# Special Grand Unification

Naoki Yamatsu
Maskawa Institute for Science and Culture
Kyoto Sangyo University

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This talk is based on arXiv:1704.08827 [1].

# Purpose of this talk

**I will propose a new-type grand unified theory (GUT) based on an $SU(16)$ GUT gauge group broken to its $SO(10)$ special subgroup.**

- 4D gauge anomaly cancellation restricts the minimal number of generations of the 4D SM Weyl fermions.

- In an SU(16) GUT on 6D orbifold space, three generations are allowed by the 6D and 4D gauge anomaly cancellation on the bulk and fixed points without exotic 4D chiral fermions.

# Purpose of this talk

**I will propose a new-type grand unified theory (GUT) based on an $SU(16)$ GUT gauge group broken to its $SO(10)$ special subgroup.**

Content of this talk:

① What is a **special subgroup**?

② 4D anomaly cancellation

③ 6D $SU(16)$ special GUT

# Motivation for Grand Unification [2, 3, R.Slansky'81;...]

The idea of grand unification has attractive features; e.g.,

- Unification of the SM gauge bosons
- Unification of the SM Weyl fermions
- 4D SM gauge anomaly cancellation
- Charge quantization for quarks and leptons

...

# GUT gauge groups (broken to regular subgroups)

4D GUTs based on GUT gauge groups e.g.,

$SU(5)$ [4, H.Georgi,S.L.Glashow'74], $SU(6)$ [5, K.Inoue,A.Kakuto,Y.Nakano'77], $SO(10)$ [6, H.Fritzsch,P.Minkowski'75], $E_6$ [7, F.Gursey,P.Ramond,P.Sikivie'76].

5D GUTs based on GUT gauge groups e.g.,

$SU(5)$ [8,9, K.Kojima et al.'11], $SU(6)$ [10,11, G.Burdman,Y.Nomura'03], $SO(10)$ [12,13, H.D.Kim,S.Raby'03;...], $E_6$ [14,15, Y.Kawamura,T.Miura'13], $SO(11)$ [16–20, Y.Hosotani,N.Yamatsu'15;...].

Usually, GUT gauge groups are broken to regular subgroups; e.g.

$$E_6 \supset SO(10) \supset SU(5) \supset G_{\rm SM}.$$

However, there are other subgroups: **special subgroup**.

# Regular and special subgroups [2, 3, 21–23, E.Dynkin'57;...]

Lie group $SU(3)$

**Regular subgroup**

$SU(2) \times U(1)$

Branching rules

$$\mathbf{3} = (\mathbf{2})(1) \oplus (\mathbf{1})(-2)$$

$$\overline{\mathbf{3}} = (\mathbf{2})(-1) \oplus (\mathbf{1})(2)$$

$$\mathbf{8} = (\mathbf{3})(0) \oplus (\mathbf{2})(3) \oplus (\mathbf{2})(-3) \oplus (\mathbf{1})(0)$$

**Special subgroup**

$SU(2)$

Branching rules

$$\mathbf{3} = (\mathbf{3})$$

$$\overline{\mathbf{3}} = (\mathbf{3})$$

$$\mathbf{8} = (\mathbf{5}) \oplus (\mathbf{3})$$

# Motivation for Special GUTs [1, N.Yamatsu'17]

There are several features of special GUTs. E.g.,

- Special embeddings reduce rank. E.g, $SU(16) \supset SO(10)$, $SU(27) \supset E_6$, $USp(56) \supset E_7$, $SO(248) \supset E_8$,
- Almost all unnecessary $U(1)$s are eliminated. E.g., $SO(32) \supset SU(16) \times U(1) \supset SO(10) \times U(1) \supset G_{\rm SM} \times U(1) \times U(1)$.
- For $SO(32) \supset SU(16) \times U(1) \supset SO(10) \times U(1)$, $SO(32)$ vector representation $\mathbf{32}$ is decomposed into $SO(10)$ spinor representations $\mathbf{16}$ and $\overline{\mathbf{16}}$.

# Special embedding $SU(16) \supset SO(10)$ [3, N.Yamatsu'15]

Lie group $SU(16)$

**Regular subgroup**

$SU(15) \times U(1)$

Branching rules

$$\mathbf{16} = (\mathbf{15})(1) \oplus (\mathbf{1})(-15)$$
$$\overline{\mathbf{16}} = (\overline{\mathbf{15}})(-1) \oplus (\mathbf{1})(15)$$
$$\mathbf{255} = (\mathbf{224})(0) \oplus (\mathbf{15})(16) \oplus (\overline{\mathbf{15}})(-16) \oplus (\mathbf{1})(0)$$

**Special subgroup**

$SO(10)$

Branching rules

$$\mathbf{16} = (\mathbf{16})$$
$$\overline{\mathbf{16}} = (\overline{\mathbf{16}})$$
$$\mathbf{255} = (\mathbf{210}) \oplus (\mathbf{45})$$

# Branching rules of $SU(16) \supset SO(10)$ [3, N.Yamatsu'15]

$$\mathbf{16} = (\mathbf{16}), \quad \overline{\mathbf{16}} = (\overline{\mathbf{16}})$$

$$\mathbf{120} = (\mathbf{120}), \quad \overline{\mathbf{120}} = (\mathbf{120})$$

$$\mathbf{255} = (\mathbf{210}) \oplus (\mathbf{45})$$

$$\mathbf{5440} = (\mathbf{4125}) \oplus (\overline{\mathbf{1050}}) \oplus (\mathbf{54}) \oplus (\mathbf{210}) \oplus (\mathbf{1})$$

- $\mathbf{16}$ of $SU(16)$ can be identified with the SM Weyl fermions.
- $\mathbf{120}$ and $\overline{\mathbf{120}}$ of $SU(16)$ is real under $SO(10)$ subgroup.
- $\mathbf{5440}$ of $SU(16)$ can break $SU(16)$ to $SO(10)$.

# 4D $SU(n)$ chiral gauge theory [3, e.g.,N.Yamatsu'15]

- A 4D Weyl fermion in $(\mathbf{10} \oplus \overline{\mathbf{5}})$ of $SU(5)$ has zero 4D anomaly coefficient: $A(\mathbf{10} \oplus \overline{\mathbf{5}}) = 0$.

- A 4D Weyl fermion in $(\mathbf{15} \oplus 2 \times \overline{\mathbf{6}})$ of $SU(6)$ has $A(R) = 0$

- In general, a 4D Weyl fermion in $\left(\frac{\mathbf{n(n-1)}}{\mathbf{2}} \oplus (n-4) \times \overline{\mathbf{n}}\right)$ of $SU(n)$ has $A(R) = 0$.

**A 4D Weyl fermion in $\left(\mathbf{120} \oplus 12 \times \overline{\mathbf{16}}\right)$ of $SU(16)$ has zero anomaly coefficient: $A\left(\mathbf{120} \oplus 12 \times \overline{\mathbf{16}}\right) = 0$.**

# 4D $SU(16)$ special GUT [1, N.Yamatsu'17]

- A 4D Weyl fermion in $\left(12 \times \mathbf{16} \oplus \overline{\mathbf{120}}\right)$ of $SU(16)$ is OK from the viewpoint of 4D anomaly cancellation.

- However, once $SU(16)$ is broken to $SO(10)$, the number of generations of the SM chiral fermions is 12.

- Fortunately, since $\overline{\mathbf{120}}$ of $SU(16)$ is real under the $SO(10)$ subgroup, a 4D Weyl fermion in $\mathbf{120}$ of $SO(10)$ is not chiral.

**It seems impossible to realize three generations of the SM fermions in 4D. How about higher dimensions?**

# 6D $SU(16)$ special GUT [1, N.Yamatsu'17]

- We consider 6D orbifold spacetime $M^4 \times T^2/\mathbb{Z}_2$:

$$ds^2 = e^{-2\sigma(y)}(\eta_{\mu\nu}dx^\mu dx^\nu + dv^2) + dy^2.$$

- The $\mathbb{Z}_2$ parity reflection around each fixed point $(j = 1, 2, 3, 4)$ is

$$P_j : \ (x_\mu, y_j + y, v_j + v) \ \to \ (x_\mu, y_j - y, v_j - y).$$

- Orbifold BCs for e.g., a 6D Weyl fermion in $\mathbf{16}$ of $SU(16)$ are given by [24, Y.Hosotani,N.Yamatsu'17]

$$\Psi_{\mathbf{16}\pm}(x, y_j - y, v_j - v) = \eta_j(-i\Gamma^5\Gamma^6)P_{j\mathbf{16}}\Psi_{\mathbf{16}\pm}(x, y_j + y, v_j + v).$$

# Symmetry breaking pattern [1, N.Yamatsu'17]

$SU(16) \to G_{\rm SM}$ can be realized e.g., by the Higgs mechanism;

$$
\begin{aligned}
SU(16) \quad &\underset{\rm BCs}{\longrightarrow} \quad SU(16) \\
&\underset{\langle\phi_{\mathbf{5440}}\rangle\neq 0}{\longrightarrow} \quad SO(10) \\
&\underset{\langle\phi_{\mathbf{16}}\rangle\neq 0}{\longrightarrow} \quad SU(5) \\
&\underset{\langle\phi_{\mathbf{255}}\rangle\neq 0}{\longrightarrow} \quad SU(3)\times SU(2)\times U(1).
\end{aligned}
$$

Note that the nonvanishing VEVs of scalars in $\mathbf{5440}$, $\mathbf{16}$, $\mathbf{255}$ of $SU(16)$ are responsible for breaking $SU(16)$ to $G_{\rm SM}$.

# 4D SM Weyl fermions [1, N.Yamatsu'17]

| 6D Bulk field | $\Psi^{(a)}_{\mathbf{16}+}$ | $\Psi^{(b)}_{\mathbf{16}+}$ | $\Psi^{(c)}_{\mathbf{16}-}$ | $\Psi^{(d)}_{\mathbf{16}-}$ |
|---|---|---|---|---|
| $SU(16)$ | $\mathbf{16}$ | $\mathbf{16}$ | $\mathbf{16}$ | $\mathbf{16}$ |
| $SO(5,1)$ | $\mathbf{4}_+$ | $\mathbf{4}_+$ | $\mathbf{4}_-$ | $\mathbf{4}_-$ |
| Orbifold BC | $\begin{pmatrix} - & - \\ - & - \end{pmatrix}$ | $\begin{pmatrix} + & + \\ - & - \end{pmatrix}$ | $\begin{pmatrix} + & - \\ - & + \end{pmatrix}$ | $\begin{pmatrix} - & + \\ - & + \end{pmatrix}$ |

- We can identify the zero modes of 6D positive Weyl fermions $\Psi^{(a)}_{\mathbf{16}+}$ $(a = 1, 2, 3)$ with three generations of the SM fermions.
- 6D anomaly in the bulk is canceled because of vectorlike.
- 4D anomaly on a fixed point is not canceled by 6D bulk fermions.

# 4D gauge anomaly cancellation [1, N.Yamatsu'17]

| 4D Brane field | $\psi_{\overline{\mathbf{120}}}$ |
|---|---|
| $SU(16)$ | $\overline{\mathbf{120}}$ |
| $SL(2,\mathbb{C})$ | $(1/2,0)$ |
| Spacetime $(y,v)$ | $(0,0)$ |

- We introduce a 4D Weyl brane fermion in $\overline{\mathbf{120}}$ of $SU(16)$.
- The brane fermion cancels 4D anomaly generated by 6D bulk fermions at the fixed point $(y,v)=(0,0)$.
- The 4D $SU(16)$ $\overline{\mathbf{120}}$ Weyl brane fermion is not chiral at the ~~$SU(16)$~~ vacuum because $\mathbf{120}$ of $SO(10)$ is real.

# Matter content in the special GUT [1, N.Yamatsu'17]

| 6D Bulk field | $A_M$ | $\Psi^{(a)}_{\mathbf{16}+}$ | $\Psi^{(b)}_{\mathbf{16}+}$ | $\Psi^{(c)}_{\mathbf{16}-}$ | $\Psi^{(d)}_{\mathbf{16}-}$ |
|---|---|---|---|---|---|
| $SU(16)$ | $\mathbf{255}$ | $\mathbf{16}$ | $\mathbf{16}$ | $\mathbf{16}$ | $\mathbf{16}$ |
| $SO(5,1)$ | $\mathbf{6}$ | $\mathbf{4}_+$ | $\mathbf{4}_+$ | $\mathbf{4}_-$ | $\mathbf{4}_-$ |
| Orbifold BC | | $\begin{pmatrix} - & - \\ - & - \end{pmatrix}$ | $\begin{pmatrix} + & + \\ - & - \end{pmatrix}$ | $\begin{pmatrix} + & - \\ - & + \end{pmatrix}$ | $\begin{pmatrix} - & + \\ - & + \end{pmatrix}$ |

| 5D Brane field | $\Phi_{\mathbf{5440}}$ | $\Phi_{\mathbf{255}}$ | $\Phi_{\mathbf{16}}$ |
|---|---|---|---|
| $SU(16)$ | $\mathbf{5440}$ | $\mathbf{255}$ | $\mathbf{16}$ |
| $SO(4,1)$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{1}$ |
| Orbifold BC | $\begin{pmatrix} + \\ + \end{pmatrix}$ | $\begin{pmatrix} + \\ + \end{pmatrix}$ | $\begin{pmatrix} + \\ + \end{pmatrix}$ |
| Spacetime | $y=0$ | $y=0$ | $y=0$ |

| 4D Brane field | $\psi_{\overline{\mathbf{120}}}$ |
|---|---|
| $SU(16)$ | $\overline{\mathbf{120}}$ |
| $SL(2,\mathbb{C})$ | $(1/2,0)$ |
| Spacetime $(y,v)$ | $(0,0)$ |

# Summary

**I proposed 6D $SU(16)$ special GUT on $M^4 \times T^2/\mathbb{Z}_2$ by using a special embedding $SU(16) \supset SO(10)$.**

Results

- Three generations of the SM fermions can be identified with the zero modes of 6D positive Weyl fermions $\Psi^{(a)}_{\mathbf{16}+}$ $(a = 1, 2, 3)$.
- 6D and 4D gauge anomalies on the bulk and fixed points are canceled by 6D bulk and 4D brane Weyl fermions.
- There are no 4D exotic chiral fermions.

# Comments

- Special embeddings such as $SU(10) \supset SU(5)$, $SU(15) \supset SU(6)$, $SU\left(\frac{n(n-1)}{2}\right) \supset SU(n)$, $SU\left(\frac{n(n+1)}{2}\right) \supset SU(n)$, $(n \geq 8)$ seem useless to construct special GUTs because the SM fermions cannot be embedded into their lower dimensional representations.

- For $SO(32) \supset SU(16) \times U(1) \supset SO(10) \times U(1)$, an $SO(32)$ vector representation $\mathbf{32}$ is decomposed into $SO(10)$ spinor representations $\mathbf{16}$ and $\overline{\mathbf{16}}$.

- A 4D $SU(27) \supset E_6$ special GUT has $252$ generations from the constraints of anomaly and absence of non-SM chiral fermions.

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