Search for dS vacua and inflation in type IIA strings on rigid Calabi–Yau manifolds

Based on

S. Alexandrov, S. Ketov, YW, arXiv:1607.05293 (JHEP)

YW, S. Ketov, arXiv: 1703.08993 (to be published in PTEP)

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General Motivation

- Is it possible for us to use string theory for our Universe?
- Cosmology may give us an opportunity.
- To make this encounter a reality, string theory has to achieve
  1. Moduli Stabilization,
  2. Produce dS vacua, and
  3. Provide inflationary models.
- However, several “no-go” theorems forbid dS vacua and Inflation.

[Maldacena *et al*, (2001), Ivanov *et al*, (2001), Hertzberg *et al*, (2007), Gómez-Reino *et al*, (2009), ⋯]

General Motivation

- To avoid the "no-go" theorems, one has to include either
  1. Quantum corrections (both perturbative and non-perturbative), or
  2. Non-geometric fluxes.
- We take the 1st option by including some perturbative and non-perturbative quantum corrections in the supergravity LEEA of IIA/CY string theory with fluxes.
- Though IIA/CY strings have $N = 2$ SUSY in 4D and are not realistic, they have non trivial quantum corrections that can be explicitly calculated from first principles.

Setup

To achieve our purpose, we use

1. The $N=2$ *gauged* supergravity from type IIA/CY strings with fluxes.
    - So our model has $N=2$ vector- and hyper-multiplets representing matter in 4D.
2. Compactification on *rigid* Calabi–Yau manifolds
    - So that there are no complex moduli.
    - The universal hypermultiplet metric is a solution to the integrable 3D Toda equation.

[Ketov, (2001)]

3. Ignore back-reaction (assumption)
4. Take D-instantons with "mutually local charges" $\langle\gamma,\gamma'\rangle=0$.
    - This makes explicit calculations possible.
5. Ignore NS5-instantons in some range of g_s
    - They can contribute weakly $\sim\mathrm{e}^{-1/g_s^2}$ than D-instantons $\sim\mathrm{e}^{-1/g_s}$.

[Witten, (1985)]

Setup (remarks)

- Quantum corrections (in the presence of D-instantons) break all continuous isometries of the classical universal hypermultiplet quaternionic moduli space $\frac{SU(2,1)}{SU(2)\times U(1)}$, except a single (abelian) isometry. We gauge this isometry to obtain the gauged supergravity by using a gauge field in an abelian vector supermultiplet or a gravi-photon in $N = 2$ supergravity multiplet.
- Tadpole cancellation is achieved with NS-flux H_3, RR-fluxes F_2 and F_4 (without Roman's mass) which are the only fluxes we use.

Outline

Based on these settings, we;

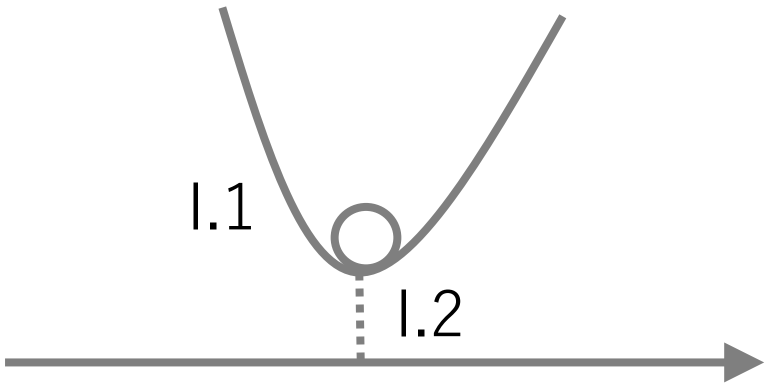
I. derive the non-perturbative scalar potential,

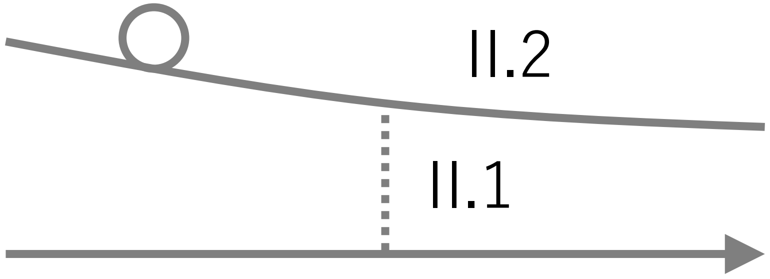
II. Search for dS vacua,

1. Achieve moduli stabilization partially, and
2. Potential positivity

III. Search for Inflation,

1. Achieve potential positivity, and
2. Compute slow-roll parameters





Non-perturbative scalar potential

The scalar potential in the gauged $N = 2$ supergravity is fixed as

$$V = 4e^{\mathcal{K}}\boldsymbol{k}_I^u\boldsymbol{k}_J^v g_{uv}X^I\bar{X}^J + e^{\mathcal{K}}\left(\mathcal{K}^{i\bar{\jmath}}D_iX^ID_{\bar{\jmath}}\bar{X}^J - 3X^I\bar{X}^J\right)\left(\vec{\mu}_I\cdot\vec{\mu}_J\right)$$

The special Kähler metric $\mathcal{K}^{i\bar{\jmath}}$ in the vector multiplet moduli space is given by the below exact prepotential

[Candelas *et al*, (1991), Hosono *et al*, (1995)]

$$F(X) = F^{\mathrm{cl}}(X) + \chi_{\mathfrak{Y}}\frac{\mathrm{i}\zeta(3)(X^0)^2}{16\pi^3} - \frac{\mathrm{i}(X^0)^2}{8\pi^3}\sum_{k_i\gamma^i\in H_2^+(\mathfrak{Y})} n_k^{(0)}\mathrm{Li}\left(\mathrm{e}^{2\pi\mathrm{i}k_iX^i/X_0}\right)$$

1-loop perturbative correction

world sheet instanton corrections

Universal hypermultiplet metric

The quatanion-Kähler metric g_{uv} in the universal hypermultiplet moduli space was found recently, as

[Alexandrov and Banerjee, (2015)]

$$\mathrm{d}s^2 = \frac{2}{r^2}\left[\left(1-\frac{2r}{\mathcal{R}^2\mathbf{U}}\right)\left((\mathrm{d}r)^2+\frac{\mathcal{R}^2}{4}\left|\mathcal{Y}\right|^2\right)\right.$$
$$\left.+\frac{1}{64}\left(1-\frac{2r}{\mathcal{R}^2\mathbf{U}}\right)^{-1}\left(\mathrm{d}\sigma+\tilde{\zeta}\mathrm{d}\zeta-\zeta\mathrm{d}\tilde{\zeta}+\mathcal{V}_{(\sigma)}\right)^2\right]$$

where $r = \mathrm{e}^{\phi} = g_s^{-2}$. It includes the classical contributions, all perturbative contributions (no α'-correction, the 1-loop g_s-correction is exact), and all D-instanton corrections.

Axion stabilization

Given the flux charges satisfying a relation

$$e_0 = \left(n\tilde{h} - \ell^i e_i\right)/2, \quad n, \ell^i \in \mathbb{Z}$$

We found that there is a stable solution for the **axions** due to D-instantons $\zeta = n/2, \quad b^i = \ell^i/2$ with integers n and ℓ^i i.e.

$$\partial_{\psi^I} V\Big|_{\substack{\zeta=n/2 \\ b^i=\ell^i/2}} = 0, \quad \psi^I = (\zeta, b^i).$$

The remaining subspace is $\phi^I = (r, t^i)$.

$$\partial\partial V = \begin{pmatrix} \partial_{\phi^I}\partial_{\phi^J} V & 0 \\ 0 & \partial_{\psi^I}\partial_{\psi^J} V \end{pmatrix}$$

this sign can be adjusted.

Effective Potential (after Axions stabilization)

The effective potential is

$$V^{(\phi)}(r,t^i) \equiv V\big|_{\substack{\zeta=n/2 \\ b^i=\ell^i/2}} = \frac{e^{\mathcal{K}}}{4r^2}\left[\frac{4r(et)^2}{\mathcal{R}^2 M - 2r} - e^{-\mathcal{K}}\hat{N}^{ij}e_i e_j + \frac{4\tilde{h}^2\mathcal{R}^2}{e^{\mathcal{K}}N_{ij}t^i t^j - 1} - \frac{16\tilde{h}^2 r}{M}\right]$$

We study the equations

$$\partial_r V^{(\phi)} = 0, \quad \partial_{t^i} V^{(\phi)} = 0$$

both *analytically* and *numerically* (by Mathematica) on below strategy;

1. first by neglecting all non-perturbative corrections
2. then adding worldsheet and D-brane instanton corrections.

Comments

There are some physical restrictions.

For t^i,

1. $e^{-\mathcal{K}} > 0$ (from the condition that t^i belong to the Kähler cone)
2. $\mathrm{Im}\,\mathcal{N}_{IJ} < 0$ (from the definition of $\mathcal{N}_{IJ}$)

For r,

1. $$r > r_{\rm cr}\,, \quad r_{\rm cr} = -2c, \quad c = -\frac{\chi_{\mathfrak{Y}}}{192\pi}$$
   (to avoid the curvature singularity because of the lack of the NS5-brane instanton corrections)

Perturbative approximation (1-loop g_s-correction $\sim\chi_{\mathrm{CY}}$ is included)

Potential is simplified;

$$V^{(\phi)} \approx \frac{e^{\mathcal{K}}}{8r^2}\left[\frac{16\tilde{h}^2}{\lambda_2}\frac{(1-\gamma)r+2c}{1+\gamma}+\frac{4r(et)^2}{r+2c}-e^{-\mathcal{K}}\kappa^{ij}e_ie_j\right]$$

$$\gamma = \frac{3\chi_{\mathfrak{Y}}}{4\pi^3}\zeta(3)e^{\mathcal{K}}$$

We find

1. One modulus case (analytic) → has critical points, but not local minima
2. Two modulus case (numeric, difficult to treat analytically) → no local minima (by 3)
3. Generic case (numeric) → no local minima (below)

Perturbative approximation (in generic case i.e. for all $h_{1,1}$)

Rotate ∂_{t^i} by $\quad \mathbf{m}_1^j = t^j, \quad \mathbf{m}_2^j = n^j \equiv \frac{\kappa^{jk} e_k}{e^{\mathcal{K}}(et)}, \quad \mathbf{m}_3^j = \cdots$

$\mathbf{M}^{(1)}$ $\mathbf{M}^{(2)}$ $\mathbf{M}^{(3)}$

$$\mathbf{M} = \begin{pmatrix} \partial_r^2 V^{(\phi)} & t^i \partial_{t^i} \partial_r V^{(\phi)} & n^i \partial_{t^i} \partial_r V^{(\phi)} & \\ t^i \partial_{t^i} \partial_r V^{(\phi)} & t^i t^j \partial_{t^i} \partial_{t^j} V^{(\phi)} & n^i t^j \partial_{t^i} \partial_{t^j} V^{(\phi)} & \cdots \\ n^i \partial_{t^i} \partial_r V^{(\phi)} & n^i t^j \partial_{t^i} \partial_{t^j} V^{(\phi)} & n^i n^j \partial_{t^i} \partial_{t^j} V^{(\phi)} & \\ & \vdots & & \ddots \end{pmatrix}$$

From Sylvester's criterion;

$$\mathbf{M} \text{ is positive definite}$$

$$\Longrightarrow \Delta_k \equiv \det \mathbf{M}^{(k)} > 0 \quad (k = 1, 2, 3)$$

Perturbative approximation (in generic case i.e. for all $h_{1,1}$)

- Δ_k are expressed by r and γ only on $\partial V = 0$.
- All parameters λ_2, κ_{ijk}, the fluxes e_i and $\tilde{h}$ do not change the signs of the leading principal minors Δ_k.

⬇

Three minors can not be positive in any physical regions in γ-r plane.

⬇

The matrix $\mathbf{M}$ *cannot* be positive definite, and the perturbative potential *cannot* have local minima for any number of Kähler moduli.

The Instanton contributions

We studied the D-instanton corrections in one modulus case only, by simplifying the equations;

$$\begin{cases} \partial_{\mathcal{R}} V^{(\phi)} = 0 \\ \partial_{t^i} V^{(\phi)} = 0 \end{cases} \Longrightarrow \begin{cases} \mathscr{Q}(t,\mathcal{R}) = 0 \\ (et)^2 = \tilde{h}^2 \mathscr{E}(t,\mathcal{R}) \end{cases} \quad \text{and} \quad \partial\partial V \Longrightarrow \tilde{h}^2 \begin{pmatrix} \Phi_{IJ}(t,\mathcal{R}) & 0 \\ 0 & \Psi_{IJ}(t,\mathcal{R}) \end{pmatrix}$$

And demanding

$$\mathscr{Q}(t,\mathcal{R}) = 0, \quad \mathscr{E}(t,\mathcal{R}) > 0, \quad \Psi_{IJ}(t,\mathcal{R}) > 0, \quad \Phi_{IJ}(t,\mathcal{R}) > 0$$

with the Gopakumar–Vafa invariants $n_k^{(0)}$ chosen *randomly*.

The result also do not lead to appearance of dS vacua.

Search for Inflationary solutions

- We searched for inflation with the perturbative scalar potential in one modulus case.

⬇

- We found the constraints on the flux charges which make the potential positive. ☺
- However, the slow-roll parameters for the canonically normalized fields $t = e^{\sqrt{1/6}\chi}$ and $\mathrm{r} = e^{\sqrt{2}\varphi}$ are found to be

$$\epsilon > \frac{13}{6} \quad \text{and} \quad \eta > 2. \quad ☹$$

So our potential does not allow slow roll inflation.

Conclusion

- There is *a gap* in the literature for a possible existence of dS vacua and inflation in IIA/CY strings with fluxes, with the *simultaneous* presence of both $N = 2$ vector- and hyper-multiplets.
- We closed this gap in the case of *rigid* CY.
- Thus, we extended the existing "no-go" theorems to the case when the perturbative and D-instanton corrections are taken into account.
- The case with NS5-instanton corrections remains open.

Thank you for your attention.