Effective field theory of inflation in Lifshitz regime of gravity

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Lifshitz regime of gravity

Non-reormalizability of GR

$$M_{pl}^2 \int d^4x \sqrt{-g} R \longrightarrow M_{pl}^2 \int d^4x [(\partial h)^2 + \underline{h(\partial h)^2} \cdots]$$

$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$

irrelevant op.

Isotropic scaling $t \to b^{-1} t,\ x^i \to b^{-1} x^i$ ⟹ $h \to b\, h$

Power counting renormalizability

Horava(09)

$$S \sim \int dt d^3x (\dot{h}^2 - h(-\Delta)^z h + \cdots)$$

Lifshitz scaling $t \to b^{-z} t,\ x^i \to b^{-1} x^i$ ⟹ $h \to b^{(3-z)/2} h$

e.g. z=3, scaling dim. of h =0 $\quad h^p \times h(-\Delta)^3 h$ relevant

Effective field theory

Write all the possible terms compatible w/Lifshitz scaling

gravity sector

$$\mathcal{L}_{HG} = N\sqrt{h}\Big\{\frac{M_*^2}{2}\Big[\frac{1}{\alpha_1}K_{ij}K^{ij} - \frac{1}{\alpha_2}K^2 + \frac{1}{\alpha_3}R + a_i a^i\Big] - \frac{1}{2}\Big[\frac{R_{ij}R^{ij}}{\beta_1} + \frac{R^2}{\beta_2} - \frac{R\nabla_i a^i}{\beta_3} + \frac{a_i \Delta a^i}{\beta_4}\Big] - \frac{1}{2M_*^2}\Big[\frac{(\nabla_i R_{jk})^2}{\gamma_1} + \frac{(\nabla_i R)^2}{\gamma_2} + \frac{\Delta R \nabla_i a^i}{\gamma_3} - \frac{a_i \Delta^2 a^i}{\gamma_4}\Big] + \ldots$$

M_* :Lorentz violating scale

$$\alpha \equiv \frac{M_*^2}{M_{pl}^2} \ll 1$$

Looks messy?

Effective field theory

Write all the possible terms compatible w/Lifshitz scaling

gravity sector

$$\mathcal{L}_{HG} = N\sqrt{h}\Big\{\frac{M_*^2}{2}\Big[\frac{1}{\alpha_1}K_{ij}K^{ij} - \frac{1}{\alpha_2}K^2 + \frac{1}{\alpha_3}R + a_i a^i\Big] - \frac{1}{2}\Big[\frac{R_{ij}R^{ij}}{\beta_1} + \frac{R^2}{\beta_2} - \frac{R\nabla_i a^i}{\beta_3} + \frac{a_i \Delta a^i}{\beta_4}\Big] - \frac{1}{2M_*^2}\Big[\frac{(\nabla_i R_{jk})^2}{\gamma_1} + \frac{(\nabla_i R)^2}{\gamma_2} + \frac{\Delta R \nabla_i a^i}{\gamma_3} - \frac{a_i \Delta^2 a^i}{\gamma_4}\Big] + \ldots$$

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Looks messy?

Yet, soft limit in Lifshitz regime shows universal behaviour.

$M_* << \omega/a << H_{\rm inf}$

Lifshitz scalar w/o metric perturbations

$$\mathcal{L}_\varphi \sim \frac{a^3}{2}\left(\dot{\varphi}^2 - \frac{\omega^2}{a^2}\varphi^2\right)$$

Mukohyama(09),…

dispersion relation $\omega^2 \sim p^2\left(\frac{p}{aM_*}\right)^{2(z-1)}$ z=1,2, …

at $\omega_p/a_p \sim H_p$ $P_{\rm LS}(p) \sim \frac{1}{a_p^2}\frac{1}{2\omega_p} \sim \frac{M_*^{3(z-1)/z}}{p^3} H_p^{\frac{3}{z}-1}$

GW spectrum

$$P_{\rm GW}(p) \sim \frac{1}{p^3}\left(\frac{H_p}{M_{pl}}\right)^{\frac{3}{z}-1} \qquad n_t \simeq -\frac{3-z}{z}\varepsilon_{1,p}$$

no-tilt for z=3

Including gravity

4D Diff. → Foliation preserving Diff t → t'(t)

- Khronon (Scalar graviton): DOF for time foliation
- Inflaton

Spatial metric $h_{ij} = a^2 e^{2\mathcal{R}} \delta_{ij}$ $\mathcal{R}$:Khronon

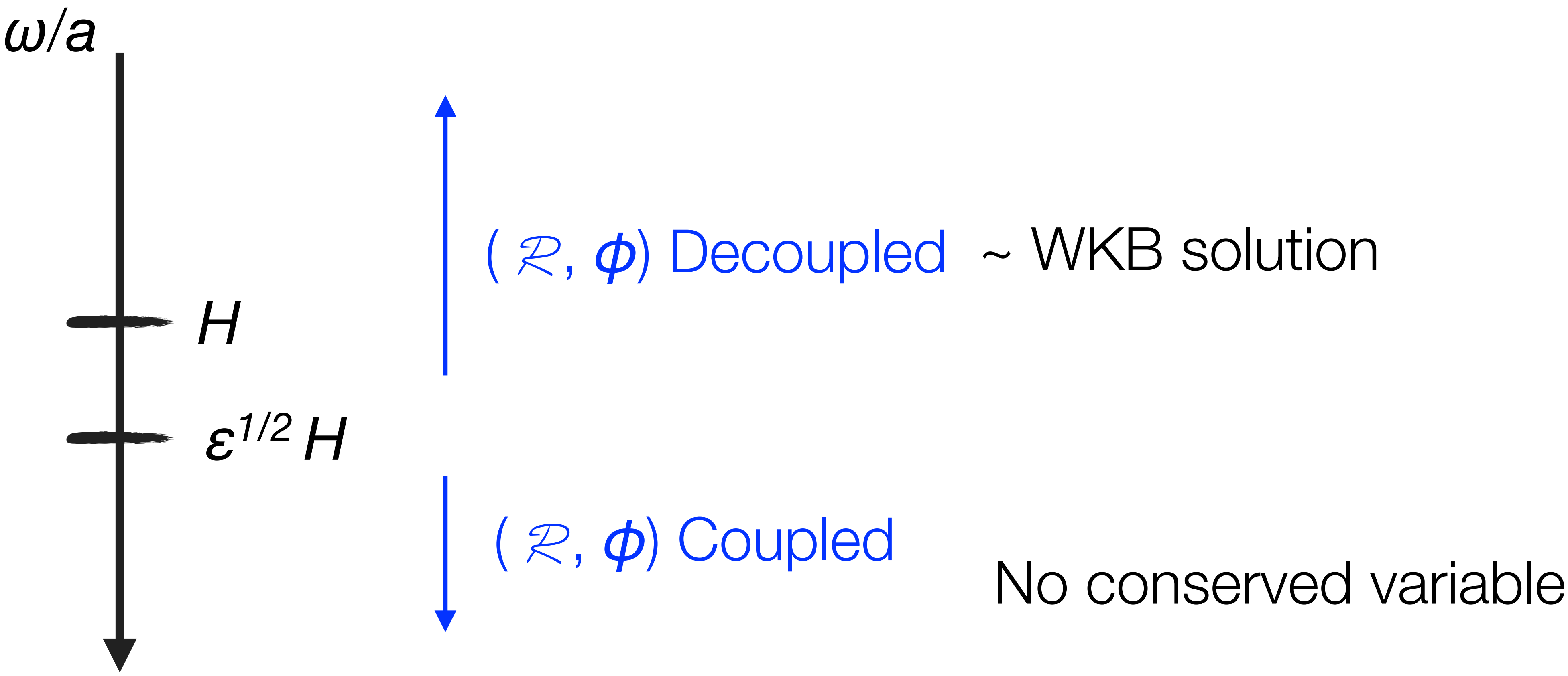
Dilatation sym. $\boldsymbol{x} \to e^s \boldsymbol{x}$ $\mathcal{R} - s$

Naive guess Two massless fields (ϕ, $\mathcal{R}$) ?

- Absence of conserved variable at large scales?
- Model dependent prediction?

Projectable version ($z \neq 1$) $N = N(t)$

Khronon $\mathcal{R}$ and Inflaton ϕ starts to be coupled, where gravity becomes important. *Arai, Sibiryakov, Y.U.*



No robust predictions

Non-projectable version $N = N(t, x^i)$

Arai, Sibiryakov, Y.U.

EoM for $\mathcal{R}$ becomes non-local, since ***N*** is non-local.

Taking de Sitter limit

$$\mathcal{L}_{\mathcal{R}} \sim M_{pl}^2 a^3 \left[\frac{\alpha(\omega/aH)^2}{1+\alpha^2(\omega/aH)^2} \dot{\mathcal{R}}^2 - \frac{\omega^2}{a^2}\left(C + (\omega/aH)^2\right)\mathcal{R}^2 \right]$$

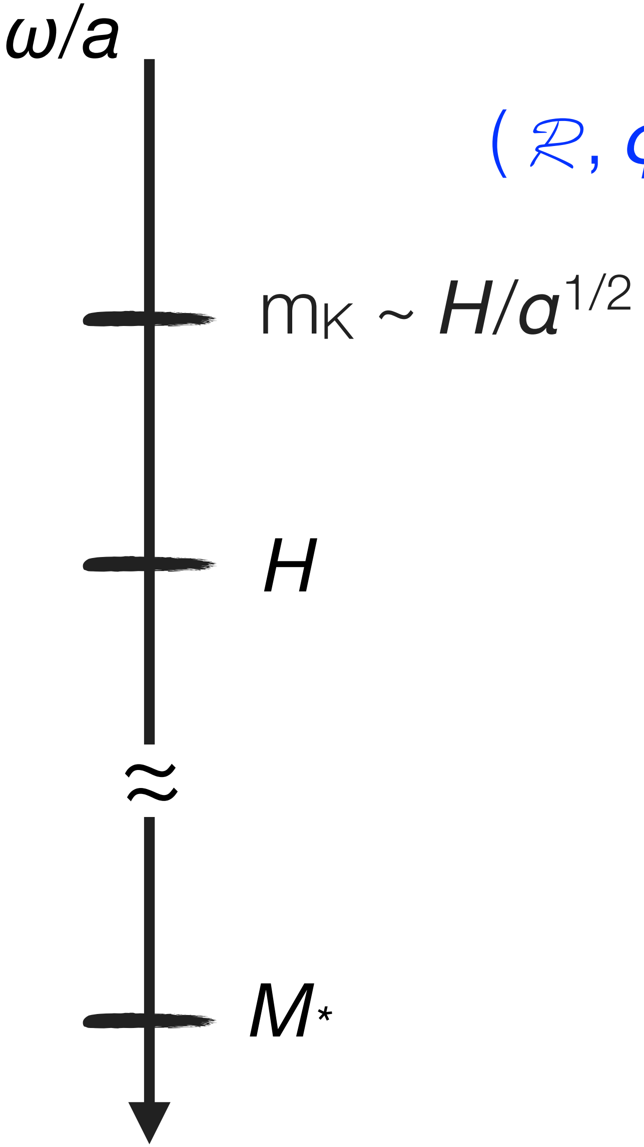
<u>dispersion relation</u> $\omega^2 \sim p^2 \left(\frac{p}{aM_*}\right)^{2(z-1)}$

- Lifshitz scaling regime, i.e., $H > M_*$ C=O(1)

For $\omega/a < H/a^{1/2}$, Khronon is gapped by $m_K \sim H/a^{1/2}$

Non-projectable version 2

Arai, Sibiryakov, Y.U.



($\mathcal{R}$, ϕ) Decoupled ~ WKB solution

renormalizability $\alpha \equiv \frac{M_*^2}{M_{pl}^2} \ll 1$

Non-projectable version 2

Arai, Sibiryakov, Y.U.

ω/a

$(\mathcal{R}, \phi)$ Decoupled ~ WKB solution

$m_K \sim H/\alpha^{1/2}$

renormalizability $\alpha \equiv \frac{M_*^2}{M_{pl}^2} \ll 1$

Khronon decoupled from inflaton sector

H

$^\exists$Conserved variable $\zeta \equiv \mathcal{R} - \frac{H}{\dot{\phi}}\varphi$

$\approx$

$$P_\zeta(p) \simeq \frac{1}{\varepsilon_1 M_{pl}^2} P_{\rm LS}(p) \simeq \frac{1}{\varepsilon_1 p^3}\left(\frac{H_p}{M_{pl}}\right)^{\frac{3}{z}-1}$$

M_*

Primordial power spectrum

Arai, Sibiryakov, Y.U.

Spectral index

$$n_s - 1 = -\frac{3-z}{z}\varepsilon_1 - \varepsilon_2 + \mathcal{O}(\varepsilon^2)$$

Tensor to scalar ratio

$$r = 16\varepsilon \left(\frac{\varkappa_z}{\varkappa_{\gamma,z}}\right)^{\frac{3}{2z}+\frac{\varepsilon}{2(z-\varepsilon)}\left(\frac{3}{z}-1\right)}$$

$$\omega_\phi^2 \sim \kappa_z\, p^2 \left(\frac{p}{aM_*}\right)^{2(z-1)}$$

$$\omega_\gamma^2 \sim \kappa_{\gamma,z}\, p^2 \left(\frac{p}{aM_*}\right)^{2(z-1)}$$

Consistency relation

$$n_t = -\frac{3-z}{z-\varepsilon}\frac{r}{16}\left(\frac{\varkappa_{\gamma,z}}{\varkappa_z}\right)^{\frac{3}{2z}+\frac{\varepsilon}{2(z-\varepsilon)}\left(\frac{3}{z}-1\right)}$$

Creminelli et al.(14)

N.B. ∃4D Diff $n_t = -\dfrac{r}{8c_s}$ for $c_s < 1$, $-n_t > r/8$

Violation of consistency relation → $c_s > 1$ or ~~4D Diff~~.

Anti-friction of Khronon

For $H > M_*$, $\omega/a < m_K$

Arai, Sibiryakov, Y.U.

$$\mathcal{L}_{\mathcal{R}} \sim M_*^2 a^3 \left(\frac{\omega}{aH}\right)^2 \left[\dot{\mathcal{R}}^2 - m_K^2 \mathcal{R}^2\right] \qquad \omega^2 \sim p^2 \left(\frac{p}{aM_*}\right)^{2(z-1)}$$

- Khronon $\mathcal{R}$, canonically normalised in UV

  Grows due to anti-friction as $\mathcal{R} \propto a^{\frac{2z-3}{2}} e^{-im_K t}$

- Canonical field in IR is $\mathcal{R}_c \sim M_* \dfrac{\omega}{aH} \mathcal{R}$

  No exponential growth

Changing foliation removes the anti-friction.

Summary

- Examined inflation in Lifshitz regime of gravity with $H > M_*$

- In non-projectable ver. ($H >> M_*$), Khronon gets gapped with $m_K >> H$ before the Hubble crossing.

↓

$P_\zeta(p)$, $P_{GW}(p)$ are robustly determined by the scaling.

- In projectable ver. and non-projectable ver. ($H < M_*$), need to solve mixed system w/ two light scalar fields.

- Violation of consistency relation indicates either super-luminal propagation or violation of 4D Diff.