Axion Inflation with Dissipation and Thermalization

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[1] In collaboration with Ricardo Z.Ferreira, and Konrad Tywoniuk

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  - New dynamics? (not based on flat potentials)

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Inflation with Axial coupling

- ϕ coupled to U(1) gauge fields, “axion-like":

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- Derivative coupling $\implies$ No corrections to $V(\phi)$

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  (conformal time $ad\tau = dt$, $\pm$ positive (negative) helicity) :

$$A''_{\pm} + \left(k^2 \mp \frac{k\phi'}{f} \right) A_{\pm} = 0 \, ,$$

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- One helicity is unstable: gauge fields becomes quickly large

Constant $\dot\phi$ and de Sitter

- (Sorbo & Anber '09) assumed: $\dot\phi = const$ in de Sitter $a(t) = -\frac{1}{H\tau}$

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- Solution at $\tau \to 0^-$ (future):

$$A_+ \approx \frac{1}{\sqrt{2k}} \left(\frac{k|\tau|}{2\xi}\right)^{1/4} e^{\pi\xi - 2\sqrt{2\xi k|\tau|}}$$

Consequences

- (Sorbo & Anber '09) estimated:

$$\frac{\langle F\tilde{F}\rangle}{4} = \frac{1}{2a^4}\int \frac{d^3k}{(2\pi)^3}\, k\, \frac{d\left[|A_+|^2 - |A_-|^2\right]}{d\tau} \approx \left(\frac{H}{\xi}\right)^4 e^{2\pi\xi}$$

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    - Contribution to 2-point function $\langle\delta\phi\delta\phi\rangle_{loop}$
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    - Contribution to tensors
  - At large ξ: Backreaction on ϕ dynamics

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Outline

Thermalized Axion Inflation

- Include scattering of gauge bosons [3]:
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  - $\implies$ Thermalization during Inflation, with $T > H$
  - Very efficient if Standard Model gauge fields: $\gamma\gamma \leftrightarrow \ell^+\ell^-$
- New phenomenology:
  - Moves power of gauge fields to UV $k/a \sim T$, more inside the horizon
  - Expect: suppression of fluctuations in $\langle\delta\phi\delta\phi\rangle_{loop}$, $\langle\delta\phi\delta\phi\delta\phi\rangle_{loop}$ at Hubble scale

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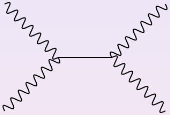
$$\begin{cases} N_\gamma(k) \simeq \quad 0, & k/a \gg H \\ N_\gamma(k) \simeq \frac{e^{2\pi\xi}}{8\pi\xi}, & k/a \ll H \end{cases}$$

Scatterings

Scatterings are enhanced by powers of N_γ



$$\frac{dN_\gamma(k)}{d\tau} = S(k)$$

$$S = \frac{1}{\omega(k)} \int \prod_{i=2}^{4} \left(\frac{d^3 p_i}{(2\pi)^3 (2E_i)} \right) |M_n|^2 (2\pi)^4 \delta^{(4)} \left(k^\mu + p_2^\mu - p_3^\mu - p_4^\mu \right) \cdot$$

$$\cdot N_\gamma(k) N_\gamma(p_2) \left[1 + N_\gamma(p_3)\right] \left[1 + N_\gamma(p_4)\right]$$

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$$N_+ \gg \sqrt{\beta_S \frac{Hf^4}{\omega^5}} \quad \underset{\omega \approx H}{\Longrightarrow} \quad \xi \gtrsim 0.45 \ln\left(\frac{f}{H}\right) + 2.7 \,,$$

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- Expectation: thermal bath of photons with temperature

$$T \approx \rho_{\gamma,\text{initial}}^{1/4} \approx 0.1 H e^{\pi\xi/2}$$

Boltzmann-like equations

- We rewrite the eom as a Boltzmann-like eq. $(g \equiv A'_+(k,\tau)/A_+(k,\tau))$:

$$\frac{dN_{\gamma_+}(k)}{d\tau} = -\frac{4k\xi}{\tau}\frac{\mathrm{Re}\,[g]}{|g|^2+k^2}\left(N_{\gamma_+}(k)+\frac{1}{2}\right)$$

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- Full system $(\gamma_+, \gamma_-, \phi)$

$$\begin{cases} N'_+ = -\frac{4k\xi}{\tau}\frac{\mathrm{Re}[g]}{|g|^2+k^2}\left(N_+ + \frac{1}{2}\right) + S^{++} + S^{+\phi} + D^{+\phi} + S^{+-} \,, \\ N'_u = -S^{+\phi} - D^{+\phi} \,, \\ N'_- = -S^{+-} \,, \end{cases}$$

(u$\equiv a\delta\phi$)

Numerical results

- Discretize: $\mathcal{O}(10)$ modes of comoving momentum: $k \in [1, \mathcal{O}(10)]H$.
- Duration of simulation: $\mathcal{O}(1)$ e-fold, $\{\tau_0 = -2, \tau_f = -1\}$

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- Duration of simulation: $\mathcal{O}(1)$ e-fold, $\{\tau_0 = -2, \tau_f = -1\}$
- Distribution of particles approaches Bose-Einstein distribution at ξ, f in agreement with estimations.

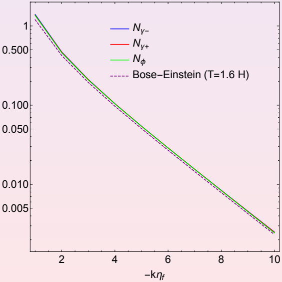
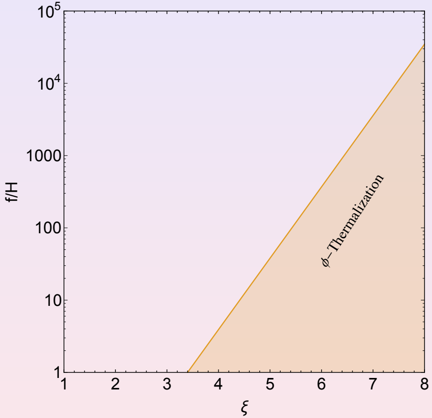


Figure: $\xi = 2$, $f/H = 0.1$

Thermalization



$$\Gamma_s \gg H \qquad \Longrightarrow \qquad \xi \gtrsim 0.45 \ln\left(\frac{f}{H}\right) + 2.7\,,$$

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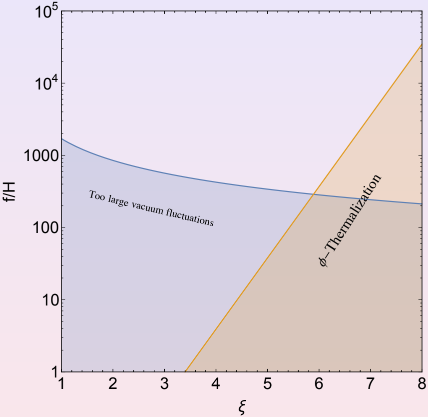
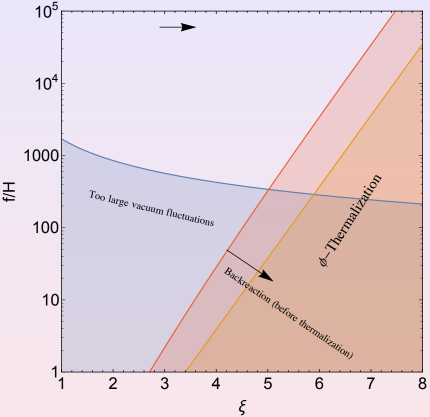


Vacuum fluctuations:

$$P_\zeta^{\mathrm{vac}} = \frac{H_*^4}{4\pi^2 \dot{\phi}_*^2} = \frac{H^2}{16\pi^2 f^2 \xi^2} \leq 2.2 \times 10^{-9} \qquad \Longrightarrow \qquad \frac{f}{H} \gtrsim \frac{2 \times 10^3}{\xi}\,.$$

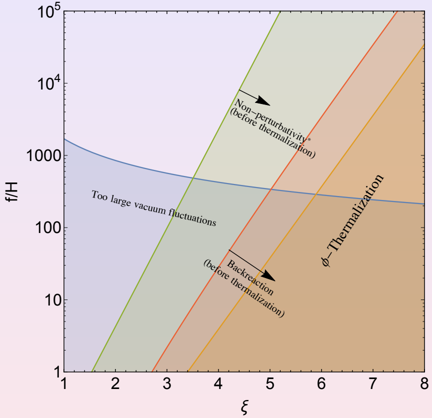
Thermalization



Backreaction on ϕ (before reaching thermalization):

$$\frac{\langle F\tilde{F}\rangle}{f} \gtrsim V'(\phi) \simeq 3H\dot{\phi} \qquad \Longrightarrow \qquad f/H \lesssim 4\times 10^{-3} e^{\pi\xi}/\xi^{5/2}.$$

Thermalization



* Requiring perturbativity on loop expansion for cosmological correlators, in absence of thermalization (Ferreira *et al.*,JCAP 1604 (2016)):

$$\frac{H^2}{f^2}\,\frac{e^{2\pi\xi}}{16\pi^2 I} < 1,$$

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  Requirement for thermalization ($\Gamma_s \gg H$) :

$$N_\gamma H \ll \frac{\alpha_{EM}}{\omega^2} \cdot H^3 N_\gamma^2 \implies \xi \gtrsim 2.9$$

SM Thermalization

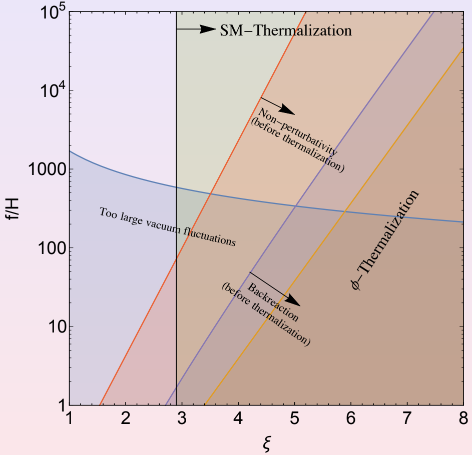

Figure: Summing over all $U_Y(1)$ charged particles in SM.

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- When $m_T \geq \xi H$ completely shields the instability band ($\omega^2 > 0$)
- Expect $T_{eq} \approx \frac{\xi H}{g}$ (or maybe oscillations?)
- New regime: linear in ξ, not exponential: $T_{eq} \ll (e^{2\pi\xi} H^4)^{1/4}$!

At equilibrium temperature

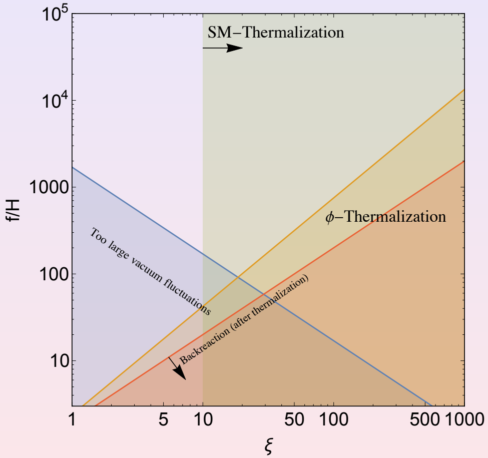

Figure: Using $T_{eq} \approx \frac{\xi H}{g}$, with $g = 0.5$.

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$$\begin{cases} P_\zeta^{\text{therm}} = \frac{\xi}{g}\frac{H_*^4}{2\pi^2\dot{\phi}_*^2} = \frac{H^2}{8\pi^2 f^2 \bar{g}\xi}, \\ n_s - 1 \equiv = -6\epsilon_H + 2\eta + \frac{\dot{\xi}}{H\xi} = -4\epsilon_H + \eta . \end{cases}$$

New regime at T_{eq}

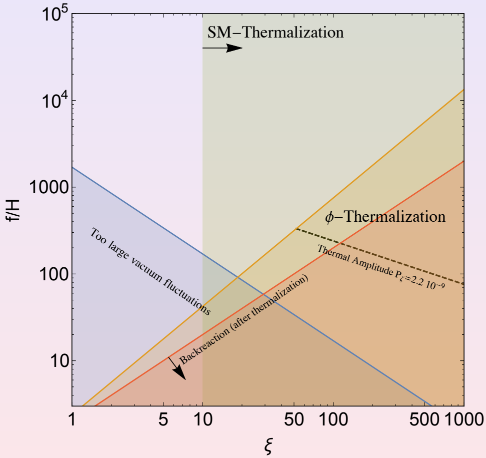

Figure: $P_\zeta = P_\zeta^{vac} \cdot \frac{2T_*}{H_*} = P_\zeta^{vac} \cdot \frac{2\xi}{g}$, $(g = 0.5.)$

Phenomenology in the thermal regime

- Loop effects on ζ drastically modified!
- Thermalization shifts gauge fields from horizon size to UV. At horizon crossing N_γ is reduced.
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  - Instead of non-thermal case: $f_{NL} \propto e^{4\pi\xi}$!
  - Constraints on ξ become weaker and (maybe) allow for the backreacting regime? (Work in progress)

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$$r \equiv \frac{P_T}{P_\zeta^{\text{therm}}} = 16\,\epsilon \frac{H_*}{2T_*}\,.$$

- Suppressed by $\frac{H_*}{2T_*} = \frac{g}{2\xi}$
- At least $\mathcal{O}(10^{-2})$ suppression.

Summary

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- Reheating is automatic and fixed (when $\rho_\gamma > V(\phi)$)
- P_ζ can be thermal
- Work in progress: Inclusion of Thermal masses, Non-gaussianity, Backreaction

Outline

Backreaction regime

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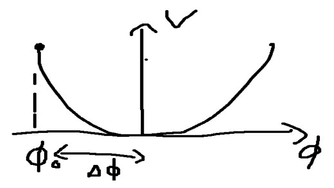
- Possible if f much smaller than field excursion $\Delta\phi$



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- Do we actually get $\dot{\phi} \approx$ *const* and $H \approx$ *const* ?

Our work

- We solved[4] numerically for $A_\pm$ from an initial condition:

$$\begin{aligned} A(0) &= \frac{1}{\sqrt{2k}}\,, \qquad \dot{A}(0) = \frac{ik}{\sqrt{2k}}\,, \\ \phi(0) &= \phi_0\,, \qquad \dot{\phi}(0) = 0\,. \end{aligned}$$

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- For $k < f$ (cutoff of the effective theory)

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Flat spacetime case: Results

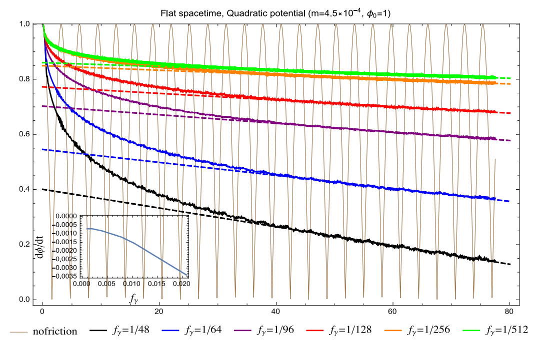
- The simplest case $H = 0$:
- The A_- and $\langle \tilde{F}F \rangle$ grow faster than the free evolution if

$$f_\gamma \ll \Delta\phi \implies \text{Friction}$$

- Independently on $V(\phi)$

Flat spacetime case: Results

ϕ evolution



$$\dot{\phi} \approx f_\gamma^2$$

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Inflationary case: Results

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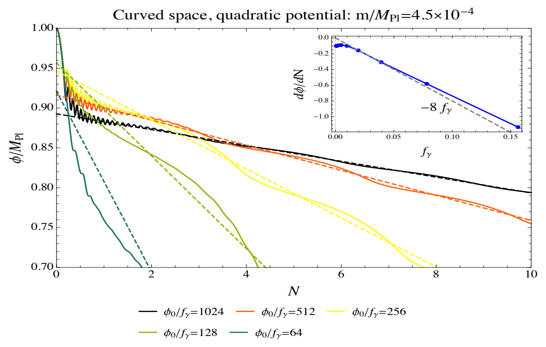
- So, we confirm $N_{TOT} \approx \frac{\Delta\phi}{f}$

FLRW case: Results

ϕ evolution



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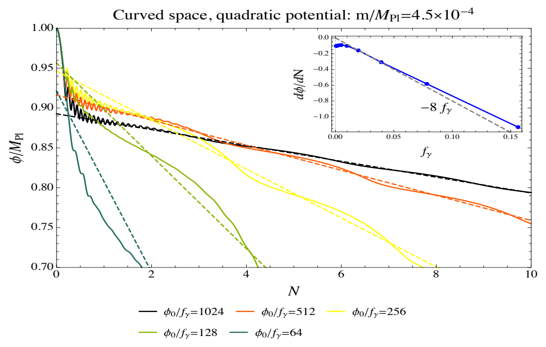

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Plus oscillations

FLRW case: Results

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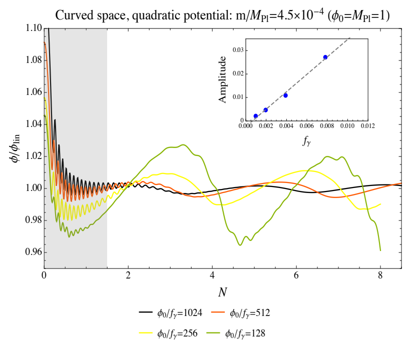

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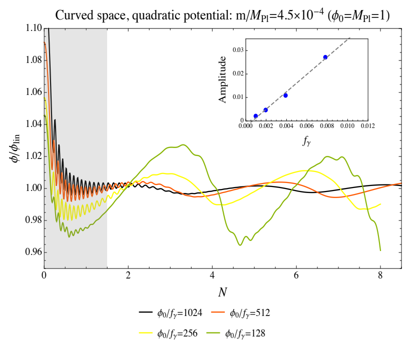
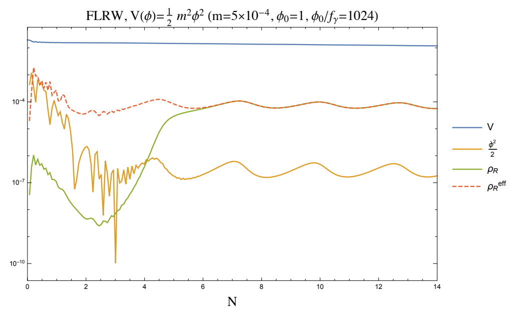


- Period of about 4 efolds
- Amplitude $\propto f$

FLRW case: Results

Energy densities:



$$V \gg \rho_R \gg \dot{\phi}^2/2$$

Reheating

- There is no separation between reheating and inflation
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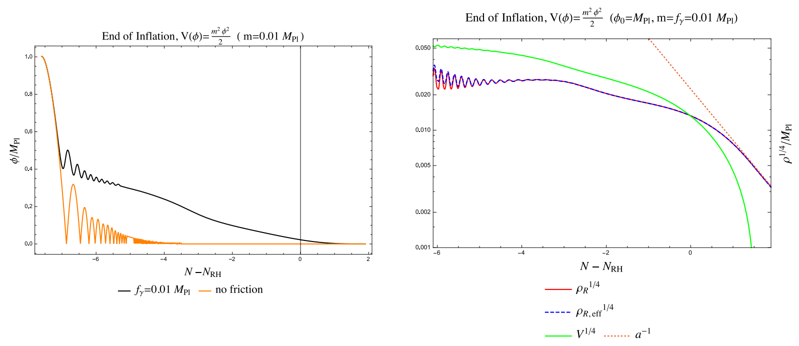

EXTRA SLIDES

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- It is possible: independent parameters.

FLRW case: Results

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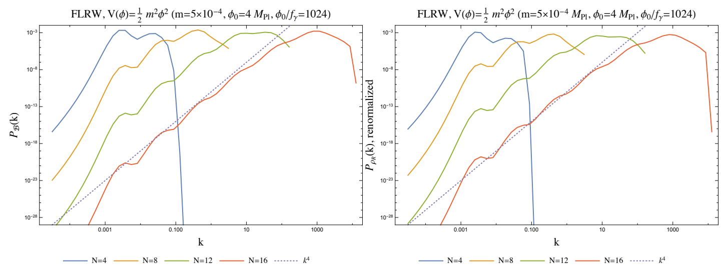

k^4 behavior

(suppression on large scales due to a^{-4})

Analytical example: linear potential

- Consider a constant force $V_{,\phi} \equiv -V_p < 0$. Free solution:

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