Holography for spin-2 fields in rotating black holes

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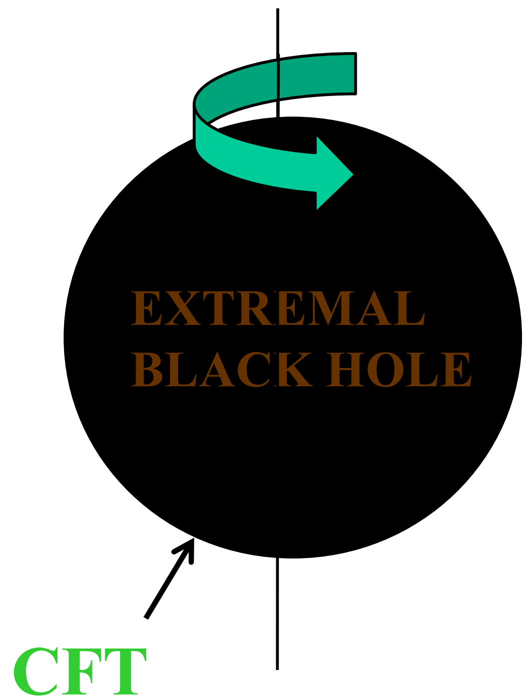
PASCOS 2017
June 22

Based on:

JHEP 08 (2009) 045
Class. Quantum Grav. 30 (2013) 135005
Phys. Rev. D 89, 024017 (2014)
Gen. Relativ. Gravit. (2016) 48:102

KERR/CFT CORRESPONDENCE



The near-horizon states and physical quantities of an extremal rotating BH (4D or higher) could be identified with and obtained from a certain chiral CFT living on the boundary of BH.
Generalizing the idea to non-extremal rotating black holes also is possible by finding a local conformal invariance (known as hidden conformal symmetry) in the solution space of the wave equation for the propagating fields in the background of rotating BH.

The near horizon geometry of rotating extremal black holes consists of a copy of AdS

Example: near horizon geometry of Kerr-Sen black hole

$$
\begin{aligned}
ds^2 &= \{M^2(1+\cos^2\theta)+\frac{1}{4}(-\varrho^2\sin^2\theta-4\varrho M\cos^2\theta)\}\{\frac{-dt^2+dy^2}{y^2}+d\theta^2+ \\
&+ \frac{4M^2\sin^2\theta}{(\frac{1}{2}\varrho\sin^2\theta+M(1+\cos^2\theta))^2}(d\phi+\frac{dt}{y})^2\}
\end{aligned}
$$

AdS_2

a S^1 bundle over AdS_2

This geometry has a SL(2,R) isometry as well as a rotational U(1) isometry generated by the Killing vector ∂_φ

The u(1) rotational isometry can be enhanced to a Virasoro algebra with a non-trivial central charge!

$$[L_m, L_n] = (m-n)L_{m+n} + \frac{J}{\hbar}m(m^2-1)\delta_{m+n,0} \longrightarrow c = \frac{12J}{\hbar}$$

The Cardy formula gives the entropy of the two dimensional CFT

$$S = 2\pi\sqrt{\frac{cL}{6}}$$

$L \rightarrow$ **Energy**

$c \rightarrow$ **Central charge**

First Law of Thermodynamics

$$dL = TdS \rightarrow dS = \pi\sqrt{\frac{c}{6L}}TdS \rightarrow \sqrt{L} = \pi\sqrt{\frac{c}{6}}T \quad \Rightarrow \quad S = \frac{\pi^2}{3}cT$$

Frolov-Thorne temperature of the near horizon region ~Temp. of left moving CFT

$$T_{F.T.} \equiv T_L = \frac{1}{2\pi}$$

$$\Rightarrow \quad S_{microscopic} = 2\pi J \quad \checkmark$$

This is exactly equal to the macroscopic Bekenstein-Hawking entropy

$$S_{BH} = 2\pi J$$

Extremal Kerr/CFT Conclusions:

- The perfect match of the macroscopic Bekenstein-Hawking entropy of rotating black holes with the Cardy entropy for CFT

- Super-radiant scattering off the black hole: The bulk scattering amplitudes are in precise agreement with the CFT results

- Real-time correlators of various perturbations in even near-extremal black hole could be computed directly from the bulk

- The near-horizon states of an extremal black hole could be identified with a certain chiral CFT on the horizon

- The corresponding Virasoro algebra is generated with a class of diffeomorphisms that preserves an appropriate boundary condition on the near-horizon geometry.

- The black hole near-horizon geometry consists of a certain AdS structure; the central charges of dual CFT can be obtained by analyzing the asymptotic symmetry group (ASG)

How about generic non-extremal Black Holes?

If Kerr/CFT correspondence is correct, then energy excitations of CFT should correspond to generic non-extremal black hole.

Problem:
Away from the extremality, there is no AdS structure for the near horizon geometry. In fact the near horizon geometry is Rindler space with no known associated CFT.

Solution: Existence of conformal invariance in a near-horizon geometry is not a necessary condition for the interactions to exhibit conformal invariance.

Instead the existence of a local conformal invariance (known as hidden conformal symmetry) in the solution space of the wave equation for the propagating field is sufficient to ensure a dual CFT description.

This hidden conformal symmetry is a sufficient condition the scattering amplitudes exhibit conformal invariance though the space on which the field propagates doesn't have the conformal symmetry

Can we obtain the two-point functions of vector (and spin 2 fields) fields in the near horizon of Kerr BH, similar to what has already been obtained in AdS/CFT correspondence?

$$\exp(-I_{AdS}) \equiv \left\langle \exp\left(\int d^d x \, J_j(\mathbf{x}) A_{0,j}(\mathbf{x}) \right) \right\rangle$$

conformal operators on the boundary of AdS

the Maxwell's fields on the boundary of AdS

Kerr BH

$$ds^2 = -\frac{\Delta}{\rho^2}\left(dt - a\sin^2\theta d\phi\right)^2 + \frac{\rho^2}{\Delta}dr^2 + \rho^2 d\theta^2 + \frac{\sin^2\theta}{\rho^2}\left(adt - \left(r^2 + a^2\right)d\phi\right)^2$$

$$\rho^2 = r^2 + a^2\cos^2\theta \qquad \Delta = r^2 + a^2 - 2Mr$$

Set of four null vectors in Newman-Penrose formalism (a tetrad formalism with special choice of null basis vectors; useful to reveal the inherent symmetries of space-time)

$$l^\mu = \Delta^{-1}\left(r^2 + a^2, \Delta, 0, a\right),$$

$$n^\mu = \frac{1}{2\rho^2}\left(r^2 + a^2, -\Delta, 0, a\right),$$

$$m^\mu = \frac{1}{\bar{\rho}\sqrt{2}}\left(ia\sin\theta, 0, 1, \frac{i}{\sin\theta}\right), \qquad \bar{\rho} = r + ia\cos\theta$$

Differential operators that are generalization of operators made of null Newman-Penrose vectors

$$\mathcal{D}_n = \frac{\partial}{\partial r} + \frac{iK}{\Delta} + 2n\left(\frac{r-M}{\Delta}\right) \quad , \quad \mathcal{D}_n^\dagger = \frac{\partial}{\partial r} - \frac{iK}{\Delta} + 2n\left(\frac{r-M}{\Delta}\right),$$

$$\mathcal{L}_n = \frac{\partial}{\partial \theta} + Q + n\cot\theta \quad , \quad \mathcal{L}_n^\dagger = \frac{\partial}{\partial \theta} - Q + n\cot\theta,$$

completely angular operators

completely radial operators

$$K = -\left(r^2 + a^2\right)\omega + am$$

$$Q = -a\omega\sin\theta + m\left(\sin\theta\right)^{-1}.$$

special cases: $l = \mathcal{D}_0 \qquad n = -\dfrac{\Delta}{2\rho^2}\mathcal{D}_0^\dagger$

$$m = \frac{1}{\bar{\rho}\sqrt{2}}\mathcal{L}_0^\dagger \quad \bar{m} = \frac{1}{\bar{\rho}^*\sqrt{2}}\mathcal{L}_0$$

$$S = S_{EH} + S_{GH},$$ **Gibbons-Hawking boundary action**

$$S_{EH} = \tfrac{1}{16\pi}\int_{\mathcal{M}} d^4x\sqrt{-g}R$$

We re-write the bulk action such that it depends only on the metric field and the first order derivatives of the metric field

$$S_{EH} = S_1 + \frac{1}{16\pi}\int_{\partial M} d^3x\sqrt{-g}\{g^{\mu\nu}\Gamma^{\sigma}_{\nu\sigma} - g^{\sigma\nu}\Gamma^{\mu}_{\sigma\nu}\},$$

$$S_1 = \tfrac{1}{16\pi}\int_{\mathcal{M}} d^4x\sqrt{-g}g^{\mu\nu}\{\Gamma^{\tau}_{\mu\nu}\Gamma^{\sigma}_{\tau\sigma} - \Gamma^{\tau}_{\mu\sigma}\Gamma^{\sigma}_{\tau\nu}\}$$

The boundary action

Kerr background

$$S_B = \frac{1}{16\pi}\int_{r=r_B} d^3x\sqrt{-g}\{g^{r\nu}\Gamma^{\sigma}_{\nu\sigma} - g^{\sigma\nu}\Gamma^{r}_{\sigma\nu}\},$$

$$g_{\mu\nu} = g^{(0)}_{\mu\nu} + \epsilon\tilde{h}_{\mu\nu}$$

Boundary of near-NHEK geometry

Spin 2 gravitational perturbation

$$\sqrt{-g} = \sqrt{-g^{(0)}}\{1 + \frac{\epsilon}{2}\tilde{h} + \frac{\epsilon^2}{8}(\tilde{h}^2 - h_{\mu\nu}\tilde{h}^{\mu\nu}) + O(\epsilon^3)\},$$

$$\Gamma^{\rho}_{\mu\nu} = \Gamma^{(0)\rho}_{\mu\nu} + \epsilon\Gamma^{(1)\rho}_{\mu\nu} + \epsilon^2\Gamma^{(2)\rho}_{\mu\nu} + O(\epsilon^3),$$

$$\begin{aligned}
\Gamma^{(0)\rho}_{\mu\nu} &= \frac{1}{2}g^{(0)\rho\lambda}\{\partial_\mu g^{(0)}_{\lambda\nu} + \partial_\nu g^{(0)}_{\mu\lambda} - \partial_\lambda g^{(0)}_{\mu\nu}\}, \\
\Gamma^{(1)\rho}_{\mu\nu} &= \frac{1}{2}\{g^{(0)\rho\lambda}(\partial_\mu\tilde{h}_{\lambda\nu} + \partial_\nu\tilde{h}_{\mu\lambda} - \partial_\lambda\tilde{h}_{\mu\nu}) - \tilde{h}^{\rho\lambda}(\partial_\mu g^{(0)}_{\lambda\nu} + \partial_\nu g^{(0)}_{\mu\lambda} - \partial_\lambda g^{(0)}_{\mu\nu})\}, \\
\Gamma^{(2)\rho}_{\mu\nu} &= -\frac{1}{2}\tilde{h}^{\rho\lambda}\{\partial_\mu\tilde{h}_{\lambda\nu} + \partial_\nu\tilde{h}_{\mu\lambda} - \partial_\lambda\tilde{h}_{\mu\nu}\}.
\end{aligned}$$

The only relevant terms in the boundary action that lead to the proper two-point function:

$$S^{(2)}_B = \int_{r=r_B} d^3x\sqrt{-g^{(0)}}\{g^{(0)r\nu}\Gamma^{(2)\sigma}_{\nu\sigma} + \Gamma^{(1)\sigma}_{\nu\sigma}(\frac{1}{2}g^{(0)r\nu}\tilde{h} - \tilde{h}^{r\nu}) + \Gamma^{(0)\sigma}_{\nu\sigma}(-\frac{1}{2}\tilde{h}\tilde{h}^{r\nu} + \frac{g^{(0)r\nu}}{8}(\tilde{h}^2 - \tilde{h}_{\rho\lambda}\tilde{h}^{\rho\lambda}))\}$$

Different components of the affine connections:

$$\Gamma^{(0)\sigma}_{r\sigma} = \frac{1}{2} g^{(0)\sigma\lambda} g^{(0)}_{\lambda\sigma,r},$$

$$\Gamma^{(1)\sigma}_{r\sigma} = \frac{1}{2}\{g^{(0)\sigma\lambda}(\partial_r \tilde{h}_{\lambda\sigma} + \partial_\sigma \tilde{h}_{r\lambda} - \partial_\lambda \tilde{h}_{r\sigma}) - \tilde{h}^{\sigma\lambda} g^{(0)}_{\lambda\sigma,r}\},$$

$$\Gamma^{(2)\sigma}_{r\sigma} = -\frac{1}{2}\tilde{h}^{\sigma\lambda}\partial_r \tilde{h}_{\lambda\sigma}.$$

Writing $\tilde{h}_{\alpha\beta}$ **as** $\frac{1}{2}(h_{\alpha\beta}e^{-i\omega t+im\phi} + h^*_{\alpha\beta}e^{i\omega t-im\phi})$**, we find**

$$S^{(2)}_B = \int_{r=r_B} d^3x\sqrt{-g}\{h^{*\alpha\beta}\Psi_{\alpha\beta\gamma\delta}h^{\gamma\delta} + c.c.\}$$

$$\begin{aligned}\Psi_{\alpha\beta\gamma\delta} \;=\; & -\frac{g^{rr}}{8}\{\partial_r(g_{\alpha\gamma}g_{\beta\delta}) + g_{\alpha\gamma}g_{\beta\delta}\partial_r\} + \frac{g^{rr}}{16}g_{\alpha\beta}\{g_{\gamma\delta}\partial_r - g_{\delta\gamma,r}\} - \frac{1}{8}g^r_\alpha g^r_\beta\{g_{\gamma\delta}\partial_r - g_{\delta\gamma,r}\} \\ & -\frac{1}{16}g^{\sigma\lambda}g_{\sigma\lambda,r}g_{\alpha\beta}g^r_\gamma g^r_\delta + \frac{g^{rr}}{64}g^{\sigma\lambda}g_{\sigma\lambda,r}(g_{\alpha\beta}g_{\gamma\delta} - g_{\alpha\delta}g_{\beta\delta}).\end{aligned}$$

The gravitational fields can be written as:

$$h^{\mu\nu} = \sum_{i,j=+,-} f_{ij}^{\mu\nu}(r,\theta) P_i(r) S_j(\theta),$$ **Teukolsky angular functions**

$$f_{ij}^{\mu\nu} = f_{ij}^{\mu\nu(-1)} \Delta^{-1} + f_{ij}^{\mu\nu(0)} \Delta^0 + \cdots$$

Teukolsky radial functions

where $P_+ = \Delta^2 R_{+2}, P_- = R_{-2}.$

$$\begin{aligned}\left(\Delta \mathcal{D}_1 \mathcal{D}_2^\dagger - 6i\omega r\right) R_{+2} &= \bar{\lambda} R_{+2}, & \left(\mathcal{L}_{-1}^\dagger \mathcal{L}_2 + 6a\omega\cos\theta\right) S_{+2} &= -\bar{\lambda} S_{+2}, \\ \left(\Delta \mathcal{D}_{-1}^\dagger \mathcal{D}_0 + 6i\omega r\right) R_{-2} &= \bar{\lambda} R_{-2}, & \left(\mathcal{L}_{-1} \mathcal{L}_2^\dagger - 6a\omega\cos\theta\right) S_{-2} &= -\bar{\lambda} S_{-2}.\end{aligned}$$

$$\Psi_0^{(1)} = R_{+2}(r) S_{+2}(\theta),$$

First order corrections to Weyl scalars due to gravitational perturbation

$$\Psi_4^{(1)} = \frac{1}{(\bar{\rho}^*)^4} R_{-2}(r) S_{-2}(\theta),$$

Each $f_{ij}^{\mu\nu}$ **has a structure as** $\dfrac{\mathcal{A}+\mathcal{C}\Delta+\mathcal{E}\Delta^2}{\mathcal{B}+\mathcal{F}\Delta+\mathcal{G}\Delta^2}$

We find a completely separable structure for the gravitational perturbation by setting:

$\rho_B = \rho(r = r_B)$ **and** $\Delta_B = \Delta(r = r_B)$

$$h^{\mu\nu} = \sum_{i,j=+,-} \chi_i \sum_{\aleph} f_{1ij\aleph}^{\overline{\mu\nu}}(r) f_{2ij\aleph}^{\overline{\mu\nu}}(\theta) R_i(r) S_j(\theta) \qquad \chi_+ = \Delta_B,\ \chi_- = 1$$

$$\begin{aligned} f_{1++1}^{rr}(r) &= 24\sqrt{2}aQM\omega r^2 \\ f_{1++2}^{rr}(r) &= 6\sqrt{2}a^2\omega Q r^3 \end{aligned}$$

$$\begin{aligned} f_{2++1}^{rr}(\theta) &= \frac{-\sin^3\theta\cos^2\theta}{\rho_B^4(3a^2\omega^2\sin^2\theta\cos^2\theta + Qa\omega\sin\theta(1-3\cos^2\theta) + Q^4\sin^2\theta - Q^2)} \\ f_{2++2}^{rr}(\theta) &= \frac{-\sin^3\theta\cos\theta}{\rho_B^4(3a^2\omega^2\sin^2\theta\cos^2\theta + Qa\omega\sin\theta(1-3\cos^2\theta) + Q^4\sin^2\theta - Q^2)} \end{aligned}$$

We find the gravitational perturbation has an expansion in terms of boundary fields:

$$h^{\mu\nu} = \frac{h^{\mu\nu}_{B+}}{R_{B+}} R_+(r) + \frac{h^{\mu\nu}_{B-}}{R_{B-}} R_-(r)$$

$R_\pm(r_B)$.

$$\frac{h^{\mu\nu}_{B+}}{R_{B+}} = \sum_{\aleph, j=+,-} \Delta_B f^{\overline{\mu\nu}}_{1+j\aleph}(r_B) f^{\overline{\mu\nu}}_{2+j\aleph}(\theta) S_j(\theta)$$

$$\frac{h^{\mu\nu}_{B-}}{R_{B-}} = \sum_{\aleph, j=+,-} f^{\overline{\mu\nu}}_{1-j\aleph}(r_B) f^{\overline{\mu\nu}}_{2-j\aleph}(\theta) S_j(\theta).$$

or

$$h^{\mu\nu} = \frac{h^{\mu\nu}_{B+}}{R_{B+}} R_+(r) + \frac{1}{\Delta_B} F^{-+}_{++}(\theta) R_-(r) (\frac{h^{\mu\nu}_{B+}}{R_{B+}})_+ + \frac{1}{\Delta_B} F^{--}_{+-}(\theta) R_-(r) (\frac{h^{\mu\nu}_{B+}}{R_{B+}})_-$$

These two terms are dominant terms in the boundary action

$$F^{-+}_{++}(\theta) = \frac{\sum_{\aleph} f^{\overline{\mu\nu}}_{1-+\aleph}(r_B) f^{\overline{\mu\nu}}_{2-+\aleph}(\theta)}{\sum_{\aleph} f^{\overline{\mu\nu}}_{1++\aleph}(r_B) f^{\overline{\mu\nu}}_{2++\aleph}(\theta)}$$

$$F^{--}_{+-}(\theta) = \frac{\sum_{\aleph} f^{\overline{\mu\nu}}_{1--\aleph}(r_B) f^{\overline{\mu\nu}}_{2--\aleph}(\theta)}{\sum_{\aleph} f^{\overline{\mu\nu}}_{1+-\aleph}(r_B) f^{\overline{\mu\nu}}_{2+-\aleph}(\theta)}$$

The boundary action:

$$\begin{aligned} S^{(2)}_{B} \;&=\; \frac{1}{\Delta^2_B} \int_{r=r_B} d^3x \sqrt{-g} \{ [F^{-+*}_{++}(\theta)(\frac{h^{\alpha\beta *}_{B+}}{R^*_{B+}})_+ + F^{--*}_{+-}(\theta)(\frac{h^{\alpha\beta *}_{B+}}{R^*_{B+}})_-][F^{-+}_{++}(\theta)(\frac{h^{\gamma\delta}_{B+}}{R_{B+}})_+ + F^{--}_{+-}(\theta)(\frac{h^{\gamma\delta}_{B+}}{R_{B+}})_-] \\ &\quad R^*_-(r)\Psi_{\alpha\beta\gamma\delta}R_-(r) + c.c.\}. \end{aligned}$$

$$\begin{aligned} \Psi_{\alpha\beta\gamma\delta} \;&=\; -\frac{g^{rr}}{8}\{\partial_r(g_{\alpha\gamma}g_{\beta\delta}) + g_{\alpha\gamma}g_{\beta\delta}\partial_r\} + \frac{g^{rr}}{16}g_{\alpha\beta}\{g_{\gamma\delta}\partial_r - g_{\delta\gamma,r}\} - \frac{1}{8}g^r_\alpha g^r_\beta\{g_{\gamma\delta}\partial_r - g_{\delta\gamma,r}\} \\ &\quad -\frac{1}{16}g^{\sigma\lambda}g_{\sigma\lambda,r}g_{\alpha\beta}g^r_\gamma g^r_\delta + \frac{g^{rr}}{64}g^{\sigma\lambda}g_{\sigma\lambda,r}(g_{\alpha\beta}g_{\gamma\delta} - g_{\alpha\delta}g_{\beta\delta}). \end{aligned}$$

The derivatives of radial Teukolsky functions:

$$\begin{aligned}\partial_r R_+(r) &= (\beta - \frac{5}{2})\frac{1}{r}R_+(r) - 2\beta\mathcal{B}_+ N_+ \tau_H^{-in/2+\beta+1/2} r^{-\beta-7/2} \\ \partial_r R_-(r) &= (\beta + \frac{3}{2})\frac{1}{r}R_-(r) - 2\beta\mathcal{B}_- N_- \tau_H^{-in/2+\beta+1/2} r^{-\beta+1/2}\end{aligned}$$

Order of $\tau_H^{\beta+1/2}$

For extremal black hole $\tau_H \to 0$

Moreover, we can write the decomposition of the boundary fields as:

$$h^{\alpha\beta}_{B+}(t,\theta,\phi) = \mathcal{H}^{\overline{\alpha\beta}}_{B+}(t,\phi)\Theta^{\overline{\alpha\beta}}_{\pm}(\theta)$$

$$\Theta^{\overline{\alpha\beta}}_{\pm} = \sum_{\aleph}[f^{\overline{\alpha\beta}}_{1+\pm\aleph}(r_B) f^{\overline{\alpha\beta}}_{2+\pm\aleph}(\theta)]S_{\pm}(\theta)$$

We get the final result for the boundary action as:

$$S_B^{(2)} = \frac{1}{\Delta_B^2}\int_{r=r_B} d^3x\sqrt{-g}\{[F_{++}^{-+*}(\theta)\Theta_+^{\overline{\alpha\beta}*} + F_{+-}^{--*}(\theta)\Theta_-^{\overline{\alpha\beta}*}](\Psi_B)_{\alpha\beta\gamma\delta}(r_B,\theta)[F_{++}^{-+}(\theta)\Theta_+^{\overline{\gamma\delta}} + F_{+-}^{--}(\theta)\Theta_-^{\overline{\gamma\delta}}]$$
$$\times \frac{R_-^*(r_B)R_-(r_B)}{R_{B+}^*R_{B+}}\mathcal{H}_{B+}^{\overline{\alpha\beta}*}(t,\phi)\mathcal{H}_{B+}^{\overline{\gamma\delta}}(t,\phi) + c.c.\},$$

$$\frac{\mathcal{N}r_B^8\{|\mathcal{A}_-|^2 + (\mathcal{A}_-\mathcal{B}_-^* + \mathcal{A}_-^*\mathcal{B}_-)(\frac{r_B}{\tau_H})^{-2\beta} + |\mathcal{B}_-|^2(\frac{r_B}{\tau_H})^{-4\beta}\}}{n^2(n^2-4)(n^2+1)^2|\mathcal{A}_+|^2}$$

$$\mathcal{N} = \bar{\lambda}^4 + 4\bar{\lambda}^3 + (4+10m^2)\bar{\lambda}^2 + 36m^2\bar{\lambda} + 9m^2(m^2+4).$$

$$x = \frac{r-r_+}{r_+}$$

$$R_{+2} = A_+x^{\beta-5/2} + B_+x^{-\beta-\frac{5}{2}},$$
$$R_{-2} = A_-x^{\beta+\frac{3}{2}} + B_-x^{-\beta-\frac{3}{2}},$$

The radial Teukolsky functions in the matching region: $r_+(1+\tau_H) << r << 2r_+$

$$A_\pm = \frac{\Gamma(2\beta)\Gamma(1\mp 2 - in)}{\Gamma\left(\frac{1}{2}+\beta-i(n-m)\right)\Gamma\left(\frac{1}{2}+\beta\mp 2 - im\right)}\tau_H^{-\beta+\frac{1}{2}-i\frac{n}{2}} = \mathcal{A}_\pm\tau_H^{-\beta+\frac{1}{2}-i\frac{n}{2}}$$
$$B_\pm = \frac{\Gamma(-2\beta)\Gamma(1\mp 2 - in)}{\Gamma\left(\frac{1}{2}-\beta-i(n-m)\right)\Gamma\left(\frac{1}{2}-\beta\mp 2 - im\right)}\tau_H^{\beta+\frac{1}{2}-i\frac{n}{2}} = \mathcal{B}_\pm\tau_H^{\beta+\frac{1}{2}-i\frac{n}{2}}.$$

$$n = \frac{\omega - m\Omega_H}{2\pi T_H}$$

Inspired by AdS/CFT correspondence: The two-point function of the boundary energy-momentum tensor operators $\mathcal{O}_{\alpha\beta}$ is equal to functional derivative of the boundary action with respect to the rescaled boundary gravitational fields

$$\hat{\mathcal{H}}^{\alpha\beta}_{B+} = \frac{\mathcal{H}^{\alpha\beta}_{B+} + \mathcal{H}^{\alpha\beta *}_{B+}}{2\Delta_B r_B^{\beta-4}}$$

$$< \mathcal{O}_{\alpha\beta}\mathcal{O}_{\gamma\delta} >= \frac{\delta^2 S_B^{(2)}}{\delta\hat{\mathcal{H}}^{\alpha\beta}_{B+}\delta\hat{\mathcal{H}}^{\gamma\delta}_{B+}}$$

We find

the ratio of last term to the third term is of the order of $(\frac{T_H}{M})^{2\beta}$

$$< \mathcal{O}_{\alpha\beta}\mathcal{O}_{\gamma\delta} >= M^8 r_B^{2\beta-8}\mathcal{F}_{\alpha\beta\gamma\delta}\{1 + M^{-2\beta}\frac{\mathcal{N}}{\mathcal{C}}G_R^* + M^{-2\beta}\frac{\mathcal{N}}{\mathcal{C}^*}G_R + \mathcal{N}M^{-4\beta}|G_R|^2\}$$

Constant term

Drop this term to have a complex two-point function

$$G_R = T_R^{2\beta}\frac{\Gamma(-2\beta)\Gamma(\beta - 3/2 - im)\Gamma(1/2 + \beta - i(n-m))}{\Gamma(2\beta)\Gamma(5/2 - \beta - im)\Gamma(1/2 - \beta - i(n-m))}$$

$$\mathcal{C} = (\bar{\lambda} - 2m^2 - 2im\beta)(\bar{\lambda} - 2m^2 - 2im\beta + 2)$$

$$
\begin{aligned}
\mathcal{F}_{\alpha\beta\gamma\delta} &= \int_0^\pi d\theta \sin(\theta)\{[F_{++}^{-+*}(\theta)\Theta_+^{\overline{\alpha\beta}*} + F_{+-}^{--*}(\theta)\Theta_-^{\overline{\alpha\beta}*}](\Psi_B)_{\alpha\beta\gamma\delta}(r_B,\theta)[F_{++}^{-+}(\theta)\Theta_+^{\overline{\gamma\delta}} + F_{+-}^{--}(\theta)\Theta_-^{\overline{\gamma\delta}}] \\
&+ [F_{++}^{-+}(\theta)\Theta_+^{\overline{\alpha\beta}} + F_{+-}^{--}(\theta)\Theta_-^{\overline{\alpha\beta}}](\Psi_B)_{\alpha\beta\gamma\delta}(r_B,\theta)[F_{++}^{-+*}(\theta)\Theta_+^{\overline{\gamma\delta}*} + F_{+-}^{--*}(\theta)\Theta_-^{\overline{\gamma\delta}*}]\}\rho_B^2.
\end{aligned}
$$

We conclude

$< \mathcal{O}_{\alpha\beta}\mathcal{O}_{\gamma\delta} >$ **can be described by** $G_R\mathcal{F}_{\alpha\beta\gamma\delta}$

Any agreement with CFT-side-calculation of two-point functions for spin-2 operators ?
according to proposed Kerr/CFT correspondence

AdS/CFT Correspondence: The two point functions for the spin-2 operators that are coupled to the gravitational perturbation on the boundary of AdS

$$
\langle \mathcal{T}_{\alpha\beta}(\vec{x})\mathcal{T}_{\gamma\rho}(\vec{y})\rangle = \frac{V_{\alpha\beta\gamma\rho}(\vec{x}-\vec{y})}{|x-y|^{2d}}
$$

Due to separability of indices in our result for two-point functions, we may find the field theory-calculated G_R from CFT. Let's consider a finite temperature CFT with two-point function

conformal weights

$$\langle \mathcal{O}\mathcal{O} \rangle \sim \left(\frac{\pi T_R}{\sinh\left(\pi T_R t_{12}\right)} \right)^{2h_R} \left(\frac{\pi T_L}{\sinh\left(\pi T_L \bar{t}_{12}\right)} \right)^{2h_L}$$

$$t_1 = x_0 + i x_1$$
$$t_2 = y_0 + i y_1$$
$$t_{12} = t_1 - t_2$$

temperatures of right and left sectors

The Fourier transform of $\langle \mathcal{O}\mathcal{O} \rangle$

$$\mathcal{F}\langle \mathcal{O}\mathcal{O} \rangle \sim$$

$$\int_0^{1/T_R} dt_{12} e^{i\omega t_{12}} \left(\frac{\pi T_R}{\sin\left(\pi T_R t_{12}\right)} \right)^{2h_R} \int_0^{1/T_L} d\bar{t}_{12} e^{i\omega \bar{t}_{12}} \left(\frac{\pi T_L}{\sin\left(\pi T_L \bar{t}_{12}\right)} \right)^{2h_L}$$

$$\mathcal{F}(\langle \mathcal{O}\mathcal{O} \rangle) \sim T_R{}^{2\beta} \frac{\Gamma\left(1-2h_R\right)\Gamma\left(1-2h_L\right)}{\Gamma\left(1-h_R-in_R\right)\Gamma\left(1-h_R+in_R\right)\Gamma\left(1-h_L-in_L\right)\Gamma\left(1-h_L+in_L\right)}$$

By choosing the appropriate conformal weight

$$h_R = \beta + 1/2 \qquad h_L = \beta - 3/2$$

1

We find

$$\mathcal{F}(\langle \mathcal{O}\mathcal{O} \rangle) \sim T_R{}^{2\beta} \frac{\Gamma\left(-2\beta\right)\Gamma\left(1/2+\beta-in_R\right)\Gamma\left(\beta-3/2-in_L\right)}{\Gamma\left(1/2-\beta-in_R\right)\Gamma\left(-\beta+5/2-in_L\right)\Gamma\left(2\beta\right)}.$$

Choosing $n_L = m$

$n_R = n - m$

We find the field theoretic two-point function is in agreement with the CFT two-point function

$$\mathcal{F}\left\langle \mathcal{O}\mathcal{O} \right\rangle \sim G_R$$

Thank you for your attention!