

Electroweak monopoles and the electroweak phase transition¹

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PASCOS, June 2017

¹S. Arunasalam and A. Kobakhidze, arXiv: 1702.04068 (2017)

Outline

- 1 Motivation
- 2 Kibble mechanism
- 3 Cho-Maison monopoles
- 4 The effect on the electroweak phase transition
- 5 Nucleosynthesis constraint

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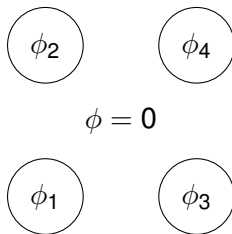
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- The suppression of these processes in the broken phase requires a first order phase transition with $\frac{\phi_c}{T_c} \gtrsim 1$
- The EWPT is second order in the standard model
- The energy density of monopoles can contribute to the energy of the broken phase, enhancing the phase transition while satisfying nucleosynthesis constraints.

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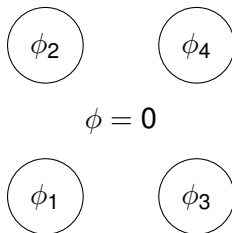
The Kibble Mechanism

- At $T = T_c$, domains of the broken phase will appear



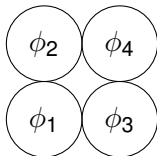
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- At $T = T_c$, domains of the broken phase will appear
- The higgs field in each domain takes independent directions on the vacuum manifold



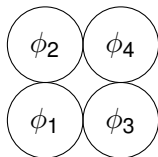
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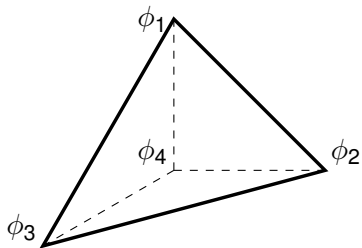
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- Consider an intersection of four of these domains:



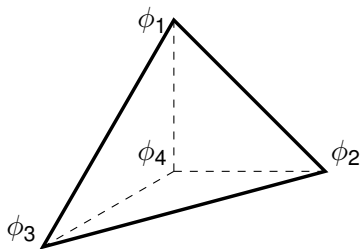
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- In field space, these points form the vertices of a tetrahedron.



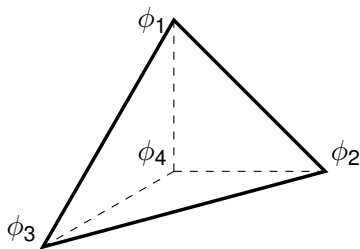
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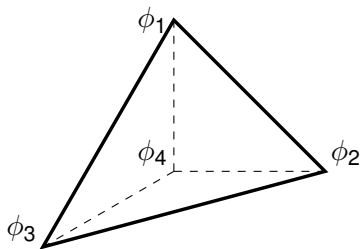
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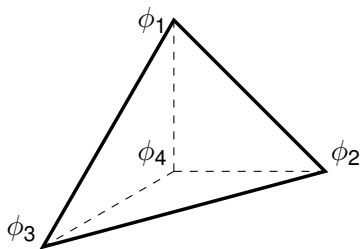
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- The tetrahedron is homotopically equivalent to S^2 .
- Therefore, $\pi_2(M_{\text{vac}}) \neq 0$ implies the existence of monopoles



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- No electroweak monopoles?

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The Ansatz

- Cho and Maison (1997) found electroweak monopoles through the ansatz:

$$\phi = \frac{1}{\sqrt{2}}\rho\xi$$

$$\rho = \rho(r)$$

$$\xi = i \begin{pmatrix} \sin(\theta/2)e^{-i\varphi} \\ -\cos(\theta/2) \end{pmatrix}$$

$$A_\mu = \frac{1}{g}A(r)\partial_\mu t \hat{r} + \frac{1}{g}(f(r) - 1)\hat{r} \times \partial_\mu \hat{r}$$

$$B_\mu = -\frac{1}{g'}B(r)\partial_\mu t - \frac{1}{g'}(1 - \cos\theta)\partial_\mu\varphi$$

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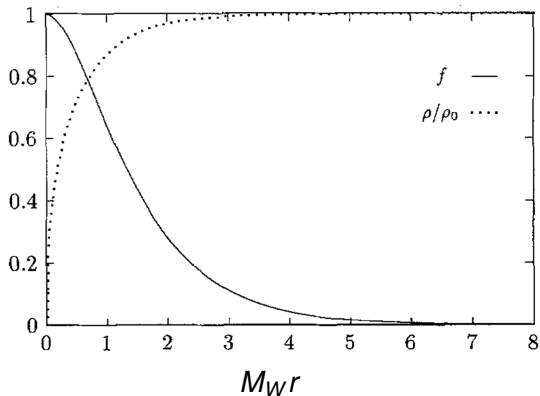
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- $M_{\text{vac}} = SU(2)/U(1) \cong \mathbb{C}P^1$
- $\pi_2(M_{\text{vac}}) = \mathbb{Z}$

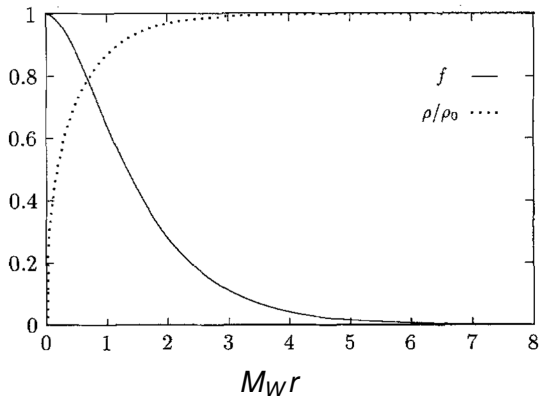
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- $h = \frac{4\pi}{e}$



The energy

$$E = E_0 + E_1$$

$$E_0 = 4\pi \int_0^\infty \frac{dr}{2r^2} \left\{ \frac{1}{g'^2} + \frac{1}{g^2} (f^2 - 1)^2 \right\}$$

$$E_1 = 4\pi \int_0^\infty dr \left\{ \frac{1}{2} (r\dot{\rho})^2 + \frac{1}{g^2} \left(\dot{f}^2 + \frac{1}{2} (r\dot{A})^2 + f^2 A^2 \right) \right. \\ \left. + \frac{1}{2g'^2} (r\dot{B})^2 + \frac{\lambda r^2}{8} (\rho^2 - \rho_0^2)^2 \right. \\ \left. + \frac{1}{4} f^2 \rho^2 + \frac{r^2}{8} (B - A)^2 \rho^2 \right\}$$

- The first term of E_0 is divergent at the origin.

Regularisation

- Cho, Kim and Yoon(2015) proposed a regularisation of the form:

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- However, g' becomes non-perturbative as $\phi \rightarrow 0$.
- This is undesirable in an EFT framework.
- We instead propose a Born-Infeld modification for the $U(1)_Y$ kinetic term.

Born-Infeld modification

- We regularise the $U(1)_Y$ kinetic term by replacing it with:

$$\begin{aligned} & \beta^2 \left[1 - \sqrt{-\det \left(\eta_{\mu\nu} + \frac{1}{\beta} B_{\mu\nu} \right)} \right] \\ &= \beta^2 \left[1 - \sqrt{1 + \frac{1}{2\beta^2} B_{\mu\nu} B^{\mu\nu} - \frac{1}{16\beta^4} (B_{\mu\nu} \tilde{B}^{\mu\nu})^2} \right] \end{aligned}$$

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- The corresponding energy is

$$\begin{aligned} & \int_0^\infty dr \beta^2 \left[\sqrt{(4\pi r^2)^2 + \left(\frac{4\pi}{g'\beta} \right)^2} - 4\pi r^2 \right] \\ &= \frac{4\pi^{5/2}}{3\Gamma\left(\frac{3}{4}\right)^2} \sqrt{\frac{\beta}{g'^3}} \end{aligned}$$

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- Hence, β acts as a mass parameter for the monopoles.

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The electroweak phase transition

- The Gibbs free energy:

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- Assuming $T \ll M$, the monopoles are decoupled and $E_{\text{monopoles}} = M \times n_M$

The initial density

- $n_M \approx \frac{1}{d^3}$ where d is the separation of two uncorrelated monopoles.

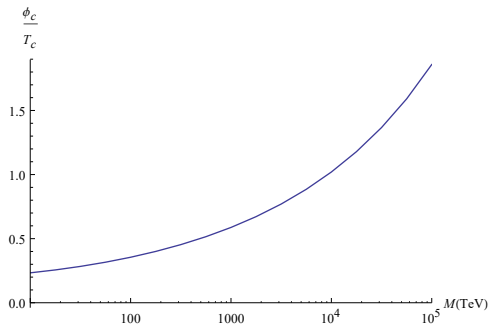
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- Hence, $n_M \approx \left(\frac{4\pi}{h^2}\right)^3 T^3$

Results



- Sphaleron processes are suppressed for $M > 0.9 \cdot 10^4$ TeV.

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- $\frac{n}{T^3} \Big|_{T=1\text{MeV}} < \frac{1\text{MeV}}{M}$
- Hence, the evolution of the number density over time must be considered.

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- This yields a mean free path of:

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- $B = \frac{3}{4\pi^2} \zeta(3) \sum_i (hq_i/4\pi)^2$

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- For $T < T_f$, the monopole density simply dilutes as $n \propto T^3$.

Nucleosynthesis constraint

- Solving the Boltzmann equation, one obtains (Preskill, 1979)

$$\frac{n}{T^3} = \frac{1}{Bh^2} \left(\frac{4\pi}{h^2} \right)^2 \frac{M}{CM_{pl}}, \quad (T > T_f)$$

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- This constrains the mass of the monopole to $M \lesssim 2.3 \cdot 10^4$ TeV.

Summary and future work

- Sphaleron mediated processes can be made ineffective in the broken phase while remaining under the nucleosynthesis constraints.

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- This occurs for monopoles with a mass of $(0.9 - 2.3) \cdot 10^4 \text{TeV}$.
- This could lead to a new mechanism for baryogenesis.