On the robustness of the primordial power spectrum in Higgs inflation

Javier Rubio

based on 1706.05007, F. Bezrukov, M. Pauly, J. Rubio

UNIVERSITÄT HEIDELBERG
Zukunft. Seit 1386.

INSTITUTO DE FISICA TEORICA UAM-CSIC
PASCOS 2017
19-23 JUNE, MADRID

Higgs inflation at tree level

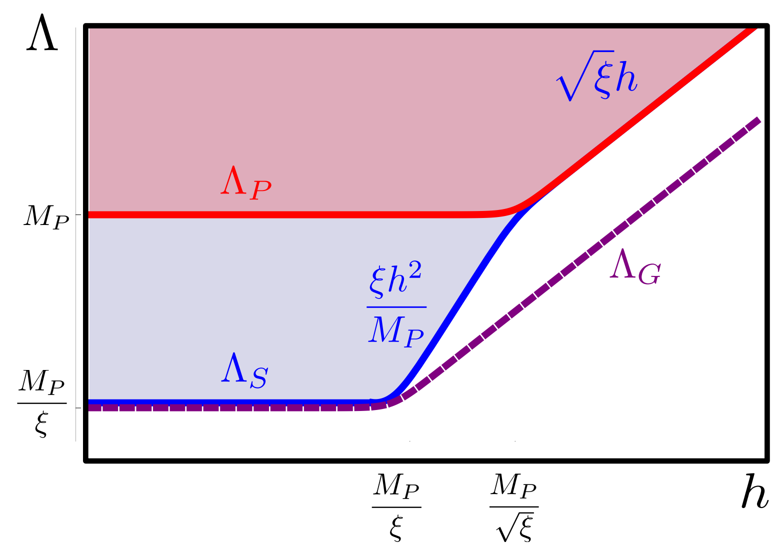
$$S = \int d^4x \sqrt{-g} \left[\frac{M_P^2 + \xi h^2}{2} R - \frac{1}{2} (\partial h)^2 - \frac{\lambda}{4} (h^2 - v_{\rm EW}^2)^2 \right]$$

- Scale invariant at $h > M_P/\sqrt{\xi}$
- h becomes Goldstone boson & effectively decouples

- SM + Gravity $\longrightarrow$ Non-renormalizable

$$\Lambda_S(h) = \frac{M_P^2 + \xi(1+6\xi)h^2}{\xi\sqrt{M_P^2 + \xi h^2}}$$

$$\Delta\mathcal{L}_S = \sum_n \frac{c_n \mathcal{O}_n(h)}{[\Lambda_S(h)]^{n-4}} ,$$



F. L. Bezrukov and M. Shaposhnikov Phys. Lett. B659 (2008) 703–706 , C. P. Burgess, H. M. Lee and M. Trott JHEP 09 (2009) 103, J. L. F. Barbon and J. R. Espinosa Phys. Rev. D79 (2009) 081302, M. P. Hertzberg JHEP 11 (2010) 023, F. Bezrukov, A. Magnin, M. Shaposhnikov, and S. Sibiryakov JHEP 1101 (2011) 016

Higgs inflation in Einstein-frame

$$V(\phi) = \frac{\lambda}{4} F^4(\phi) \quad \text{with} \quad F(\phi) \simeq \begin{cases} \phi & \text{for} \quad \phi < \sqrt{\frac{2}{3}} \frac{M_P}{\xi} \\ \frac{M_P}{\sqrt{\xi}} \left(1 - e^{-\sqrt{\frac{2}{3}}\frac{\phi}{M_P}}\right)^{\frac{1}{2}} & \text{for} \quad \phi > \sqrt{\frac{2}{3}} \frac{M_P}{\xi} \end{cases}$$

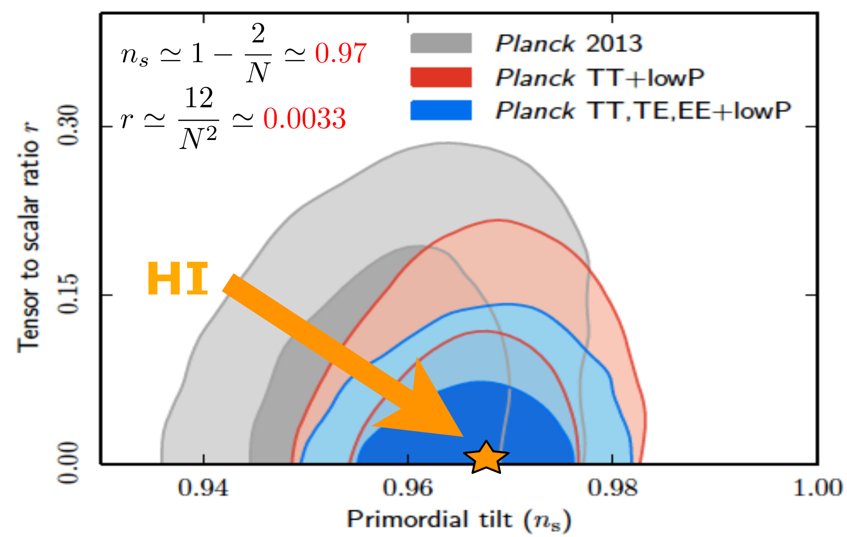
Due to quadratic pole in h *See Linde's talk*

Note that the transition is not instantaneous $\sqrt{\frac{2}{3}}\frac{M_P}{\xi} < \phi < \sqrt{\frac{3}{2}} M_P$

Scale invariance $h > M_P/\sqrt{\xi}$

↓ ↓

Shift symmetry $\phi > \sqrt{3/2} M_P$



so far so good...but the night is dark and full of terrors

HI is non-renormalizable

- **Quantum corrections should be introduced by interpreting the theory as an EFT in which a particular set of higher dim. operators are included.**
- **We will take a minimalistic approach and add only the higher dimensional operators generated by radiative corrections (i.e. those needed to make theory finite at every order in PT).**

Fixed by the divergencies → A_n **Arbitrary** → B_n

$$\delta\mathcal{L}_{ct} = \left(\frac{A_n}{\epsilon} + B_n\right)\mathcal{O}_n$$

(Partially) controllable link
between the low and high energy parameters of the model

F. Bezrukov, A. Magnin, M. Shaposhnikov, and S. Sibiryakov JHEP 1101 (2011) 016,
See also talks by Andrey Shkerin & Pawel Olszewski

Higgs inflation as an EFT

$$= \frac{1}{2}\,\mathrm{Tr}\ln\left[\Box - \left(\frac{\lambda}{4}(F^4)''\right)^2\right]$$

$$V(\phi) = \frac{\lambda}{4}F^4(\phi) \quad + \text{counterterms to cancel divergencies}$$

$$\delta\mathcal{L}_{\rm ct} = \left(-\frac{2}{\bar\epsilon}\frac{9\lambda^2}{64\pi^2} + \delta\lambda_a\right)\left(F'^2 + \frac{1}{3}F''F\right)^2 F^4$$

new

At low energies

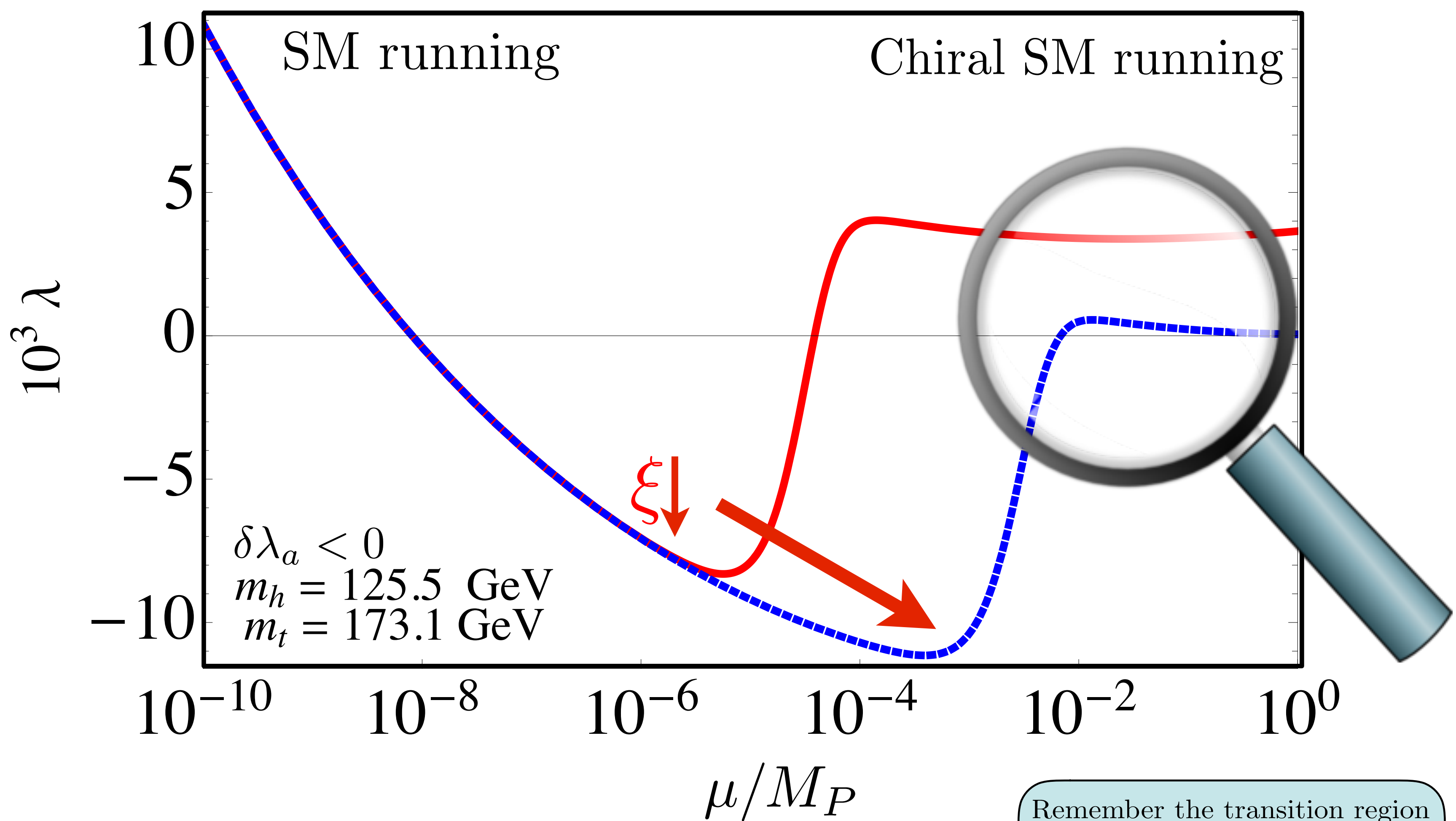
$F = \phi \quad F'(0) = 1$

At high energies

$F = const. \quad F'(\phi_0) = 0$

F. L. Bezrukov, J. Rubio, M. Shaposhnikov Phys.Rev. D92 (2015) no.8, 083512

Asymptotics are not modified



Remember the transition region

$$\sqrt{\frac{2}{3}}\frac{M_P}{\xi} < \phi < \sqrt{\frac{3}{2}}\,M_P$$

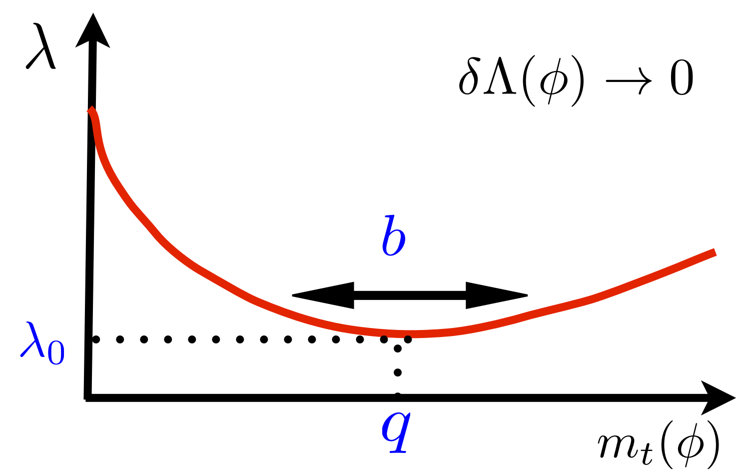
F. L. Bezrukov, J. Rubio, M. Shaposhnikov Phys.Rev. D92 (2015) no.8, 083512

The RGE potential

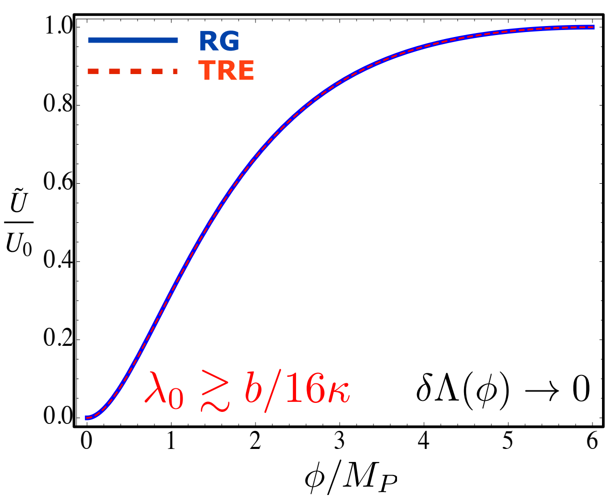
$$V(\phi) = \frac{\lambda(\phi)}{4} F^4(\phi)$$

$$\lambda(\phi) = \lambda_0 + b \log^2\left(\frac{m_t(\phi)}{q}\right) + \delta\Lambda(\phi)$$

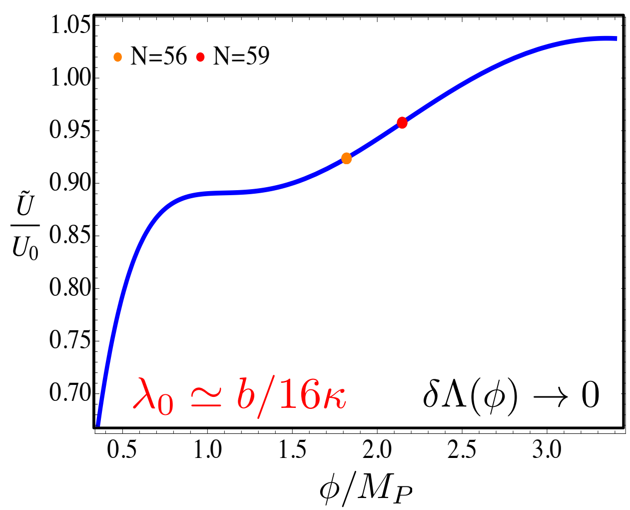
$$\frac{m_t}{q} = \alpha \cdot \frac{y_t}{\sqrt{2}} \frac{h}{\Omega(h)} \frac{1}{q} \equiv \frac{\sqrt{\xi} F(\phi)}{\kappa M_P} \,,$$



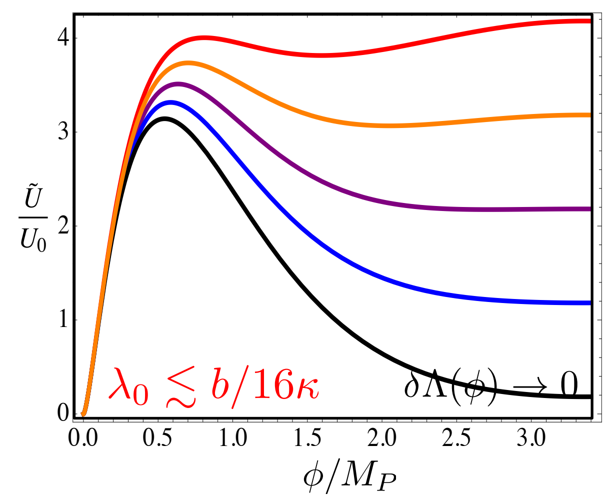
UNIVERSALITY



CRITICALITY



NO INFLATION

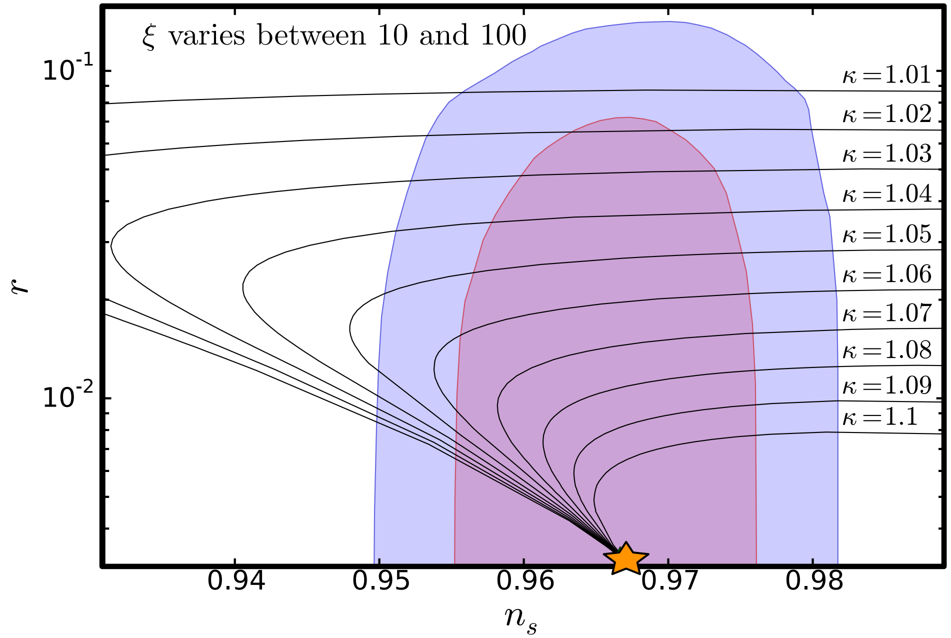


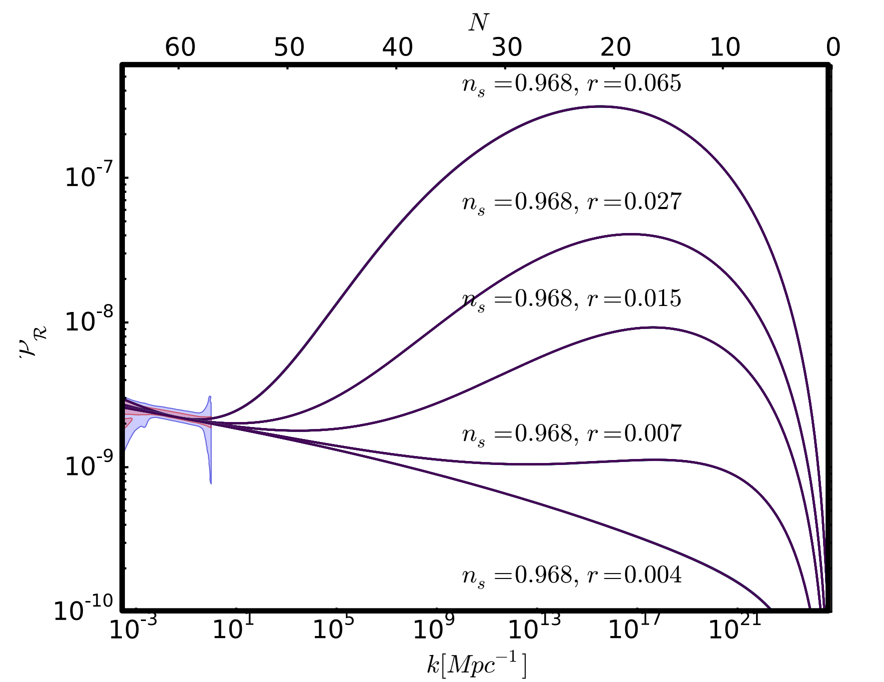
Sudden threshold approximation

$$V(\phi) = \frac{\lambda(\phi)}{4} F^4(\phi) \quad \text{with} \quad \lambda(\phi) = \lambda_0 + b \log^2\left(\frac{\sqrt{\xi} F(\phi)}{\kappa M_P}\right)$$

For the SM, $b = 2.3 \times 10^{-5}$

3 observational inputs for 3 unknown parameters

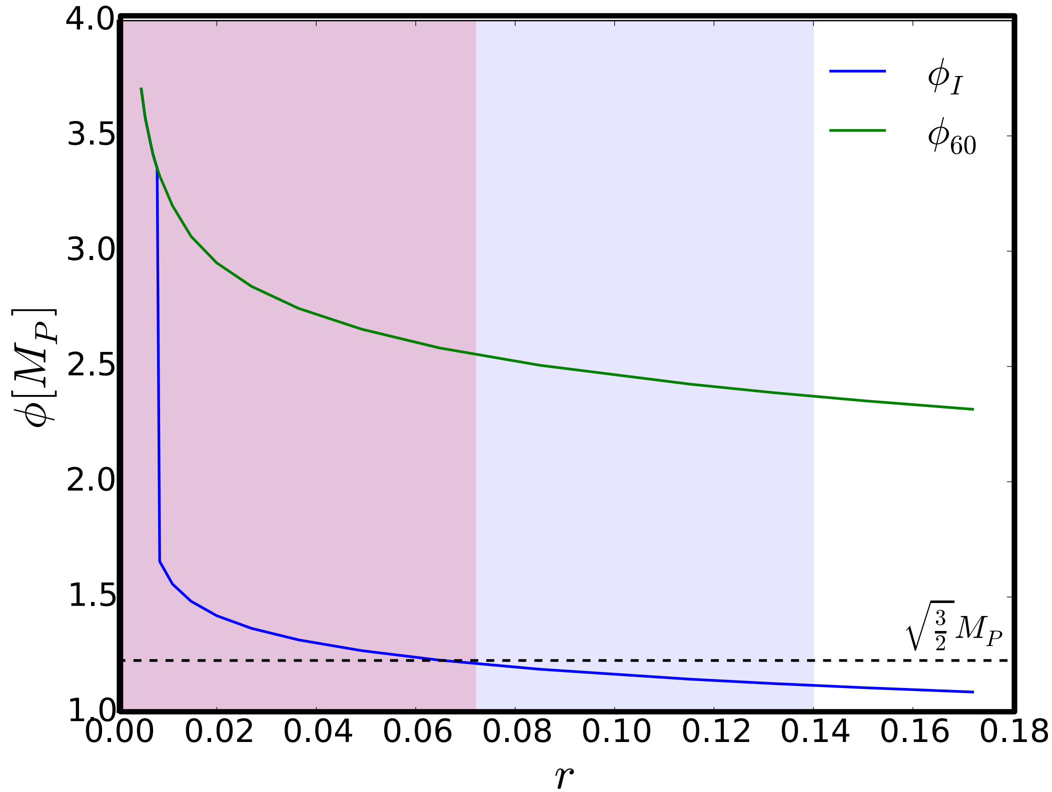




F. Bezrukov and M. Shaposhnikov Phys.Lett. B734 (2014) 249-254

Y. Hamada, H. Kawai, K.-y. Oda, and S. C. Park Phys.Rev.Lett. 112 (2014) no.24, 241301

The (quasi-) inflection point



How robust is CHI?

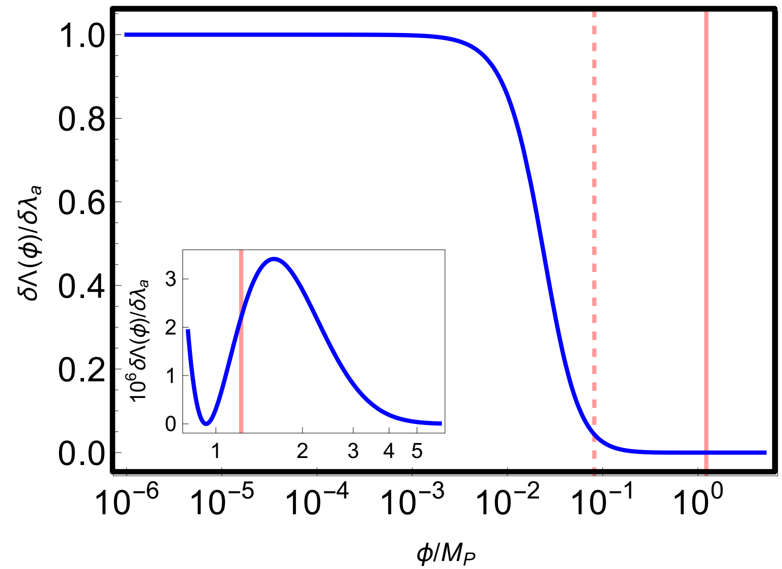
For large r, the inflection point is within the transition region

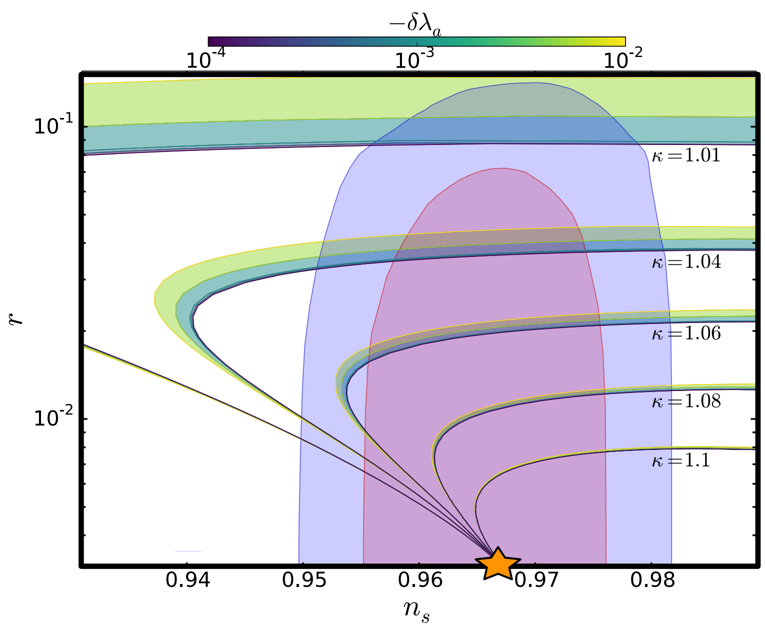
$$\sqrt{\frac{2}{3}}\frac{M_P}{\xi} < \phi < \sqrt{\frac{3}{2}}M_P$$

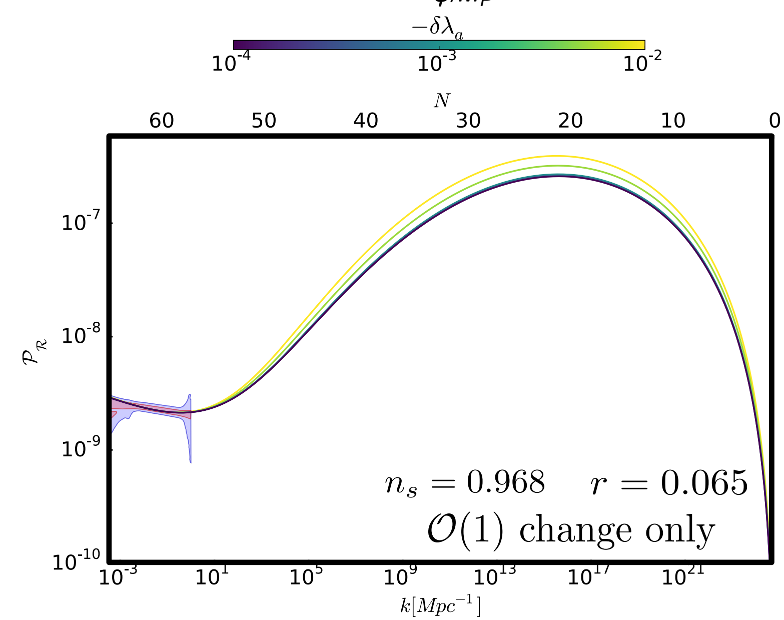
1-loop threshold approximation

- Assume $\delta\lambda_a \sim \mathcal{O}(\lambda^2)$
- Neglect the running of $\delta\lambda_a$

$$\delta\Lambda(\phi) = \delta\lambda_a \left(F'^2 + \frac{1}{3} F'' F \right)^2$$



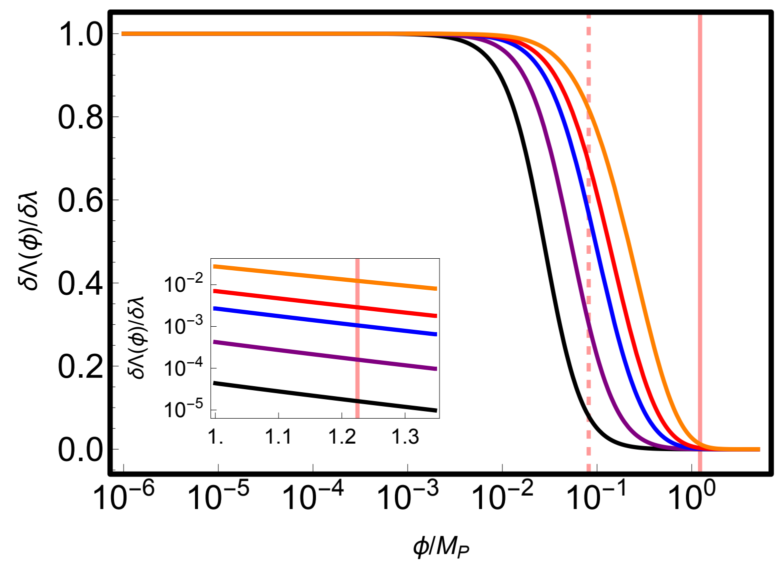


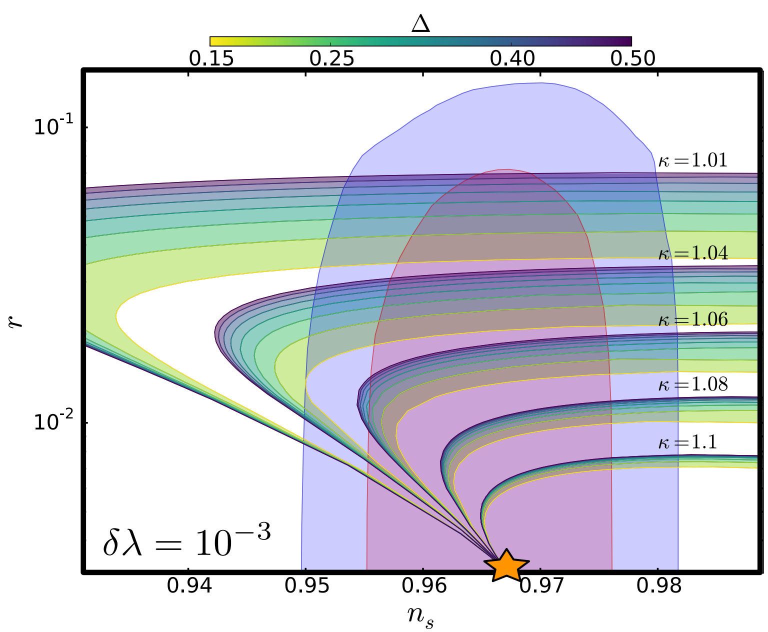


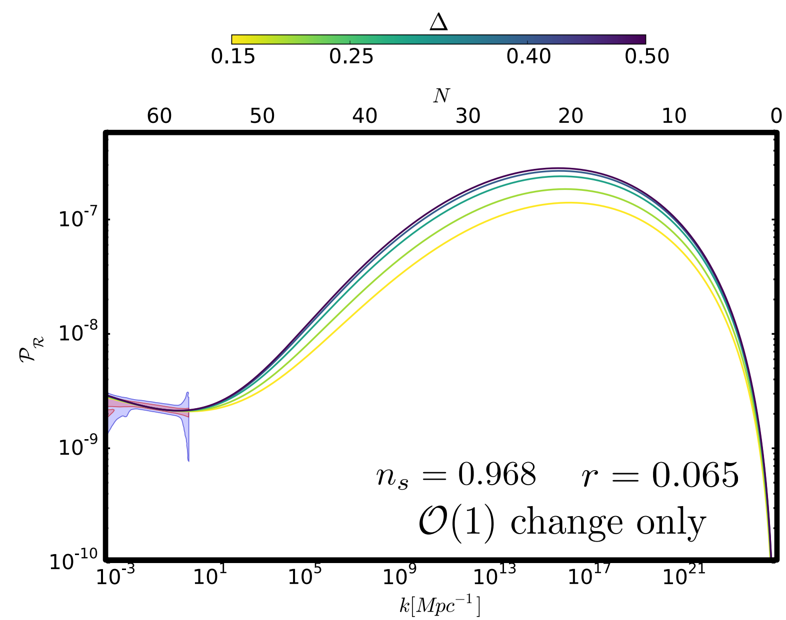
Collective threshold effects

- Running of finite parts?
- Higher order operators?

$$\delta\Lambda(\phi) = \delta\lambda \frac{\left(1 - F^2/F_\infty^2\right)^4}{\left(1 + \Delta \cdot 6\,\xi\, F^2/F_\infty^2\right)^2}$$

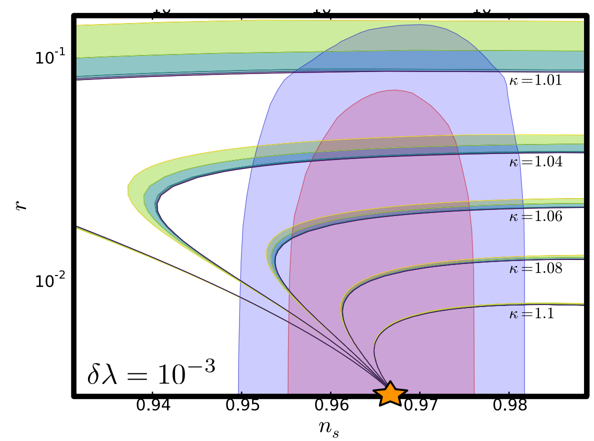




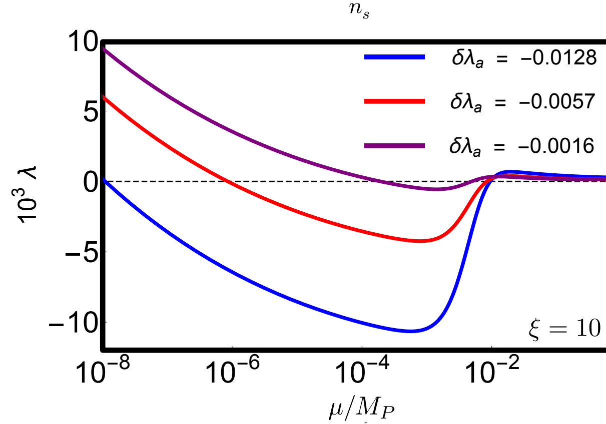


CONCLUSIONS

✓ **Non-critical HI is "fireproof": it provides robust & universal predictions** ☆



✓ **Precise predictions in critical HI must be complemented by a particular UV completion of the Standard Model coupled to gravity**



✓ **The relation of inflationary predictions to LE observables contains an irreducible theoretical uncertainty —> UV completion ?**

BACKUP SLIDES

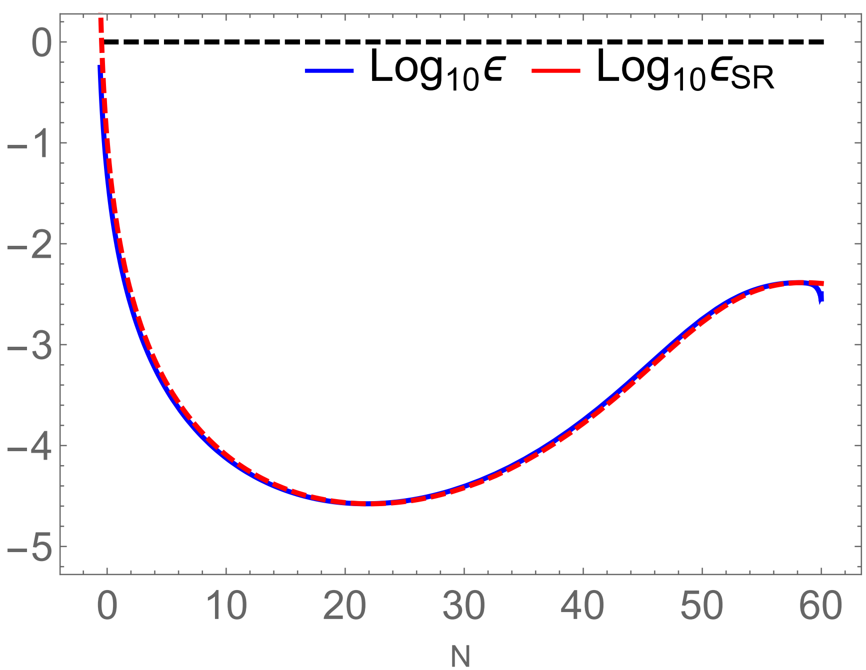
Slow-roll consistency

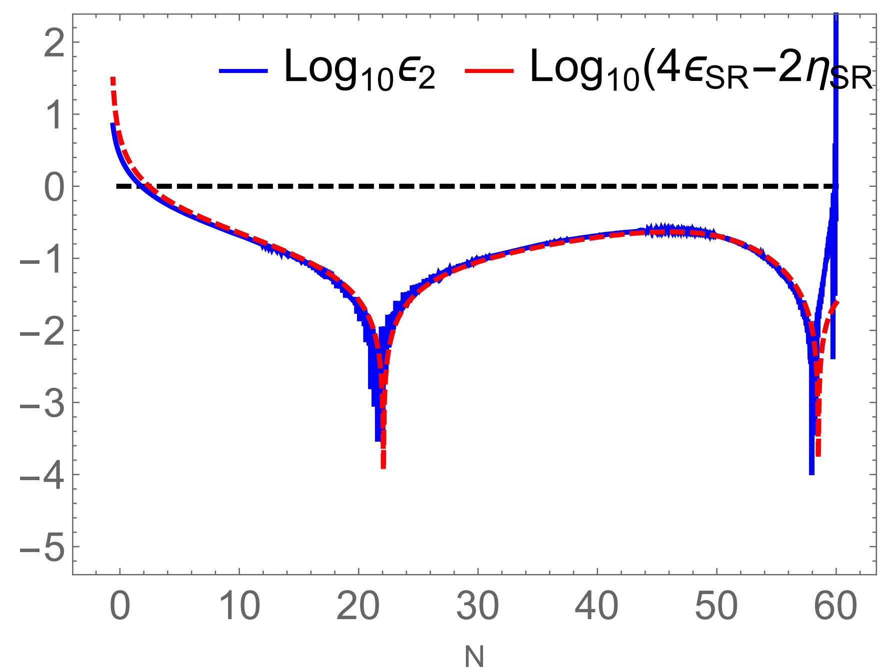
In general

$$\epsilon \equiv -\frac{\dot{H}}{H^2} \qquad \epsilon_2 \equiv \frac{\dot{\epsilon}}{H\epsilon}$$

In a slow-roll attractor

$$\epsilon \simeq \epsilon_{\rm SR} \qquad \epsilon_2 \simeq 4\epsilon_{\rm SR} - 2\eta_{\rm SR}$$





The self-consistent approach

A self-consistent approach is to define the cutoff from the theory itself by considering all possible reactions between the SM constituents....

.... and add all kind of operators suppressed by these cutoffs...

1. Compute the quadratic lagrangian $\phi = \bar{\phi} + \delta\phi \quad g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$

$$\mathcal{L}^{(2)} = -\frac{M_P^2 + \xi\bar{\phi}^2}{8}\left(h^{\mu\nu}\Box h_{\mu\nu} + 2\partial_\nu h^{\mu\nu}\partial^\rho h_{\mu\rho} - 2\partial_\nu h^{\mu\nu}\partial_\mu h - h\Box h\right) + \frac{1}{2}(\partial_\mu\delta\phi)^2 + \xi\bar{\phi}\left(\Box h - \partial_\lambda\partial_\rho h^{\lambda\rho}\right)\delta\phi\ ,$$

2. Get rid of the mixings in the quadratic action

$$\delta\phi = \sqrt{\frac{M_P^2 + \xi\bar{\phi}^2}{M_P^2 + \xi\bar{\phi}^2 + 6\xi^2\bar{\phi}^2}}\,\delta\hat{\phi}\ ,$$

$$h_{\mu\nu} = \frac{1}{\sqrt{M_P^2 + \xi\bar{\phi}^2}}\,\hat{h}_{\mu\nu} - \frac{2\xi\bar{\phi}}{\sqrt{(M_P^2 + \xi\bar{\phi}^2)(M_P^2 + \xi\bar{\phi}^2 + 6\xi^2\bar{\phi}^2)}}\,\bar{g}_{\mu\nu}\,\delta\hat{\phi}$$

3. Read out the cutoff from higher order operators

$$\frac{\xi\sqrt{M_P^2 + \xi\bar{\phi}^2}}{M_P^2 + \xi\bar{\phi}^2 + 6\xi^2\bar{\phi}^2}(\delta\hat{\phi})^2\Box\hat{h}$$

1-loop truncation

Finite parts should be promoted to new couplings with own RGE

$$\frac{d\lambda}{d\log\mu} = \beta_\lambda(\lambda, \lambda_a, \ldots) \qquad \frac{d\lambda_1}{d\log\mu} = \beta_{\lambda_1}(\lambda, \lambda_a, \ldots) \qquad \ldots$$

The set of RG equations is not closed

The one-loop diagrams associated to the new counterterms are given by

$$\sim \delta\lambda[(F'^2 + \frac{1}{3}F''F)^2F^4]''\lambda(F^4)''$$

$\delta\lambda[(F'^2 + \frac{1}{3}F''F)^2F^4]''$

+ other diagrams

The two-loop contributions generated by the original Lagrangian

$\lambda(F^4)'''$  $\lambda(F^4)''' \sim (\lambda(F^4)''')^2\lambda(F^4)''$,

+ other diagrams

In order to be able to truncate the expansion, the finite parts must be of the same order (in power counting) than the loops producing them.

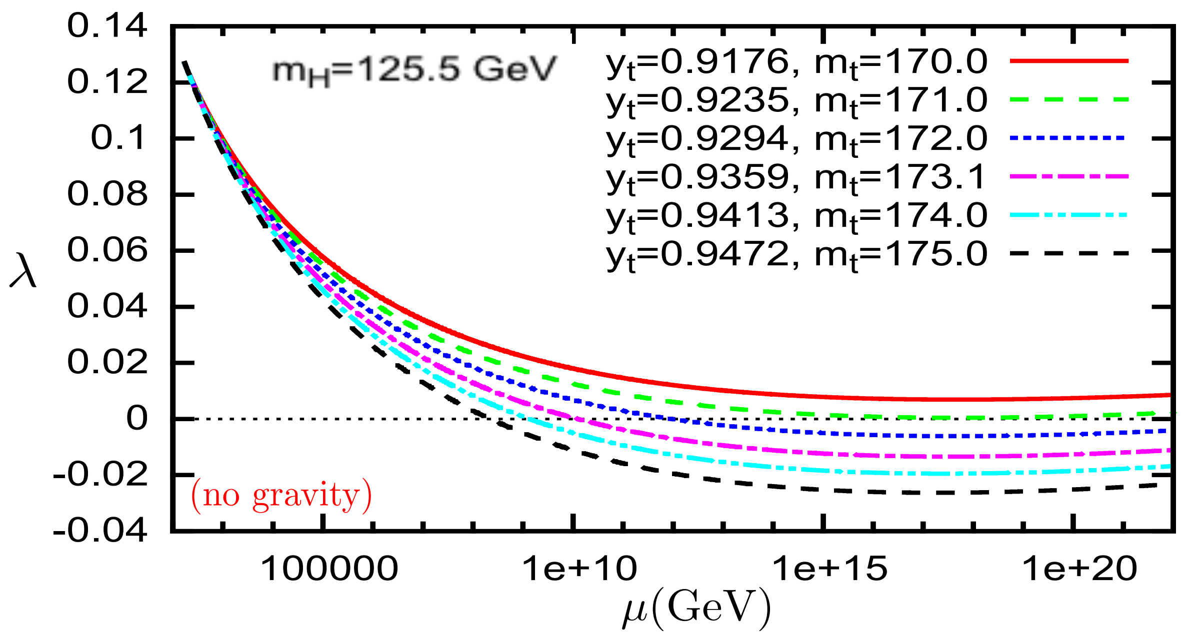
$$\delta\lambda \sim \mathcal{O}(\lambda^2, y^4) \qquad \delta y \sim \mathcal{O}(y^3, y\lambda) \qquad \lambda \sim \mathcal{O}(y^2)$$

On the edge of stability

- ***SM remains perturbative all the way up till the inflat./Planck scale***

$$\mu \frac{d\lambda(\mu)}{d\log(\mu)} = +\#\lambda^2 + \ldots - \#y_t^4$$

Non trivial interplay between Higgs self-coupling and top quark Yukawa coupling

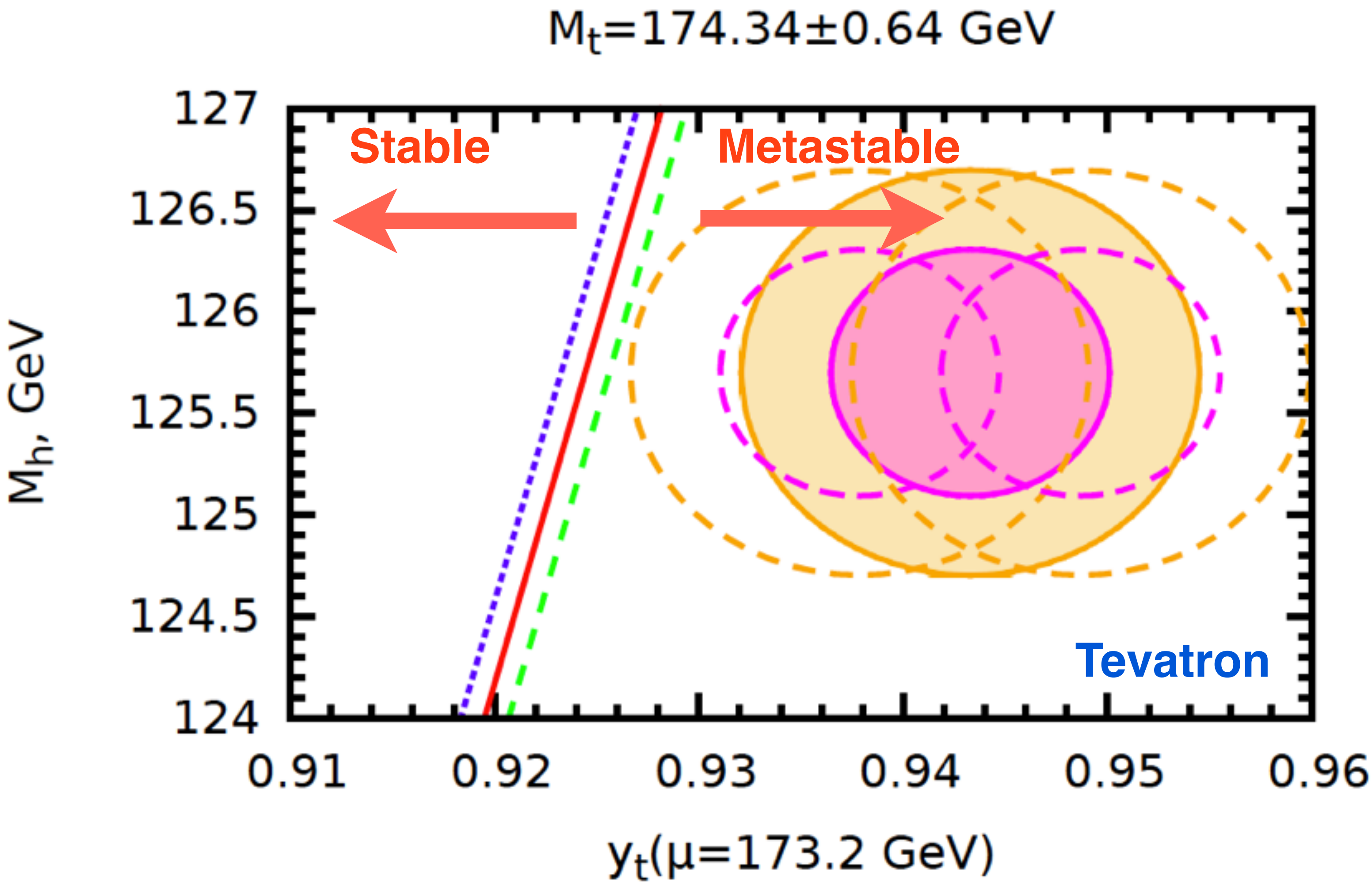


Is there a reason for that? M. Lindner, M. Sher, and H. W. Zaglauer, Phys.Lett., B228, 139 (1989), C. Froggatt and H. B. Nielsen, Phys.Lett., B368,96 (1996), M. Shaposhnikov and C. Wetterich, Phys.Lett., B683, 196 (2010), M. Veltman, Acta Phys.Polon., B12, 437 (1981), B683, 196 (2010), M. Holthausen, K.S.Lim, M.Lindner and references therein....

Gravitational corrections? Lalak,Lewicki, Olszewki arXiv 14.02.3826, Branchina, Massina Phys.Rev.Lett. 111 (2013) 241801 etc...

Top quark & vac. instability

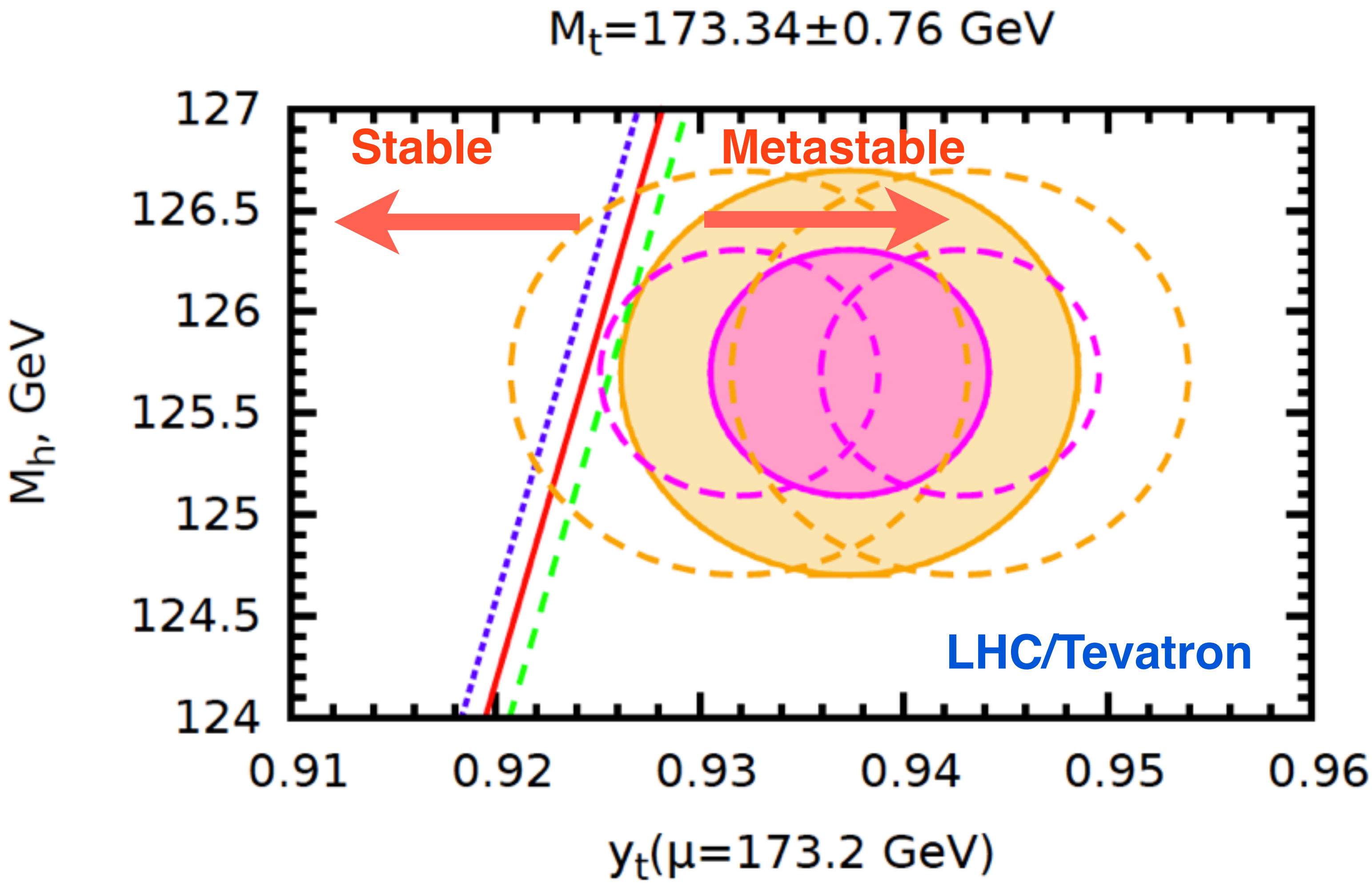
$$y_t^{\rm crit} = 0.9223 + 0.00118\left(\frac{\alpha_s - 0.1184}{0.0007}\right) + 0.00085\left(\frac{M_h - 125.03}{0.3}\right)$$



See F. Bezrukov, M. Shaposhnikov J.Exp.Theor.Phys. 120 (2015) 335-343 and references therein

Top quark & vac. instability

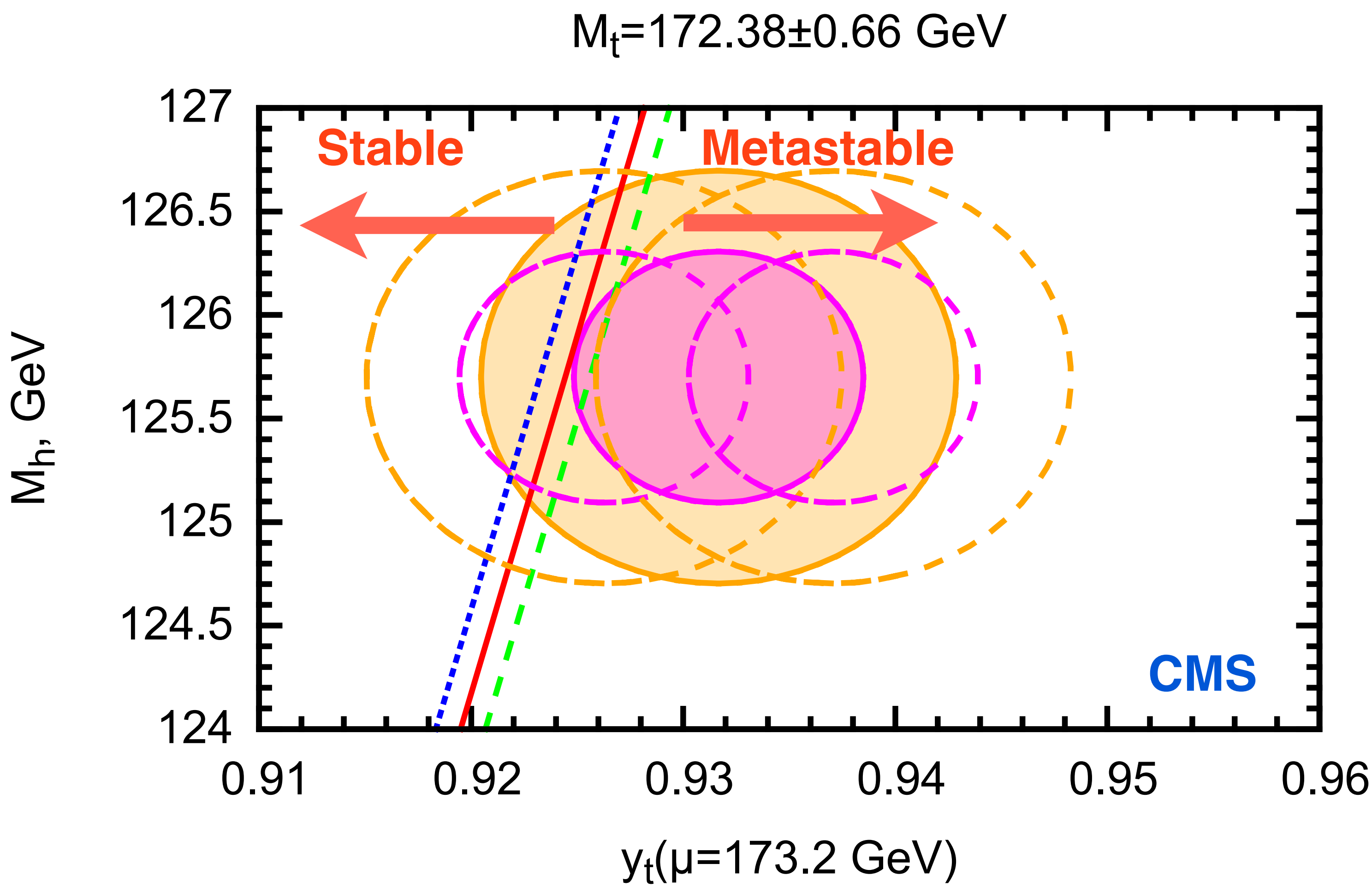
$$y_t^{\rm crit} = 0.9223 + 0.00118\left(\frac{\alpha_s - 0.1184}{0.0007}\right) + 0.00085\left(\frac{M_h - 125.03}{0.3}\right)$$



See F. Bezrukov, M. Shaposhnikov J.Exp.Theor.Phys. 120 (2015) 335-343 and references therein

Top quark & vac. instability

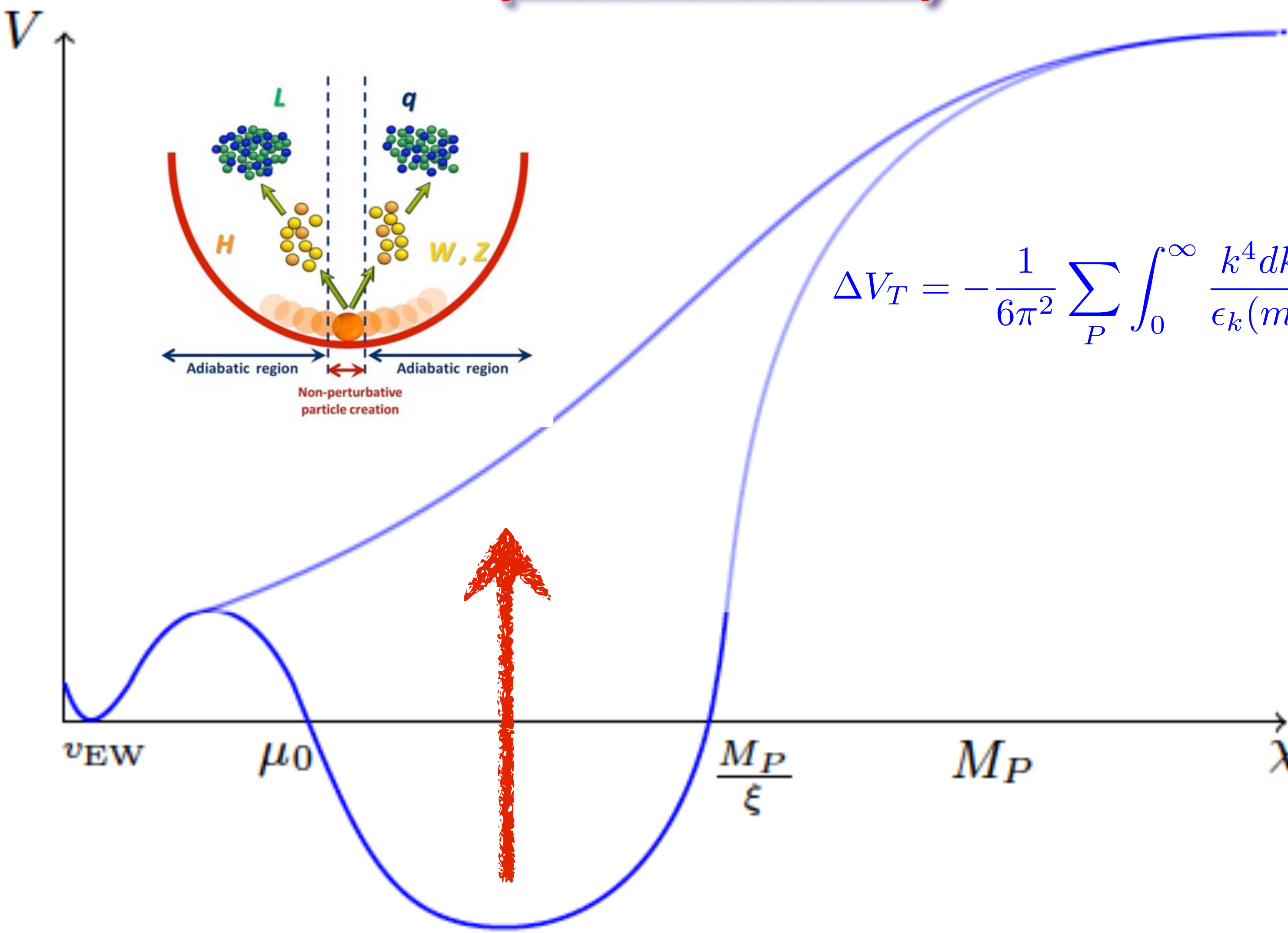
$$y_t^{\rm crit} = 0.9223 + 0.00118\left(\frac{\alpha_s - 0.1184}{0.0007}\right) + 0.00085\left(\frac{M_h - 125.03}{0.3}\right)$$



See F. Bezrukov, M. Shaposhnikov J.Exp.Theor.Phys. 120 (2015) 335-343 and references therein

Sketch of effective potential

(not to scale!)



$$\Delta V_T = -\frac{1}{6\pi^2}\sum_P \int_0^\infty \frac{k^4 dk}{\epsilon_k(m)} \frac{1}{e^{\epsilon_k(m)T} \mp 1}$$

F. L. Bezrukov, J. Rubio, M. Shaposhnikov Phys.Rev. D92 (2015) no.8, 083512