On the vacuum structure of F-theory compactifications

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Moduli stabilization in string theory

(Perturbative) superstring theory predicts the extra 6D space.

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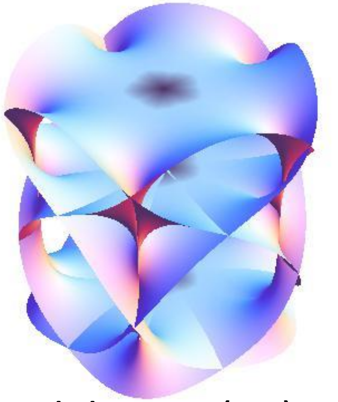


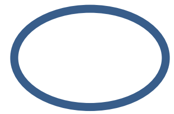
$R_{ij} = 0$

Extra 6D space should be compactified to be consistent with the observational and experimental data.

→ Stabilization of the extra dimensional space
<u>Moduli stabilization</u>

Two types of moduli fields

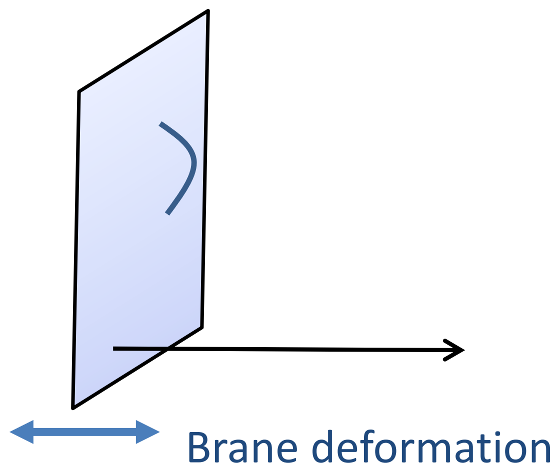
①Closed string moduli



Complex structure moduli
Kähler moduli
Dilaton
Kalb-Ramond field

②Open string moduli

D-brane



Flux compactification in type IIB on CY

①Closed string moduli

$$W_{\text{flux}} \propto \int_{\Gamma,\partial\Gamma=0} \Omega$$

[Gukov-Vafa-Witten '99]

GVW superpotential is calculated by the closed mirror symmetry.
→Stabilization of complex structure moduli and dilaton

[Giddings-Kachru-Polchinski '01,..

②Open string moduli

$$W_{\text{brane}} \propto \int_{\Gamma,\partial\Gamma\neq 0} \Omega$$

Witten *'97*

Brane superpotential is calculated by the open mirror symmetry.
→Stabilization of open string moduli

In this talk, we study their stabilization based on F-theory

Outline

- 4D N=1 effective potential

i) Flux-superpotential

ii) Brane superpotential

- Flux compactification in F-theory

- Conclusion

Flux superpotential in type IIB [Gukov-Vafa-Witten '99]

$$W_{\text{flux}}(\psi,\tau) = \int_M G_3(\tau) \wedge \Omega(\psi) = N_a \int_{\Gamma_a, \partial\Gamma_a = 0} \Omega$$

Ω : holomorphic three-form
ψ : Complex structure (CS) moduli
τ : Axio-dilaton

$G_3 = F_3 - \tau H_3$: Background three-form fluxes

The background flux generate the potential of the CS moduli and dilaton.
(Potential of closed string moduli)

Brane Superpotential:

$$W_{\mathrm{brane}} = \widehat{N_b} \int_{\Gamma_b, \partial\Gamma_{\mathrm{b}} = \mathrm{C}} \Omega$$

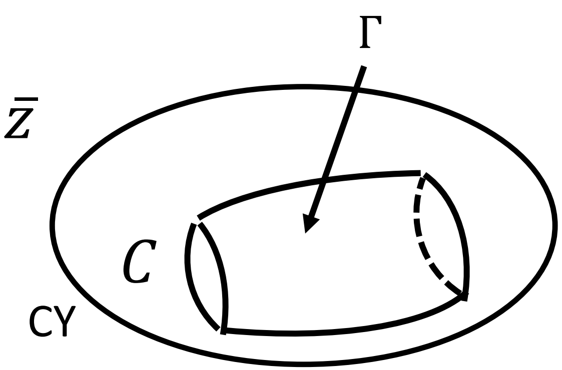
When branes wrap on the whole CY,
open string partition function is given by holomorphic CS theory

[Witten '92]

$$W = \int_{CY} \Omega \wedge \mathrm{Tr}[\mathrm{A} \wedge \bar{\partial} A + \frac{2}{3} A \wedge A \wedge A]$$

Brane superpotential can be derived from dimensional reduction $A \to \phi$
When branes wrap on holomorphic submanifold C, [Aganagic-Vafa '00]

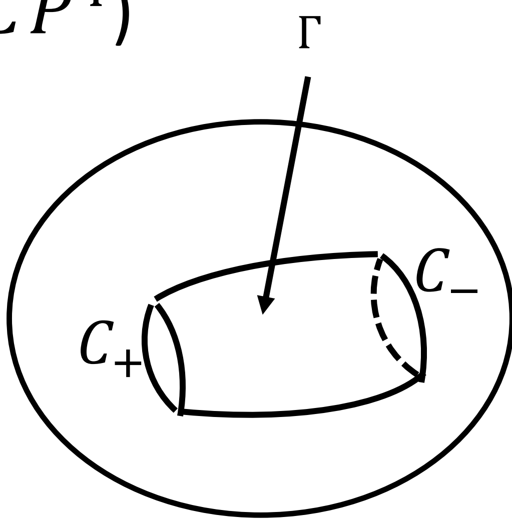
$$W_{\mathrm{brane}}(\psi, \phi) = \int_C \Omega_{ijz}\, \phi^i \bar{\partial}_z \phi^j dz d\bar{z}$$
$$= \int_{\Gamma, \partial\Gamma = \mathrm{C}} \Omega$$



Brane superpotential (Open mirror symmetry)

Mirror Quintic CY3 (degree 5 hypersurface in CP^4)

$$P(\psi) = \sum_{i=1}^{5} y_i^5 - 5\psi y_1 y_2\, y_3\, y_4\, y_5 = 0$$



Mirror Quintic CY3

Let us consider holomorphic 2-cycles
where the brane wraps [Morrison-Walcher '07]

$$C_{\pm}: y_1 + y_2 = 0, y_3 + y_4 = 0, y_5^2 \pm \sqrt{5\psi}\, y_1 y_3 = 0$$

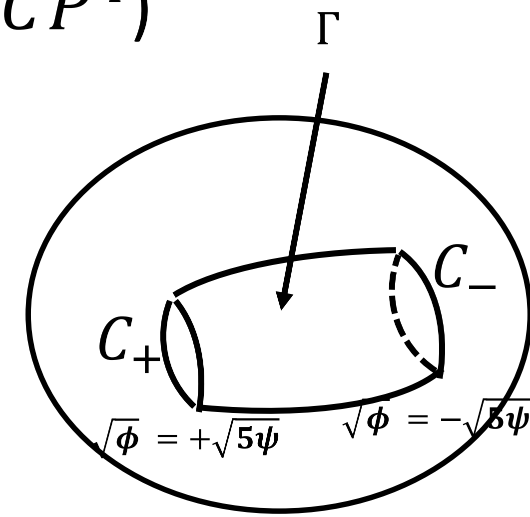
$$W = \int_{\Gamma} \Omega$$

No moduli dependence at fixed $C_{\pm}$

→<u>Brane deformation : $\partial\Gamma$ into non-holomorphic curve</u>

Brane superpotential (Open mirror symmetry)

Mirror Quintic CY3 (degree 5 hypersurface in CP^4)



Mirror Quintic CY3

$$P(\psi) = \sum_{i=1}^{5} y_i^5 - 5\psi y_1 y_2\, y_3\, y_4\, y_5 = 0$$

Continuous deformation of $C_{\pm}$:
(Holomorphic divisor)

$$\mathrm{Q}(\phi) = y_5^4 - 5\phi y_1 y_2 y_3 y_4 = 0$$

Brane deformation

Brane superpotential:

$$W_{\mathrm{brane}}(\psi,\phi) = \int_{\Gamma} \Omega(\psi,\phi) = \int_{\hat{\Gamma},\partial\hat{\Gamma}=Q(\phi)} F \wedge \Omega$$

which is related to D7-brane with magnetic flux F [Grimm-Ha-Klemm-Klevers '09]

● Elliptically fibered CY fourfold

○In the toric language, CY3 + brane deformation can be uplifted to an elliptically fibered CY4 without brane.

Toric charge :

Quintic CY3 : $l = (-5,1,1,1,1,1,0,0)$

Brane deformation : $\hat{l} = (-1,0,0,0,0,1,1,-1)$

⇩

Compact CY4 (CY3 fibered over CP^1):

$$l_1 = (-4,0,1,1,1,1,-1,-1,0)$$
$$l_2 = (-1,1,0,0,0,0,1,-1,0)$$
$$l_3 = (0,-2,0,0,0,0,0,1,1)$$

$l_1 + l_2$: Quintic CY3

l_2: brane deformation

l_3: base CP^1

<u>An elliptically fibered mirror CY4 in the B-model side</u>

[Berglund-Mayr '98, Grimm-Ha-Klemm-Klevers '09]

Elliptically fibered CY fourfold

○GVW superpotential + brane superpotential in type IIB
=G_4-flux superpotential in F-theory [Grimm-Ha-Klemm-Klevers '09]

$$W = \int_{\mathrm{CY4}} G_4 \wedge \Omega$$

Imaginary self-dual three-form fluxes in type IIB
=Self-dual G_4-fluxes

$$G_4 = * G_4$$

○By solving the Picard Fuchs equation,
we can determine the holomorphic 4-form Ω of CY4.

Outline

- 4D N=1 effective potential

i) Flux-superpotential

ii) Brane superpotential

- Flux compactification in F-theory

- Conclusion

F-theory on elliptically fibered CY4 →4D N=1 supergravity

In 4D N=1 SUGRA

Kähler potential:

$$K = -\ln \int_{\mathrm{CY4}} \Omega \wedge \overline{\Omega} - 2\ln V$$

Superpotential:

$$W = \int_{\mathrm{CY4}} G_4 \wedge \Omega$$

Ω: Holomorphic 4-form of CY4

G_4: Background four-form fluxes

V: CY3 volume

We consider the large complex structure point of CY4.

F-theory compactification on elliptically fibered CY4

z: Complex structure modulus

S: Dilaton

z_1: Open string modulus

n_i: Quantized fluxes

Kähler potential:

$$K = -\ln\left[-i(S-\overline{S})\right] - \ln\left[\frac{5i}{6}(z-\overline{z})^3 + \boxed{\frac{i}{S-\overline{S}}\left(-\frac{1}{6}(z_1-\overline{z_1})^4 + \frac{5}{12}(z-\overline{z})^4\right)}\right] - 2\ln\mathcal{V}$$

Deviation from the weak coupling limit

Superpotential:

$$W = n_{11} + n_{10}S + n_8 z + n_6 S z + \frac{5}{2}\left(\frac{n_5}{5} + \frac{2n_6}{5}\right)z^2 - \frac{5n_4}{6}z^3 - n_2\left(\frac{5}{2}Sz^2 + \frac{5}{3}z^3\right) - n_9 z_1 - \frac{n_7}{2}z_1^2 - \frac{2n_3}{3}z_1^3 + n_1\left(\frac{5}{6}Sz^3 + \frac{5}{12}z^4 - \frac{1}{6}z_1^4\right)$$

F-theory compactification on elliptically fibered CY4

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Deviation from the weak coupling limit

Superpotential:

$$W = n_{11} + n_6 S z + \frac{5}{2}\left(\frac{n_5}{5} + \frac{2n_6}{5}\right)z^2 - \frac{n_7}{2}z_1^2 + n_1\left(\frac{5}{6}Sz^3 + \frac{5}{12}z^4 - \frac{1}{6}z_1^4\right)$$

The self-dual G_4 fluxes

Vacuum structure of F-theory

After imposing the self-dual condition to G_4 fluxes, we find that minimum of all the moduli fields:

$$D_S W = D_z W = D_{z_1} W = 0$$

z: CS modulus
S: Dilaton
z_1: Open string modulus
n_i: Quantized fluxes

VEVs

$$\mathrm{Re}z = \mathrm{Re}z_1 = \mathrm{Re}S = 0$$

$$\mathrm{Im}z = \left(\frac{6n_{11}}{5n_1}\right)^{1/4} \frac{2\sqrt{n_6}}{(8n_6(n_5+n_6)-5n_7^2)^{1/4}},$$

$$\mathrm{Im}z_1 = \left(\frac{30n_{11}}{n_1}\right)^{1/4} \frac{\sqrt{n_7}}{(8n_6(n_5+n_6)-5n_7^2)^{1/4}},$$

$$\mathrm{Im}S = \left(\frac{6n_{11}}{5n_1}\right)^{1/4} \frac{n_5}{\sqrt{n_6}(8n_6(n_5+n_6)-5n_7^2)^{1/4}}$$

Vacuum structure of F-theory

Although the fluxes are constrained by the tadpole condition,

$$\frac{\chi}{24} = n_{D3} + \frac{1}{2}\int_{CY4} G_4 \wedge G_4$$

χ=1860: Euler number of CY4
n_{D3}: # of D3

we find the consistent F-theory vacuum, e.g.,

$$n_1 = 1, n_5 = 15, n_6 = 10, n_7 = 2, n_{11} = 28$$

$$n_{\rm D3} = 0$$

All the moduli fields can be stabilized at the LCS point of CY fourfold

$$\mathrm{Re}z = \mathrm{Re}z_1 = \mathrm{Re}S = 0,$$
$$\mathrm{Im}z \simeq 2.28, \quad \mathrm{Im}z_1 \simeq 1.14, \quad \mathrm{Im}S \simeq 1.71$$

Conclusion

○Mirror symmetry techniques can be applied to the F-theory compactifications.

○We explicitly demonstrate the moduli stabilization around the large complex structure point of the F-theory fourfold.

○All the complex structure moduli can be stabilized at the Minkowski minimum.

Discussion

○Quantum corrections to the moduli potential
○Other CY4
○Stabilization of Kähler moduli
→LARGE volume scenario or KKLT