Thank for the invitation

ANGULAR MOMENTUM
OF DARK MATTER
BLACK HOLES

Paul H. Frampton

Visiting Professor
University of Salento at Lecce

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Introduction

One of the central questions in both particle phenomenology and theoretical cosmology is the identity of the dark matter. In this paper, we show that the angular momentum of Kerr black holes strongly affects the accretion of normal matter and hence the emission of X-rays. This in turn shows that the dark matter Kerr black holes are consistent with the severe empirical limits on distortion of the Cosmic Microwave Background (CMB).

The Milky Way galaxy in which we reside lies within a large approximately spherical halo of dark matter (DM) which does not experience the strong or electromagnetic interactions, nor as we shall assume here the weak interactions. The popular idea that the dark matter constituent is a WIMP with weak interactions was born out of supersymmetry which lacks any support from extensive LHC data on pp scattering which probed the energy regime where signs of SUSY were most expected. Die-hard SUSY theorists may still have hope, but it is not premature to entertain the assumption that the WIMP does not exist.

With no WIMP one is led to astrophysical MACHOs and then confronted with the constraint from BBN that no more than 20% of the DM can be baryonic. This means that to make 100% of the DM we cannot use compact objects such as white dwarfs, neutron stars, brown dwarfs and unassociated planets. Nor is it possible to use black holes which are the result of gravitational collapse of baryonic stars.

There is, however, a second type of black hole which is formed primordially (PBH) during the radiation era. To form 100% of the DM we must therefore use PBHs. Since the resultant black holes of the two types are indistinguishable, can we use, say, 20% of the gravitational collapse variety and 80% of PBHs? The surprising answer is no. One result of the present talk is that the vast majority, well over 99%, must be PBHs from a study of X-rays and the related CMB distortion.

Focusing on the Milky Way halo where we can most easily detect the PBHs, we already know from earlier searches, especially the MACHO Collaboration that masses $M \leq 20M_\odot$ can make up no more that 10% of the halo dark matter. At the high mass end, we know from Xu-Ostriker that MACHOs with $M \geq 10^5 M_\odot$ endanger disk stability. For the Milky Way halo one is led to consider intermediate mass PIMBHs in the mass range

$$25M_\odot(1y) \leq M_{PIMBH} \leq 2,500M_\odot(10y) \tag{1}$$

for the DM constituents. This leads to a *plum pudding* model for the Milky Way halo, named after Thomson's atomic model, where for the DM halo the plums are PIMBHs with masses satisfying Eq.(1) and the pudding is rarefied gas, dust and a few luminous stars.

The formation of PBHs with masses as large as Eq.(1) and much larger is known to be mathematically possible during the radiation era. An existence theorem is provided by hybrid inflationary models. One specific prediction of hybrid inflation is a sharply-peaked PBH mass function. If we need a specific PIMBH mass, we shall use a calligraphic $\mathcal{PIMBH}$ defined by $M_{\mathcal{PIMBH}} \equiv 100 M_{\odot}$ exactly. This is merely an example and extension to the whole range of Eq.(1) can also be discussed.

The cosmic time t_{PBH} at which a PBH is formed has been estimated to be

$$t_{PBH} \simeq \left(\frac{M_{PBH}}{10^5 M_\odot}\right) \quad seconds \tag{2}$$

so that the PIMBHs in Eq.(1) are formed in the time window $0.0002s \leq t_{PIMBH} \leq 1.0s$ with the special case $t_{\mathcal{PIMBH}} \simeq 0.001s$. In terms of red shift (Z), this corresponds to

$$5 \times 10^{11} \geq Z_{PIMBH} \geq 5 \times 10^9 \tag{3}$$

with the special case $Z_{\mathcal{PIMBH}} \simeq 2 \times 10^{11}$.

The formation of BHs which are not primordial, which we shall denote without an initial P or $\mathcal{P}$, necessarily occurs *after* star formation which conservatively occurs certainly only for very different redshifts satisfying

$$Z_{BH} \leq 100 \tag{4}$$

The sharp difference in the red-shifts of Eq.(3) and Eq.(4) will become important when we discuss the reasons for previous non-detection, the angular momentum of PIMBHs and BHs, and the central issue of possible CMB distortion by X-rays.

As already mentioned, by using the mathematical models, it is possible to form PBHs not only in the PIMBH mass range of Eq. (1) but also Primordial Super Massive Black Hole (PSMBHs) in the mass range

$$10^5 M_\odot \leq M_{PSMBH} \leq 10^{17} M_\odot \quad (5)$$

where the upper limit derives from the formation time t_{PSMBH} given by Eq. (2) staying within the radiation-dominated era. We shall discuss the higher mass range Eq(5) later in the paper.

Finally for this Introduction, we recall that in a microlensing experiment, *e.g.* using the LMC or SMC for convenient sources, microlensing by halo PIMBHs, and assuming a typical transit velocity $200 km.s^{-1}$, the time duration of the microlensing light curve can be estimated to be approximately

$$\tau \simeq \left(\frac{M_{PIMBH}}{25 M_{\odot}} \right)^{\frac{1}{2}} \ \ years \qquad (6)$$

which we note is close to one year and two years, respectively, for lens masses $25 M_{\odot}$ and $100 M_{\odot}$. For reference, the highest duration such light curve detected by the MACHO Collaboration which published in the year 2000 corresponded to $M_{PIMBH} \simeq 20 M_{\odot}$.

Nevertheless, if longer duration microlensing light curves can be detected of two years or more, the only known explanation will be the existence of Kerr black holes in the halo with many solar masses.

Kerr Metric and Period τ

The PIMBHs are described by a Kerr metric which has the form in Boyer-Lindquist (t, r, θ, ϕ) coordinates, after defining $\alpha = \frac{J}{M}$, $\rho^2 = r^2 + \alpha^2 \cos^2\theta$ and $\Delta = r^2 - 2Mr + \alpha^2$,

$$ds^2 = -\left(1 - \frac{2Mr}{\rho^2}\right) dt^2 - \left(\frac{4Mr\alpha \sin^2\theta}{\rho^2}\right) d\phi dt + \left(\frac{\rho^2}{\Delta}\right) dr^2 + \rho^2 d\theta^2 + \left(r^2 + \alpha^2 + \frac{2Mr\alpha^2 \sin^2\theta}{\rho^2}\right) \sin^2\theta d\phi^2 \quad (7)$$

In Eq.(7), there are two free parameters, M and J. Analytic calculations building on Eq. (7) can be difficult, usually leading to numerical techniques.

In this talk, we shall need only order-of-magnitude estimates for the rotational period τ and, in the next Section, for the angular momentum J. These will suffice to make our point about concomitant X-ray emission. The solution is axially symmetric and the radius at the pole $\theta = \frac{\pi}{2}$ is the same as the Schwarzschild radius $R = 2M$. For other values of θ the black hole radius is smaller than the static one and the rest of the static would-be sphere is filled out by an ergosphere whose equatorial radius is also $R = 2M$.

For the primordial black holes of interest, there is no reason to expect that the radiation will collapse in a spherically symmetric fashion to a static Schwarzschild black hole when the PBH formation necessarily occurs in an environment of extreme fluctuations and inhomogeneities. The black holes must be described by the Kerr metric in Eq.(7) with α having a value anything up to the maximal Kerr solution which corresponds to an equatorial speed V equal to the speed of light. The range of V is thus $0 \leq V \leq c$.

We do not know observationally any black hole which is primordial with certainty although many of the observed black holes, including those in the binary coalescences observed by LIGO, could be primordial. For illustration of black hole observations, let us consider the well-studied binary GRS1915+105 of a star and a black hole.

The black hole mass in GRS1915+105 has been established as $M \simeq 13 M_{\odot}$ and hence its Schwarzschild radius is $r_s \simeq 39$ km. Its rotation occurs $1,150$ times per second which translates to an equatorial speed $V \simeq 0.94c$, remarkably close to maximal. We mention this example to show that such high V Kerr black holes are known to exist and although we cannot derive the value of V arising from PBH formation it is to be expected that all values V up to the maximum can occur. For our present qualitative purposes, to be conservative, we employ $V = 0.1c$.

To proceed with our estimate we shall therefore take the equatorial velocity of the ergosphere to have magnitude $V = 0.1c$ and use Newtonian mechanics to estimate the rotation period τ as simply

$$\tau = \left(\frac{2\pi R}{V}\right) \tag{8}$$

For the Sun, we have $2M_\odot \simeq 3\ km$ so that for a black hole of mass $M = \eta M_\odot$ and therefore radius $R \simeq 3\eta\ km$ Eq.(8) is, for $V = 0.1c = 3 \times 10^4 km.s^{-1}$,

$$\tau = \left(2 \times 10^{-4}\pi\eta\right) \quad seconds \tag{9}$$

Astrophysical object	Mass solar masses	Period τ seconds	Angular Momentum $\mathcal{J}$ $kg.km^2.s^{-1}$
Earth	$M_\oplus = 6 \times 10^{24} kg$	24 hours	1.1×10^{27}
Sun	$M_\odot = 2 \times 10^{30} kg$	25 days	1.1×10^{36}
PIMBH	$20M_\odot$	$0.013s$	3.0×10^{37}
$\mathcal{PIMBH}$	$100M_\odot$	$0.063s$	7.2×10^{38}
PIMBH	$1000M_\odot$	$0.63s$	7.2×10^{40}
PIMBH	$10^4 M_\odot$	$6.3s$	7.2×10^{42}
PIMBH	$10^5 M_\odot$	$63s$	7.2×10^{44}
PSMBH (M87)	$6 \times 10^9 M_\odot$	$3.8 \times 10^6 s$	2.6×10^{54}

Some values of τ, estimated by this method, are shown in the third column of the Table and angular momentum J (discussed later) is in the last column.

Angular Momentum $\mathcal{J}$

Let us define the dimensionless angular momentun $\mathcal{J} \equiv J/kg.km^2.sec^{-1}$. We are interested in order of magnitude estimates of $\mathcal{J}$ for the PIMBHs and PSMBHs. The value of $\mathcal{J}$ for astrophysical objects is necessarily a large number so to set the scene we shall estimate $\mathcal{J}$ for the Earth $\mathcal{J}_\oplus$ and for the Sun $\mathcal{J}_\odot$.

The parameters for the Earth are radius $R_\oplus \simeq 6300km$, period $\tau_\oplus \simeq 86400s$, mass $M_\oplus \simeq 6 \times 10^{24}kg$, hence angular velocity $\omega_\oplus = 2\pi/\tau_\oplus$ and moment of inertia $I_\oplus = \frac{2}{5}M_\oplus R_\oplus^2$ so an estimate is $\mathcal{J}_\oplus \sim I_\oplus\omega_\oplus \simeq 1.1 \times 10^{27}$. For the Sun the similar calculation using $R_\odot \simeq 700,000km$, $\tau_\odot \simeq 25days$, $M_\odot \simeq 2 \times 10^{30}kg$ gives $\mathcal{J}_\odot \simeq 1.1 \times 10^{36}$.

For the black holes, the value of $\mathcal{J}$ is proportional to η^2 where $\eta = (M/M_\odot)$. A similar estimate to that for the Earth and Sun gives $\mathcal{J} \simeq 7.2 \times 10^{34}\eta^2$, which provides the remaining entries in the Table.

CMB Distortion

Because of rotational invariance, angular momentum is conserved. The $\mathcal{J}$ of a compact astrophysical object will not change dramatically unless there is an extremely unlikely event like a major collision. For example, the Earth and the Sun in the first two rows of our Table were formed 4.6 billion years ago. Their respective angular momenta $\mathcal{J}_{\oplus}$ and $\mathcal{J}_{\odot}$ have remained essentially constant all of that time. According to Eq.(2), the PIMBHs listed in the next five rows of our Table were all formed at time $t \leq 1s$ and their angular momenta have therefore remained roughly constant for the last 13.8 billion years since then.

In detecting the dark matter, let us focus on the special case $\mathcal{PIMBH}$ with $M = 100 M_\odot$. The $\mathcal{PIMBH}$ was formed, accordng to Eq.(2), at time $t = 10^{-3}s$ and rotates with period $t \simeq 63ms$, thus rotating ~ 16 times per second and with an absolute angular momentum $\sim 6 \times 10^{11}$ times that of the Earth and ~ 600 times that of the Sun. There is no known reason that $\mathcal{J}_{\mathcal{PIMBH}}$ would change significantly after its formation.

These remarks about angular momentum are salient to resolving the contradiction between the PIMBH dark matter proposal and the limits on halo MACHOs derived earlier by Ricotti, Ostriker and Mack (ROM) on the basis of X-ray emission and CMB distortion.

The PIMBH proposal was made that the Milky Way dark halo is a plum pudding with, as "plums", PIMBHs in the mass range of Eq.(1) making up 100% of the dark matter. On the other hand, in Figure 9 of ROM, there is displayed an upper limit of less than 0.01% of the dark matter for this mass range of MACHO. Thus, it would seem that at least one must be incorrect? The conclusion of the present talk is that ROM is correct for stellar-collapse black holes but is not applicable to a model which employs primordial black holes.

This ROM upper limit arises from the lack of any observed departure of the CMB spectrum from the predicted black-body curve or of any CMB anisotropy. ROM calculated the accretion of matter on to the MACHOs, the emission of X-rays by the accreted matter and then the downgrading of these X-rays to microwaves by cosmic expansion and more importantly by Compton scattering from electrons.

A crucial assumption made by ROM is that the accretion on to the MACHO can be modeled as if the MACHO has zero angular momentum $J = 0$. The justification for this assumption is based on earlier work by Loeb who studied the collapse of gas clouds at redshifts $200 \leq Z \leq 1400$. Such collapse can form compact objects, eventually black holes, but during the collapse angular momentum is damped out from the electrons by Compton scattering with the CMB.

From Loeb's discussion, the resultant black holes will have $J = 0$ and this appears to underly why ROM used the Bondi-Hoyle model which presumes spherical symmetry for accretion. This is justified for stellar-collapse black holes by the arguments of Loeb and therefore the upper bounds derived by ROM are applicable.

There is evidence that the Bondi-Hoyle model of accretion is not, by contrast, applicable to spinning PSMBHs, in particular the one at the centre of the large galaxy M87. In recent analyses Bondi-Hoyle was used to calculate the number of X-rays expected from the accreted material near M87. In the case of M87 the X-rays are experimentally measured. The conclusion is striking: that the measured X-rays are less by several orders of magnitude than predicted by Bondi-Hoyle theory.

This supports the idea that the SMBHs such as that in M87 are primordial, so we list PSMBH(M87) in the final row of our Table. The ROM constraints apply to black holes which originate from gravity collapse of baryonic stars. Collecting this fact, together with the ROM limit of $\leq 10^{-4}$ of the dark matter for MACHOs, implies that 99.99% of the dark matter black holes are primordial, formed during the radiation era.

Discussion

The dark matter and its explanation is a pressing problem which impacts on both high-energy physics and on cosmology. It is indisputable that over 80% of the Milky Way's mass lies in a dark approximately spherical halo surrounding the luminous more planar spiral. The results in the present Letter strongly support the model involving billions of PIMBHs.

The plum pudding model for the dark halo proposed in arose from a confluence of theoretical threads including study of the entropy of the universe and the knowledge of how to form PBHs with many solar masses as in Eqs. (1) and (5). Nevertheless it was the weakening of the argument for WIMPs which was most decisive, The strongest objection to the PIMBHs has been based on the X-rays and the CMB distortion as calculated by ROM. In the present talk we have attempted to lay this criticism to rest by noting that ROM assumed $J = 0$ and that the putative PIMBHs have not only many times the Solar mass but also many times the Solar angular momentum. This appears to us to render the ROM constraints inapplicable to the PIMBHs. On the other hand, they do apply to stellar-collapse black holes which implies that almost none ($\leq 0.01\%$) of the dark matter black holes are of that type. To decide whether dark matter really is PIMBHs will require their detection by a dedicated microlensing experiment.

Examples of PSMBHs may already have been observed in galactic cores and quasars. Other PSMBHs can play the role of dark matter in clusters and may well be detectable by other future lensing experiments. There is also the upper mass range contained in Eq.(5). Although masses of PSMBHs up to a few times $10^{10} M_{\odot}$ may have already been observed in quasars, there are what could be called Primordial Ultra Massive Black Holes (PUMBHs) with masses between 10^{11} and 10^{17} solar masses which might exist within the visible universe.

PUMBHs remain speculative but what can in the near future be examined experimentally is the existence of PIMBHs in the halo. A positive result would solve the 83-year-old problem of the dark matter and explain $\sim 26.7\%$ of the total stress-energy tensor of the visible universe. It would presumably put a stop to searches for WIMPs because the scientific community would accept that WIMPs, like low-energy supersymmetry, do not exist. Searches for axions would perhaps continue but purely within the particle physics domain with no notion that axions, if they exist, can form more than a very tiny fraction of dark matter.

The identification of the dark matter constituents as PIMBHs can revolutionize astronomy and cosmology. To give just one example, the formation of stars which takes place at redshifts $Z \leq 8$ becomes as if only a minor "afterthought" with regard to all the earlier large scale structure formation which would take place in a Universe containing *only* dark matter in the form of PIMBHs. In this sense, the result of this experiment can diminish the cosmological significance of normal matter.

CHILE

Blanco 4m telescope with DECam at Cerro Tololo.

NOAO Proposal

Submitted.
Requests ten half-nights per semester (six months) for three years.
June 15, 2017: bad news / good news

Personnel

P.H. Frampton (PI) TH
B. Carr (Co-I) TH
F. De Paolis (Co-I) EXPT
D. Minniti (Co-I) EXPT
A. Rest (Colleague) EXPT
F. Strafella (Co-I) EXPT
W. Sutherland (Co-I) EXPT

Thank you for your attention