# SUPERSYMMETRY IN DYNAMICAL M-BRANE SYSTEMS

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[arXiv:1705.09878]

❑ Our results:

❀ The supersymmetry is preserved in dynamical M2-brane background.

☞ This is similar to the result which has been obtained in 10-dimensional IIB theory (with dynamical D3-brane).

(H. Kodama & K. Uzawa, JHEP 0507 061 (2005))

## [1] Introduction

- The dynamics of geometry on the basis of unified theory.
- There is viable unified theory at present is supergravity and string theory.
- In order to study the cosmological evolution, we use the time-dependent solution in string theory.

★ Cosmological model, geometrical structure
☆ Black hole in expanding universe
◎ Supersymmetry

## membrane objects (p-brane) in gravity theory

The name "p-brane" is generally used to indicate a classical solution which is extended in p directions.
(Horowitz & Strominger, Nucl.Phys. B360 (1991) 197-209)

Then, it has p spacelike translational Killing vectors in the context of a theory containing gravity

## 👓 Time dependent solutions in gravity theory

- 4d theory

(Kastor & Traschen, Phys.Rev. D47 (1993) 5370-5375)

- Higher dimensional gravity

(Maki & Shiraishi, Class.Quant.Grav. 10 (1993) 2171-2178 )

- String theory

(Gibbons, Lu, Pope, Phys.Rev.Lett.94 (2005) 131602)
(Kodama; Uzawa, JHEP 0507 (2005) 061)
(Binetruy, Sasaki, Uzawa, Phys.Rev.D80 (2009) 026001)
(Maeda & Nozawa, Phys.Rev. D81 (2010) 044017)
(Minamitsuji & Ohta & Uzawa, Phys.Rev. D81 (2010) 126005)

✍ The dynamical D3-brane solution preserves ¼ SUSY in the conifold background.

(H. Kodama & K. Uzawa, JHEP 0507 061 (2005))

☆ Question

Do supersymmetries preserve in the dynamical M-brane background?

★Outline my talk

* Property of static M-brane

* Preserved supersymmetry

* Summary and comments

## [2] Property of static M-brane

(Duff & Stelle, Phys.Lett. B253 (1991) 113-118)
(Gibbons & Townsend, Phys. Rev. Lett., 71, 3754 (1993))
(Güven, Phys.Lett. B276 (1992) 49-55)

◆ Background

(1) The background has gravity,
4-form field strength, gravitino.
⇒ 11-dimensional supergravity

(2) Classical solutions.
ex) M2-brane, M5-brane, M-wave

# Static M2-brane solution :

(Duff & Stelle, Phys.Lett. B253 (1991) 113-118)

(1+2)-dim worldvolume spacetime

$$ds^2 = \left(1 + \frac{M}{r^6}\right)^{-2/3} \eta_{\mu\nu}(\mathrm{X}) dx^\mu dx^\nu$$

$$+ \left(1 + \frac{M}{r^6}\right)^{1/3} \left(dr^2 + r^2 d\Omega_{(7)}\right)$$

8-dim transverse space to brane

**For static background, $AdS_4 \times S^7$, the background has the full supersymmetry.**

(Freund & Rubin, Phys. Lett. B 97 (1980) 233)

# How to obtain the SUSY solutions …

## Dynamical case :

- **pp-wave**

(Matthias Blau, et al.,
JHEP 0201 (2002) 047)

(M. Sakaguchi, K. Yoshida,
JHEP 0311 (2003) 030)

- **D3-brane**

(H. Kodama & K. Uzawa,
JHEP 0507 061 (2005))

# [3] Preserved supersymmetry (11d SUGRA)

**The 11-dimensional action is invariant under local SUSY transformations :**

**$e^A{}_M$ : graviton $\Psi_M$ : gravitino,**
**$A_{MNP}$ : 3-form gauge potential**

$$\delta e^A{}_M = \bar{\varepsilon}\,\Gamma^A\,\Psi_M\,,$$

$$\delta A_{MNP} = -3\,\bar{\varepsilon}\,\Gamma_{[MN}\,\Psi_{P]}\,,$$

$$\begin{aligned}\delta\Psi_M &= D_M\,\varepsilon \\ &= \left[\nabla_M + \frac{1}{12\cdot 4!}\left(\Gamma_M\,F_{MNPQ}\Gamma^{MNPQ} - 12F_{MNPQ}\Gamma^{NPQ}\right)\right]\varepsilon\end{aligned}$$

## ★Supersymmetry in dynamical M2-brane

- The only fermionic field is the gravitino $\Psi_M$.

- Supersymmetric configuration is a nontrivial solution to the Killing spinor equation :

$$\delta\Psi_M = 0 ,$$

$$\Rightarrow \left[\nabla_M + \frac{1}{12\cdot 4!}\left(\Gamma_M F_{MNPQ}\Gamma^{MNPQ} - 12 F_{MNPQ}\Gamma^{NPQ}\right)\right]\varepsilon = 0$$

## ☆ Ansatz for fields

- 11-dim metric

(1+2)-dim worldvolume spacetime

$$ds^2 = g_{MN} dx^M dx^N = h^{-2/3}(x,r)\eta_{\mu\nu}(\mathrm{X})dx^\mu dx^\nu$$

$$+ h^{1/3}(x,r)\left(dr^2 + r^2 u_{ab}(\mathrm{Z})dz^a dz^a\right)$$

8-dim transverse space to brane

- 4-form field strength & gravitino

$$F_{r\mu\nu\rho} = \pm h^{-2}\partial_r h\, \varepsilon_{\mu\nu\rho}\,, \quad \Psi_M = 0$$

## 11-dimensional gamma matrices satisfying

$$\Gamma^M \Gamma^N + \Gamma^N \Gamma^M = 2g^{MN} \, ,$$

$$\Gamma^\mu = h^{1/3} \gamma^\mu \, , \quad \Gamma^r = h^{-1/6} \, \gamma^r \, , \quad \Gamma^a = r^{-1} h^{-1/6} \, \gamma^a$$

## and we define

$$\gamma_{(3)} = \gamma_0 \, \gamma_1 \, \gamma_2$$

## ☆ Killing spinor equation

$$\bar{\nabla}_\mu \varepsilon = \left[\partial_\mu + \frac{1}{6}\partial_\nu \ln h \gamma^\nu{}_\mu - \frac{1}{6}h^{-3/2}\partial_r h \gamma^\mu \gamma^r \left(1 \pm \gamma_{(3)}\right)\right]\varepsilon\,,$$

$$\bar{\nabla}_r \varepsilon = \left[\partial_r - \frac{1}{12}h^{-1/2}\partial_\nu h \gamma^\nu \gamma^r + \frac{1}{6}\chi h^{-1}\partial_r h \gamma_{(3)}\right]\varepsilon\,,$$

$$\bar{\nabla}_a \varepsilon = \left[{}^{Z}\nabla_a - \frac{r}{12}h^{-1/2}\partial_\nu h \gamma^\nu \gamma_a - \frac{r}{12}h^{-1}\partial_r h \gamma^r \gamma_a \left(1 \pm \gamma_{(3)}\right)\right]\varepsilon$$

- If the function h(x, r) is included in the spinor $\varepsilon = h^{-1/6}\varepsilon_0$ ( $\varepsilon_0$ : constant Killing spinor), we find …

- Solution for dynamical background

$$\partial_\mu h \gamma^\mu \varepsilon = 0 , \quad \left(1 \pm \gamma_{(3)}\right) \varepsilon = 0$$

(i) Induced effective mass for the spinor field

$$\sim h^{-1} \partial_\mu h$$

(ii) This mass scale diverges at the naked singularity where the function h vanishes.

(iii) In the region with a large warp factor, the SUSY breaking becomes negligible.

$$h(x, r) = a_\mu x^\mu + M/r^6$$

## [4] Summary and comments

(1) The dynamical M2-brane background preserved the supersymmetry.

(2) The solutions of field equations cannot give a homogeneous expansion unless supersymmetries are completely broken.

(3) The induced effective mass scale for gravitino is $h^{-1}\nabla_{\mu}h$.

(4) For dynamical M2-brane, we can set smooth initial data evolving into a curvature singularity.

(5) For dynamical brane background, how to break the supersymmetry?