Point Particle Effective Actions

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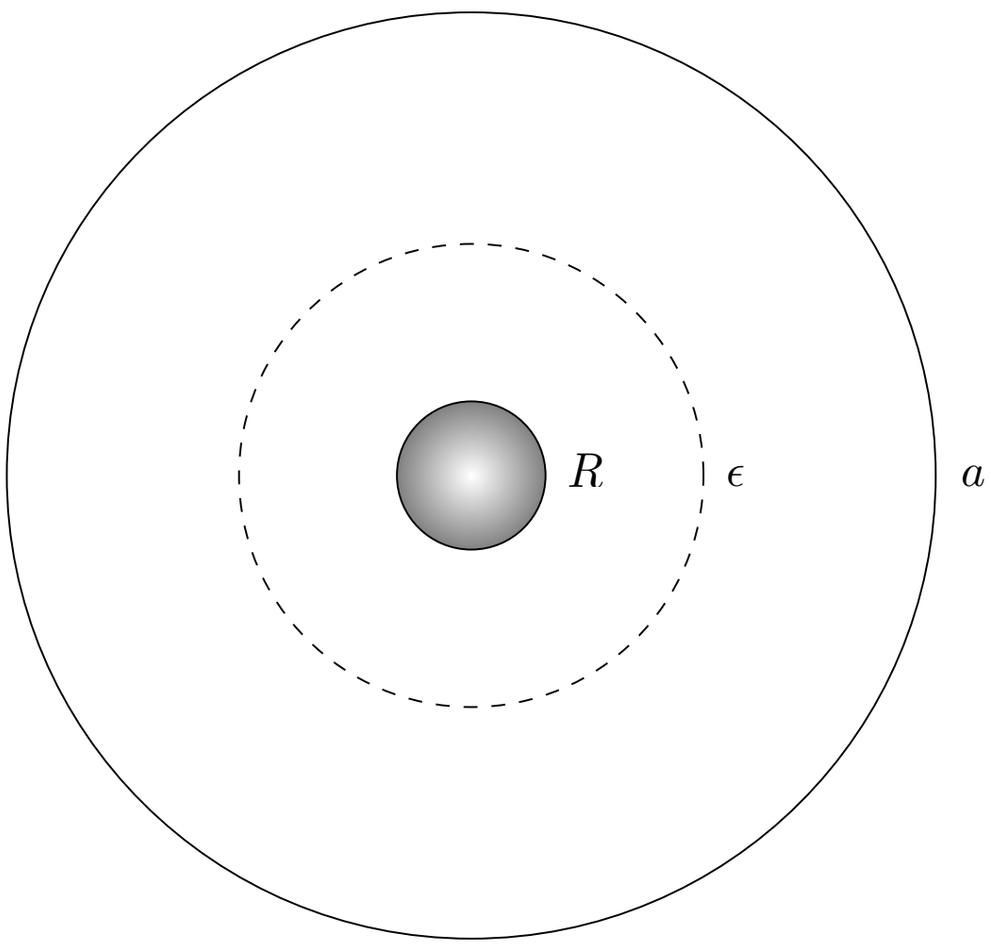
What is it

- An effective field theory (PPEFT) to describe a system of a heavy compact object and a light particle (bulk field)
- first-quantized language for the heavy compact object (localized at x = 0)
- second-quantized language for the light particle

What is it good for

- Lots of examples in nature: atoms, solar system, …
  [Goldberger, Rothstein 2006]
- Only a few properties of the compact object actually matter: observables as expansion in R/a

- Atoms
- Contact interactions (Pionic Atoms, Branes)
- Condensed Matter systems
- Monopole catalysis, Black holes



What is new

- Explicit connection between PPEFT and near-source boundary condition for bulk fields
- Recipe to calculate finite size corrections including back reaction
- Easier than full 2-quantized framework such as NRQED/ NRQCD in certain limits (bound states) [Caswell, Lepage 1986]
- By-product: Understanding of divergent potentials $(1/r^2)$ in Quantum mechanics via classical renormalization [Holstein 1996, Essin, Griffith 2006]

Action: Schrödinger field

$$S_{\rm tot} = S_{\rm Bulk} + S_{\rm brane}$$

Potential

bulk particle mass

$$S_B = \int \mathrm{d}t\, \mathrm{d}^3x \left\{ \frac{i}{2}\left(\Psi^* \partial_t \Psi - \Psi\, \partial_t \Psi^*\right) - \Psi^* \left[-\frac{1}{2m}\nabla^2 + V(\mathbf{x}) \right] \Psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right\}$$

$$S_b = -\int \mathrm{d}t\, \mathrm{d}^3x\, \delta^3(\boldsymbol{x}) \left[M - Q A_\mu \dot{y}^\mu + h \Psi^* \Psi - \tilde{h} \nabla \cdot \boldsymbol{E} + \ldots \right]$$

source mass

source charge

source world line

Contact interactions

Boundary Condition

0-th order in R (Gauss law):

$$\nabla^2 A^0 = Q\,\delta^3(\boldsymbol{x})\delta_0^\nu$$

1-st order in R:

$$\nabla^2\psi - 2m\left[V(\boldsymbol{x}) - E\right]\psi = 2m\,h\,\delta^3(\boldsymbol{x})\psi$$

Integrate over Gaussian pillbox of size ϵ :

$$\int_{\mathcal{S}} \mathrm{d}^3x\,\nabla^2\psi = \int_{\partial\mathcal{S}} \mathrm{d}^2x\,\mathbf{n}\cdot\nabla\psi = \int \mathrm{d}^2\Omega\,\left(r^2\,\frac{\partial\psi}{\partial r}\right)_{r=\epsilon} = 4\pi\epsilon^2\left(\frac{\partial\psi}{\partial r}\right)_{r=\epsilon}$$

$$\left[4\pi r^2\,\frac{\partial}{\partial r}A_0\right]_{r=\epsilon} = Q$$

$$\Rightarrow \quad A^0 = -\frac{Q}{4\pi\epsilon^2}$$

$$\left[4\pi r^2\,\frac{\partial}{\partial r}\ln\psi\right]_{r=\epsilon} = \lambda = 2m\,h$$

$\Rightarrow$ Boundary condition

Boundary condition

$$\left[4\pi r^2 \frac{\partial}{\partial r} \ln \psi\right]_{r=\epsilon} = \lambda \; = 2m\, h$$

- ψ depends on integration constants of bulk equations
- The BC relates these with the effective coupling constants like h

$\Rightarrow$ Physical observables depend on h

The potential and bulk solutions

- Consider potential $V(\mathbf{x}) = -\frac{s}{r} - \frac{g}{r^2}$
- For small r the solutions are $\psi = C_+\psi_+ + C_-\psi_-$

  with $\psi_\pm(r) \simeq (2\kappa r)^{\frac{1}{2}(-1\pm\zeta_s)}$

  with $\kappa = -2m\,E$ and $\zeta_s = \sqrt{1-8mg}$

$$\lambda = 4\pi\epsilon^2 \left(\frac{\partial}{\partial r}\,\ln\psi\right)_{r=\epsilon} = -2\pi\epsilon\left[1+\zeta_s\left(\frac{R-1}{R+1}\right)\right], \quad R := \left(\frac{C_-}{C_+}\right)(2\kappa\epsilon)^{-\zeta_s}$$

Running

Physical observables do not depend on ϵ ! $\Rightarrow \frac{d\kappa}{d\epsilon} = 0$

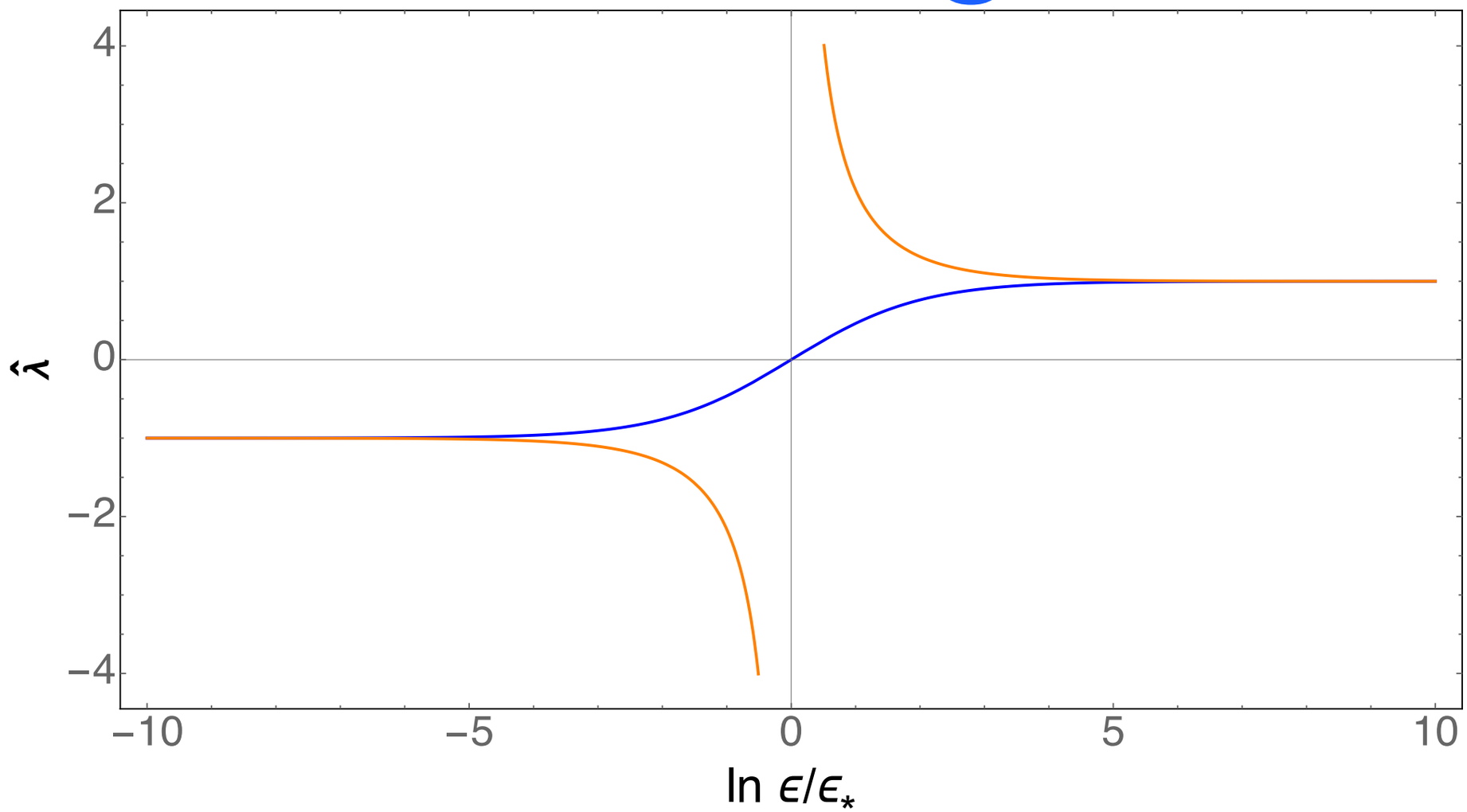
- Differentiating the boundary condition yields

$$\epsilon \frac{\mathrm{d}}{\mathrm{d}\epsilon}\left(\frac{\hat{\lambda}}{\zeta_s}\right) = \frac{\zeta_s}{2}\left[1 - \left(\frac{\hat{\lambda}}{\zeta_s}\right)^2\right]$$

  where $\hat{\lambda} := (\lambda/2\pi\epsilon) + 1$

- Fixed points: $\hat{\lambda} = \pm\zeta_s = \pm\sqrt{1 - 8mg}$
- Trivial fixed points $\lambda = 0$ only if $g = 0$ (no $1/r^2$ potential)

Running



- Klein-Gordon equation can be brought into Schrödinger form with $V(r) = -\frac{2\omega\, Z\alpha}{r} - \frac{(Z\alpha)^2}{r^2}$
- Relativistic scalars always run non-trivially!

What about fermions?

$$S_p = -\int_{\mathcal{W}} d\tau \; \left[M - Q\, A_\mu \dot{y}^\mu + c_s\, \bar{\psi}\, \psi + i c_v\, \bar{\psi}\, \gamma_\mu \psi\, \dot{y}^\mu - \tfrac{2\pi}{3} Z\alpha\, r_p^2\, \nabla \cdot \boldsymbol{E} + \cdots\right]$$

- $\delta E \sim (Z\alpha)^4\, r_p^2$ is IR fixed point of the Dirac PPEFT

  $\Rightarrow$ nothing new on the Proton radius problem

- Efficient parametrization of corrections:

$$\delta E \simeq \frac{(Z\alpha)^4 m^3}{n^3} R^2 \left\{ 2(1+2\hat{g}_1)\left[1 - (Z\alpha)^2 \left[\ln\left(\frac{2mRZ\alpha}{n}\right) + H_{n+1} + \gamma\right]\right] \right.$$
$$+ \left[4\hat{g}_3 + 5 + 8\hat{g}_1 - 2\hat{g}_1^2 + (1+2\hat{g}_1)\frac{12n^2 - n - 9}{2n^2(n+1)}\right](Z\alpha)^2$$
$$\left. + \left[2\hat{g}_2 - \frac{1}{3n^2}\left[1 + 8n^2\left(1 + \frac{3}{2}\hat{g}_1(\hat{g}_1+2)\right)\right]\right](mRZ\alpha) + \ldots \right\}$$

moments e.g.

$$(1+2\hat{g}_1)R^2 = \frac{r_p^2}{3}$$

- Checked with literature for toy charge distributions

[Friar 1979, Nickel 2013]

More applications

- Scattering for contact interactions (Deser formula) [Deser, Goldberger, Baumann, Thirring 1954]
- Vacuum polarization (Ueling potential) [Ueling 1935]
- (Mesonic) Atoms with e.g. $\pi^-, K^-, \bar{p}$ orbiting
- Exotic interactions [e.g. Tucker-Smith, Yavin 2011, Barger, Chiang, Keung, Marfatia 2011, Batell, McKeen, Pospelov 2011, Carroll, Thomas, Rafelski, Miller 2011]

Conclusions & Outlook

- PPEFT is efficient way to calculate finite size effects when source particle is approximately at rest (e.g. finite size corrections of nucleus in atoms)
- Manifests itself via boundary condition and classical renormalization
- Works in Schrödinger, Klein-Gordon and Dirac context
- Only the beginning: More applications on the way