

HIGGS MASS PREDICTION FROM THE COSMOLOGICAL CONSTANT RUNNING

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COSMOLOGICAL CONSTANT PROBLEM

Hilbert-Einstein action (neglecting higher order R- and derivative terms):

$$S_{HE} = \int d^4x \sqrt{-g} \left\{ \mathcal{L}_{SM} + \xi \varphi^\dagger \varphi R - \frac{1}{16\pi G} (R + 2\Lambda_{vac}) \right\}$$

Vacuum energy density

$$\rho_\Lambda^{vac}(\mu) \equiv \frac{\Lambda(\mu)}{8\pi G(\mu)}$$

Induced vacuum energy density

$$V_0 = -\frac{1}{2}m^2\phi^2 + \frac{1}{8}\lambda\phi^4$$

$$\rho_{ind}(\mu) \equiv V_0(\langle\phi\rangle) = -\frac{m^4(\mu)}{2\lambda(\mu)} \quad \text{Higgs condensate contribution}$$

$$\rho_{ind}(\mu = 246 \text{ GeV}) \sim 10^8 \text{ GeV}^4$$

$$\rho_{phys} = \rho_\Lambda^{vac}(\mu_c) + \rho_{ind}(\mu_c) + \dots = 10^{-47} \text{ GeV}^4$$

Today $\mu_c = \mathcal{O}(10^{-3}) \text{ eV}$



ONE NEEDS TO ACCOUNT FOR RUNNING AND FINE TUNNING (10^{55})

DECOUPLING EFFECTS

At low energies & for $m_{light} \ll \mu \ll m \rightarrow$ expectations from the dimensional analysis and the decoupling theorem (Appelquist-Carazzone):

$$\beta\left(m_{light}, \frac{\mu}{m}\right) = a m_{light}^4 + b \left(\frac{\mu}{m}\right)^2 m^4 + c \left(\frac{\mu}{m}\right)^4 m^4 + d \mu^2 R + \dots$$
$$= \mu^2 m^2 \quad \text{[it could be potentially dangerous for the CC running for large masses } m \text{]}$$

To account properly for the decoupling effects one needed to go to
the mass-dependent renormalization schemes

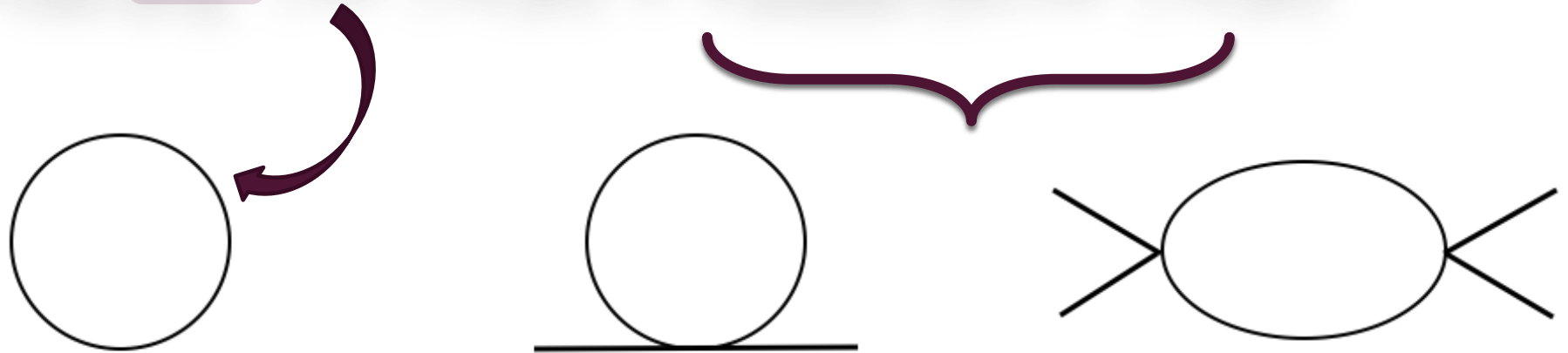
(decoupling does not hold in the mass-independent schemes like MS)

TOY MODEL – REAL SCALAR FIELD

$$L = \frac{1}{2}m^2\phi^2 + \frac{1}{8}\lambda\phi^4$$

One-loop effective potential:

$$V = V_{vac}(\rho_{\Lambda}^{\text{vac}}, m^2, \lambda, \mu) + V_{\text{scalar}}(\phi, m^2, \lambda, \mu)$$

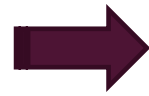
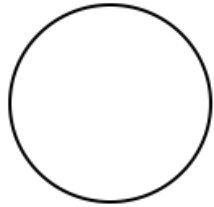


Separate pieces satisfy separate RGE equations: $\frac{dV(\langle\phi\rangle)}{d\mu} = 0$

TOY MODEL – MASS INDEPENDENT SCHEME (MS)

$$\left(\mu \frac{\partial}{\partial \mu} + \beta_\lambda \frac{\partial}{\partial \lambda} + \gamma_m m^2 \frac{\partial}{\partial m^2} + \beta_{\rho_\Lambda^{\text{vac}}} \frac{\partial}{\partial \rho_\Lambda^{\text{vac}}} \right) V_{\text{vac}}(m^2, \lambda, \rho_\Lambda^{\text{vac}}, \mu) = 0$$

vacuum part:



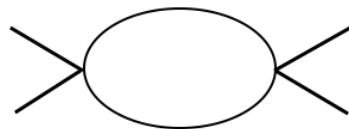
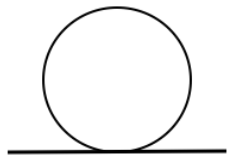
$$\mu \frac{\partial \rho_\Lambda^{\text{vac}}}{\partial \mu} \equiv \beta_{\rho_\Lambda^{\text{vac}}} = \frac{m^4}{32\pi^2}$$

\overline{MS}



$$\left(\mu \frac{\partial}{\partial \mu} + \beta_\lambda \frac{\partial}{\partial \lambda} + \gamma_m m^2 \frac{\partial}{\partial m^2} \right) V_{\text{scal}}(\langle \phi \rangle, m^2, \lambda, \mu) = 0$$

induced part:



\overline{MS}

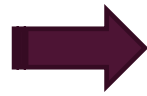
$$\mu \frac{\partial \rho_{\text{ind}}(\mu)}{\partial \mu} = \mu \frac{\partial}{\partial \mu} \left(-\frac{m^4(\mu)}{2\lambda(\mu)} \right) = \rho_{\text{ind}}(\mu) \left(2\frac{\beta_{m^2}}{m^2} - \frac{\beta_\lambda}{\lambda} \right)$$

valid **ONLY** in the UV regime ! $\mu \gg m$

TOY MODEL – MASS INDEPENDENT SCHEME (MS)

$$\left(\mu \frac{\partial}{\partial \mu} + \beta_\lambda \frac{\partial}{\partial \lambda} + \gamma_m m^2 \frac{\partial}{\partial m^2} + \beta_{\rho_\Lambda^{\text{vac}}} \frac{\partial}{\partial \rho_\Lambda^{\text{vac}}} \right) V_{\text{vac}}(m^2, \lambda, \rho_\Lambda^{\text{vac}}, \mu) = 0$$

vacuum part:



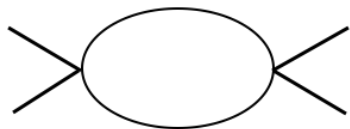
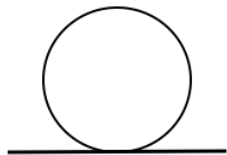
$$\mu \frac{\partial \rho_\Lambda^{\text{vac}}}{\partial \mu} \equiv \beta_{\rho_\Lambda^{\text{vac}}} = \frac{m^4}{32\pi^2}$$

\overline{MS}

HOW TO ACCOUNT FOR DECOUPLING OF MASSIVE PARTICLES FOR $m \ll \mu$??

$$\left(\mu \frac{\partial}{\partial \mu} + \beta_\lambda \frac{\partial}{\partial \lambda} + \gamma_m m^2 \frac{\partial}{\partial m^2} \right) V_{\text{scal}}(\langle \phi \rangle, m^2, \lambda, \mu) = 0$$

induced part:



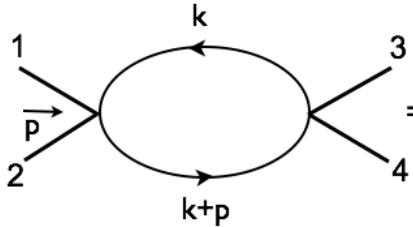
\overline{MS}

$$\mu \frac{\partial \rho_{\text{ind}}(\mu)}{\partial \mu} = \mu \frac{\partial}{\partial \mu} \left(-\frac{m^4(\mu)}{2\lambda(\mu)} \right) = \rho_{\text{ind}}(\mu) \left(2\frac{\beta_{m^2}}{m^2} - \frac{\beta_\lambda}{\lambda} \right)$$

valid ONLY in the UV regime ! $\mu \gg m$

TOY MODEL – MASS DEPENDENT SCHEME (MOM)

Example:



$$A(p^2)_{\overline{MS}} = A(p^2) + c.t. = \frac{1}{32\pi^2} \int_0^1 dx \log \left(\frac{m^2 - x(1-x)p^2}{\mu_{\overline{MS}}^2} \right)$$

$$A(p^2)_{\text{MOM}} = A(p^2) + c.t. = \frac{1}{32\pi^2} \int_0^1 dx \log \left(\frac{m^2 - x(1-x)p^2}{m^2 + x(1-x)\mu^2} \right)$$

$$\mu \frac{\partial \rho_{ind}(\mu)}{\partial \mu} = \rho_{ind}(\mu) \left(2 \frac{\beta_{m^2}}{m^2} - \frac{\beta_\lambda}{\lambda} \right) \int_0^1 \frac{x(1-x)\mu^2 dx}{m^2 + x(1-x)\mu^2}$$

MOM

Similarly one can derive the vacuum part in the MOM scheme:

$$\mu \frac{\partial \rho_\Lambda^{\text{vac}}}{\partial \mu} \Big|_{\text{MOM}} = \frac{m^4}{32\pi^2} \int_0^1 \frac{x(1-x)\mu^2 dx}{m^2 + x(1-x)\mu^2}$$

MOM

valid in the UV and IR regime !

STANDARD MODEL – MOM SCHEME

Contributions of SM particles to the effective potential:

Φ	i	n_i	κ_i	κ_i^m	c_i
W^\pm	1	6	$g^2/4$	0	5/6
Z^0	2	3	$(g^2 + g'^2)/4$	0	5/6
t	3	-12	$y_t^2/2$	0	3/2
ϕ	4	1	$3\lambda/2$	1	3/2
χ_i	5	3	$\lambda/2$	1	3/2

$$M_i^2(\phi) = \kappa_i \phi^2 - m^2 \kappa_i^m$$

background dependent mass matrix

$$\langle \phi \rangle^2 = \frac{2m^2}{\lambda}$$

$$\mu \frac{\partial \rho_\Lambda^{\text{vac}}(\mu)}{\partial \mu} \Big|_{\text{MOM}} = \frac{\langle \phi \rangle^4}{32\pi^2} \sum_i n_i \left[\frac{\kappa_i^m \lambda}{2} \right]^2 \int_0^1 \frac{x(1-x)\mu^2 dx}{(M_{\text{phys}}^2)_i + x(1-x)\mu^2}$$

$$\mu \frac{\partial \rho_{\text{ind}}(\mu)}{\partial \mu} \Big|_{\text{MOM}} = \frac{\langle \phi \rangle^4}{32\pi^2} \sum_i n_i \kappa_i [\kappa_i - \lambda \kappa_i^m] \int_0^1 \frac{x(1-x)\mu^2 dx}{(M_{\text{phys}}^2)_i + x(1-x)\mu^2}$$

$$\mu \frac{\partial (\rho_{\text{ind}} + \rho_\Lambda^{\text{vac}})}{\partial \mu} \Big|_{\text{MOM}} = \sum_i \frac{n_i}{32\pi^2} (M_{\text{phys}}^4)_i \int_0^1 \frac{x(1-x)\mu^2 dx}{(M_{\text{phys}}^2)_i + x(1-x)\mu^2}$$

STANDARD MODEL – DECOUPLING

$$\mu \frac{\partial(\rho_{ind} + \rho_{\Lambda}^{vac})}{\partial \mu} \Big|_{\text{MOM}} = \sum_i \frac{n_i}{32\pi^2} (M_{phys}^4)_i \int_0^1 \frac{x(1-x)\mu^2 dx}{(M_{phys}^2)_i + x(1-x)\mu^2}$$

$$\int_0^1 \frac{x(1-x)\mu^2 dx}{(M_{phys}^2)_i + x(1-x)\mu^2} = \begin{cases} 1 & \mu^2 \gg (M_{phys}^2)_i \text{ as before} \\ \frac{\mu^2}{6(M_{phys}^2)_i} - \frac{\mu^4}{30(M_{phys}^4)_i} & \mu^2 \ll (M_{phys}^2)_i \text{ **DECOUPLING**} \end{cases}$$

➔ Appelquist-Carazzone theorem for CC running:

$$\mu \frac{\partial(\rho_{\Lambda}^{vac} + \rho_{ind})}{\partial \mu} \approx \sum_j \frac{n_j (m_{light}^4)_j}{32\pi^2} \left[-\frac{\mu^2}{12(4\pi)^2} \left[12M_t^2 - 6M_W^2 - 3M_Z^2 - M_H^2 \right] \right] + \frac{\mu^4}{30(4\pi)^2}$$

(Goldstons cancel in the sum – only physical particles contribute)
(similar to the Veltman condition)

THIS PART IS RESPONSIBLE FOR THE CC RUNNING !

$$12M_t^2 - 6M_W^2 - 3M_Z^2 - M_H^2 \simeq 0? \quad \Rightarrow \text{ **FAILS IN THE SM !** }$$

STANDARD MODEL IN THE CONSTANT CURVATURE SPACE

$$S_{HE} = \int d^4x \sqrt{-g} \left\{ \mathcal{L}_{SM} + \xi \varphi^\dagger \varphi R - \frac{1}{16\pi G} (R + 2\Lambda_{vac}) \right\}$$

Φ	i	n_i	κ_i	κ_i^m	κ_i^R
$W^\pm(\text{ghost})$	1	-2	$g^2/4$	0	1/2
W^\pm	2	8	$g^2/4$	0	1/2
$Z^0(\text{ghost})$	3	-1	$(g^2 + g'^2)/4$	0	1/2
Z^0	4	4	$(g^2 + g'^2)/4$	0	1/2
t	5	-12	$y_t^2/2$	0	1/4
ϕ	6	1	$3\lambda/2$	1	1/2
χ_i	7	3	$\lambda/2$	1	1/2

$$\mathcal{M}_i^2(\langle\phi\rangle) = \kappa_i \langle\phi\rangle^2 - m^2 \kappa_i^m + \left(\kappa_i^R - \frac{1}{6} \right) R$$

$$\langle\phi\rangle^2 = \frac{2(m^2 - \xi R)}{\lambda} \quad (\text{vacuum is } R \text{ dependent})$$

$$\tilde{M}_i^2 = \mathcal{M}_i^2(\langle\phi\rangle) - 2\kappa_i \frac{\xi R}{\lambda} \quad (\text{masses are } R \text{ dependent})$$

➔ Appelquist-Carazzone th. for CC running in the curved space:

$$\mu \frac{\partial(\rho_{ind} + \rho_\Lambda^{vac} + \kappa R)}{\partial\mu} \simeq \sum_j \frac{n_j (\mathcal{M}_{light}^4)_j}{32\pi^2} + \frac{\mu^2}{12(4\pi)^2} \left(-12\tilde{M}_t^2 + 6\tilde{M}_W^2 + 3\tilde{M}_Z^2 + \tilde{M}_H^2 + \frac{7}{3}R \right)$$

MASSLESS THEORY

$$\mu \frac{\partial(\rho_{ind} + \rho_{\Lambda}^{\text{vac}})}{\partial\mu} \Big|_{\text{MOM}} = \sum_i \frac{n_i}{32\pi^2} (M_{phys}^4)_i \int_0^1 \frac{x(1-x)\mu^2 dx}{(M_{phys}^2)_i + x(1-x)\mu^2}$$

$$\mu \frac{\partial\rho_{\Lambda}^{\text{vac}}}{\partial\mu} \equiv \beta_{\rho_{\Lambda}^{\text{vac}}} = \frac{m^4}{32\pi^2} = 0 \quad (m = 0) \quad \Rightarrow \quad \rho_{\Lambda}^{\text{vac}} \text{ IS CONSTANT}$$

$$\mu \frac{\partial\rho_{ind}}{\partial\mu} = \frac{\langle\phi\rangle^4}{32\pi^2} \sum_i n_i \kappa_i^2 \int_0^1 \frac{x(1-x)\mu^2 dx}{(M_{phys}^2)_i + x(1-x)\mu^2} = \frac{\mu^2 \langle\phi\rangle^2}{12(4\pi)^2} \sum_i \kappa_i n_i + \frac{\mu^4}{20(4\pi)^2}$$

$M_i^2(\phi) = \kappa_i \phi^2$

[simple structure of CC running]

in massless models fine-tuning of Higgs mass and CC are linked \Rightarrow

MASSLESS THEORY: SM WITH A REAL SCALAR

[O.Antipin, M. Mojaza, F. Sannino, arXiv: 1310.0957]

$$V_0 = V_0^{SM} + \lambda_{HS} H^\dagger H S^2 + \frac{\lambda_S}{4} S^4$$

[new real scalar particle S]

Generalized Veltman condition –from the CC running ($M_H = 0$ in the massless theory):

$$12M_t^2 - 6M_W^2 - 3M_Z^2 - M_S^2 = 0 \quad \Rightarrow \quad M_S^2(\phi) \approx (550 \text{ GeV})^2$$

$$M_S^2(\phi) = \lambda_{HS} \phi^2 \quad \Rightarrow \quad \lambda_{HS}(\mu) = 6y_t^2(\mu) - \frac{9}{4}g^2(\mu) - \frac{3}{4}g'^2(\mu) \approx 4.8$$

M_H is generated by the one-loop Coleman-Weinberg mechanism:

$$M_H^2 = \frac{3}{8\pi^2} \left[\frac{1}{16} (3g^4 + 2g^2g'^2 + g'^4) - y_t^4 + \frac{1}{3}\lambda_{HS}^2 \right] v_{EW}^2 \approx (125 \text{ GeV})^2$$

prediction!

connection between the CC running and BSM physics !

CONCLUSIONS

- ❑ we have considered the RG running of the Cosmological Constant
- ❑ we have showed that only RG running of the total (induced + vacuum) CC exhibits behavior consistent with the decoupling theorem
- ❑ we have provided generalization to the constant curvature space
- ❑ we have provided a simple extension of the SM with addition of one massless real scalar where condition of absence of leading RG effect allowed us to predict the (radiative) Higgs mass correctly