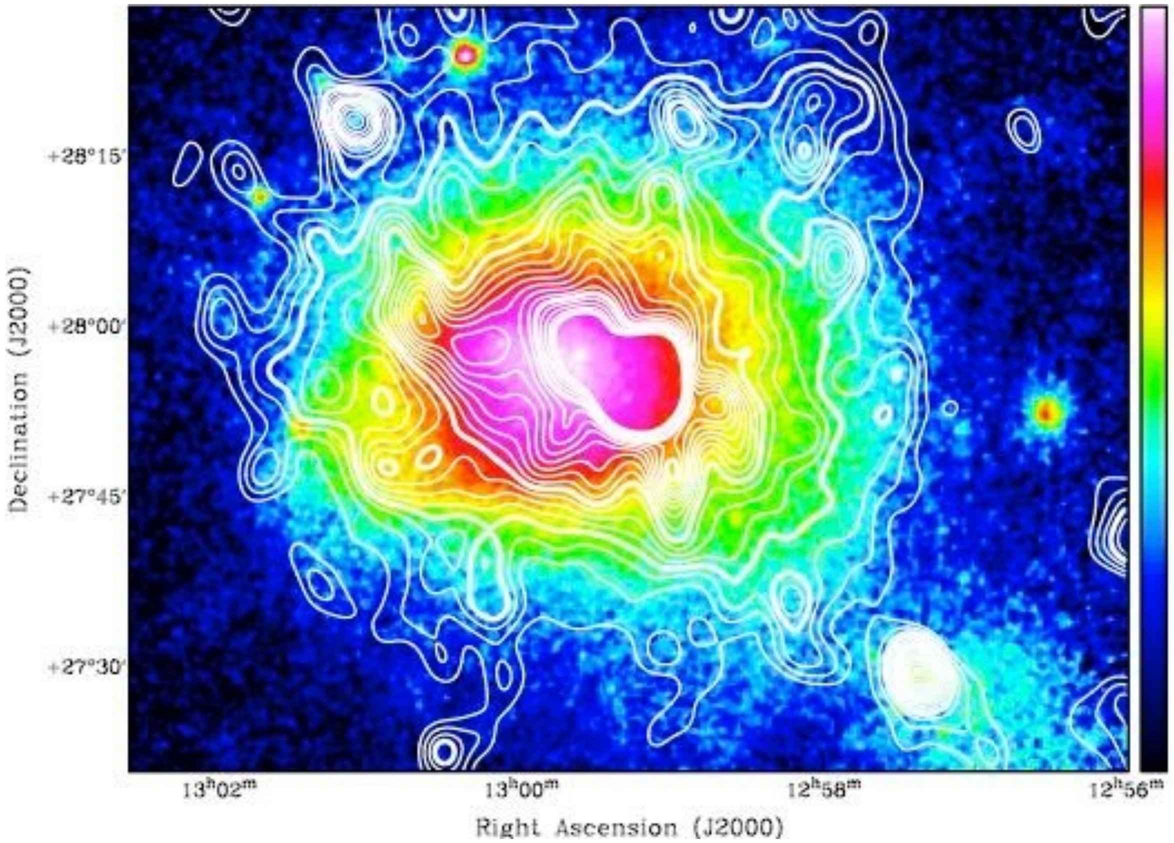


# Cosmic Ray Transport in Galaxy Clusters

S. Peng Oh
UCSB

# Why should you care about CR transport?

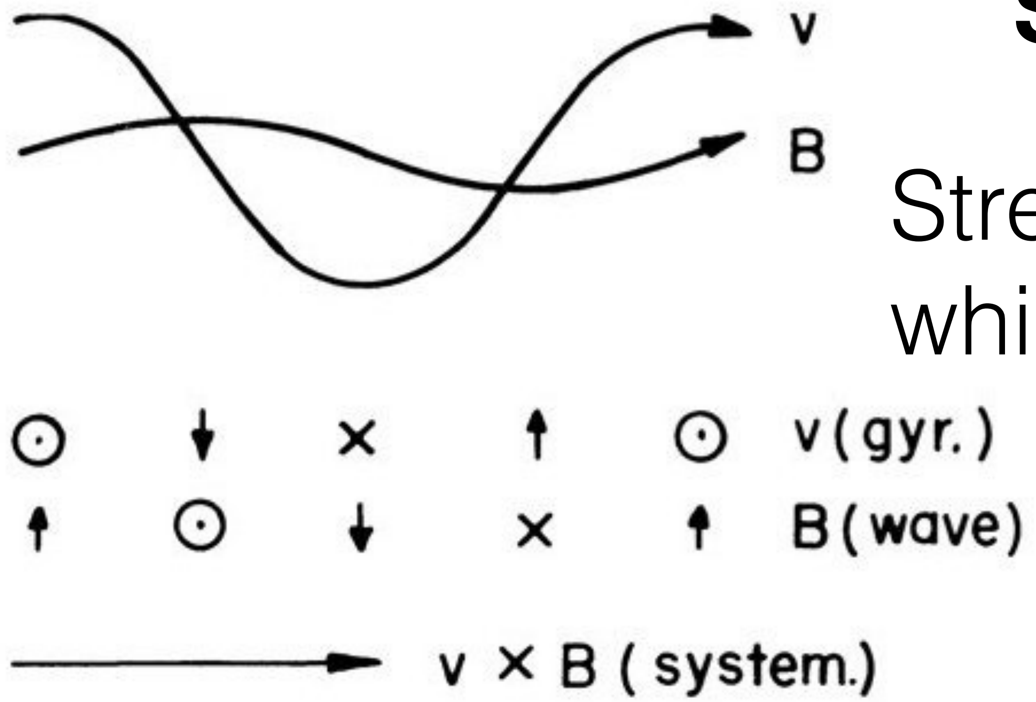
CRe lifetime << transport time

Hadronic model is mostly dead

Hard to accelerate from thermal pool — need seeds! (CRp, or AGN products)

Transport affects abundance, spatial profiles

# CR scattering



## Self-confinement

Streaming CRs amplify Alfven waves, which scatter them

Scatter in pitch angle $\delta\theta \sim \pm\frac{\delta B}{B}$

## Scattering by extrinsic turbulence

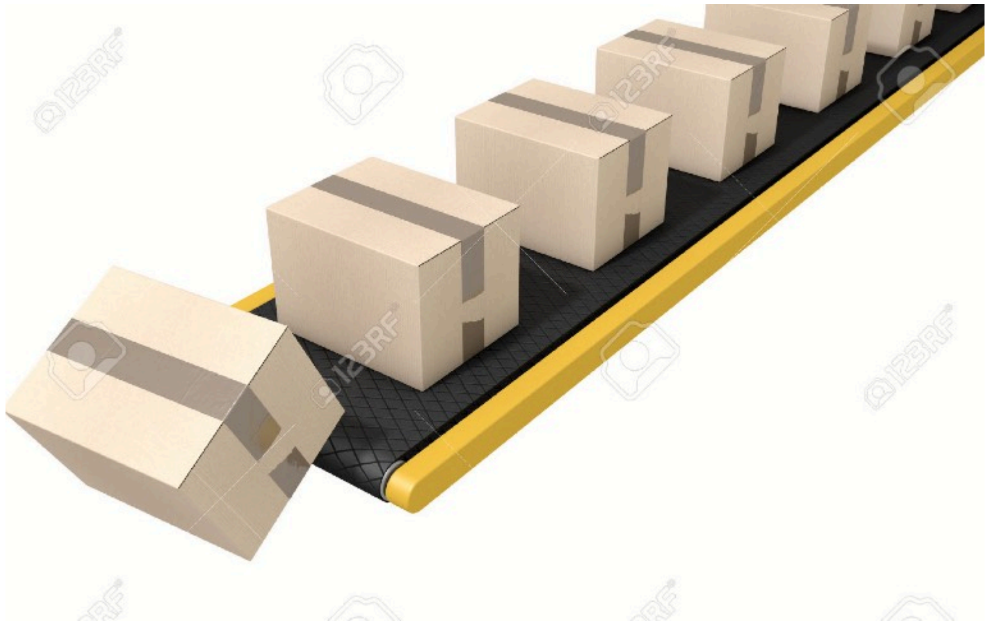
Alfven modes have wrong shape

Scattered by compressible fast modes

Particle surfing (transit time damping)

**Sign of energy transfer is opposite**

# How can we move CRs around?



**Advect, stream or diffuse?**

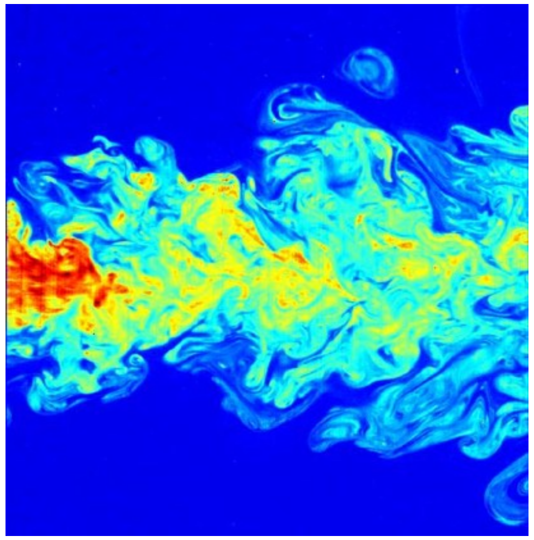
Can mean:
1) motion relative to certain frame
2) Hyperbolic (advect, stream) or parabolic (diffuse) equation

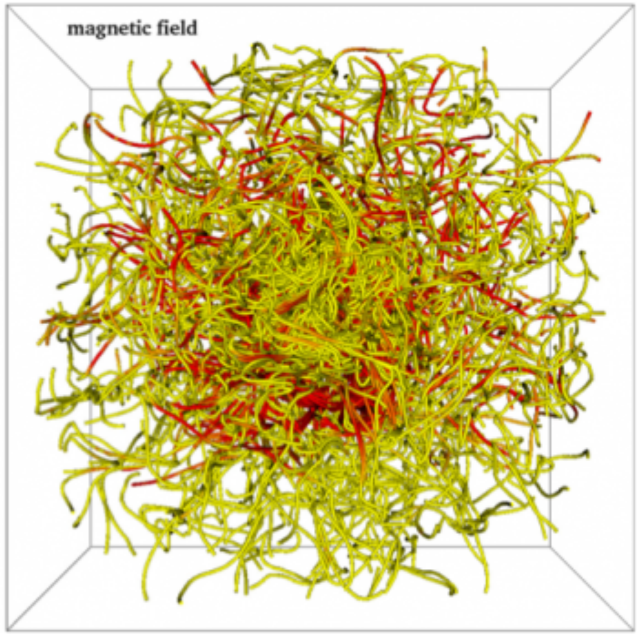
Traditionally, in self-confinement picture
— CRs scattered by Alfven waves
— advect/stream at (v_A + u)
— scattering rate finite: diffuse relative to wave frame

**In practice, not so clear cut**

**Advect** with fluid:—turbulent diffusion (parabolic!)

— buoyant rise — AGN bubbles





**Stream** with Alfven waves

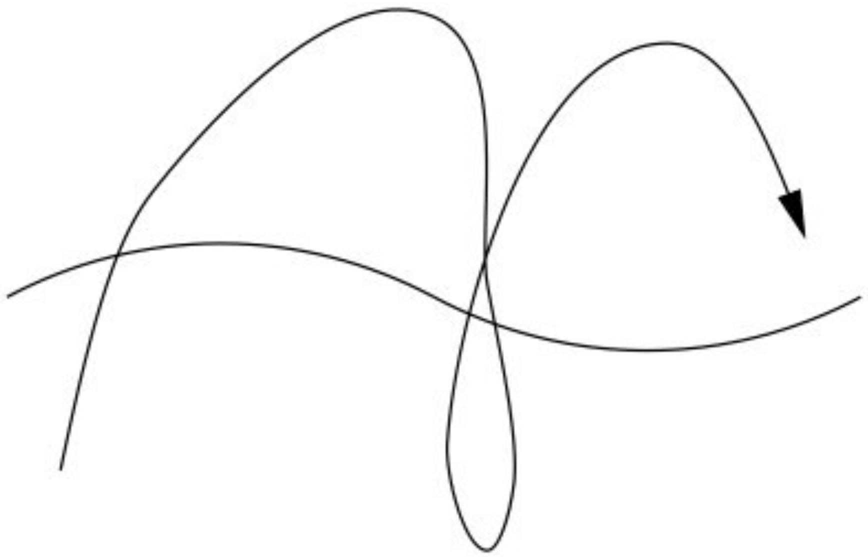
— depends on B-field geometry

— can look effectively diffusive! (field line wandering)

**Diffuse** wrt wave frame

— wave amplitude depends on balance between growth/damping

— can look mathematically like streaming!



# Self-Confinement

**Wiener, Oh, Guo, 2013, MNRAS, 434,2209**
Wiener, Oh, Zweibel, 2017, MNRAS, 467, 464
Wiener, Zweibel, Oh, 2017, MNRAS, in press

# Highly super-Alfvenic transport possible in clusters

Balance wave growth and damping

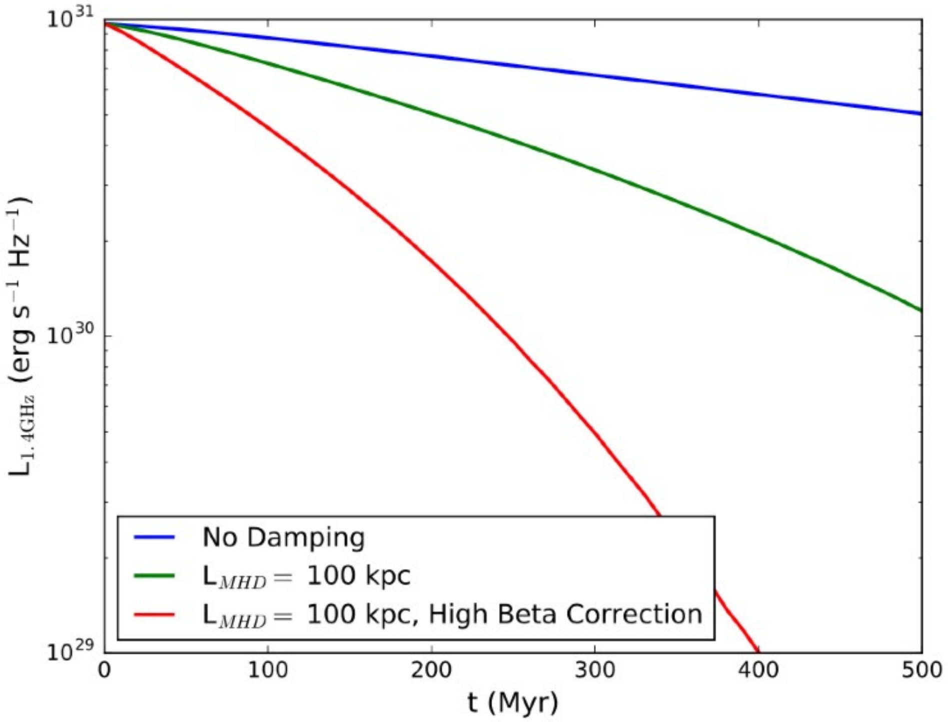
CRs have low abundance: wave growth relatively weak

Damping: for clusters,

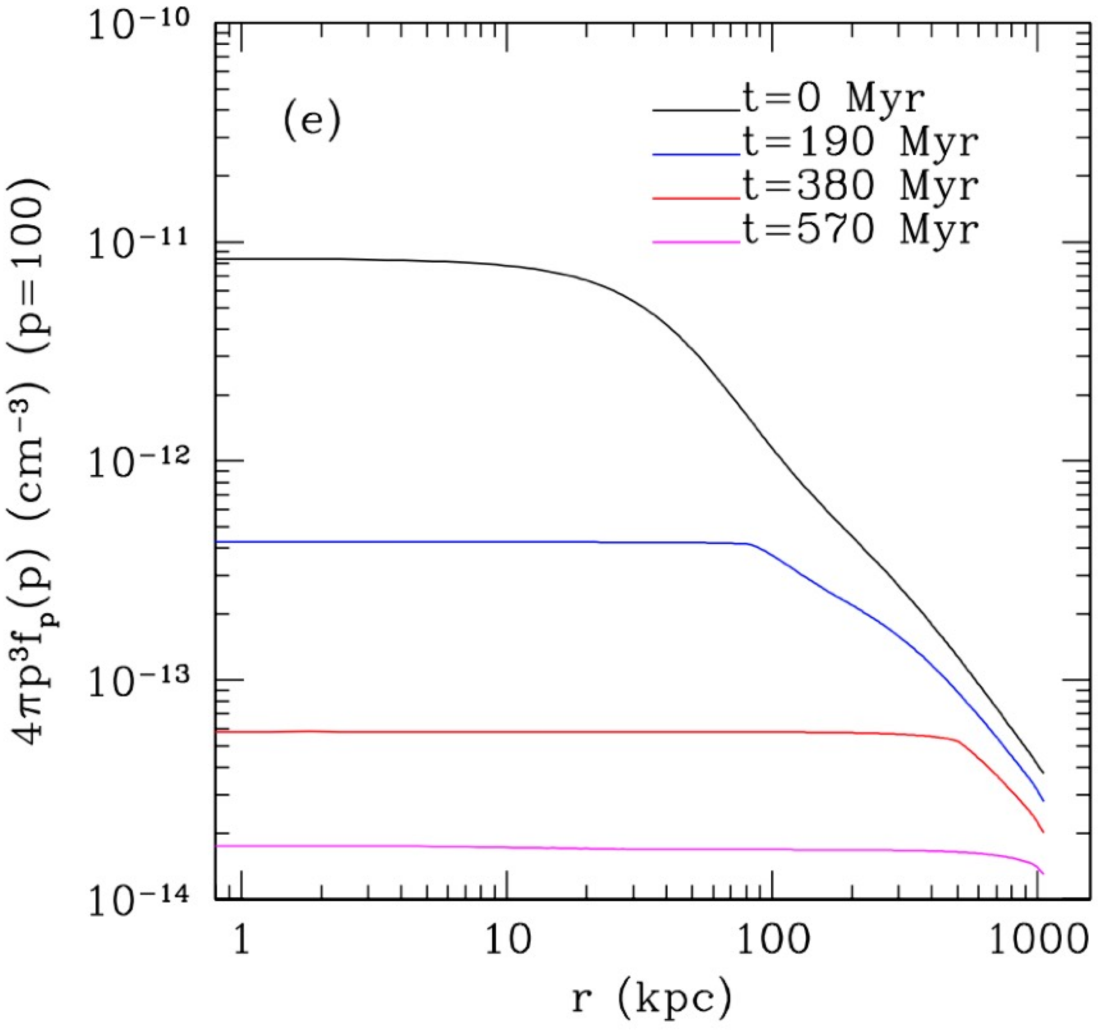
Linear Landau > turbulent damping > non-linear Landau

Wiener+17

Wiener+13



Quickly turns off hadronic radio emission



CR seeds should have a flat spatial distribution

# Slippage wrt wave frame can look like streaming

If you work it out in loving detail (Skilling 1971):

must stream down gradient

$$D(r) = \frac{1}{p^3}\nabla\cdot\left(\frac{\Gamma_{\mathrm{D}} B^2 \boldsymbol{n}}{4\pi^3 m_{\mathrm{p}} \Omega_0 v_{\mathrm{A}}}\,\frac{\boldsymbol{n}\cdot\nabla f_{\mathrm{p}}}{|\boldsymbol{n}\cdot\nabla f_{\mathrm{p}}|}\right)$$

**If damping does not depend on f** (turbulent, ion-neutral damping) **then diffusion term is independent of f, $\nabla f$ !**

This behaves essentially like streaming!

$$t_{\mathrm{stream}} \propto f_{\mathrm{p}}/\dot{f}_{\mathrm{p}} \propto f_{\mathrm{p}},$$

For non-linear damping, this doesn't happen, e.g. NLLD,

$$D \propto \Gamma \propto (\nabla f)^{1/2}$$

**For a constant diffusion coefficient, all this interesting behaviour is lost**

# Streaming is energy dependent

E.g., for turbulent damping

$$v_D = v_A \left( 1 + 1.2 \frac{B_{\mu G}^{1/2} n_{i,-3}^{1/2}}{L_{\mathrm{MHD},100}^{1/2} n_{\mathrm{CR},-10}} \gamma_{100}^{n-3.5} 10^{2(n-4.6)} \right),$$

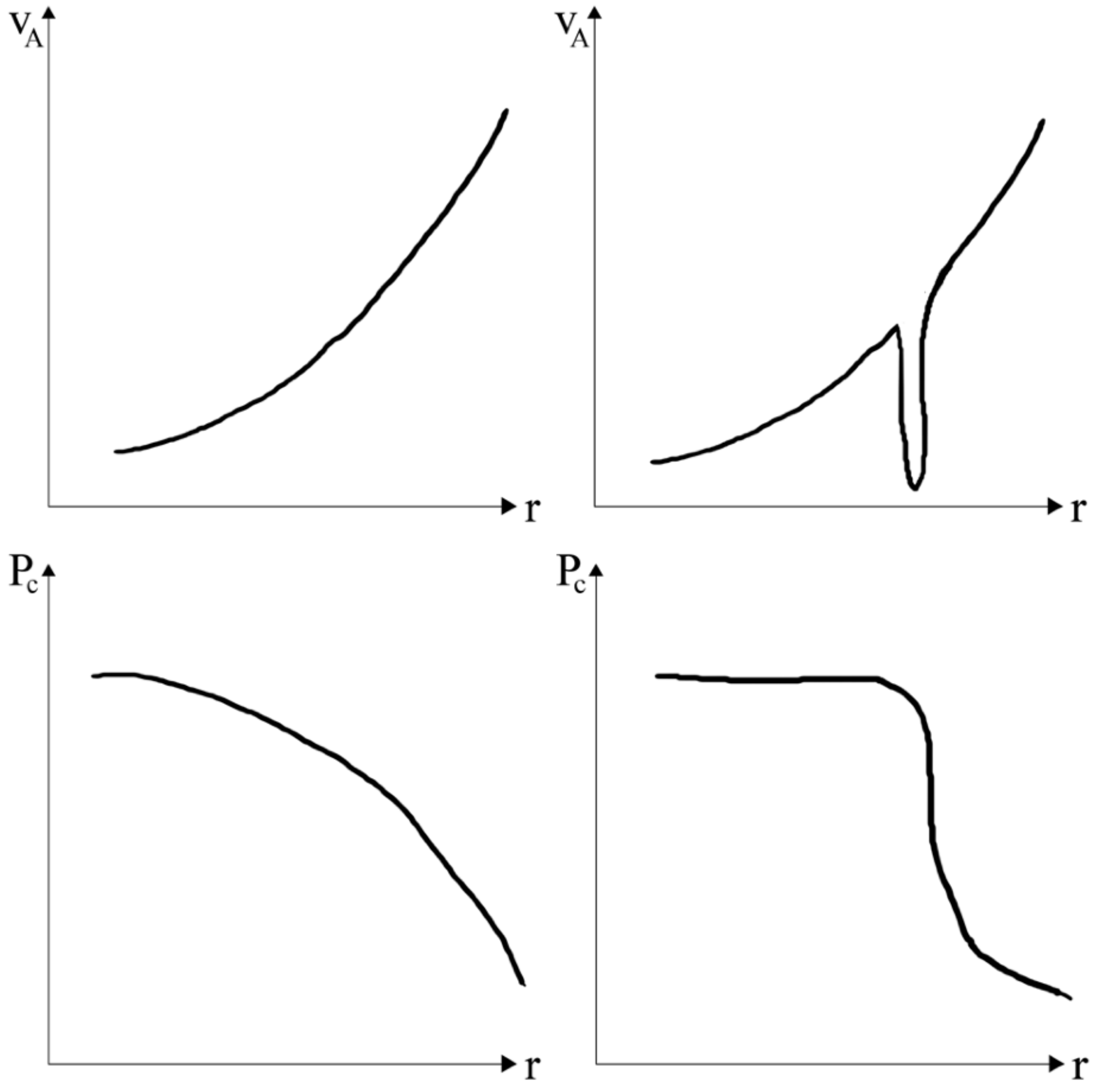
$$\propto \gamma^{1.1}$$

Distribution function will steepen

Be careful about inferences comparing CRs of different energy without taking this into account

(e.g., CR heating of radio mini-halos vs radio emission)

# Weird things can happen in multi-phase media

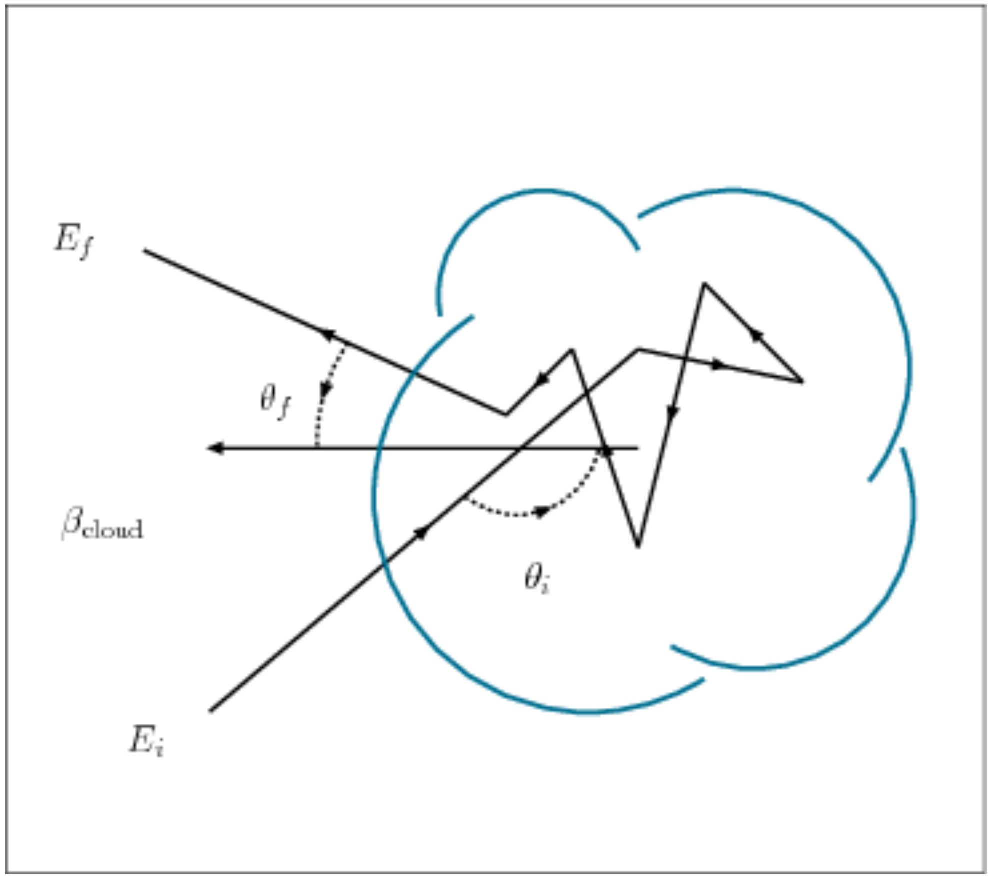


Minimum in Alfven speed creates a ‘bottleneck’

CRs are NOT coupled to upstream gas!..
because they are streaming sub-Alfvenically

changes distribution of CRs

Wiener, Oh, Zweibel 2017

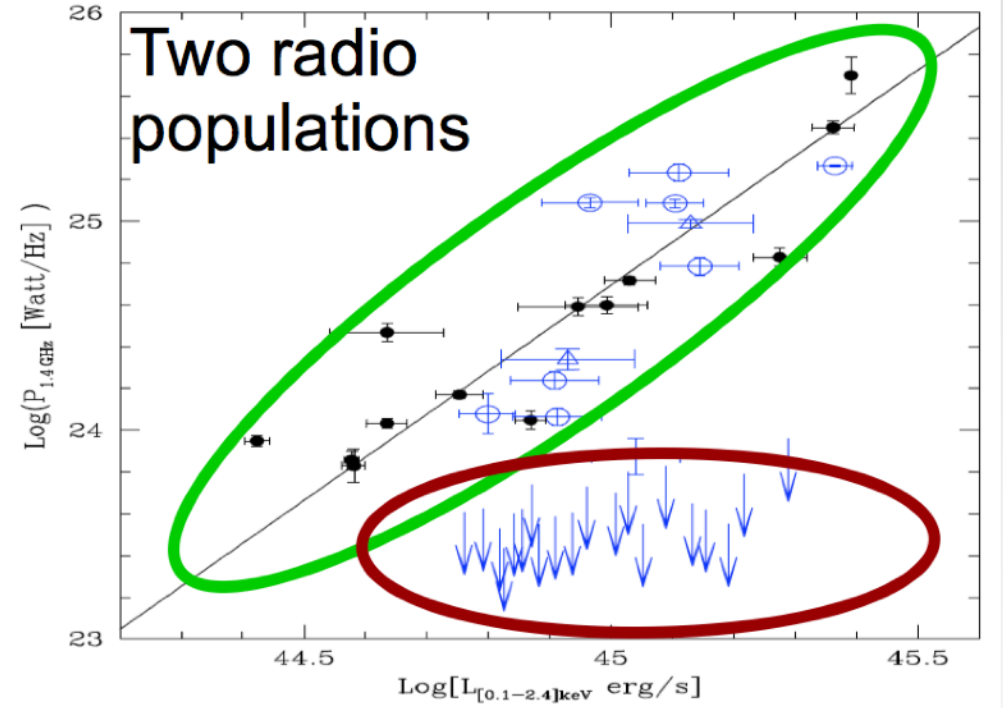


# Scattering by Extrinsic Turbulence

Pinzke, Oh, Pfrommer, 2017, MNRAS, 465, 4800

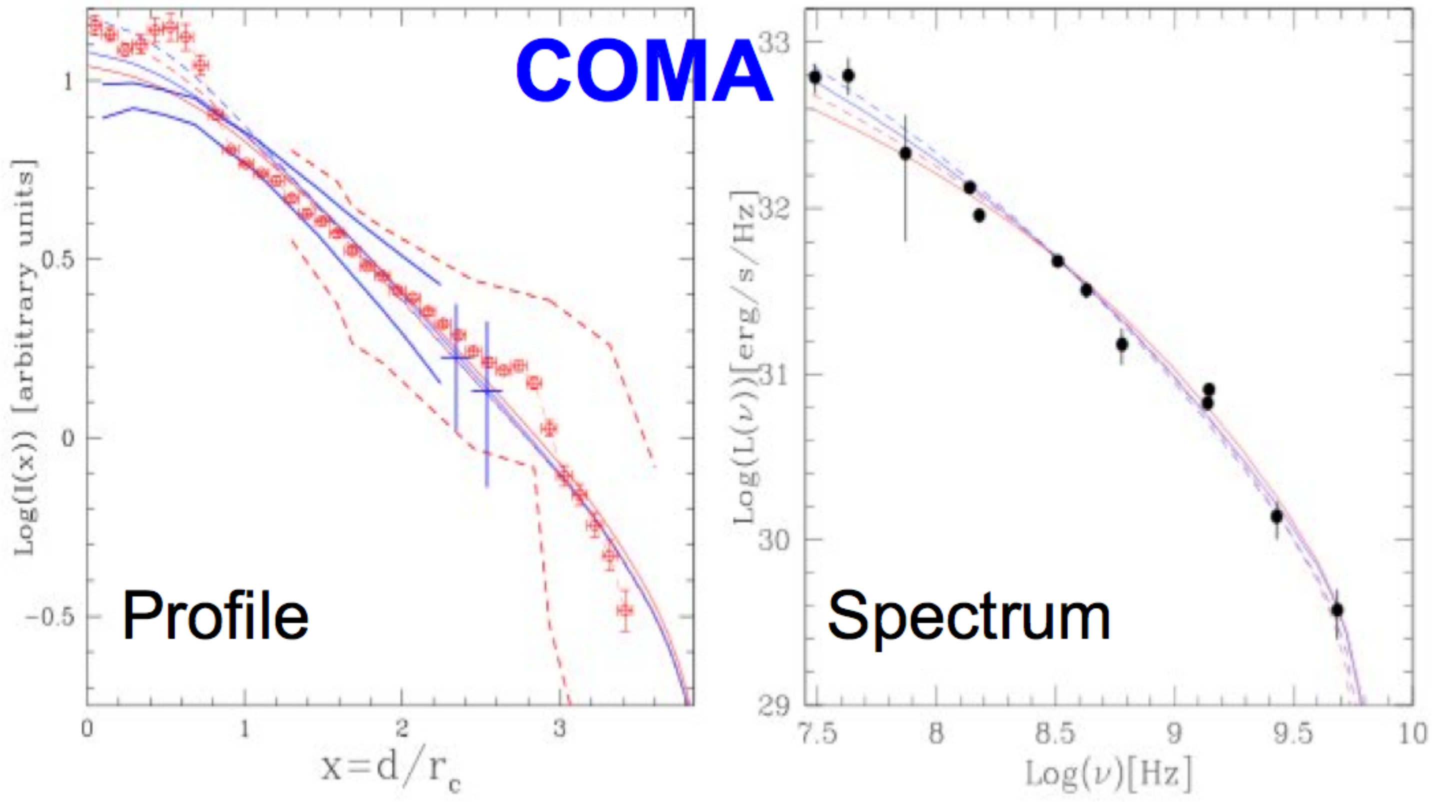
# Turbulent reacceleration is now widely accepted…

Explains bimodality naturally



*Brunetti+ 2009*

Provides a good fit to Coma data



*Brunetti and Lazarian 07,11, Brunetti+ 2012*

But these are existence proofs— let's explore parameter space

How sensitive to assumptions?

# Ingredients of a Radio Halo Model

**CR seeds profile**

— normalization, shape

— can use cosmo sims (CRp)

— sensitive to transport assumptions

**Turbulence profile**

— normalization, shape

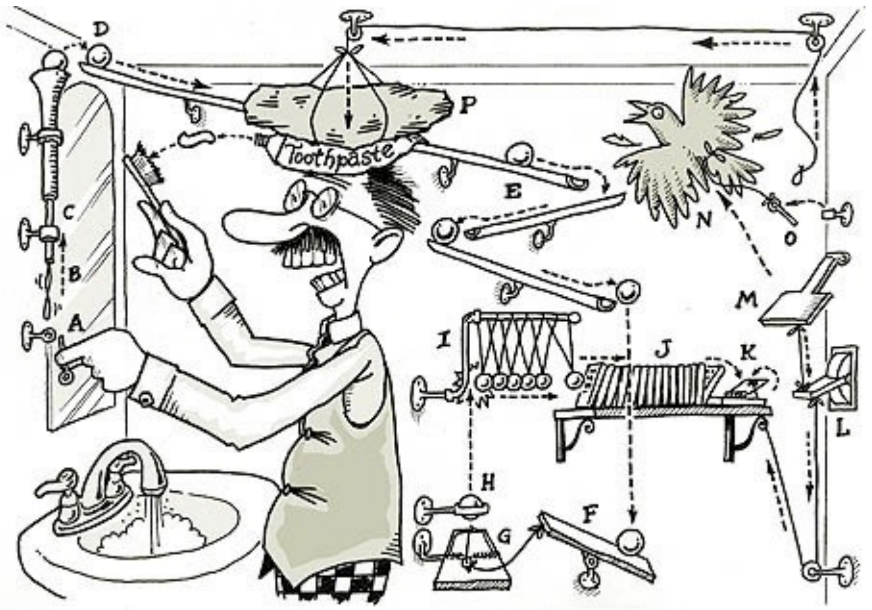
  — use cosmo sims

**Turbulent spectrum**

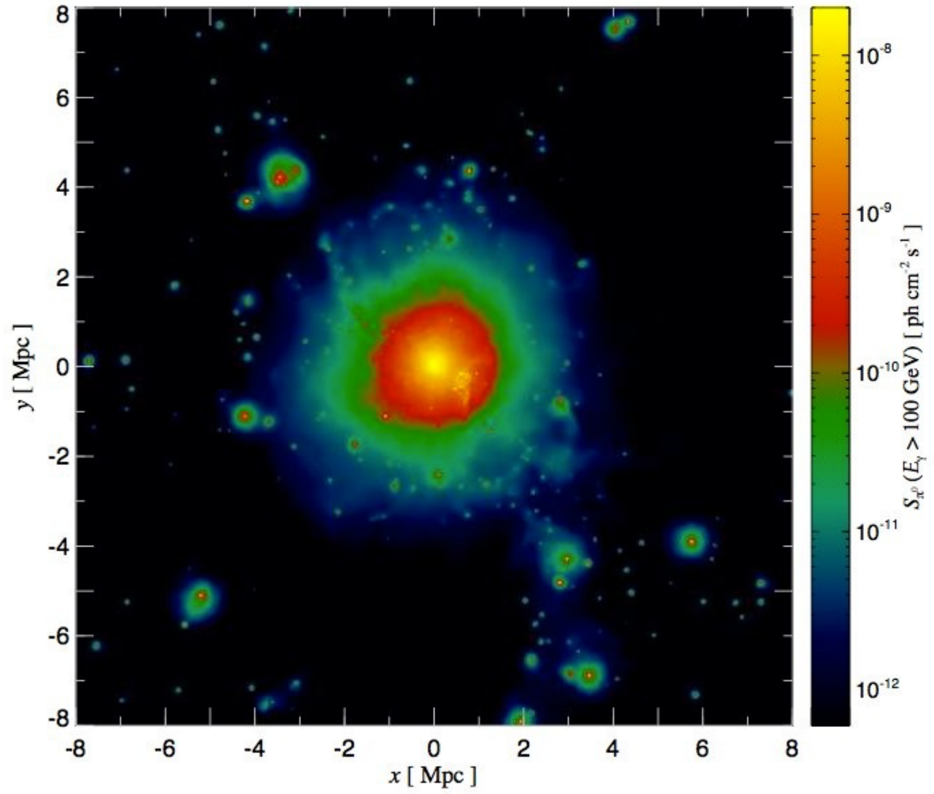
—MHD turbulence outer scale, inner scale, lifetime

— slope of power spectrum (depends on driving, B-field, damping)

# Method



Couple cosmological simulations with adiabatic CR proton + electron physics…



…to Fokker-Planck code to follow momentum diffusion

$$\begin{aligned}\frac{df_{\mathrm{e}}(p,t)}{dt} &= \frac{\partial}{\partial p}\left\{f_{\mathrm{e}}(p,t)\left[\left|\frac{dp}{dt}\right|_{\mathrm{C}} + \frac{p}{3}\left(\vec{\nabla}\cdot\vec{v}\right)\right.\right.\\ &\quad \left.\left. + \left|\frac{dp}{dt}\right|_{\mathrm{r}} - \frac{1}{p^2}\frac{\partial}{\partial p}\left(p^2 D_{pp}\right)\right]\right\} - \left(\vec{\nabla}\cdot\vec{v}\right) f_{\mathrm{e}}(p,t)\\ &\quad + \frac{\partial^2}{\partial p^2}\left[D_{pp} f_{\mathrm{e}}(p,t)\right] + Q_{\mathrm{e}}\left[p,t;f_{\mathrm{p}}(p,t)\right] . \qquad (1)\end{aligned}$$

Pinzke & Pfrommer 2010
Pinzke, Oh, Pfrommer 2013

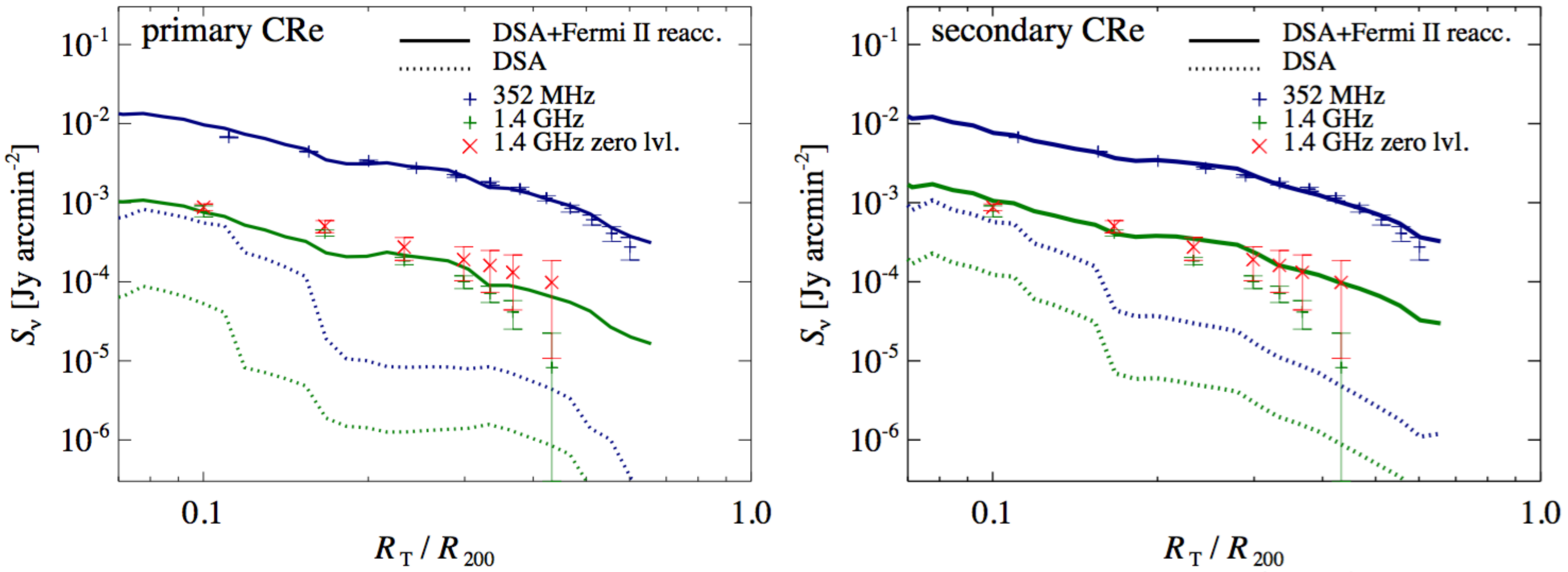
$$D_{pp}(p,t) = \frac{\pi}{16}\frac{p^2}{c\,\rho}\left\langle\frac{\beta|B_k|^2}{16\pi\,W}\right\rangle I_\theta \int_{k_{\mathrm{cut}}} \mathcal{W}(k) k\,dk\,, \quad \mathcal{W}(k) \approx \sqrt{I_0\,\rho\,\langle V_{\mathrm{ph}}\rangle}\,k^{-3/2}\,,$$

Try to reproduce Coma’s surface brightness + spectral profile

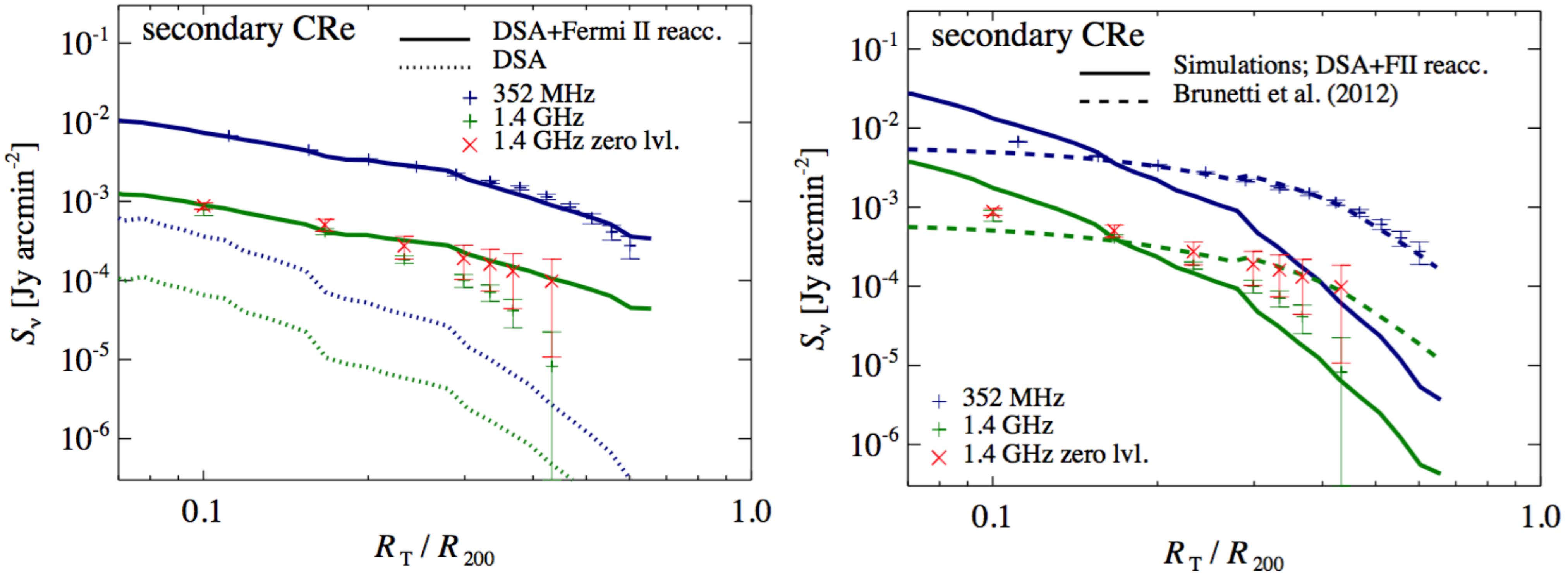
# Vanilla model needs tuning

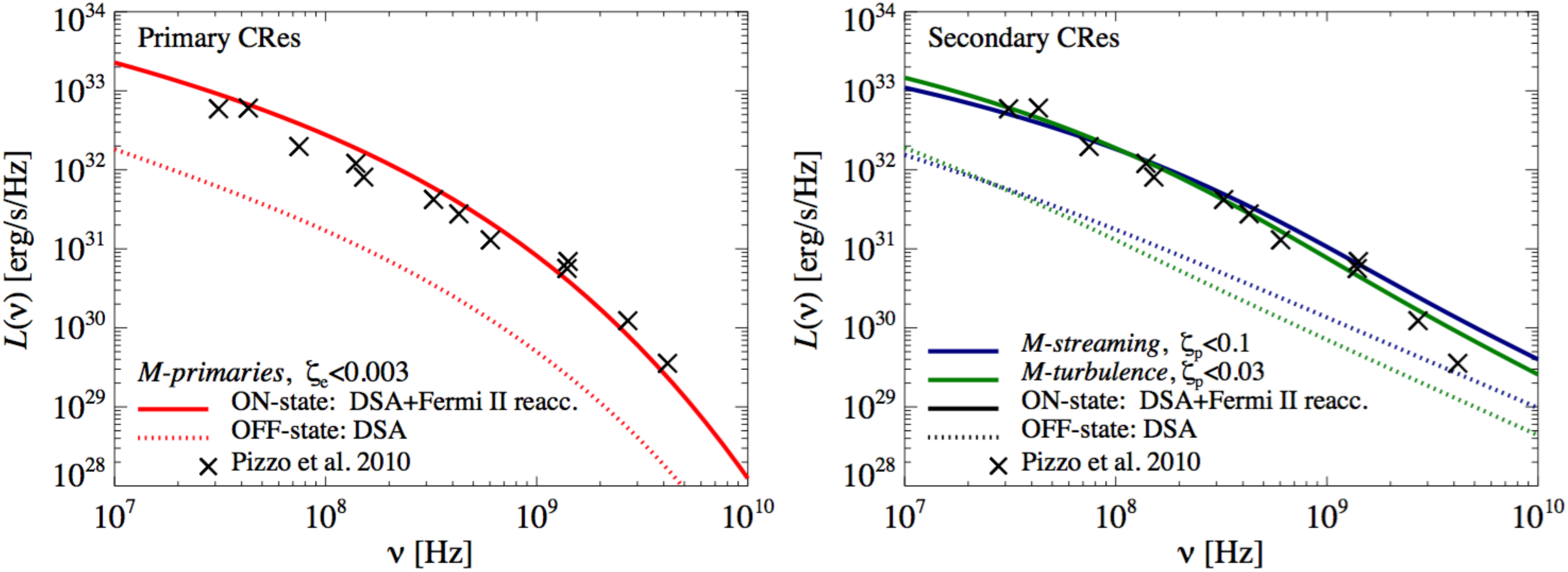
## Have to adjust turbulence and CR seed profiles

*M-primaries*: *M-streaming*:



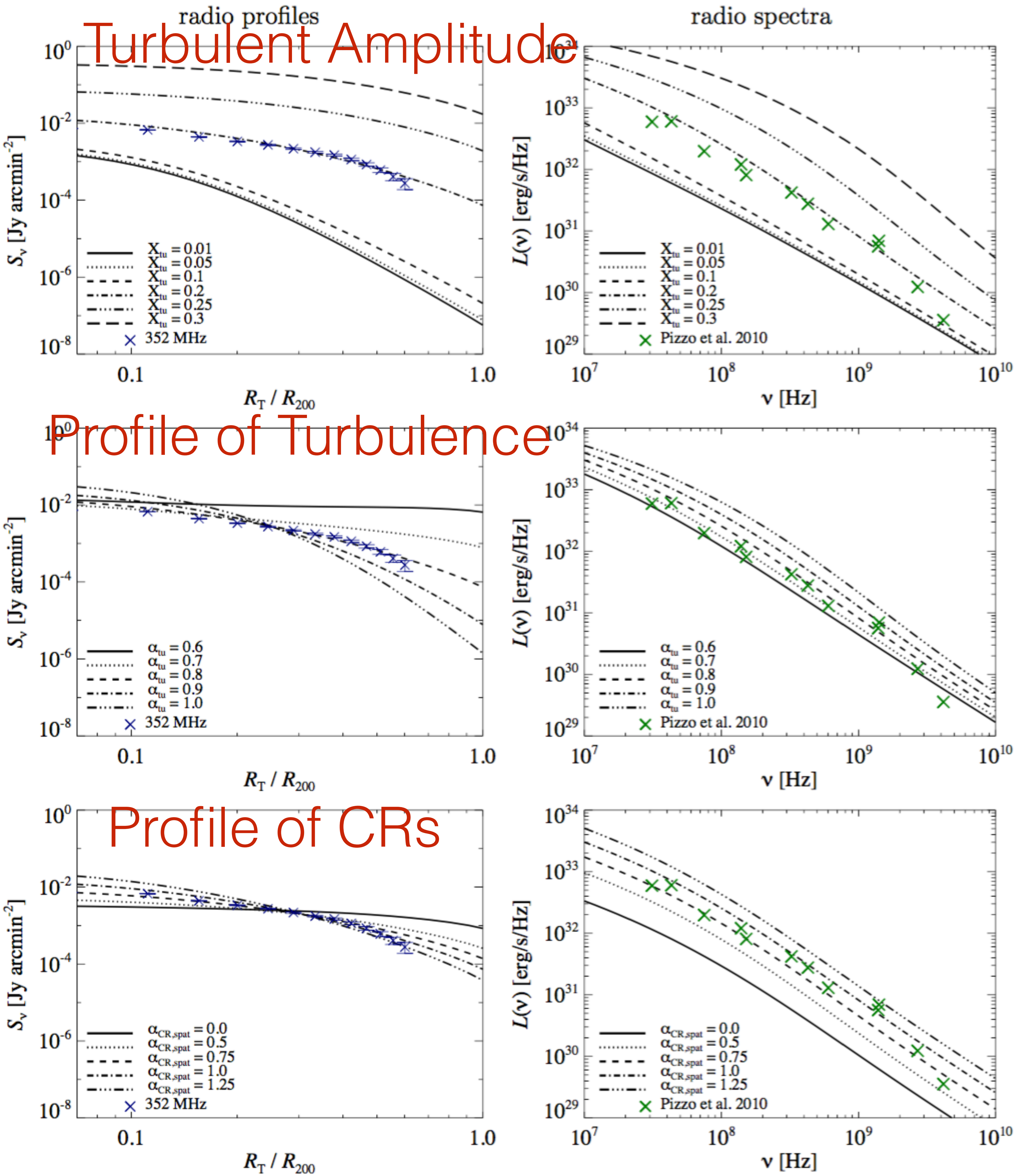
*M-turbulence*: *Brunetti et al. (2012)*:





… but this isn't so satisfying. Fine tuning?

**Do a parameter study**



Turbulent Amplitude

Profile of Turbulence

Profile of CRs

**Exponentially sensitive to level of turbulence!**

Not surprising… that's what Fermi acceleration does

$$\dot{p} = p/\tau_\mathrm{D}; \qquad p \rightarrow p \exp(\tau_\mathrm{cl}/\tau_\mathrm{D})$$

What is $\tau_\mathrm{cl}$ ?

Longest of: — driving time (merger timescale?)

— Eddy turnover time at outer scale

— Cascade time at Alfven scale

$$\tau_\mathrm{decay} = \frac{v_\mathrm{ph}}{v_k^2 k} = \frac{c_\mathrm{s}}{f_\mathrm{c} v_\mathrm{A}^2 k_\mathrm{A}} .$$

(due to wave-wave collisions)

Compare with acceleration time

$$\tau_\mathrm{D} = \frac{p^2}{4D_{pp}} = \frac{C_\mathrm{D}}{A^{1/2}} \frac{c}{k_\mathrm{A}} \frac{\beta}{f_\mathrm{c} v_\mathrm{A}^2}$$

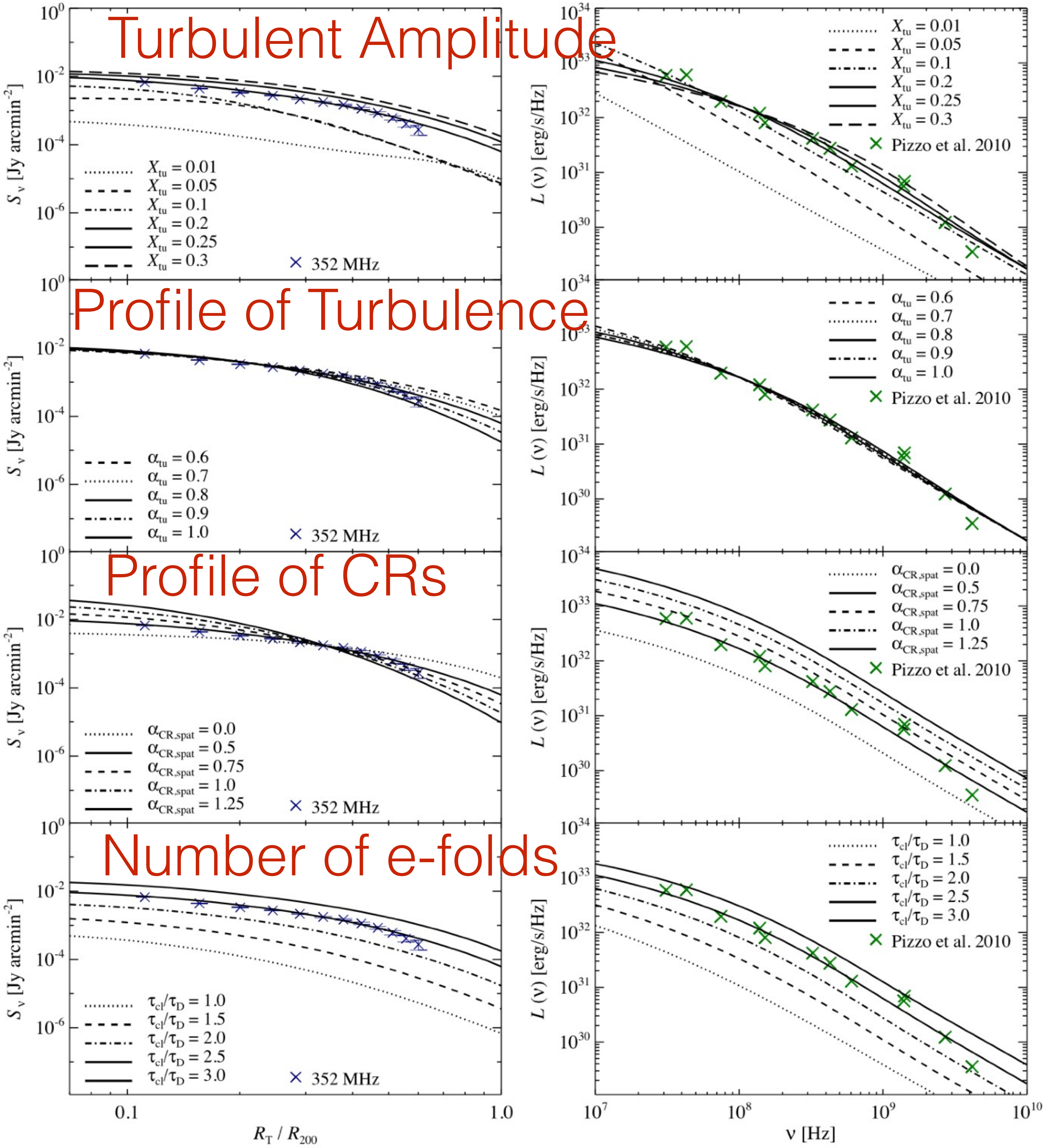
Then ratio is independent of properties of turbulence!!

$$\frac{\tau_{\rm cl}}{\tau_{\rm D}} \approx 1.6\left(\frac{\zeta}{0.2}\right).$$

Cosmic ray
TTD

Ultimately because both cascade and acceleration involve momentum space diffusion

**Above threshold, get fixed amount of Fermi II acceleration**



Turbulent Amplitude

Profile of Turbulence

Profile of CRs

Number of e-folds

Above threshold, stable to amount of and profile of turbulence!

Now sensitive to CR profile…

# What you need

✓

Relationship between cascade time and acceleration time **reduces scatter**

?

Kraichnan spectrum for fast modes $W(k) \propto k^{-3/2}$

Burgers spectrum $W(k) \propto k^{-2}$ gives inefficient acceleration

Miniati 2014

?

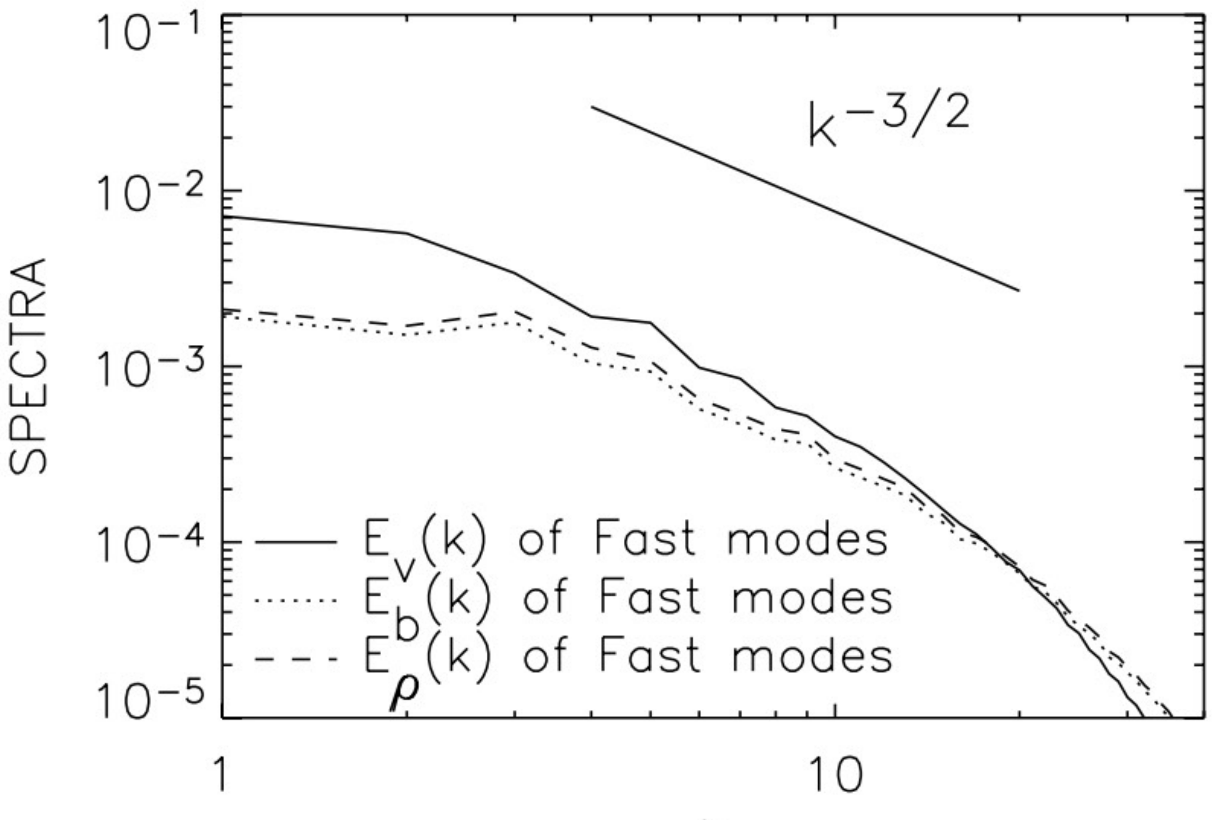
Small inner scale — suppress TTD on thermal particles

— reduce thermal particle mfp by firehose, mirror instabilities

Brunetti & Lazarian 2011

Things I don’t understand

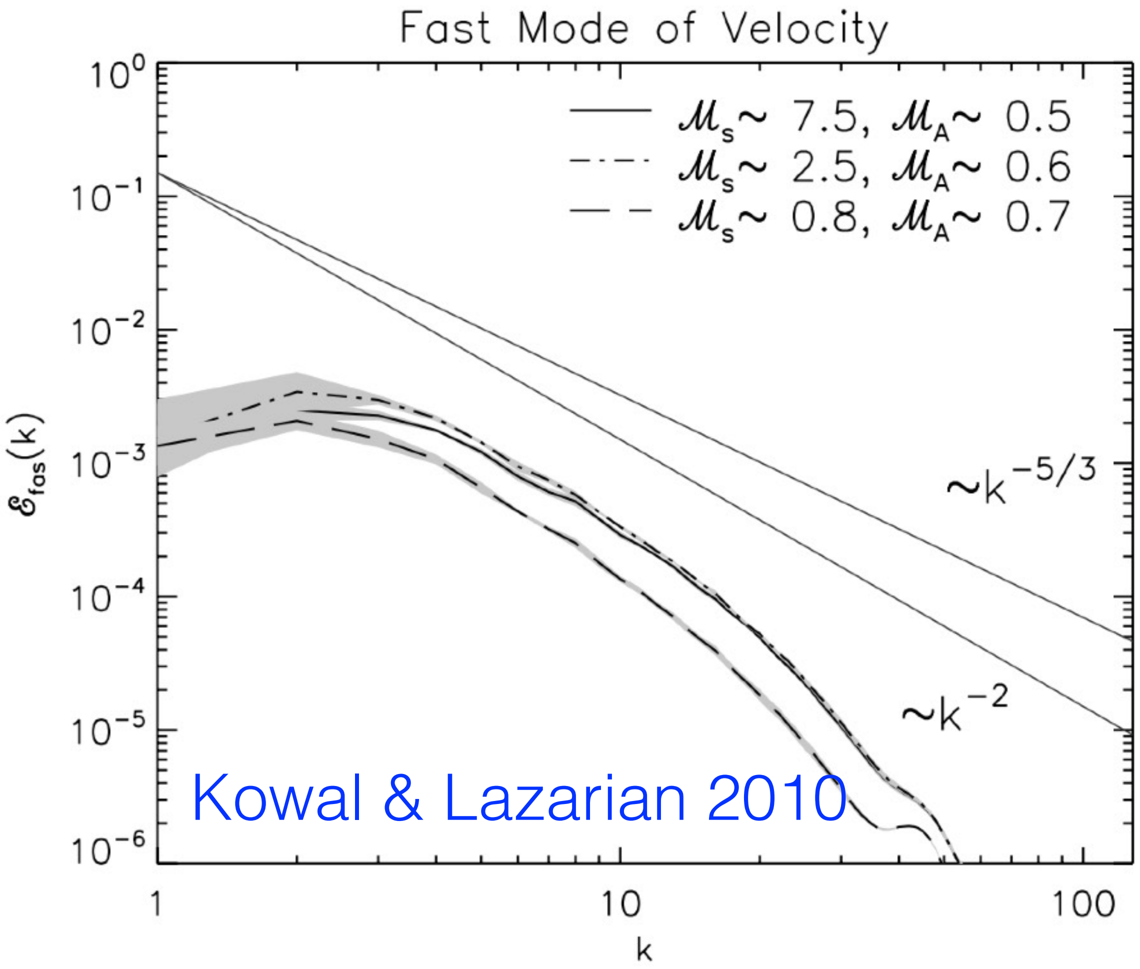
# Are we really sure about the Kraichnan spectrum?



Cho & Lazarian (2003)

There is only **one** simulation which shows this

Not a lot of dynamic range!



Kowal & Lazarian 2010

Contradicted by other work…

**We need more work on this!**

Things I don’t understand

# Do micro-instabilities really reduce thermal mfp?

Marginal stability to firehose/mirror instabilities:

$$\nu_i \sim \frac{1.45\beta_i}{\xi}\frac{U}{L} :$$

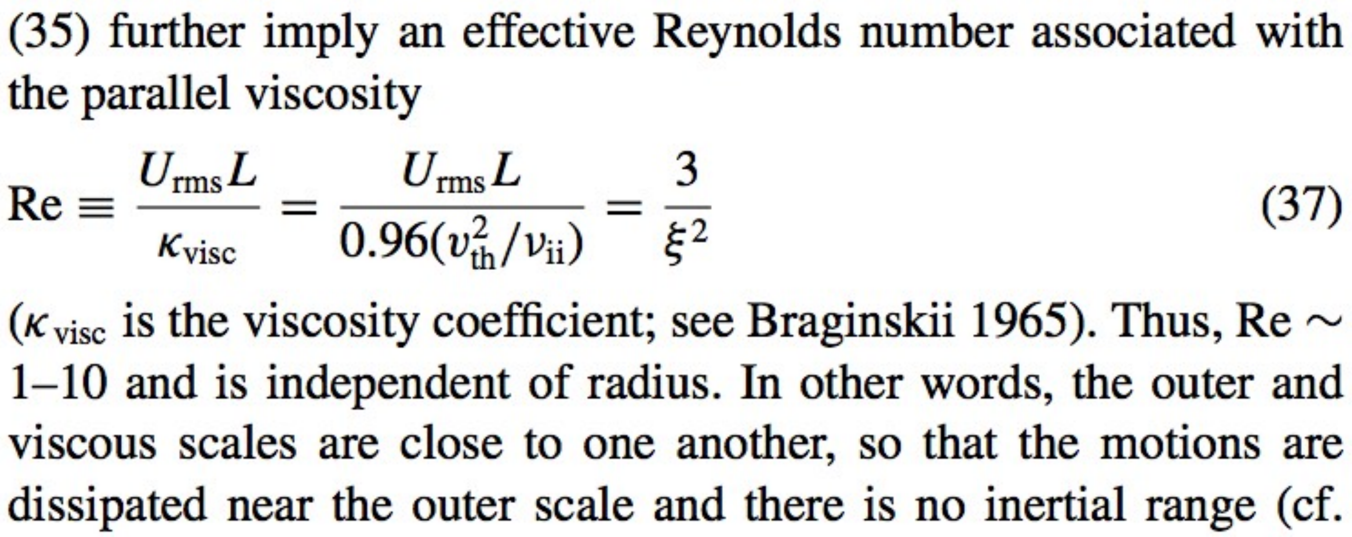
Evaluate shear at dissipation scale:

$$\frac{U_d}{L_d} = \frac{U_0}{L_0}\left(\frac{L_0}{L_d}\right)^{2/3} \sim \frac{v_i^2}{\nu_i L_d^2}$$

Get large mfp! (comparable to Coulomb)

$$\lambda_i \sim L_0 \beta_i^{-2} \mathcal{M}_0^{-3} \sim L_d \beta_i^{-1/2} \sim 14\ \mathrm{kpc}\left(\frac{L_{\mathrm{MHD}}}{100\ \mathrm{kpc}}\right)\left(\frac{\beta_i}{50}\right)^{-1/2}$$

Small Reynolds number:no parallel cascade? Wiener, Zweibel, Oh 2017



(35) further imply an effective Reynolds number associated with the parallel viscosity

$$\mathrm{Re} \equiv \frac{U_{\mathrm{rms}} L}{\kappa_{\mathrm{visc}}} = \frac{U_{\mathrm{rms}} L}{0.96(v_{\mathrm{th}}^2/\nu_{\mathrm{ii}})} = \frac{3}{\xi^2} \tag{37}$$

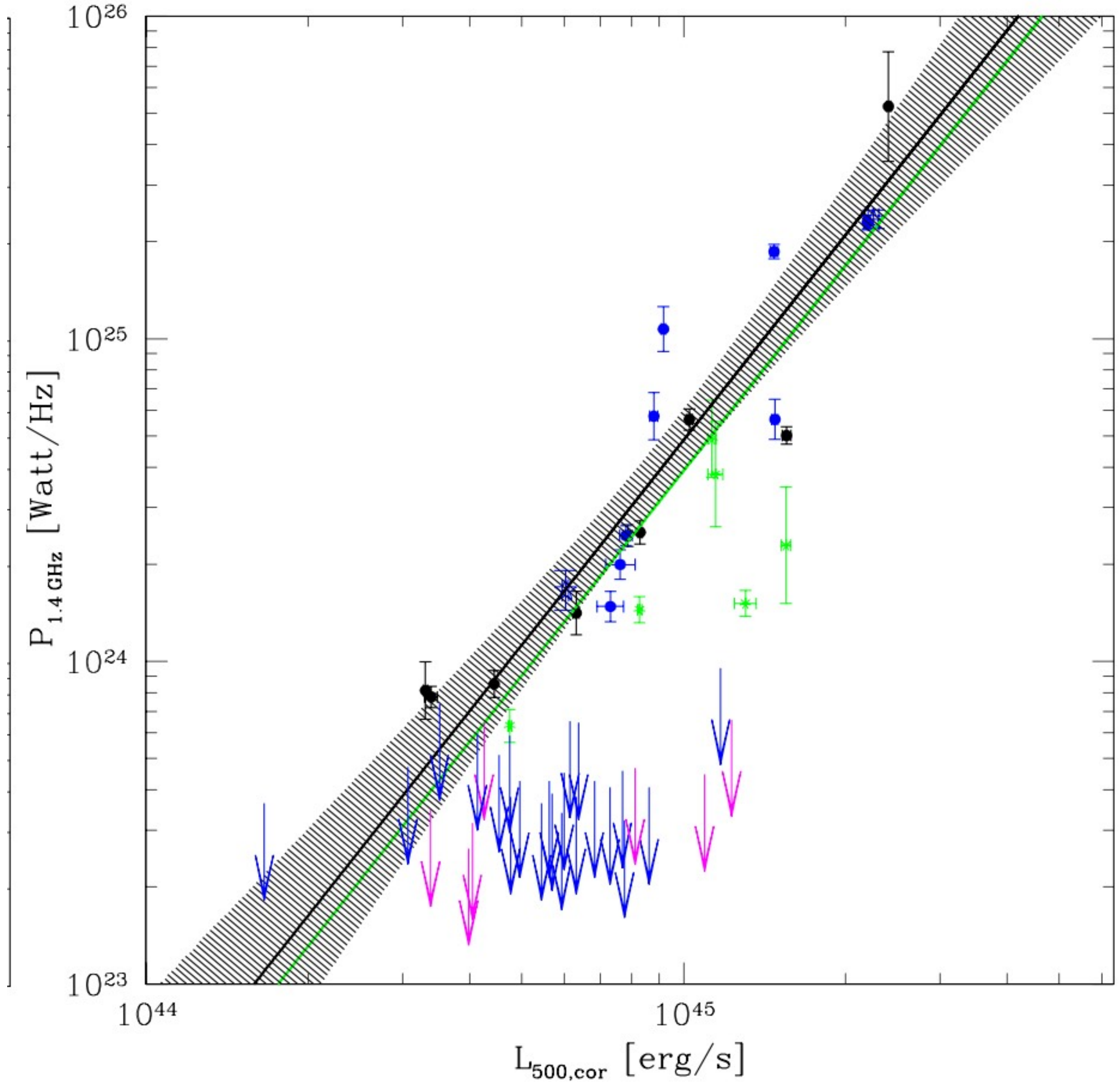
($\kappa_{\mathrm{visc}}$ is the viscosity coefficient; see Braginskii 1965). Thus, $\mathrm{Re} \sim 1$–$10$ and is independent of radius. In other words, the outer and viscous scales are close to one another, so that the motions are dissipated near the outer scale and there is no inertial range (cf.

Kunz et al 2011

**Would like to understand this better**

Things I don't understand

# The scatter in radio halo luminosities is worth studying

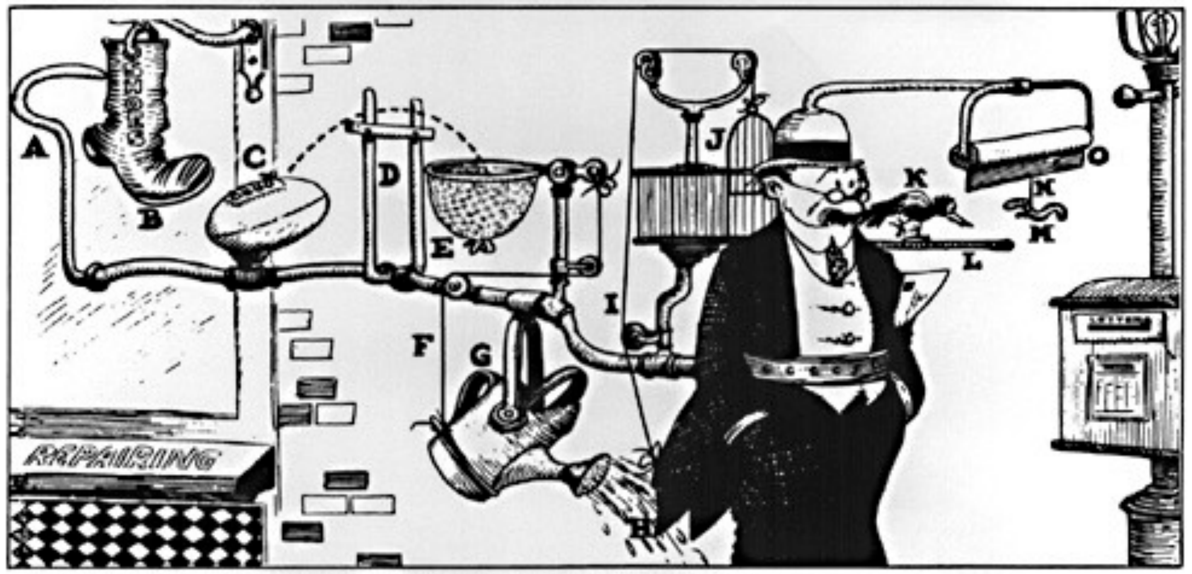


Cassano et al 2013

Most work focuses on getting the mean relation

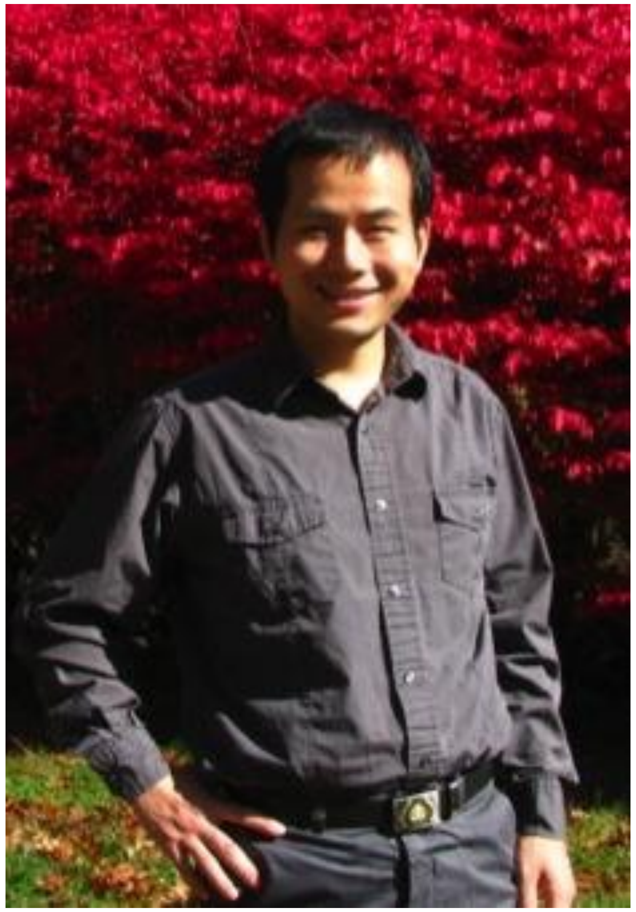
Remarkable that there isn't more scatter.

There's information there!



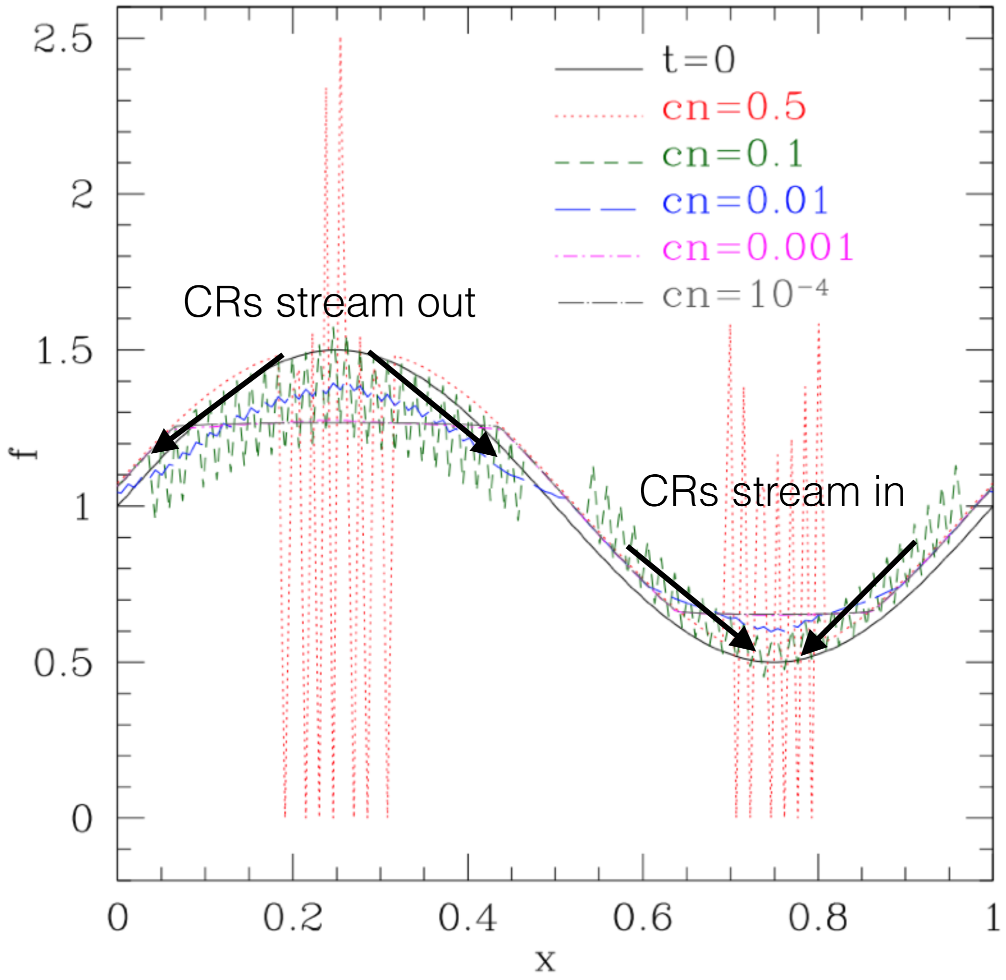
Keep You From Forgetting To Mail Your Wife's Letter RUBE GOLDBERG (tm) RGI 049

# A New Numerical Scheme for Cosmic Ray Transport



Jiang & Oh, 2017, in prep

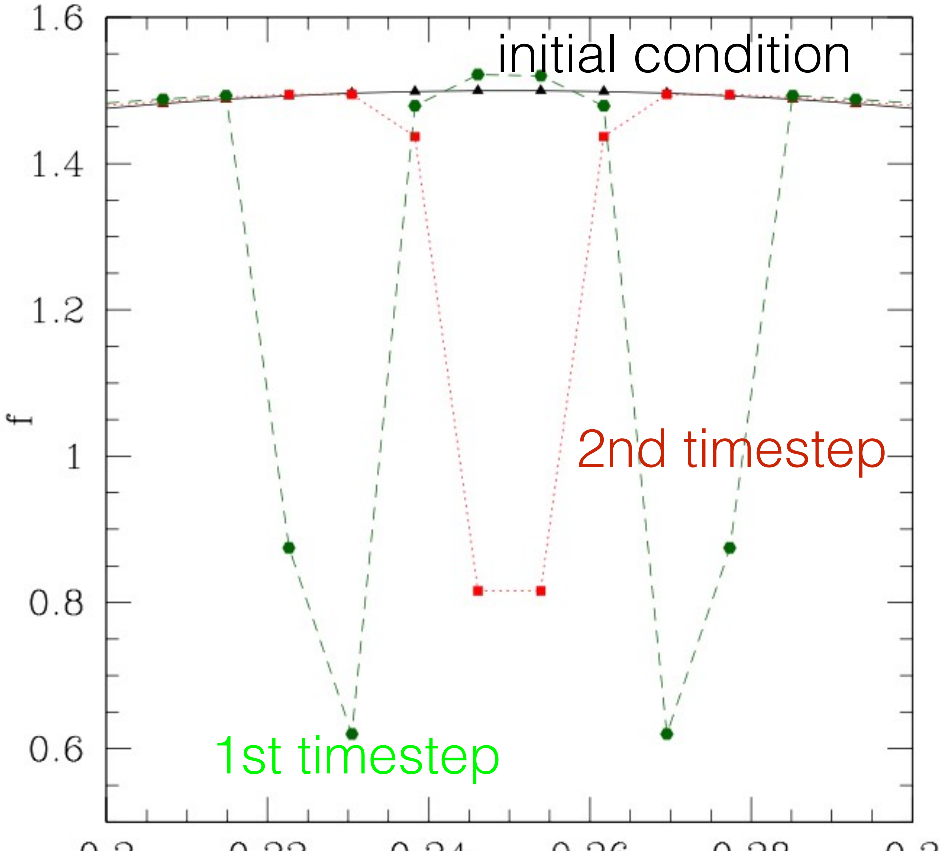
# CR streaming is a non-trivial numerical problem



Sharma et al 2010

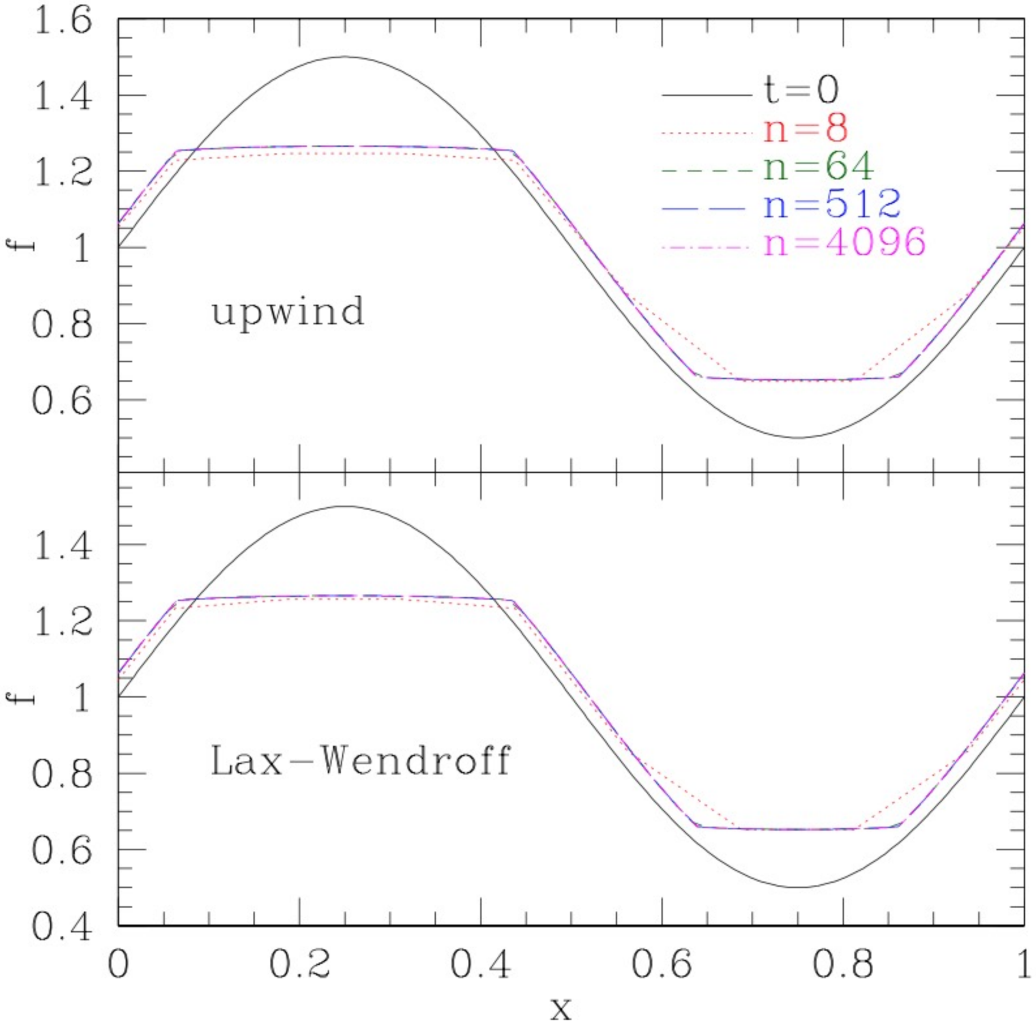
CRs can only stream down their gradient

At local extrema, overshoot and develop unphysical grid-scale oscillations

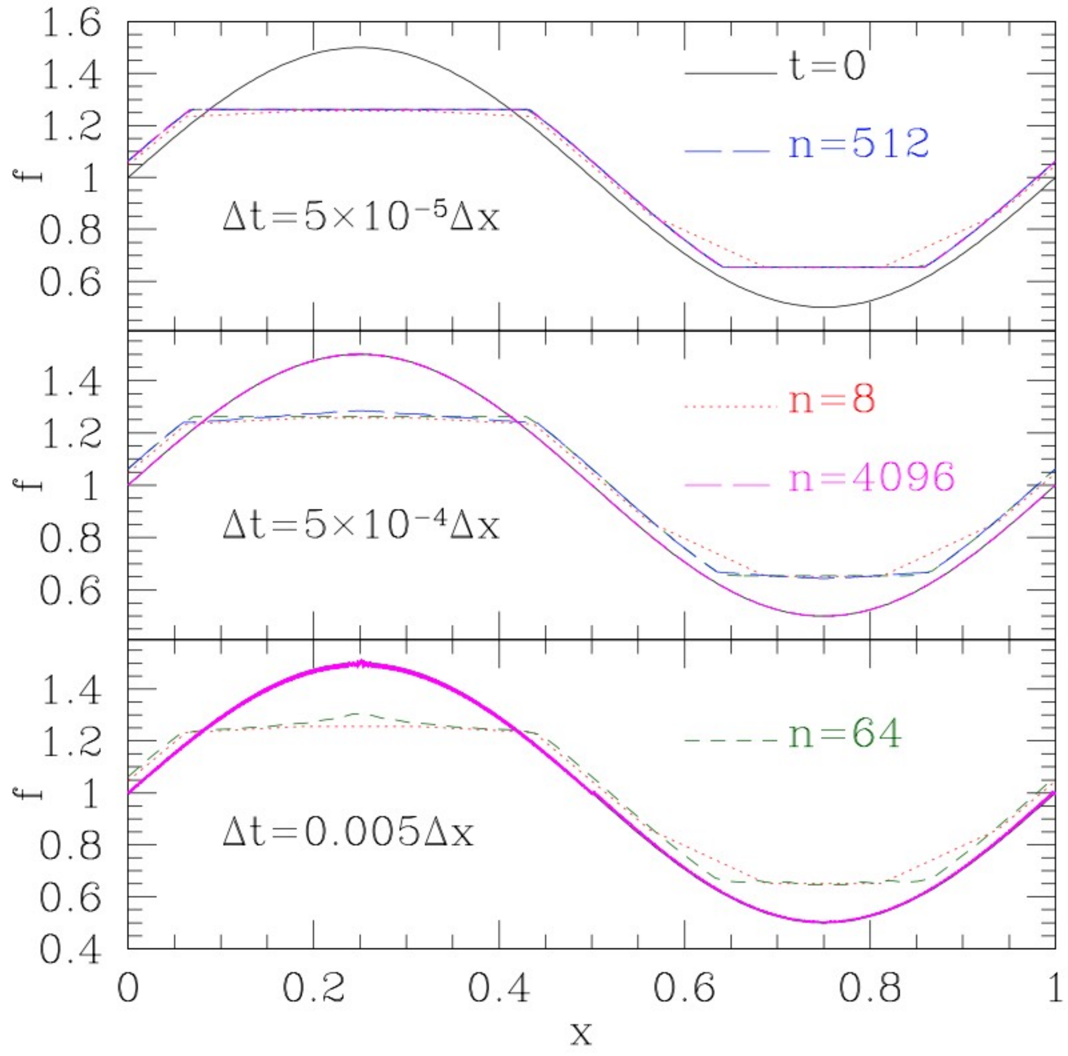


Need extremely small CFL time-step for stable solutions!

# Standard Solution: add numerical diffusion



(a) $\Delta t = \epsilon \Delta x^2/3$

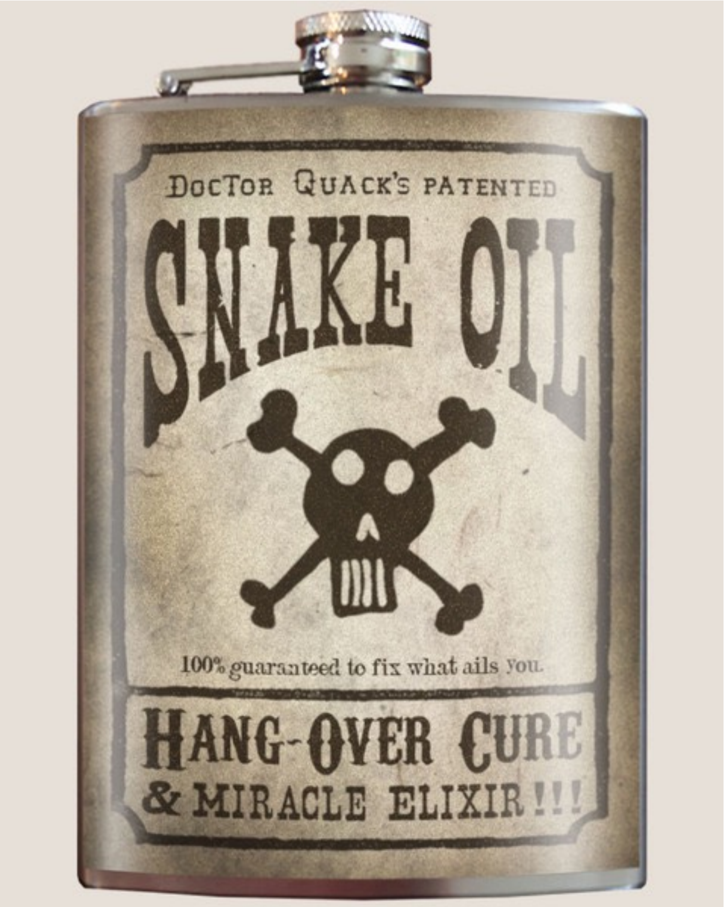


(b) linearized implicit method, $\epsilon = 0.01$

Explicit solver

Implicit solver

# But this is dodgy



Numerical diffusion masks true physical diffusion

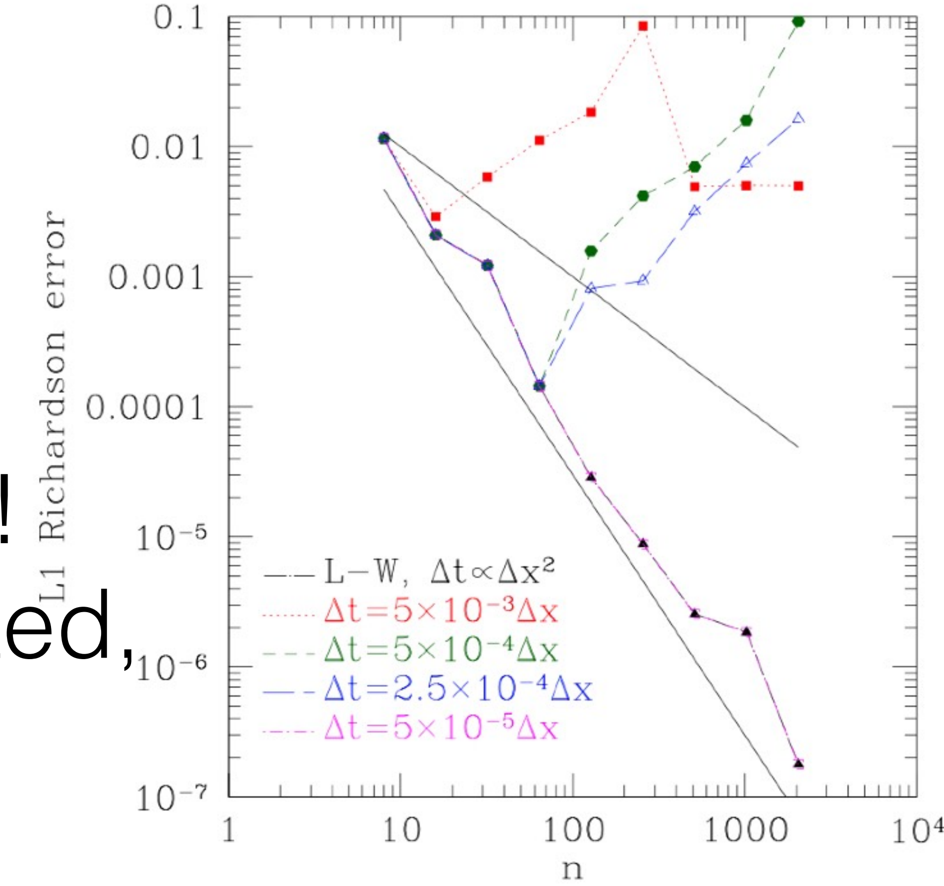
Ad-hoc, unmotivated smoothing parameter

Expensive: explicit method $\Delta t \propto (\Delta x)^2$

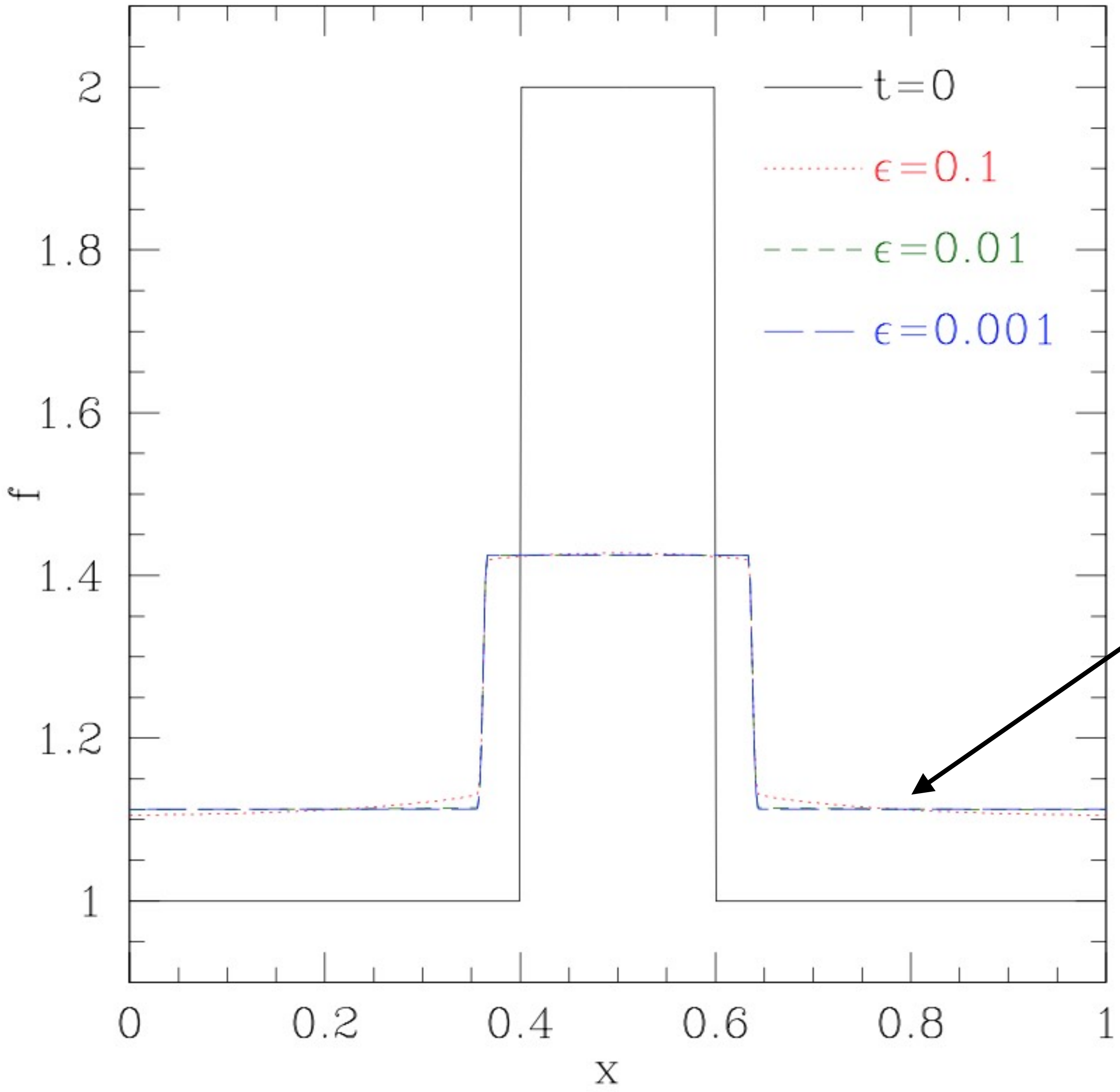
Implicit method: in principle $\Delta t \propto \Delta x$

but not really. Needs very small timestep! Also large matrix inversions — complicated, needs memory.

Sharma et al 2010



(b) linearized implicit method with $\epsilon = 0.01$



(a) $L = 1,\ n = 512$

CRs quickly populate distant tails!

Hard to interpret: highly super-Alfvenic streaming

Almost all calculations either only have streaming or diffusion.
No fully general calcs of CR transport!

# What's really the problem?

Standard approach solves the wrong set of equations

$$\frac{\partial E_c}{\partial t} = (\boldsymbol{v} + \boldsymbol{v}_s) \cdot \nabla P_c - \boldsymbol{\nabla} \cdot \boldsymbol{F}_c + Q.$$

where $$\boldsymbol{v}_s = -\boldsymbol{v}_A \frac{\boldsymbol{B} \cdot \boldsymbol{\nabla} P_c}{|\boldsymbol{B} \cdot \boldsymbol{\nabla} P_c|}.$$

But streaming velocity is undefined at extrema

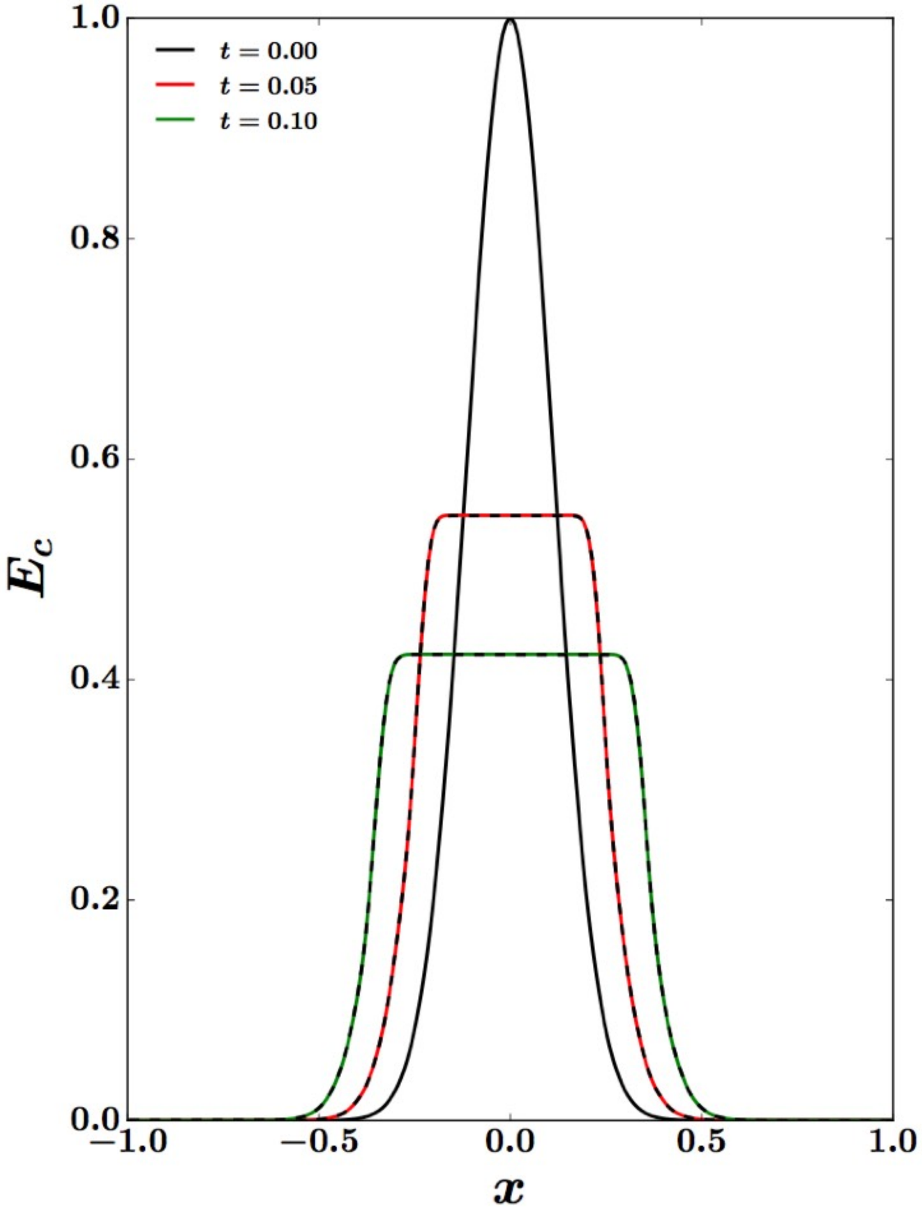
$\nabla P_c = 0 \Rightarrow$ isotropic CRs $\Longrightarrow$ no streaming instability

No CR scattering, fluid approximation fails

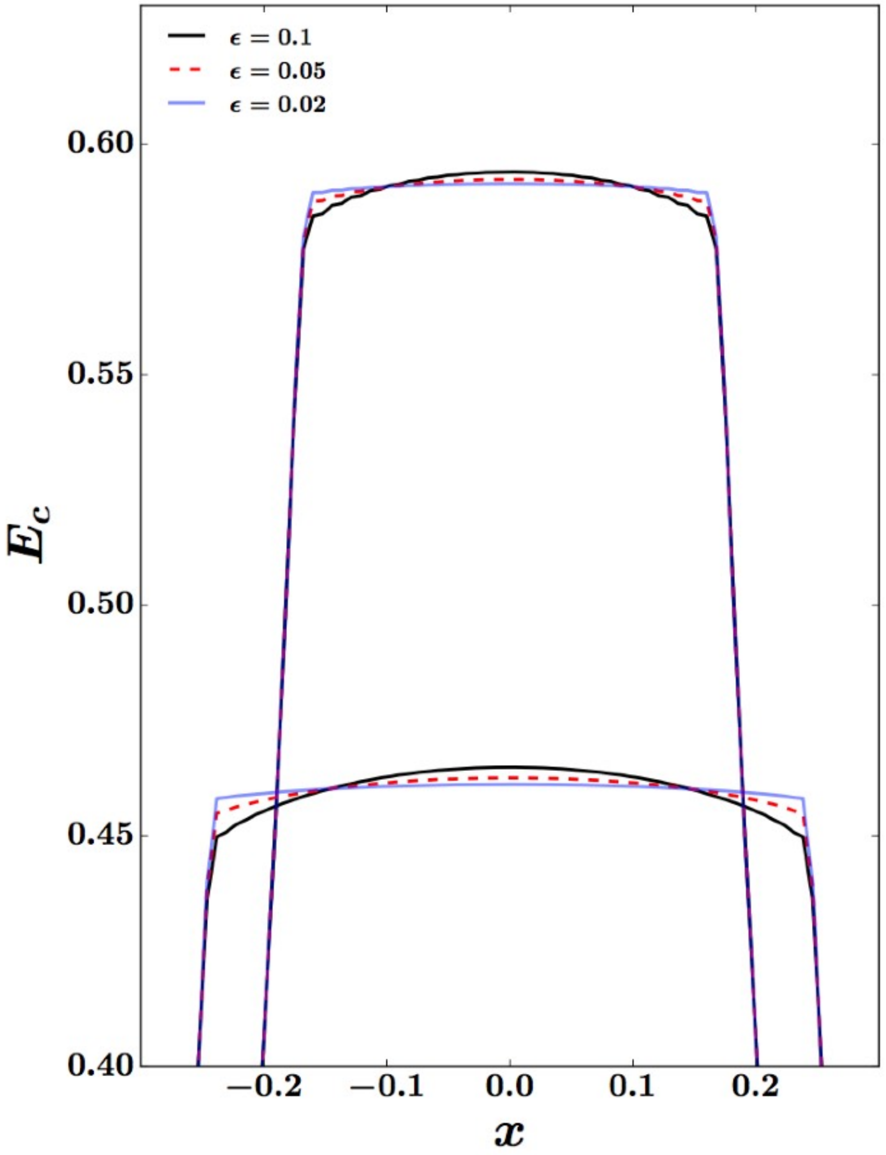
Instead, CRs are uncoupled and free stream at the speed of light at extrema

# A New Numerical Scheme

We formulate a new set of equations to take this into account



New method

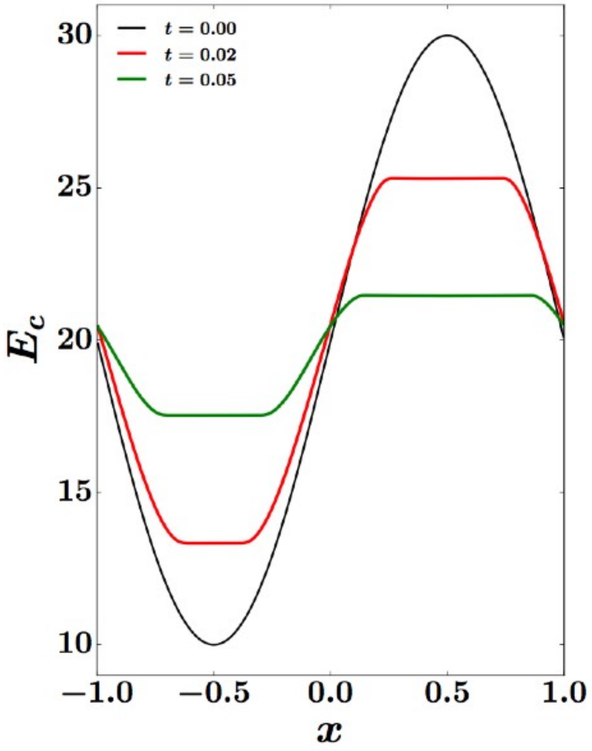


Old method

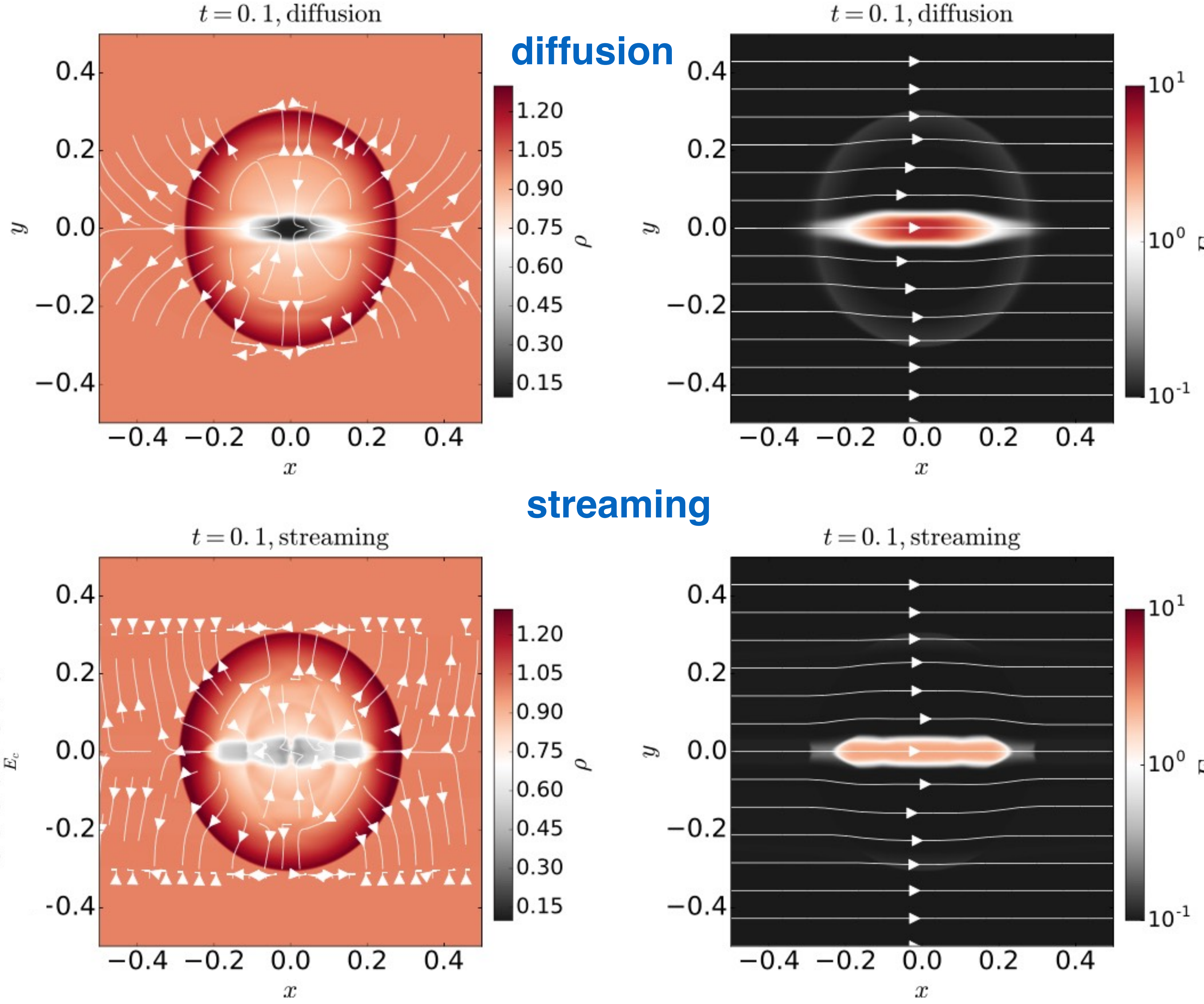
10 times faster than fastest [least smoothing] old method!

# It passes all tests we've thrown at it

sine wave



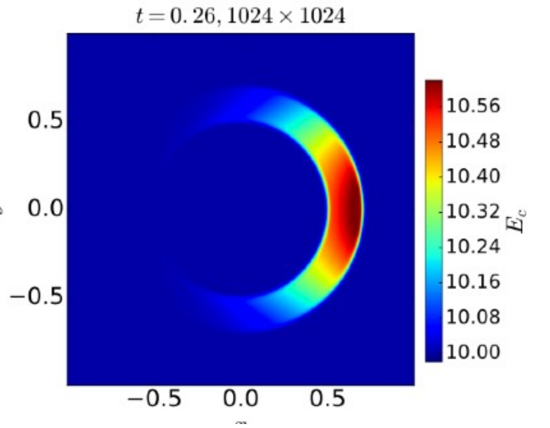
Blast wave with B-fields



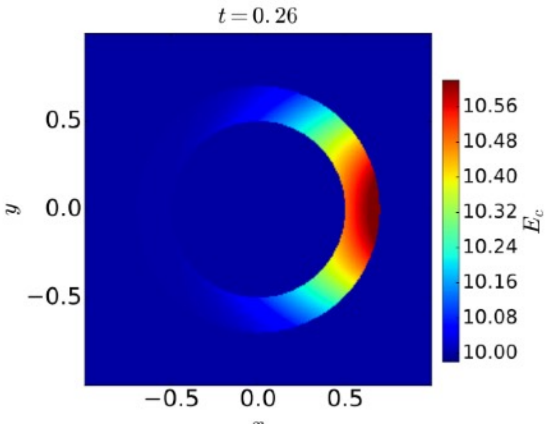
**diffusion**

**streaming**

anisotropic diffusion



**simulation**



**analytic**

# Advantages

No ad-hoc smoothing parameter

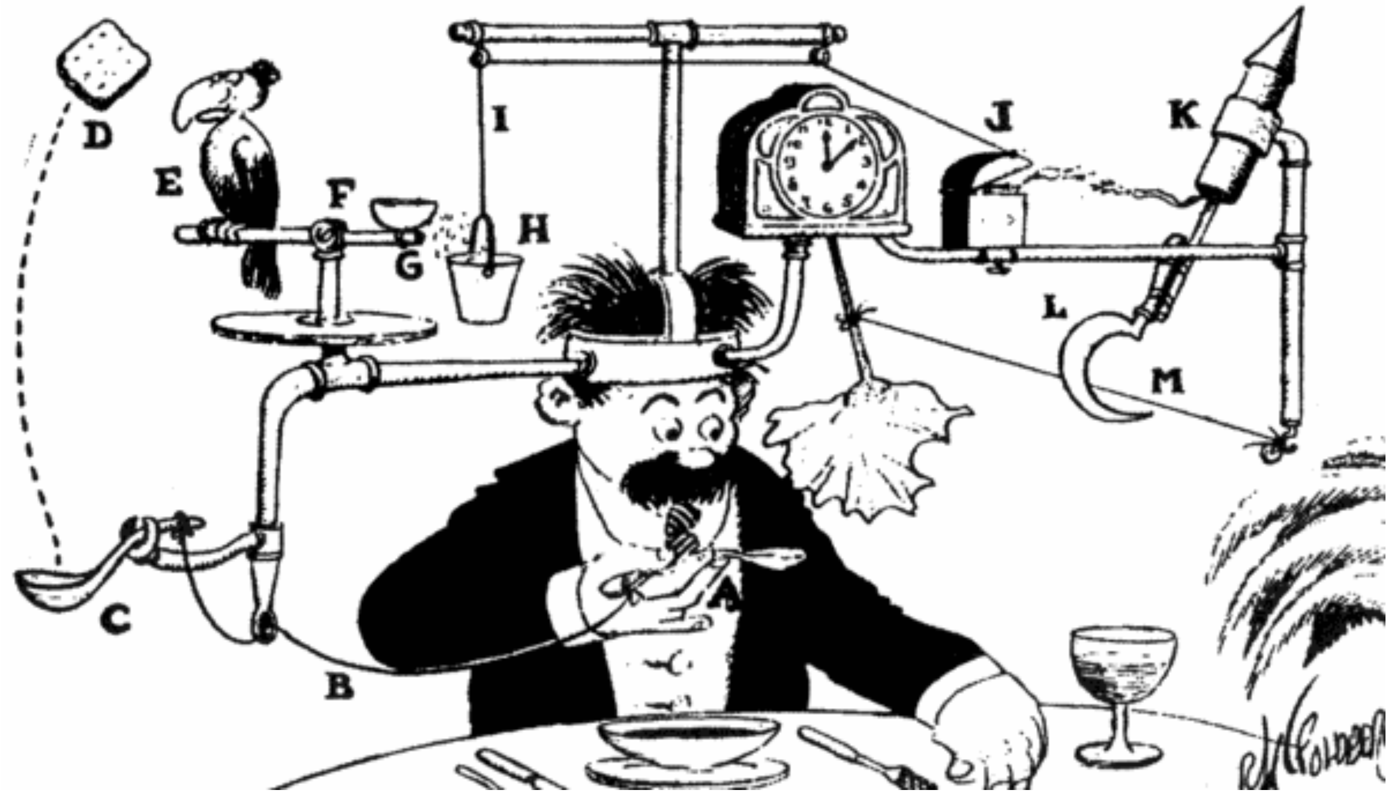
Cheap, robust. Take standard CFL time step.
Equations stay hyperbolic.

Can understand origin of fast transport.

Can calculate streaming and diffusion simultaneously with any diffusion coefficient.

Can easily do calculations where standard method chokes or is very expensive.

# Conclusions



We have a better gizmo for CR transport!

Paper out in a few weeks