The UV structure of Hořava gravity

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A. Barvinsky, D. Blas, M. H-V., S. Sibiryakov and C. Steinwachs
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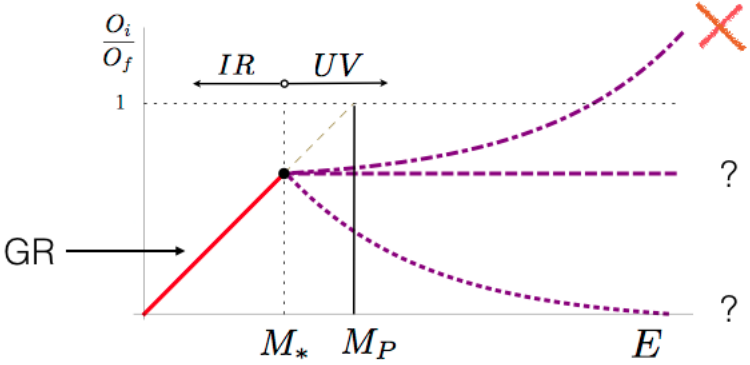
Can gravity be formulated as a pQFT?



[Figure by D. Blas]

A simple question

GR, $M_p^2\sqrt{g}R$ about a Minkowsky vacuum

$$\mathcal{L} \sim h^{\mu\nu}\frac{\mathcal{P}^{\mu\nu\alpha\beta}}{p^2}h^{\alpha\beta} + \frac{p^2}{M_p}O(h^3) + \frac{p^2}{M_p^2}O(h^4) + \ldots$$

The theory is non-renormalizable $\equiv$ infinite number of divergent diagrams

[G. 't Hooft and M. J. G. Veltman, Ann. Inst. H. Poincare Phys. Theor. A 20, 69 (1974)]

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Higher derivative gravity

$$M_p^2 R + \alpha R^2 + \beta R_{\mu\nu}R^{\mu\nu} \rightarrow \langle hh\rangle \sim \frac{M_P^2}{p^4 + p^2 M_p^2}$$

[K. S. Stelle, Phys. Rev. D 16, 953 (1977)]

However, it has a ghost

$$\langle hh\rangle \sim \frac{M_P^2}{p^4 + p^2 M_p^2} = \frac{1}{p^2} - \frac{1}{p^2 + M_p^2}$$

[Ostrogradski, M., Mem. Ac. St. Petersbourg, VI, 1850, 385]

The problem can be solved by allowing for a breaking of the boost invariance of the Lorentz group

26 Jan 2009

Quantum Gravity at a Lifshitz Point

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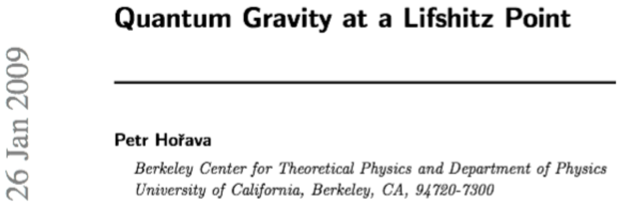

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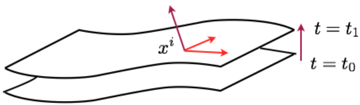
$$\langle hh \rangle \sim \frac{1}{\omega^2 + k^{2d} + c^2 k^2}$$

Anisotropic scaling $t \to b^{d} t$, $x^i \to b x^i$

$$S \sim h_{ij} \left(\partial_t^2 + \partial_i^{2d} + \dots \right) h_{ab} + O(h^3)$$

The non-linear theory



ADM formulation

$$ds^2 = N^2 dt^2 + \gamma_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$$

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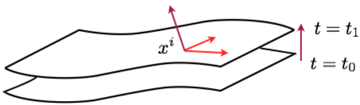

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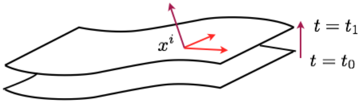

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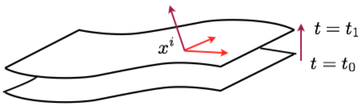

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$$S = \frac{1}{2G}\int dt d^d x\, N\sqrt{\gamma}\big(\underbrace{K_{ij}K^{ij} - \lambda K^2}_{\partial_t^2} + \underbrace{\mathcal{V}}_{\partial_i^k,\quad k\leq d}\big)$$

$$\mathcal{V} \sim \left\{R^3, \quad (a_i a^i)^3, \quad \Delta^2 R, \quad a_i a_j \Delta R^{ij}, \quad R^2, \quad R_{ijab}R^{ijab}, \quad R, \quad a_i a^i, \quad \partial_i a^i, \quad ...\right\}$$

The theory can flow to GR if $\lambda \to 1$

To be (projectable) or not to be

The role of N is particular, since there are no time derivatives of N

- Projectable version

$$N = N(t), \qquad a_i = 0$$

  Fixes time reparametrizarion invariance. It contains less terms in $\mathcal{V}$ but phenomenology is not clear. There is a mode which undergoes strong coupling in the IR.

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- Non-projectable version

$$N = N(t, x)$$

  Quantization seems more complicated due to $t \to t'(t)$.
  Passes all phenomenological tests. Phenomenology is richer.

[D. Blas, O. Pujolas and S. Sibiryakov, Phys. Rev. Lett. 104, 181302 (2010)]

What now?

- Many questions have been answered in the first burst of works (before 2016)
  - Power-counting renormalizability
  - Classical solutions
  - Black holes
  - Cosmology
  - Is the theory renormalizable (preservation of the gauge invariance to all orders in loop expansion)??

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Compute the renormalization group flow!
(in the projectable case :))

We can study the UV structure in the projectable case in $2+1$ dimensions

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$$F_i F^i$$

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We need to use BRST inspired gauges

$$F_i \frac{1}{\Delta\gamma_{ij} + \chi\partial_i\partial_j} F_j \sim \pi_i(\Delta\gamma_{ij} + \chi\partial_i\partial_j)\pi^j + \pi_i F^i$$

gives regular propagators

$$\langle \ldots \rangle \sim \frac{1}{\omega^2 + p^4}$$

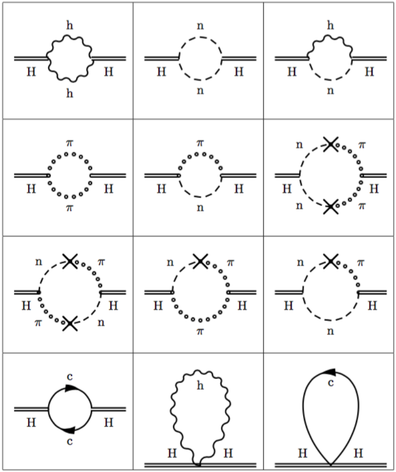
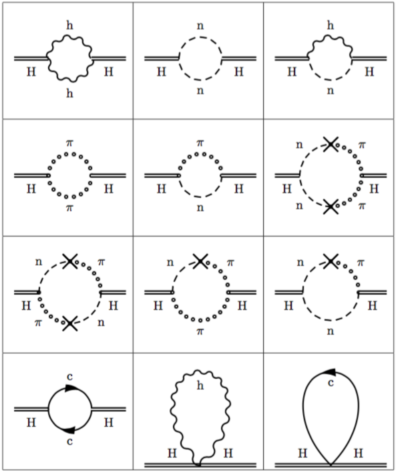


Figure 1. Feynman diagrams (bubbles and fishes) for the two point function of H_{ij}. The cross represents the mixed propagator $\langle n^i \pi^j \rangle$.

The vertices generically contain ~ 100 terms due to multi-symmetries

We use *xAct* and *FORM*



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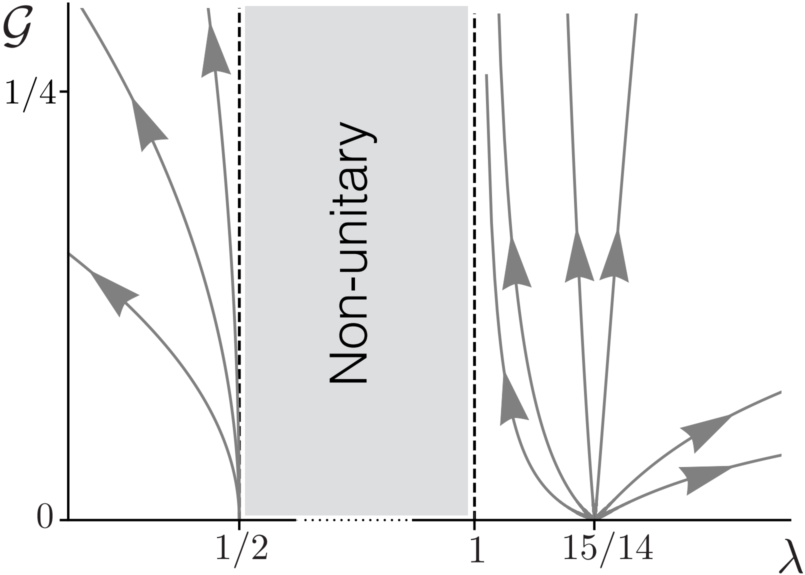
$$\beta(\lambda) = \frac{15 - 14\lambda}{64\pi} \sqrt{\frac{1-2\lambda}{1-\lambda}}\, \mathcal{G}$$

$$\beta(\mathcal{G}) = -\frac{16 - 33\lambda + 18\lambda^2}{64\pi(1-\lambda)^2} \sqrt{\frac{1-\lambda}{1-2\lambda}}\, \mathcal{G}^2$$


$\mathcal{G}$
1/4
0
Non-unitary
1/2
1
15/14
λ

In presence of matter

We add s scalar fields and we study the large s limit. We have three essential couplings λ, $\mathcal{G}$, g_1

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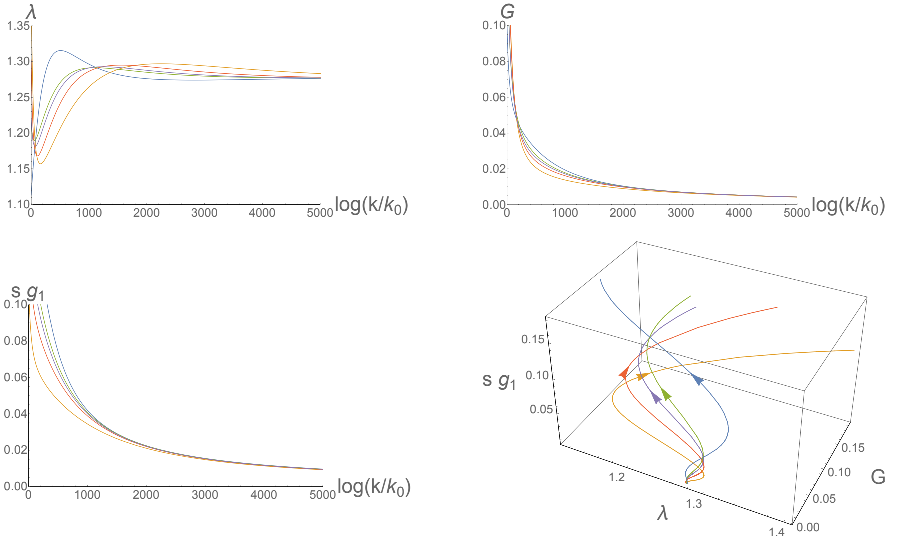

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Projectable Hořava gravity in $2+1$ dimensions is a UV *complete* theory of Quantum Gravity.

CERN-TH-2017-100, INR-TH-2017-011, FR-PHENO-2017-015

Hořava gravity is asymptotically free (in 2+1 dimensions)

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[3] *Theoretical Physics Department, CERN, CH-1211 Geneva 23, Switzerland*
[4] *Institute of Physics, LPPC, Ecole Polytechnique Fédérale de Lausanne, CH-1015, Lausanne, Switzerland*
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We compute the β-functions of marginal couplings in projectable Hořava gravity in 2+1 spacetime dimensions. We show that the renormalization group flow has an asymptotically-free fixed point in the ultraviolet (UV), establishing the theory as a UV-complete model with dynamical gravitational degrees of freedom. Therefore, this theory may serve as a toy-model to study fundamental aspects of quantum gravity. Our results represent a step forward towards understanding the UV properties of realistic versions of Hořava gravity.