \int_{beam} I_\nu(R)d\Omega$$



Observed profiles of radio flux S_ν & spectral index α_ν depend on extension angles ψ_1 & ψ_2
in addition to shock parameters (M_s, V_s, B_0)

Fitting of Radio Flux & Spectral index Profiles



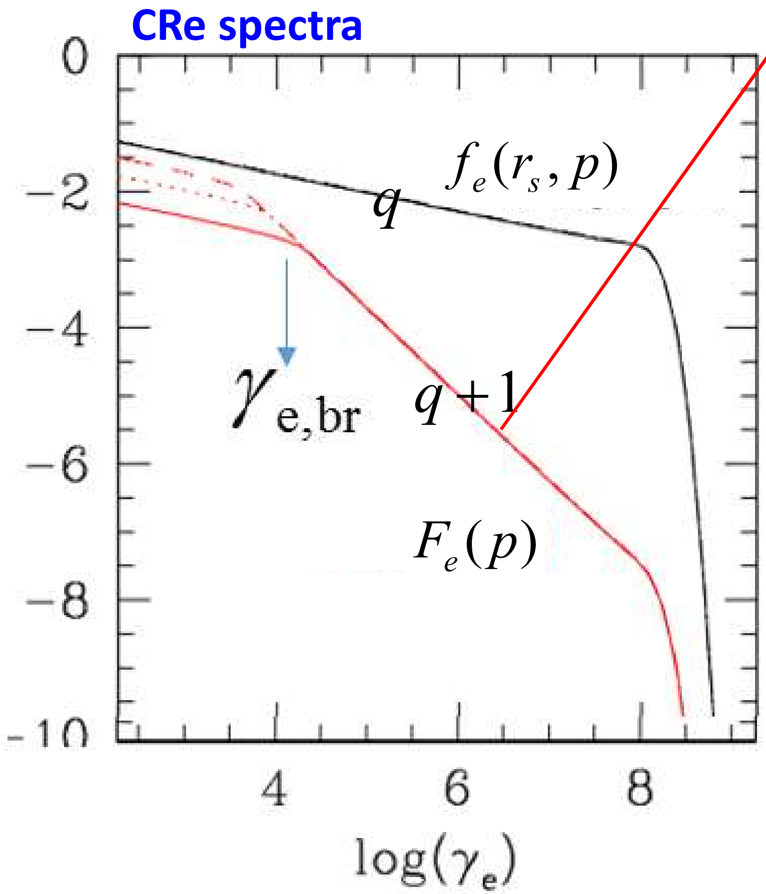
Radiative cooling only

Turbulent acceleration with $D_{pp} \approx \frac{p^2}{4\tau_{acc}}, \quad \tau_{acc} \approx 10^8$ yr

Spectral curvature due to Radiative Cooling

- test - particle power - law : $f_e(r_s, p) \propto p^{-q}$ at the shock
- volume - integrated spectrum : $F_e(p) = \int f_e(p) dV \propto p^{-(q+1)}$ for $\gamma_e > \gamma_{e,br}$

sync + iC cooling behind the shock



Steepening of volume-integrated spectrum at high energies due to cooling

$$\gamma_{e,br} \approx 10^4 \left(\frac{t_{age}}{100\mathrm{Myr}}\right)^{-1} \left(\frac{B_{e,2}}{5\ \mu\mathrm{G}}\right)^{-2}$$

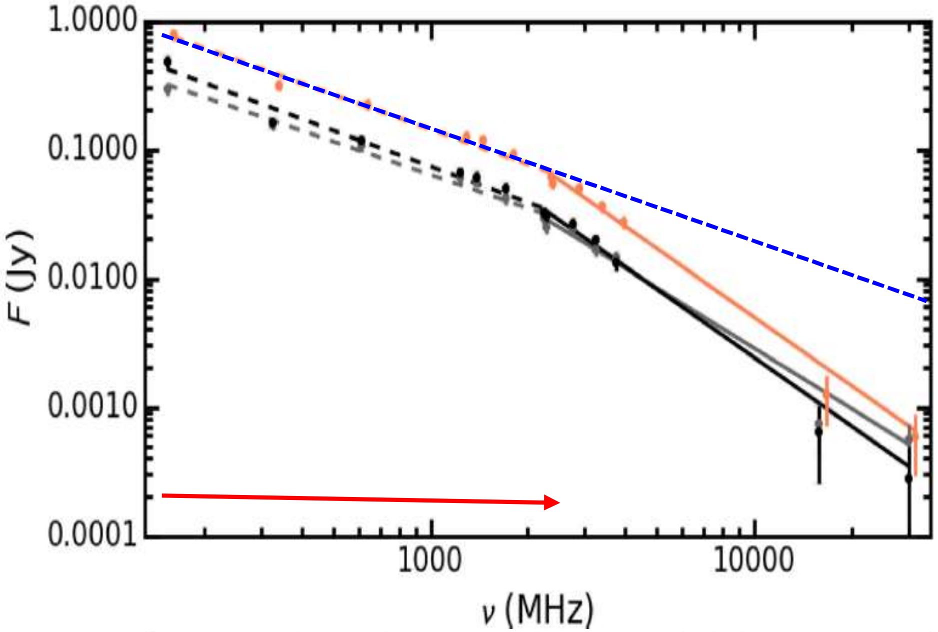
break Lorentz factor

$j_\nu(r_s) \propto \nu^{-\alpha_{sh}}$ at the shock

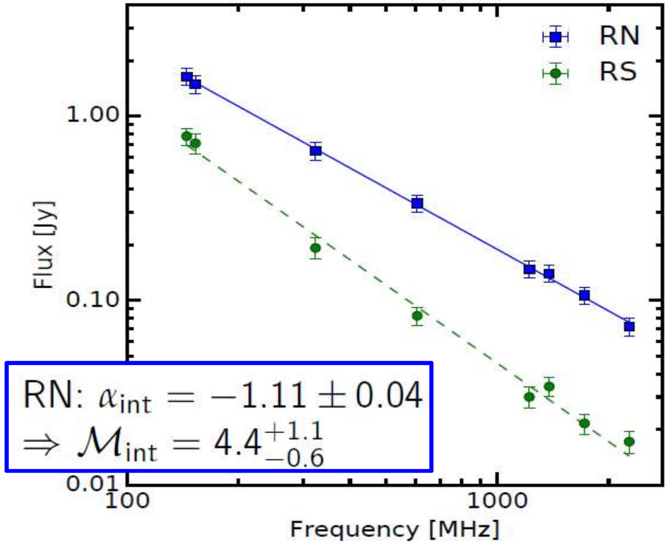
$J_\nu \propto \nu^{-(\alpha_{sh}+0.5)}$ for $\nu_e > \nu_{br}$

$$\nu_{br} \approx 0.63\mathrm{GHz} \left(\frac{t_{age}}{100\mathrm{Myr}}\right)^{-2} \left(\frac{5^2}{B_2^2 + B_{rad}^2}\right)^2 \left(\frac{B_2}{5}\right)$$

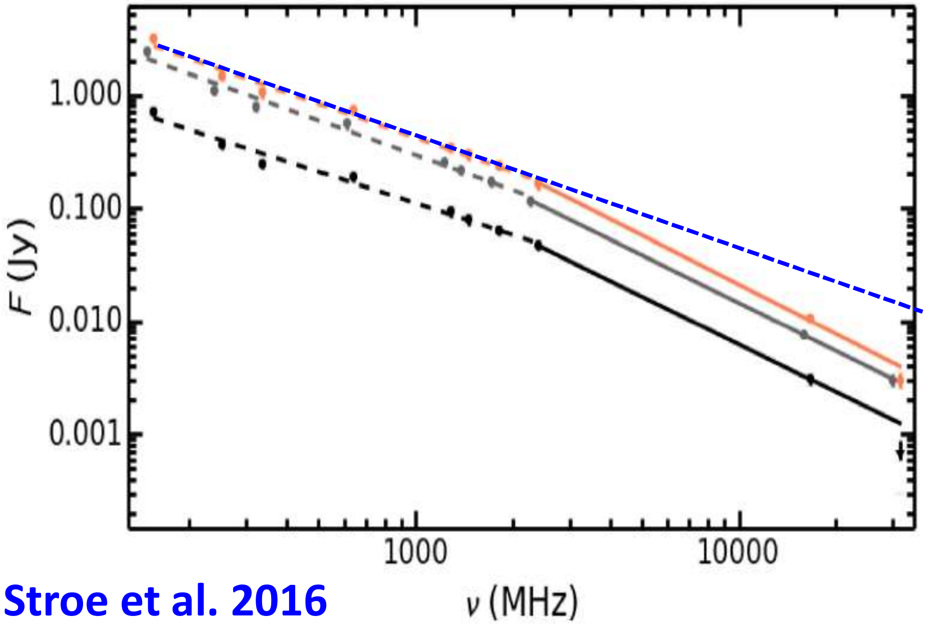
Observed integrated spectra: curvature at high frequency



Sausage



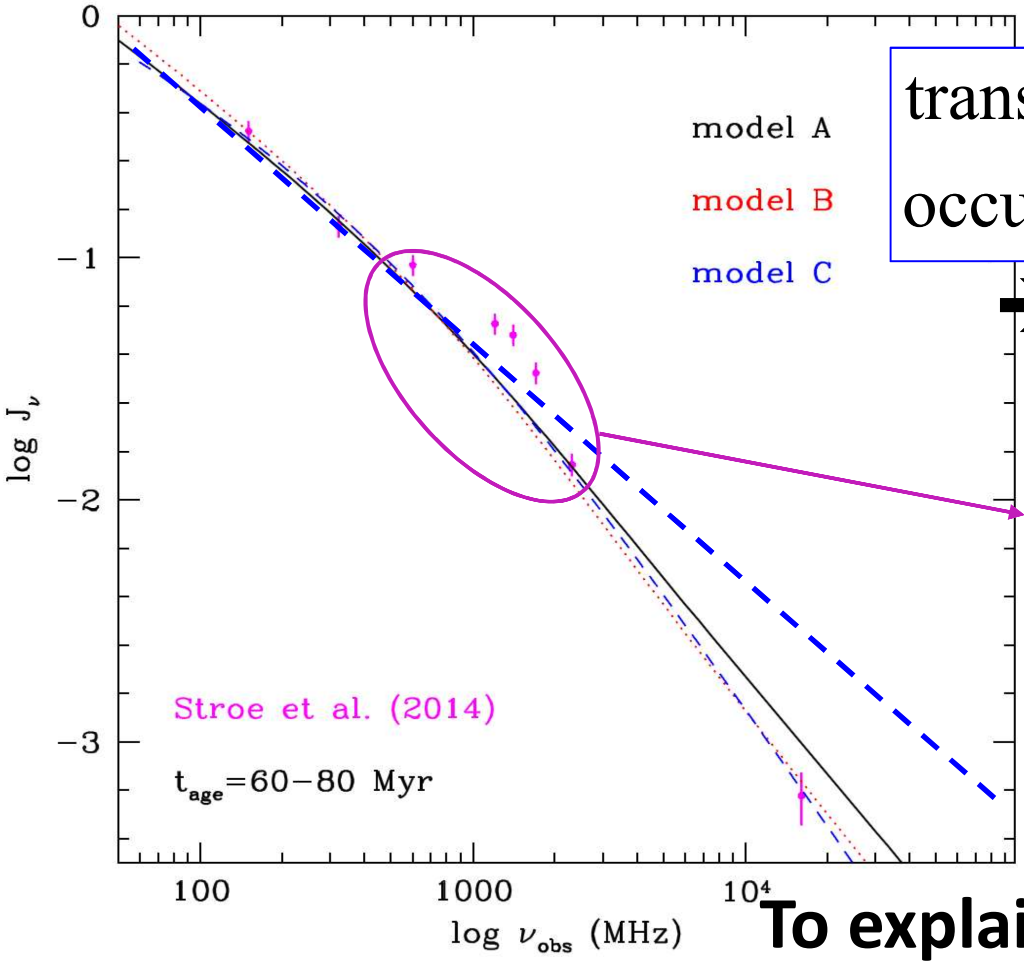
Hoang+ 2017



Toothbrush

Stroe et al. 2016

Integrated Spectrum of Sausage Relic: curvature due to cooling



Kang & Ryu 2015

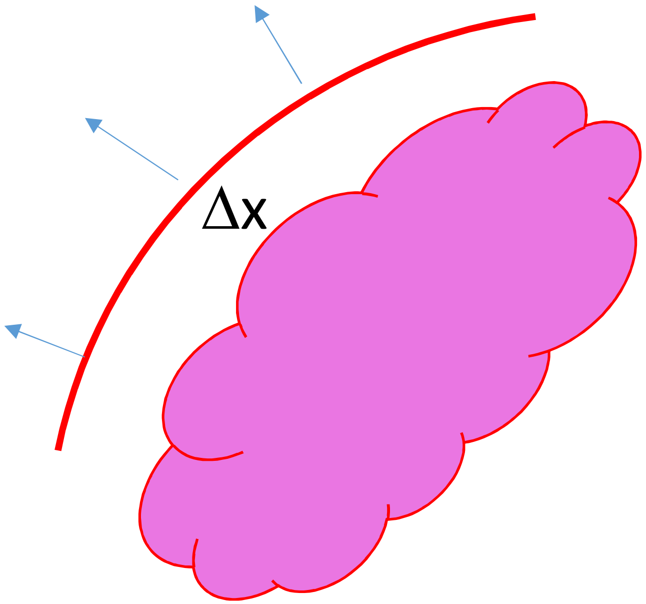
transition from α_{sh} to $\alpha_{sh} + 0.5$ occurs smothly over (0.1 - 10) ν_{br}

➔ **Curved spectrum with break frequency ν_{br} ~1GHz**

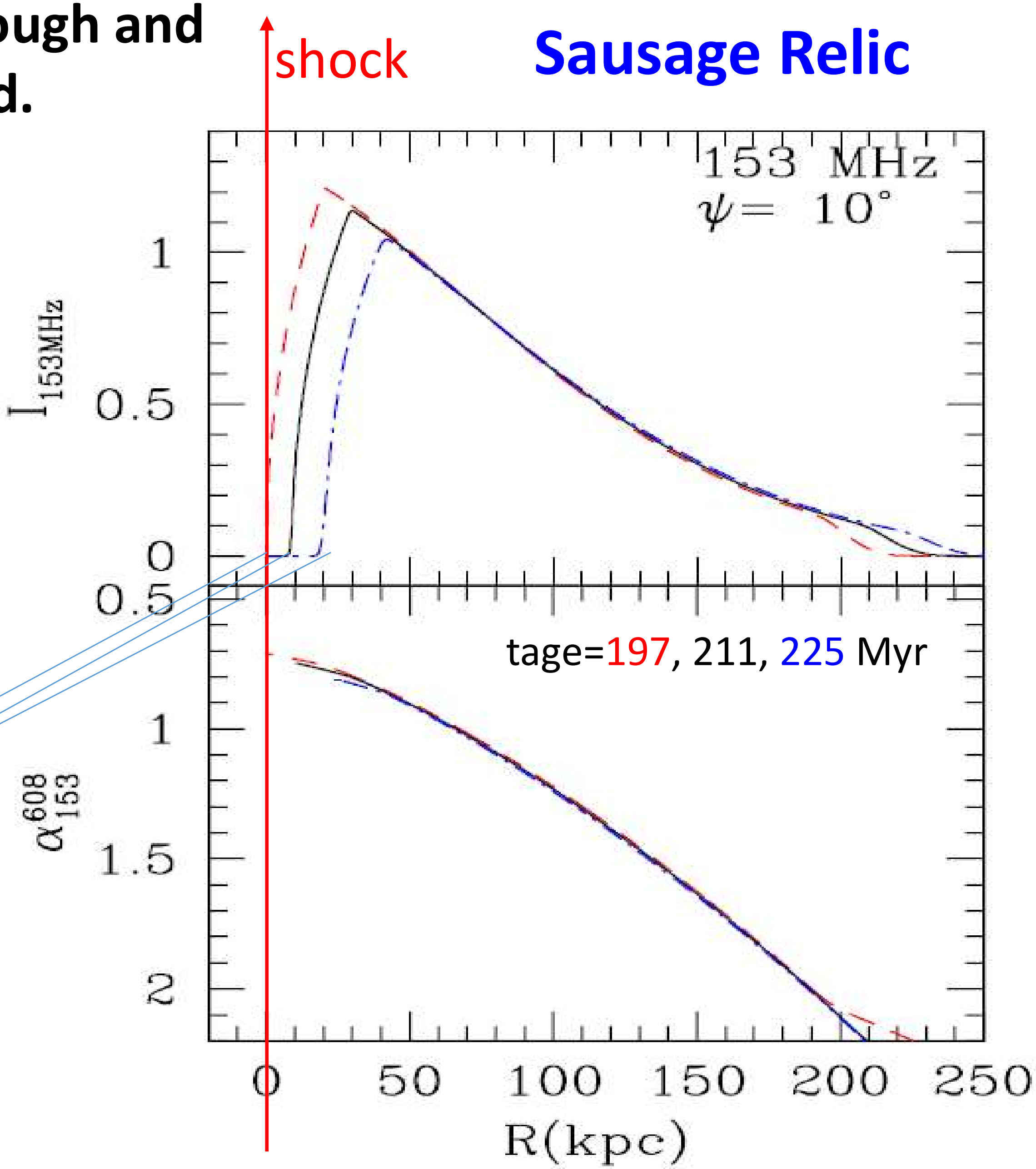
DSA + radiative cooling alone cannot explain the abrupt curvature above 1.5 GHz in the observed spectrum

To explain the observed spectral curvature we need additional reduction of high energy electrons in the downstream volume.

The shock has passed through and broken out the fossil cloud.



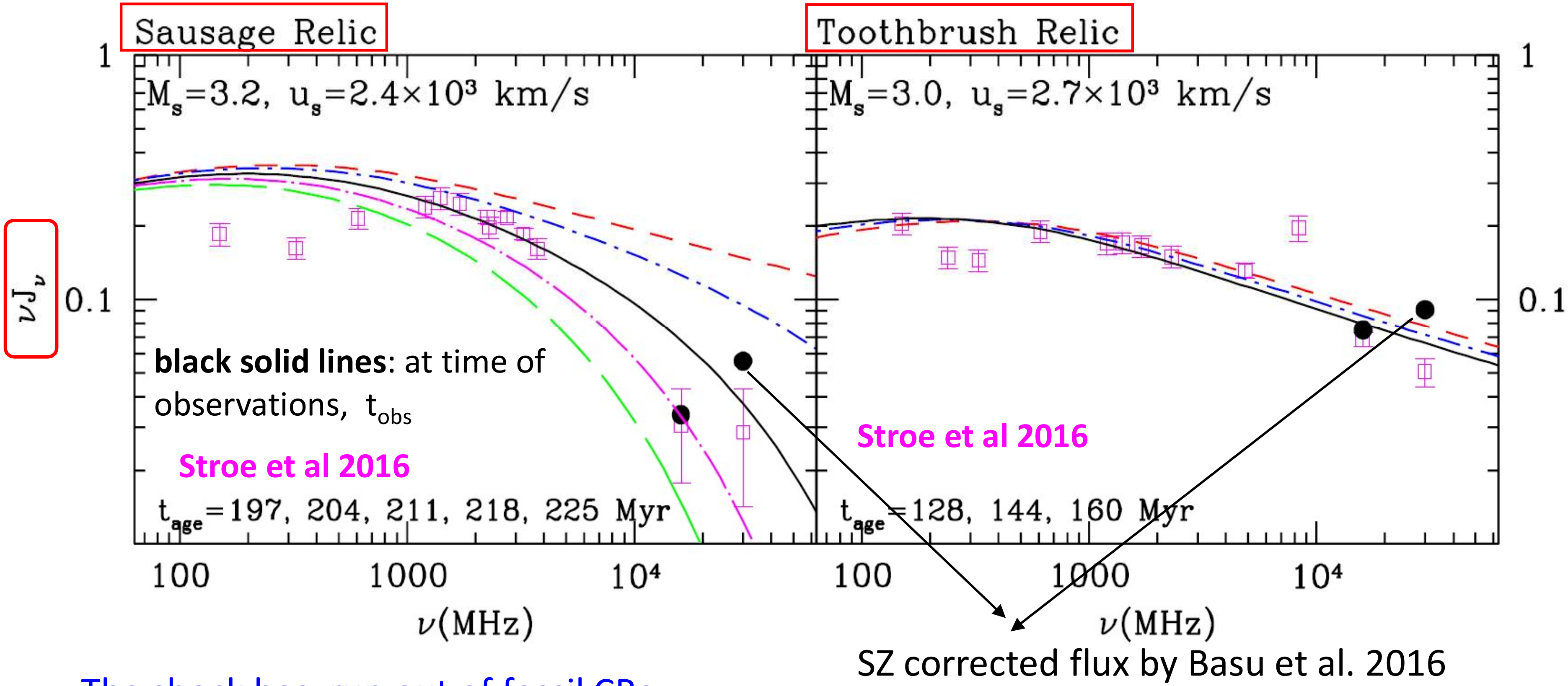
Sausage Relic



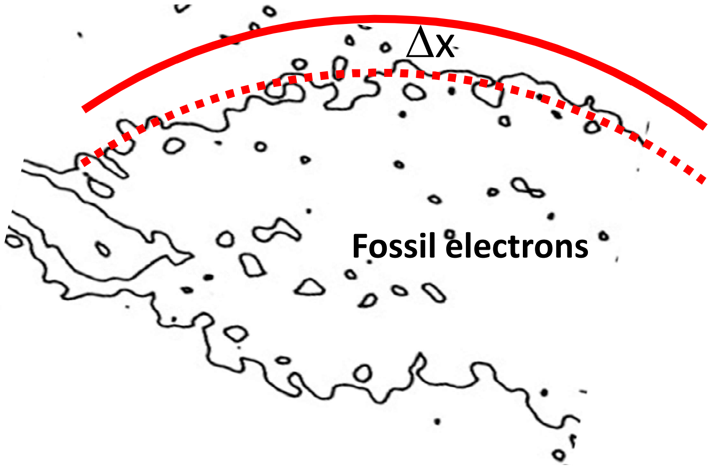
Leading edge of the relic.
Δx ~10 kpc

Kang & Ryu 2016
Kang 2017

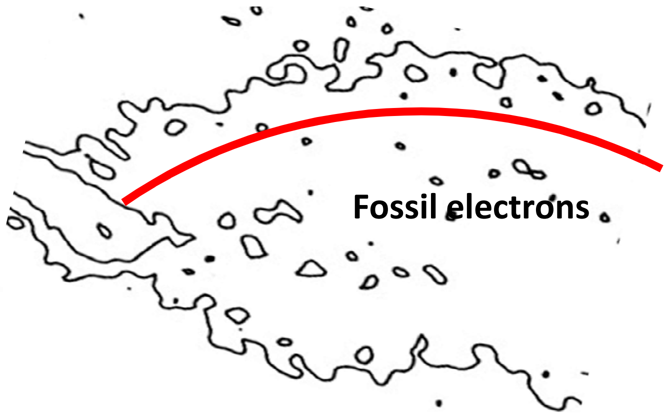
Fitting of Radio Integrated Spectra



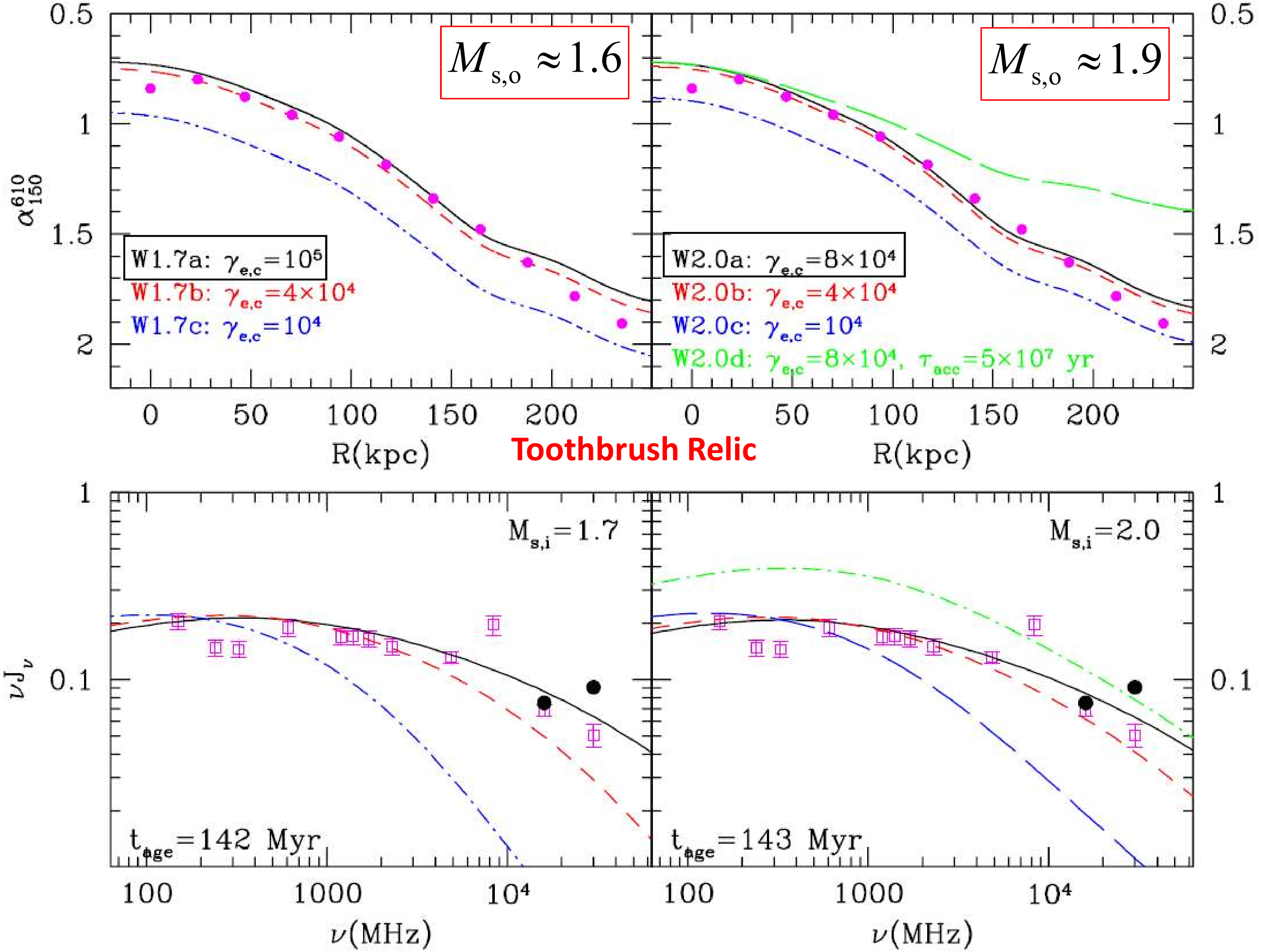
The shock has run out of fossil CRe.



The shock is inside the cloud of fossil CRe.



Weak shock models require $\gamma_{e.c} \sim 10^5$, which is unrealistically high.



DSA model parameters for Sausage & Toothbrush

fossil CRe $f(p)$

$s \sim 4.6, \gamma_{e,c} \sim 300$

$B_1 \sim 1\mu\text{G}$

shock properties

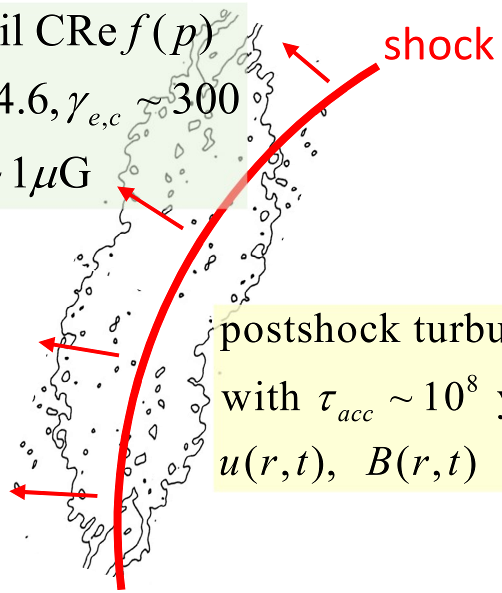
$M_s \sim 3, M_A \sim 20,$

$u_s \sim 2.5-3\times10^3$ km/s

background plasma

$n_{H,1} \sim 10^{-4}\,\text{cm}^{-3}$

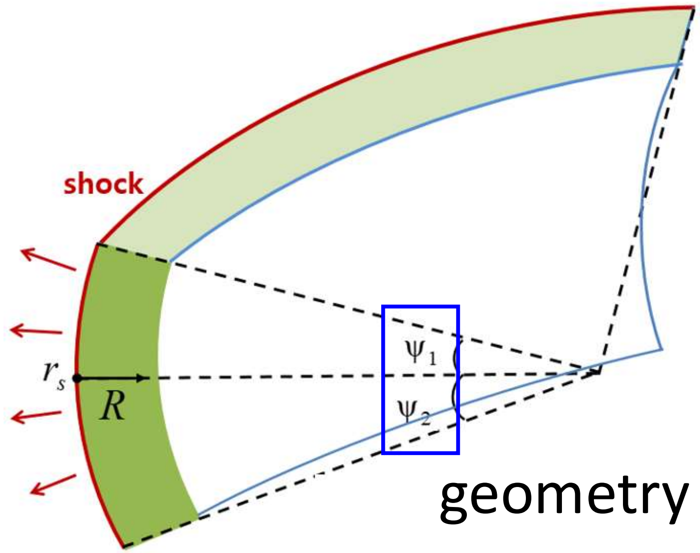
$kT_1 \sim 2-3$ keV



postshock turbulence

with $\tau_{acc} \sim 10^8$ yr

$u(r,t),\ \ B(r,t)$



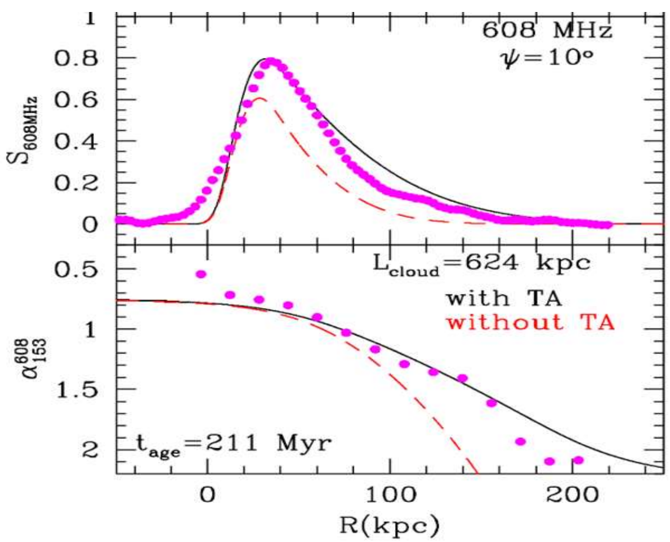
Observational Test

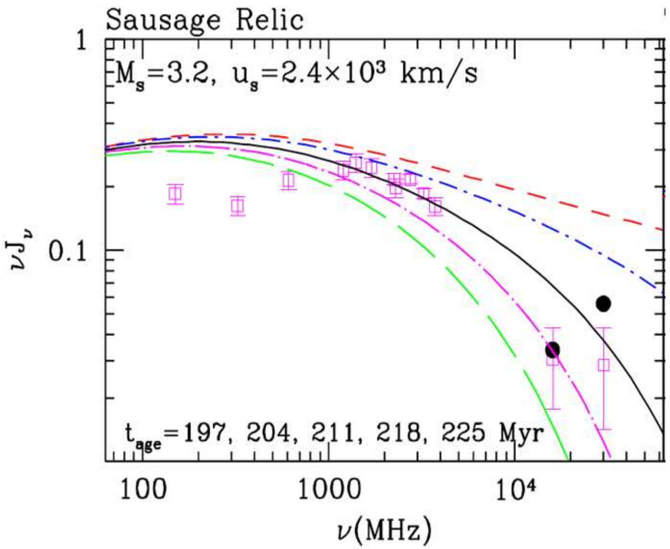
observables

$S_\nu(R)$

$\alpha_\nu(R)$

$\nu \cdot J_\nu$





-Fitting S_ν, α_ν, & $\nu\ J_\nu$ simultaneously is necessary.

Summary: DSA model for Sausage & Toothbrush relics

	M_{radio}	M_X	DSA model parameters	
Sausage	2.7	2.7	$M_{s,o} \approx 3.2$ with $\gamma_{e.c} \sim 300$	shock is outside of fossil CRe cloud
Toothbrush	2.8	1.5	**strong shock model** $M_{s,o} \approx 3.0$ with $\gamma_{e.c} \sim 300$	$M_{s,o} \neq M_X$. multiple shocks
			weak shock model $M_{s,o} \approx 1.6$ with $\gamma_{e.c} \sim 10^5$	TA: re-energizing fossil CRe

- $\tau_{acc} \sim 10^8$ yr , but need to understand better the properties of possible turbulence generated behind weak ICM shocks.

-Weak shock model cannot be used to resolve $M_{\rm radio} > M_{\rm X}$ unless re-energization of fossil CRe to $\gamma_{e,c} \sim 10^5$ is invoked.

-Radio relics may consist of multiple shocks with different Ms.