Non-Thermal Gravitino Production After Large-Field Inflation

Takahiro Terada

(JSPS research fellow, KEK)

with Y. Ema, K. Mukaida, and K. Nakayama, JHEP 1611 (2016) 184 [arXiv:1609.04716 [hep-ph]])

[Particle Physics Medal: Young Scientist Award in Theoretical Particle Physics]

June 22, PASCOS 2017, Madrid

Introduction

Inflation in Supergravity

Gravitino

Introduction

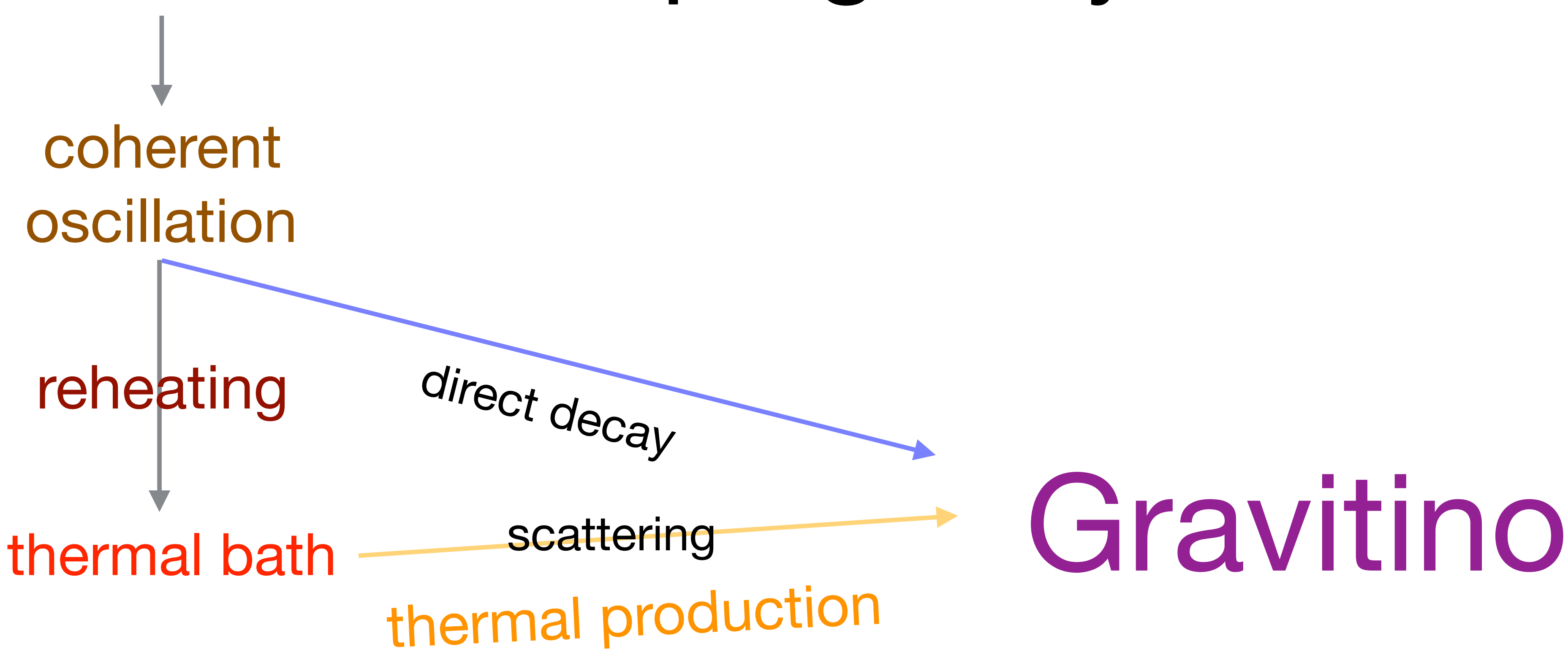
Inflation in Supergravity

coherent
oscillation

Gravitino

Introduction

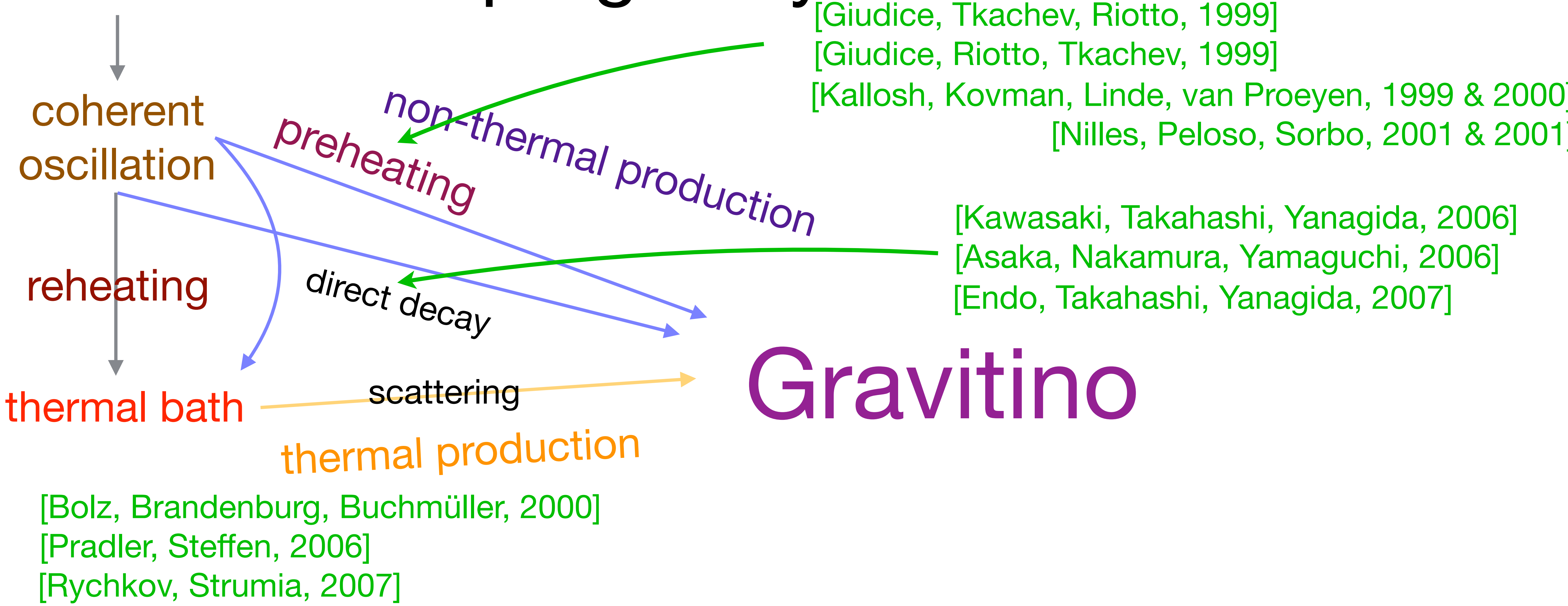
Inflation in Supergravity



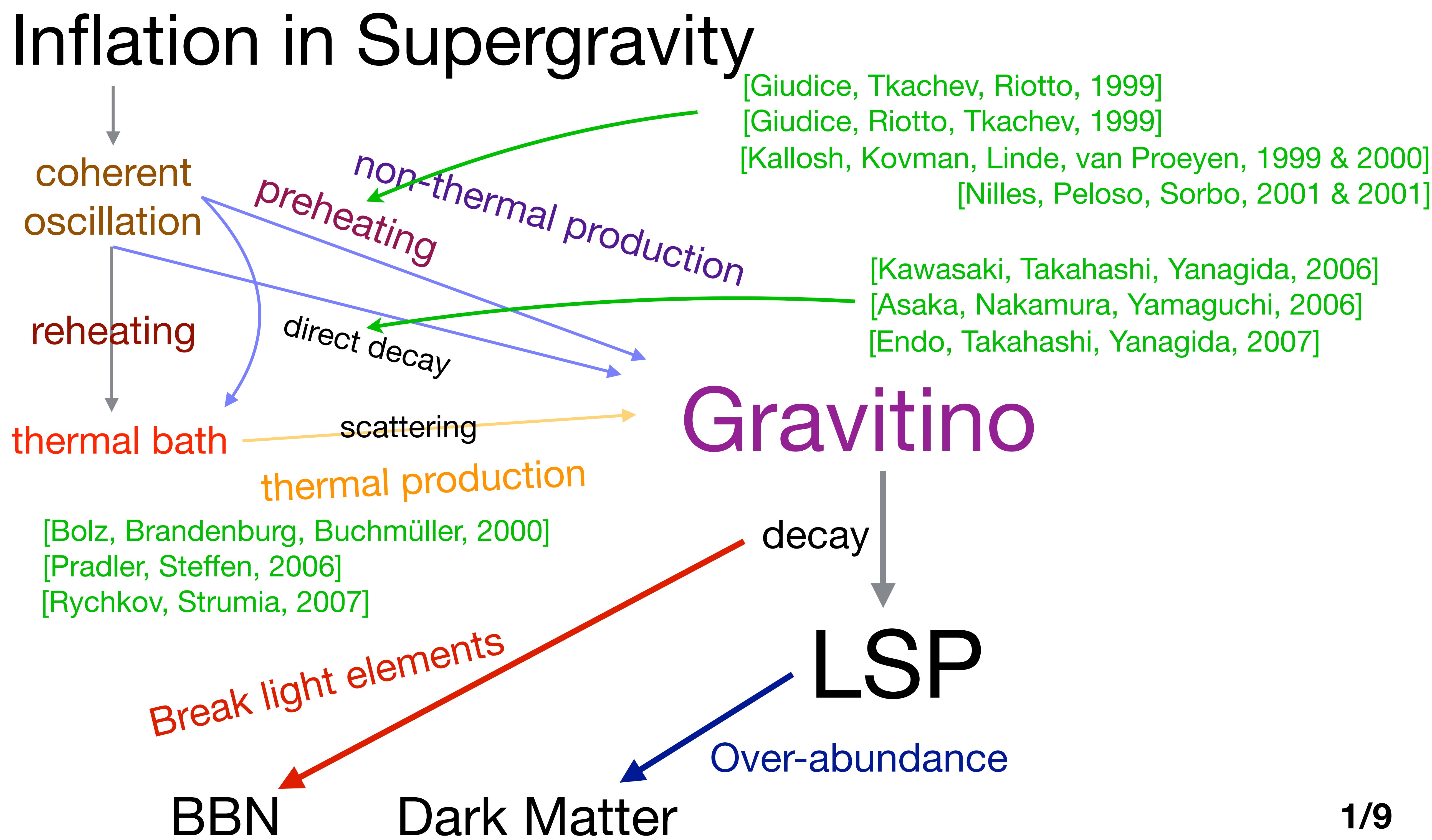
[Bolz, Brandenburg, Buchmüller, 2000]
[Pradler, Steffen, 2006]
[Rychkov, Strumia, 2007]

Introduction

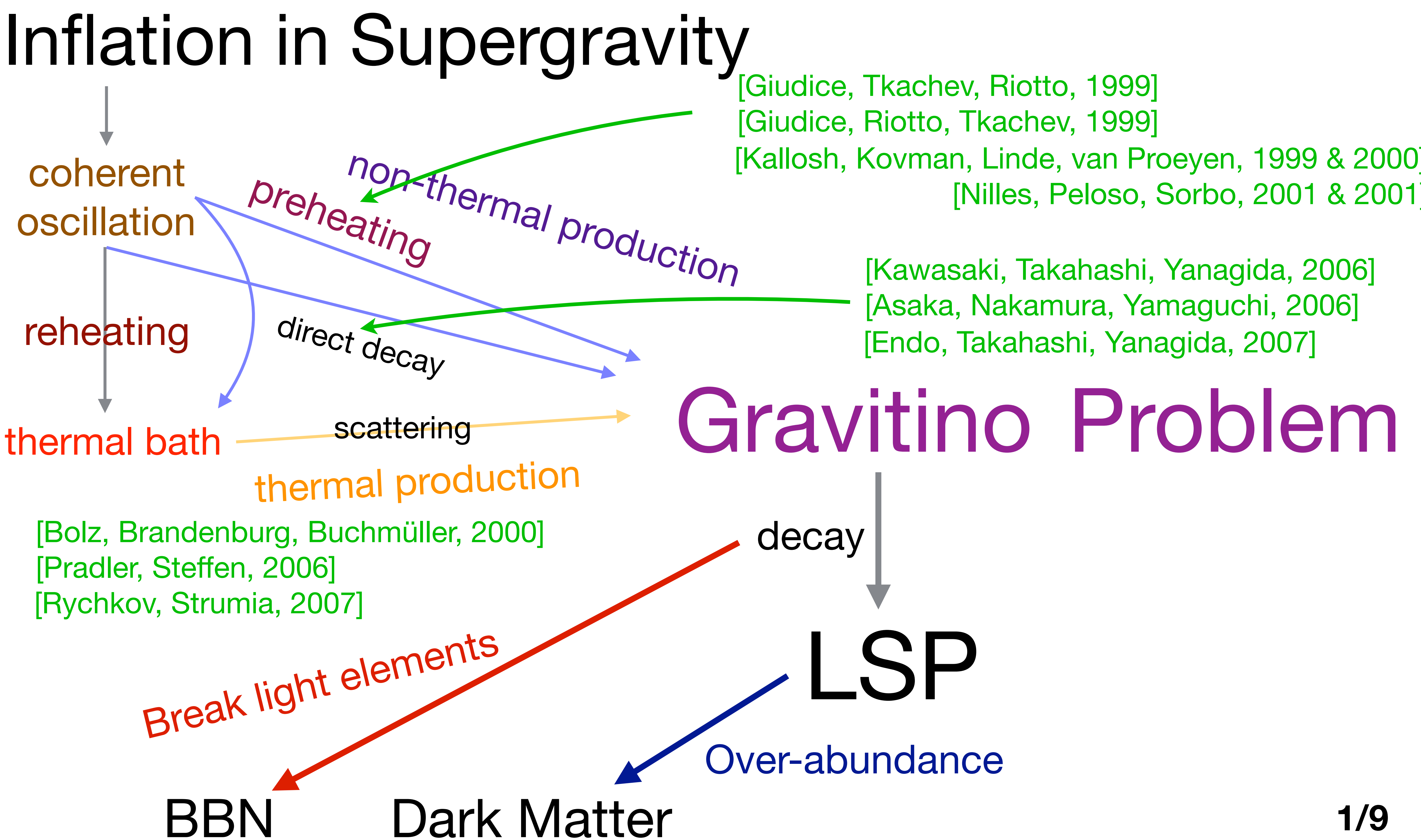
Inflation in Supergravity



Introduction



Introduction



Earlier works/ Our improvements

✗ Too simple toy models

→ ✓ Realistic models

✗ Polonyi problem

→ ✓ No Polonyi problem

✗ Irrelevant abundance

→ ✓ Relevant abundance

✗ Lack of analytic understanding

→ ✓ Analytic formulae

Earlier works/
Our improvements

inflaton

$$K = |\Phi|^2$$

$$W = \frac{1}{2} m \Phi^2$$

Earlier works/
Our improvements

inflaton

✘ Too simple
(No inflation)

$$\left[\begin{array}{l} K = |\Phi|^2 \\ W = \dfrac{1}{2} m\Phi^2 \end{array}\right.$$

Earlier works/ Our improvements

inflaton

Polonyi field

✘ Too simple (No inflation)

$$K = |\Phi|^2 + |z|^2$$

$$W = \frac{1}{2} m\Phi^2 + \mu^2 z + W_0$$

Earlier works/ Our improvements

inflaton

Polonyi field

✗ Too simple (No inflation)

$$\left[\begin{aligned} K &= |\Phi|^2 + |z|^2 \\ W &= \frac{1}{2}m\Phi^2 + \mu^2 z + W_0 \end{aligned}\right.$$

✗ Polonyi problem

Earlier works/
Our improvements

inflaton

Polonyi field

✗ Too simple (No inflation)

$$K = |\Phi|^2 + |z|^2$$

$$W = \frac{1}{2}m\Phi^2 + \mu^2 z + W_0$$

✗ Polonyi problem

✗ Irrelevant abundance

Abundance of

gravitino = inflatino

gravitino = Polonyino

Earlier works/ Our improvements

inflaton

Polonyi field

✘ Too simple (No inflation)

$$K = |\Phi|^2 + |z|^2$$

$$W = \frac{1}{2} m \Phi^2 + \mu^2 z + W_0$$

✘ Polonyi problem

✘ Irrelevant abundance

Abundance of

gravitino = inflatino

gravitino = Polonyino

✘ Lack of analytic understanding

Earlier works/
Our improvements

$$K = -\frac{1}{2}(\Phi - \bar{\Phi})^2 + |z|^2 - \frac{1}{\Lambda^2}|z|^4$$

$$W = \frac{1}{2}m\Phi^2 + \mu^2 z + W_0$$

Abundance of

gravitino = inflatino

↓

gravitino = Polonyino

Earlier works/
Our improvements

[Kawasaki, Yamaguchi, Yanagida, 2000]

✓ "Realistic"

- shift symmetry

(• stabilizer field)

$$K = -\frac{1}{2}(\Phi - \bar{\Phi})^2 + |z|^2 - \frac{1}{\Lambda^2}|z|^4$$

$$W = \frac{1}{2}m\Phi^2 + \mu^2 z + W_0$$

Abundance of

gravitino = inflatino

↓

gravitino = Polonyino

Earlier works/ Our improvements

[Kawasaki, Yamaguchi, Yanagida, 2000]

✓ "Realistic"

- shift symmetry

(· stabilizer field)

$$K = -\frac{1}{2}(\Phi - \bar{\Phi})^2 + |z|^2 - \frac{1}{\Lambda^2}|z|^4$$

$$W = \frac{1}{2}m\Phi^2 + \mu^2 z + W_0$$

✓ No Polonyi problem

[Nakayama, Takahashi, Yanagida, 2012]

Abundance of

gravitino = inflatino

gravitino = Polonyino

Earlier works/
Our improvements

[Kawasaki, Yamaguchi, Yanagida, 2000]

"Realistic"

- shift symmetry

(• stabilizer field)

$$K = -\frac{1}{2}(\Phi - \bar{\Phi})^2 + |z|^2 - \frac{1}{\Lambda^2}|z|^4$$

$$W = \frac{1}{2}m\Phi^2 + \mu^2 z + W_0$$

No Polonyi problem

[Nakayama, Takahashi, Yanagida, 2012]

Abundance of

gravitino = inflatino

gravitino = Polonyino

Analytic,

time-dependent diagonalization

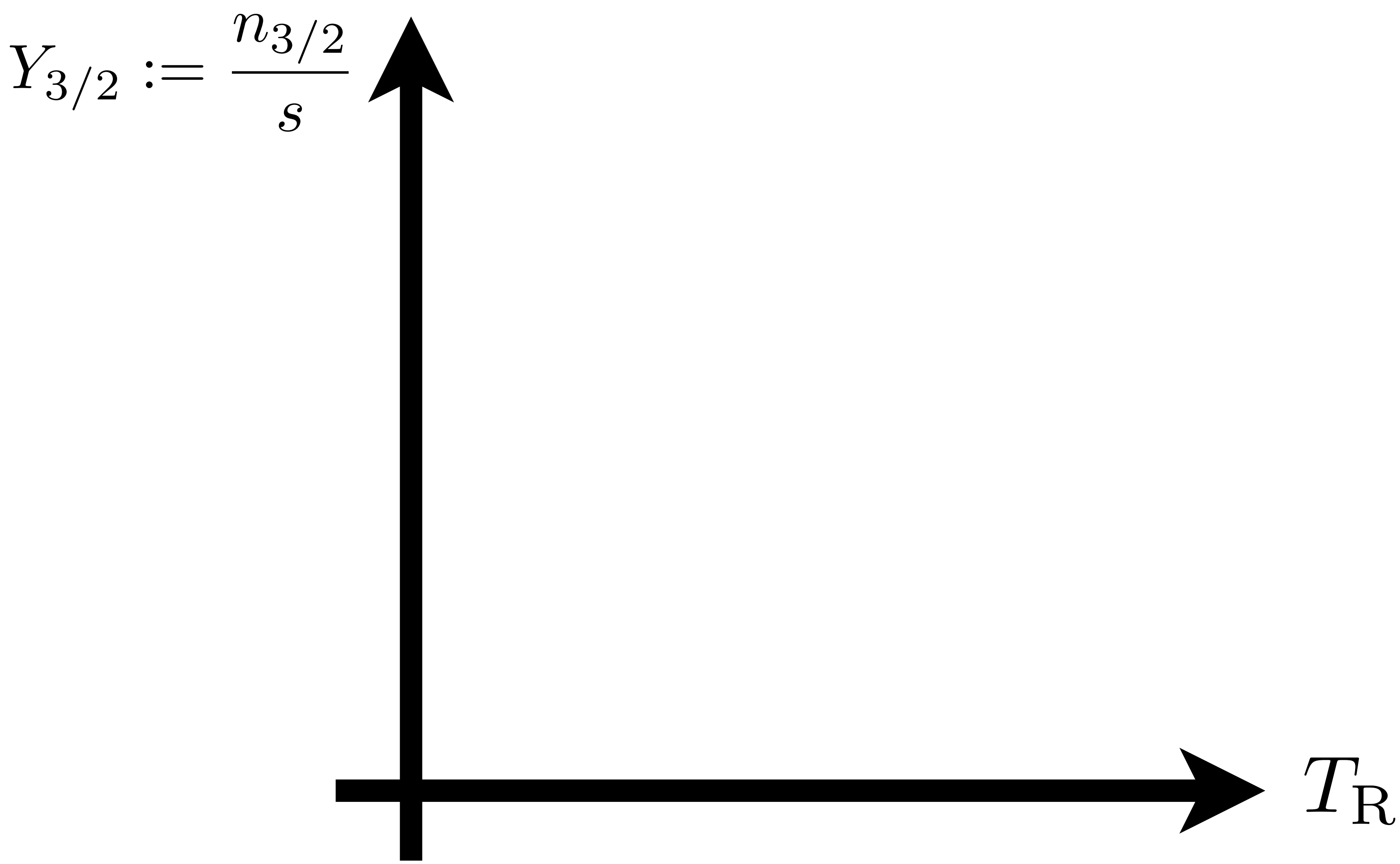
Our goal

$$Y_{3/2} = Y_{3/2}(T_{\mathrm{R}}, H_{\mathrm{inf}}, m_{3/2})$$

$$Y_{3/2} := \frac{n_{3/2}}{s}$$

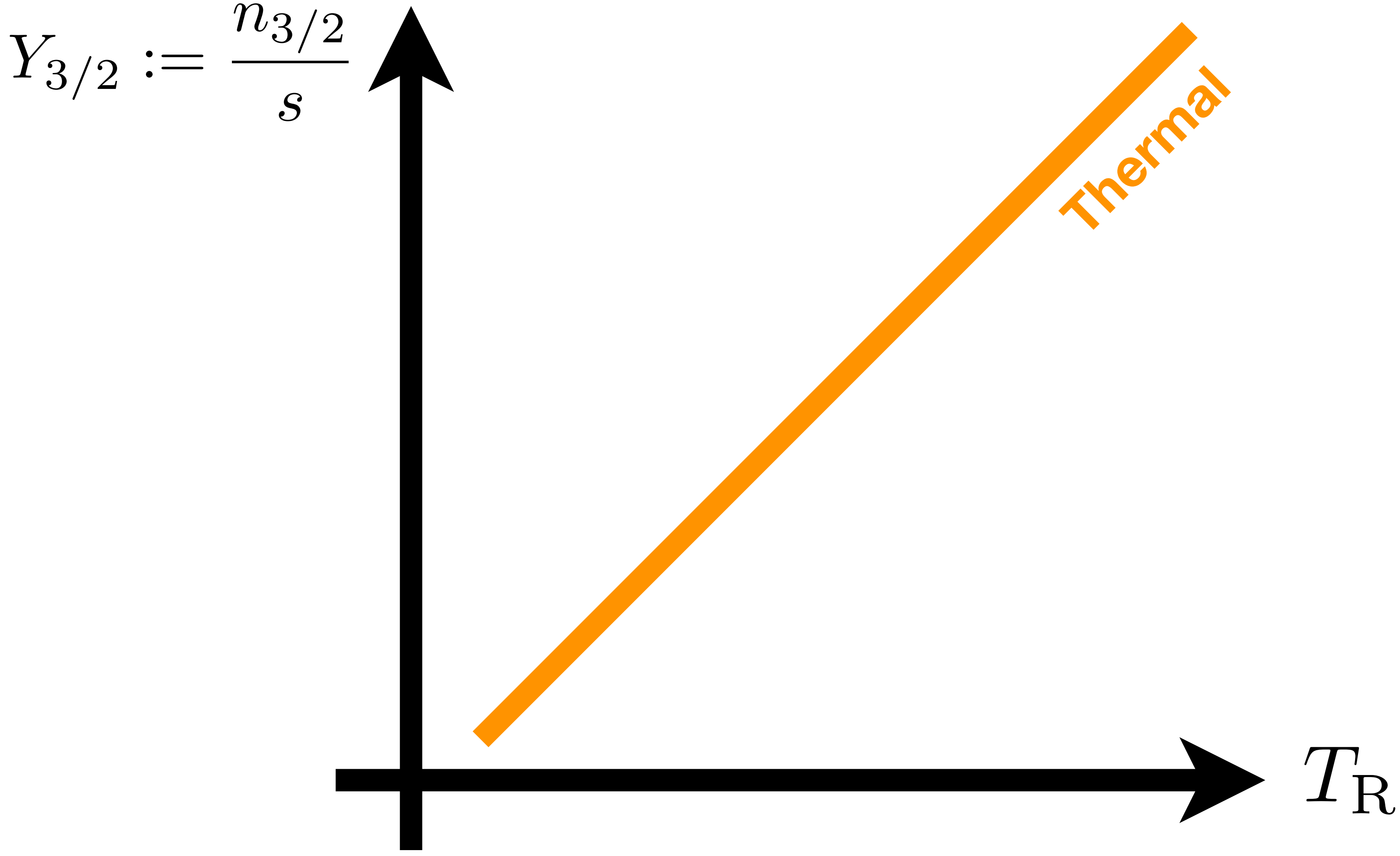
Our goal

$$Y_{3/2} = Y_{3/2}(T_\mathrm{R}, H_\mathrm{inf}, m_{3/2})$$



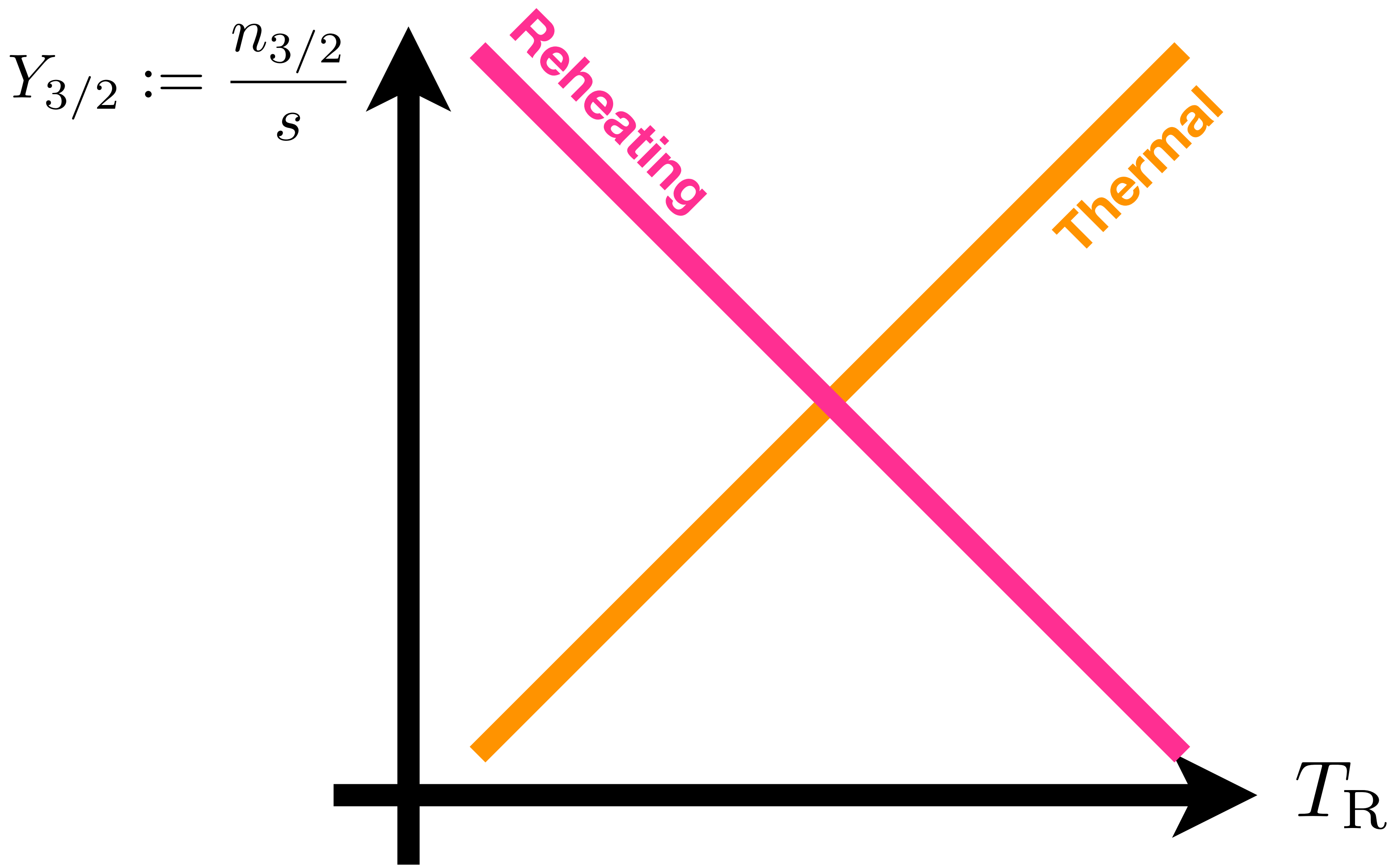
Our goal

$$Y_{3/2} = Y_{3/2}(T_{\rm R}, H_{\rm inf}, m_{3/2})$$



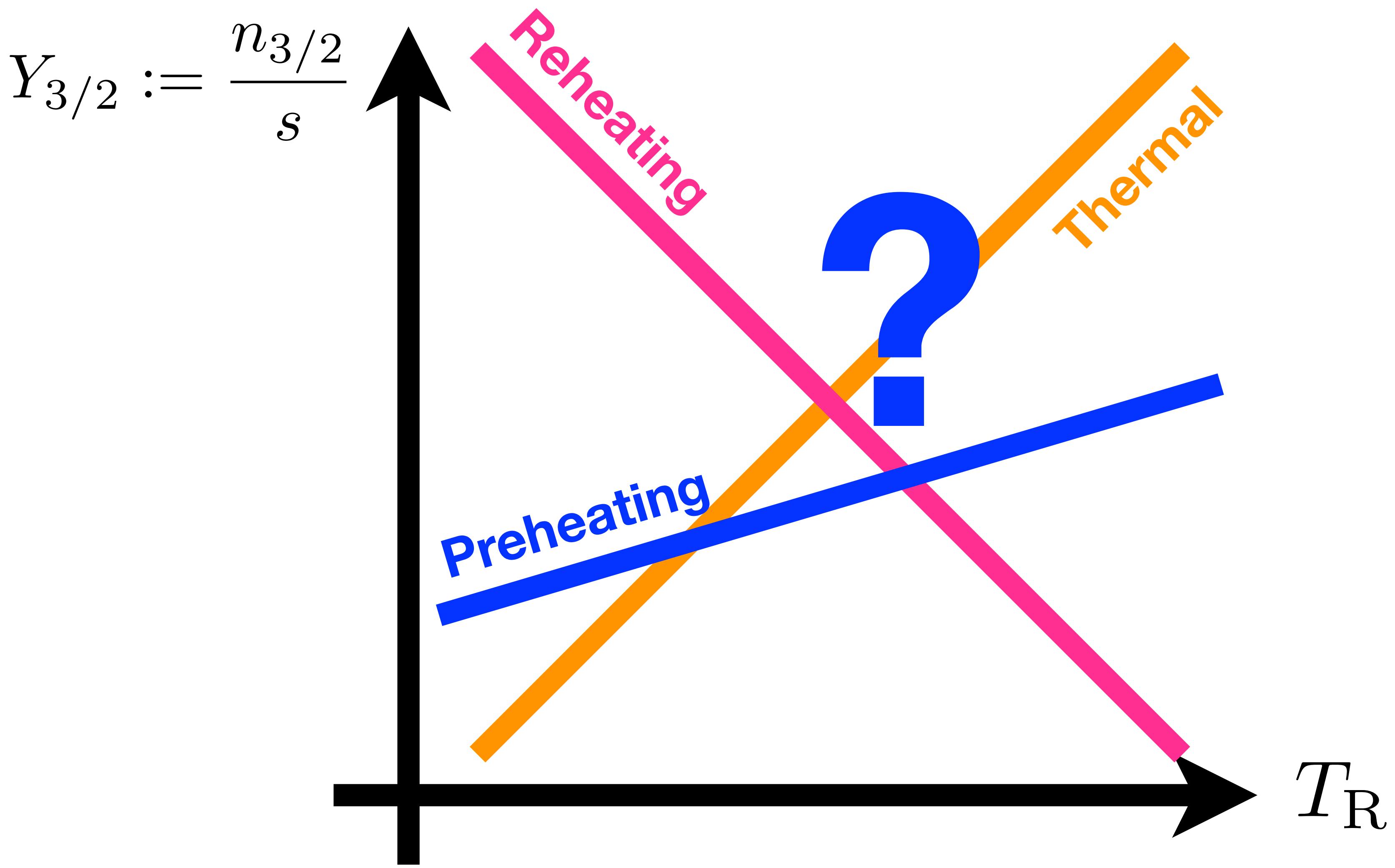
Our goal

$$Y_{3/2} = Y_{3/2}(T_{\rm R}, H_{\rm inf}, m_{3/2})$$



Our goal

$$Y_{3/2} = Y_{3/2}(T_\mathrm{R}, H_\mathrm{inf}, m_{3/2})$$



Procedure

Procedure

1. Lagrangian

Procedure

1. Lagrangian

2. Dynamical d.o.f.

Procedure

1. Lagrangian
2. Dynamical d.o.f.
3. Canonical normalization

Procedure

1. Lagrangian
2. Dynamical d.o.f.
3. Canonical normalization
4. Diagonalization

Procedure

1. Lagrangian
2. Dynamical d.o.f.
3. Canonical normalization
4. Diagonalization
5. Oscillating gravitino mass $\leftrightarrow$ Model parameters

Procedure

1. Lagrangian
2. Dynamical d.o.f.
3. Canonical normalization
4. Diagonalization
5. Oscillating gravitino mass $\leftrightarrow$ Model parameters
6. Fermion production formula

Set-up & Assumptions

Set-up & Assumptions

- Focus: $\mathbb{Z}_2$-symmetric large-field models

Set-up & Assumptions

- Focus: $\mathbb{Z}_2$-symmetric large-field models
- Kähler metric is (almost) **minimal** after inflation.

Set-up & Assumptions

- Focus: $\mathbb{Z}_2$-symmetric large-field models
- Kähler metric is (almost) **minimal** after inflation.
- Scalar field configuration is **real**.

Set-up & Assumptions

- Focus: $\mathbb{Z}_2$-symmetric large-field models
- Kähler metric is (almost) **minimal** after inflation.
- Scalar field configuration is **real**.
- D-term is **irrelevant**.

Models w/o Stabilizer

[Goncharov, Linde, 1984]
[Ketov, Terada, 2014]

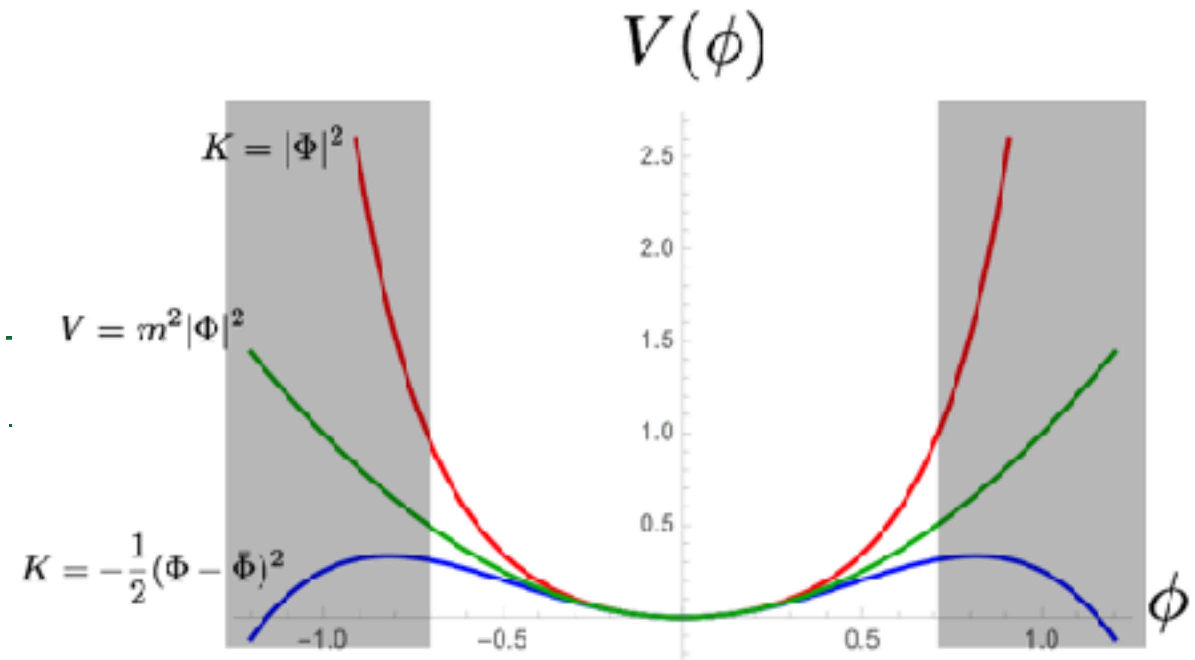
$$\left[\begin{array}{l} K = -\frac{1}{2}(\Phi - \bar{\Phi})^2 + |z|^2 - \frac{1}{\Lambda^2}|z|^4 \\ W = \frac{1}{2}m\Phi^2 + \mu^2 z + W_0 \end{array}\right.$$

Models w/o Stabilizer

[Goncharov, Linde, 1984]
[Ketov, Terada, 2014]

$$K = -\frac{1}{2}(\Phi - \bar{\Phi})^2 + |z|^2 -$$

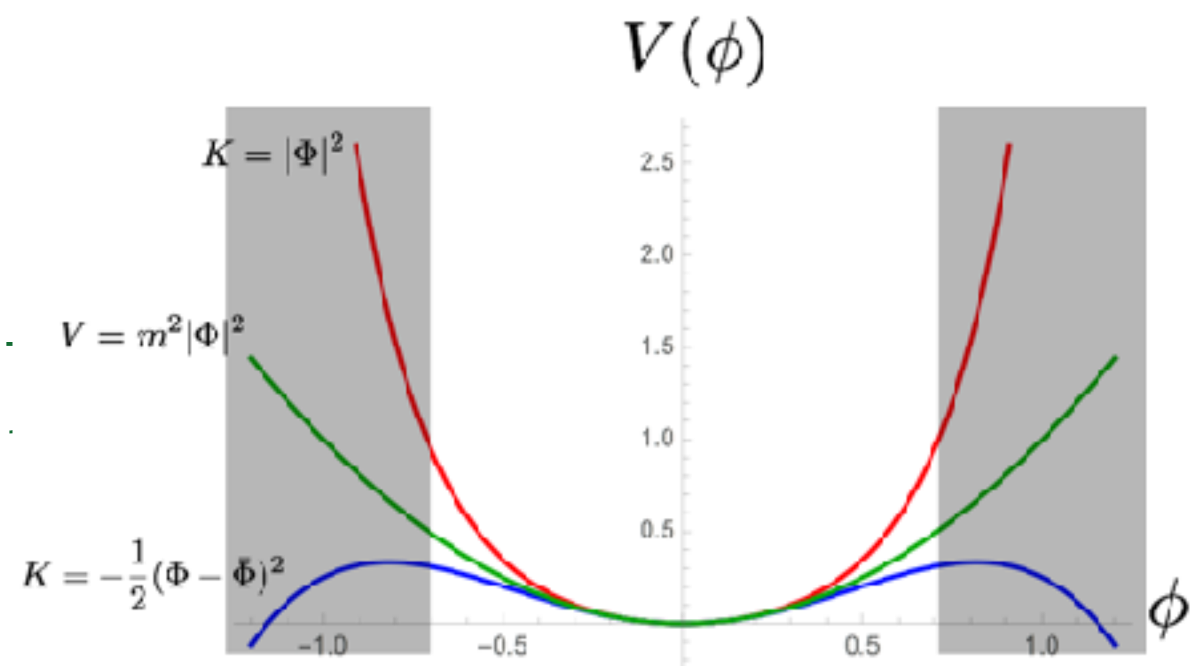
$$W = \frac{1}{2}m\Phi^2 + \mu^2 z + W_0$$



Models w/o Stabilizer

[Goncharov, Linde, 1984]
[Ketov, Terada, 2014]

$$\left[\begin{array}{l} K = -\frac{1}{2}(\Phi - \bar{\Phi})^2 + |z|^2 - \\ W = \frac{1}{2}m\Phi^2 + \mu^2 z + W_0 \end{array}\right.$$



SUSY breaking

$$|\dot{\phi}| \sim |F_\phi| \sim HM_{\rm P},$$

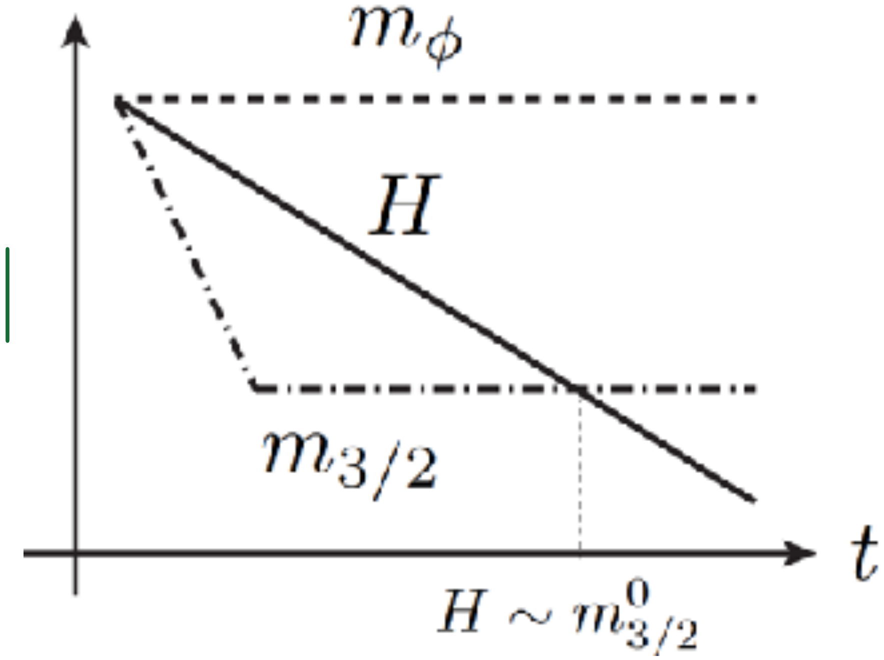
$$|F_z| \sim m^0_{3/2} M_{\rm P}$$

Models w/o Stabilizer

[Goncharov, Linde, 1984]
[Ketov, Terada, 2014]

$$K = -\frac{1}{2}(\Phi - \bar{\Phi})^2 + |z|^2 - \frac{1}{\Lambda^2}|$$

$$W = \frac{1}{2}m\Phi^2 + \mu^2 z + W_0$$



SUSY breaking

$$|\dot{\phi}| \sim |F_\phi| \sim H M_{\rm P},$$

$$|F_z| \sim m_{3/2}^0 M_{\rm P}$$

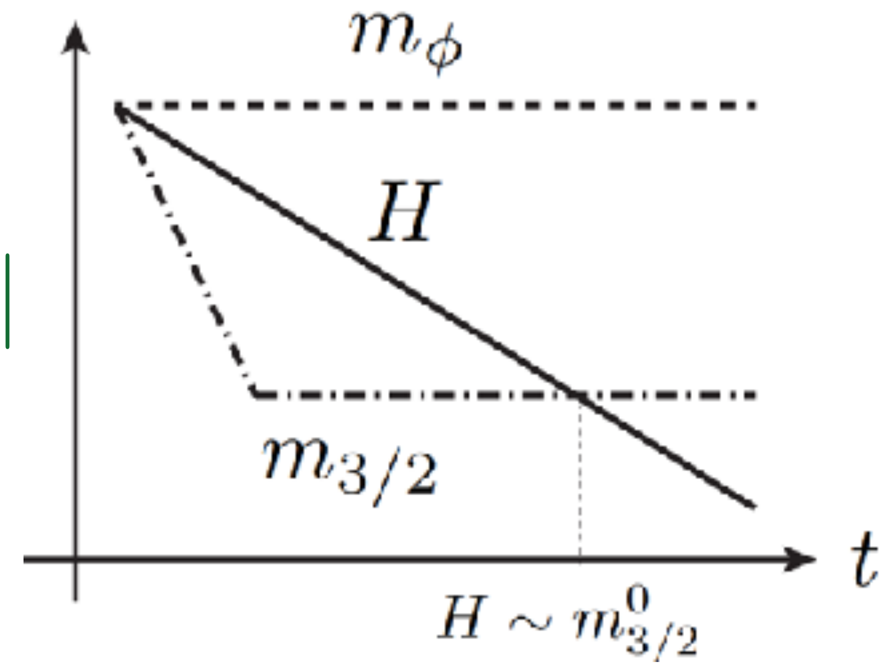
(Transverse) gravitino mass

$$m_{3/2} \simeq \frac{m_\phi \phi^2}{2M_{\rm P}^2} + m_{3/2}^0 \sim H\frac{\phi_{\rm amp}}{M_{\rm P}} + m_{3/2}^0.$$

Models w/o Stabilizer

[Goncharov, Linde, 1984]
[Ketov, Terada, 2014]

$$\left[\begin{aligned} K &= -\frac{1}{2}(\Phi - \bar{\Phi})^2 + |z|^2 - \frac{1}{\Lambda^2}| \\ W &= \frac{1}{2}m\Phi^2 + \mu^2 z + W_0 \end{aligned}\right.$$



SUSY breaking

$$|\dot{\phi}| \sim |F_\phi| \sim H M_{\rm P},$$

$$|F_z| \sim m^0_{3/2} M_{\rm P}$$

(Transverse) gravitino mass

$$m_{3/2} \simeq \frac{m_\phi \phi^2}{2M_{\rm P}^2} + m^0_{3/2} \sim H\frac{\phi_{\rm amp}}{M_{\rm P}} + m^0_{3/2}.$$

Transverse mode

$$Y^{(\rm t)}_{3/2} \simeq 8 \times 10^{-16}\mathcal{C}\left(\frac{H_{\rm inf}}{10^{13}\,{\rm GeV}}\right)\left(\frac{T_{\rm R}}{10^{10}\,{\rm GeV}}\right).$$

Models w/o Stabilizer

[Goncharov, Linde, 1984]
[Ketov, Terada, 2014]

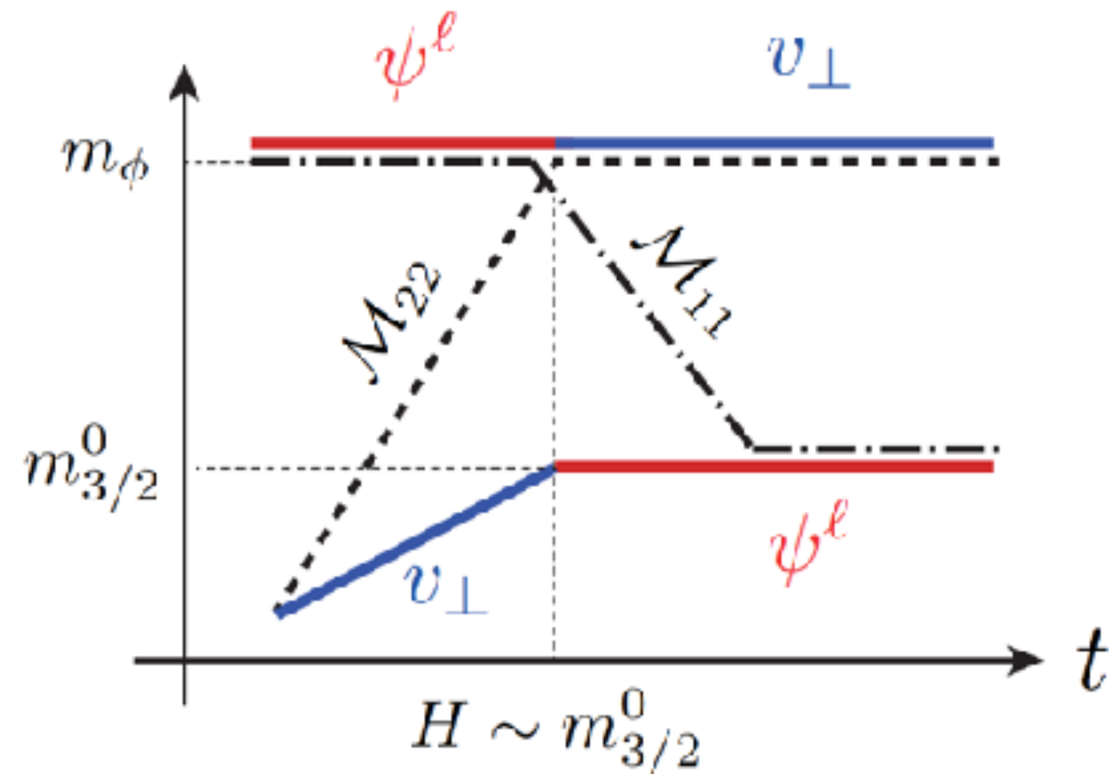
$$\left[\begin{aligned} K &= -\frac{1}{2}(\Phi - \bar{\Phi})^2 + |z|^2 \\ W &= \frac{1}{2} m \Phi^2 + \mu^2 z + W_0 \end{aligned}\right.$$

<u>**SUSY breaking**</u>

$$|\dot{\phi}| \sim |F_\phi| \sim H M_{\mathrm{P}},$$

$$|F_z| \sim m_{3/2}^0 M_{\mathrm{P}}$$

helicity ±1/2 sector



Models w/o Stabilizer

[Goncharov, Linde, 1984]
[Ketov, Terada, 2014]

$$K = -\frac{1}{2}(\Phi - \bar{\Phi})^2 + |z|^2$$

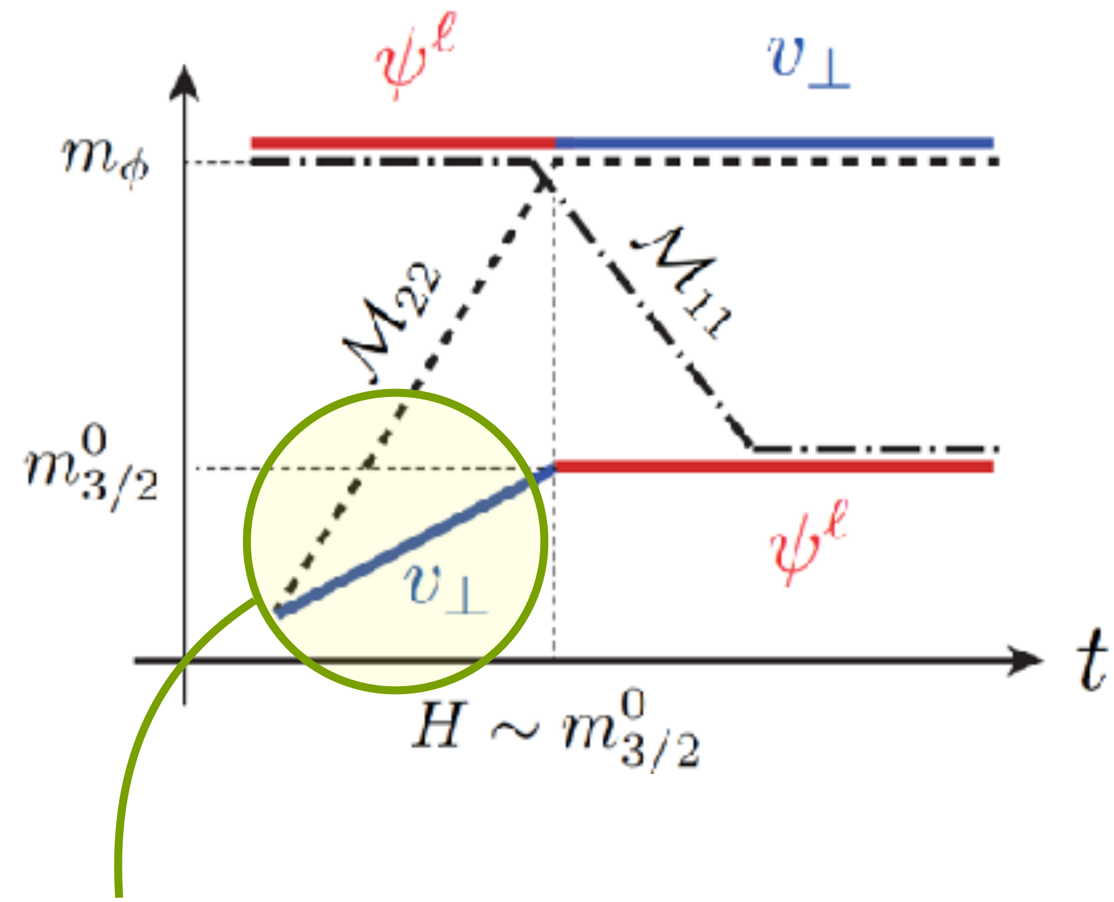
$$W = \frac{1}{2}m\Phi^2 + \mu^2 z + W_0$$

SUSY breaking

$$|\dot{\phi}| \sim |F_\phi| \sim HM_{\rm P},$$

$$|F_z| \sim m_{3/2}^0 M_{\rm P}$$

helicity ±1/2 sector



Suppression!

Models w/o Stabilizer

[Goncharov, Linde, 1984]
[Ketov, Terada, 2014]

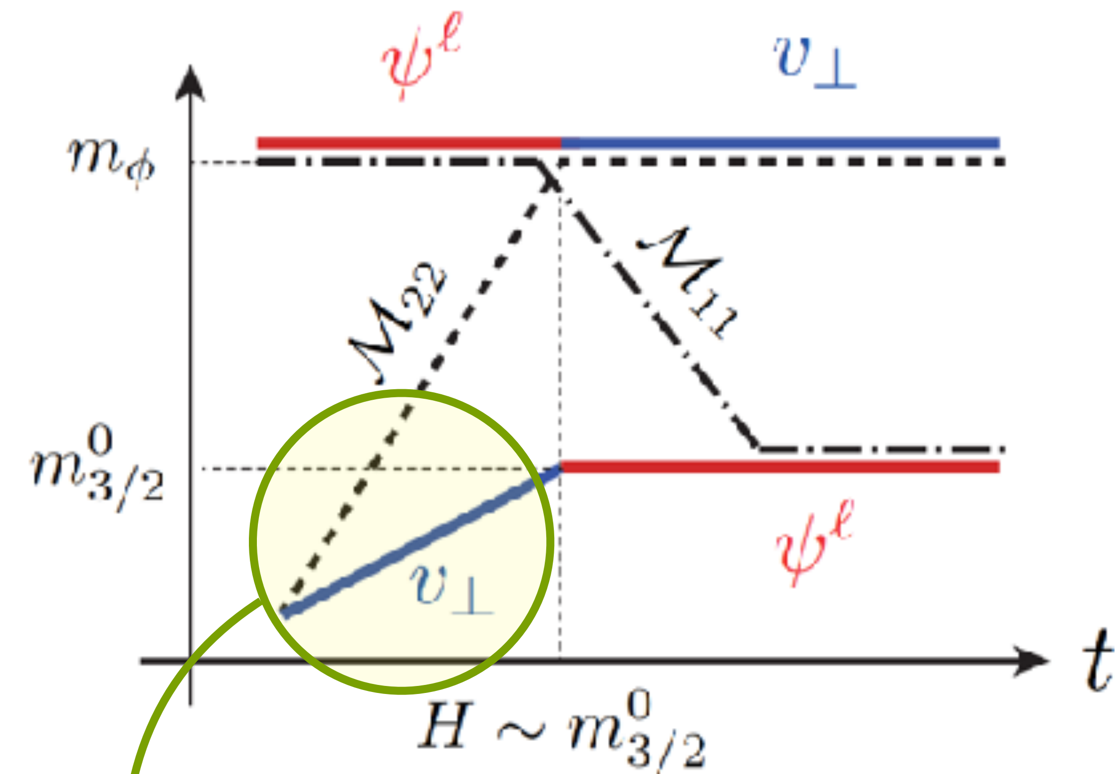
$$\left[\begin{aligned} K &= -\frac{1}{2}(\Phi - \bar{\Phi})^2 + |z|^2 \\ W &= \frac{1}{2} m\Phi^2 + \mu^2 z + W_0 \end{aligned}\right.$$

SUSY breaking

$$|\dot{\phi}| \sim |F_\phi| \sim H M_{\mathrm{P}},$$

$$|F_z| \sim m_{3/2}^0 M_{\mathrm{P}}$$

helicity ±1/2 sector



Suppression!

Longitudinal mode

$$Y_{3/2}^{(\ell)} \simeq 8 \times 10^{-23}\, \mathcal{C} \left(\frac{m_{3/2}^0}{10^6\ \mathrm{GeV}}\right) \left(\frac{T_{\mathrm{R}}}{10^{10}\ \mathrm{GeV}}\right).$$

Models w/ Stabilizer

Stabilizer superfield

$$K = -\frac{1}{2}(\Phi - \bar{\Phi})^2 + |X|^2 + |z|^2 - \frac{1}{\Lambda^2}|z|^4$$

$$W = mX\Phi + \mu^2 z + W_0$$

[Kawasaki, Yamaguchi, Yanagida, 2000]
[Kallosh, Linde, 2010]

Models w/ Stabilizer

Stabilizer superfield

$$K = -\frac{1}{2}(\Phi - \bar{\Phi})^2 + |X|^2 + |z|^2 - \frac{1}{\Lambda^2}|z|^4$$

$$W = mX\Phi + \mu^2 z + W_0$$

[Kawasaki, Yamaguchi, Yanagida, 2000]
[Kallosh, Linde, 2010]

Advantage of this type of models

$$X = 0 \quad \Rightarrow \quad W \sim mX\Phi = 0 \quad \Rightarrow \quad V > 0$$

Models w/ Stabilizer

Stabilizer superfield

$$K = -\frac{1}{2}(\Phi - \bar{\Phi})^2 + |X|^2 + |z|^2 - \frac{1}{\Lambda^2}|z|^4$$

$$W = mX\Phi + \mu^2 z + W_0$$

[Kawasaki, Yamaguchi, Yanagida, 2000]
[Kallosh, Linde, 2010]

Advantage of this type of models

$$X = 0 \quad \Rightarrow \quad W \sim mX\Phi = 0 \quad \Rightarrow \quad V > 0$$

Gravitino mass insensitive to the inflaton

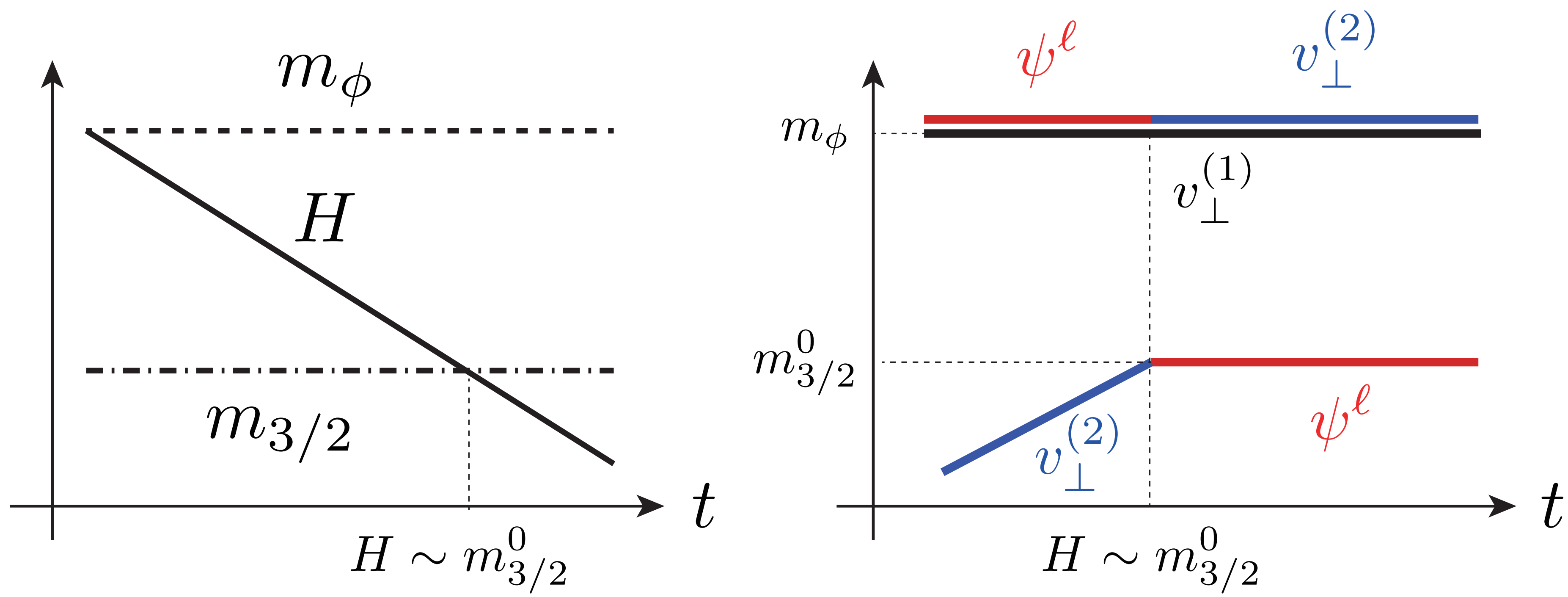
$$m_{3/2} = |e^{K/2}W| \sim 0$$

→ suppressed gravitino production

Models w/ Stabilizer

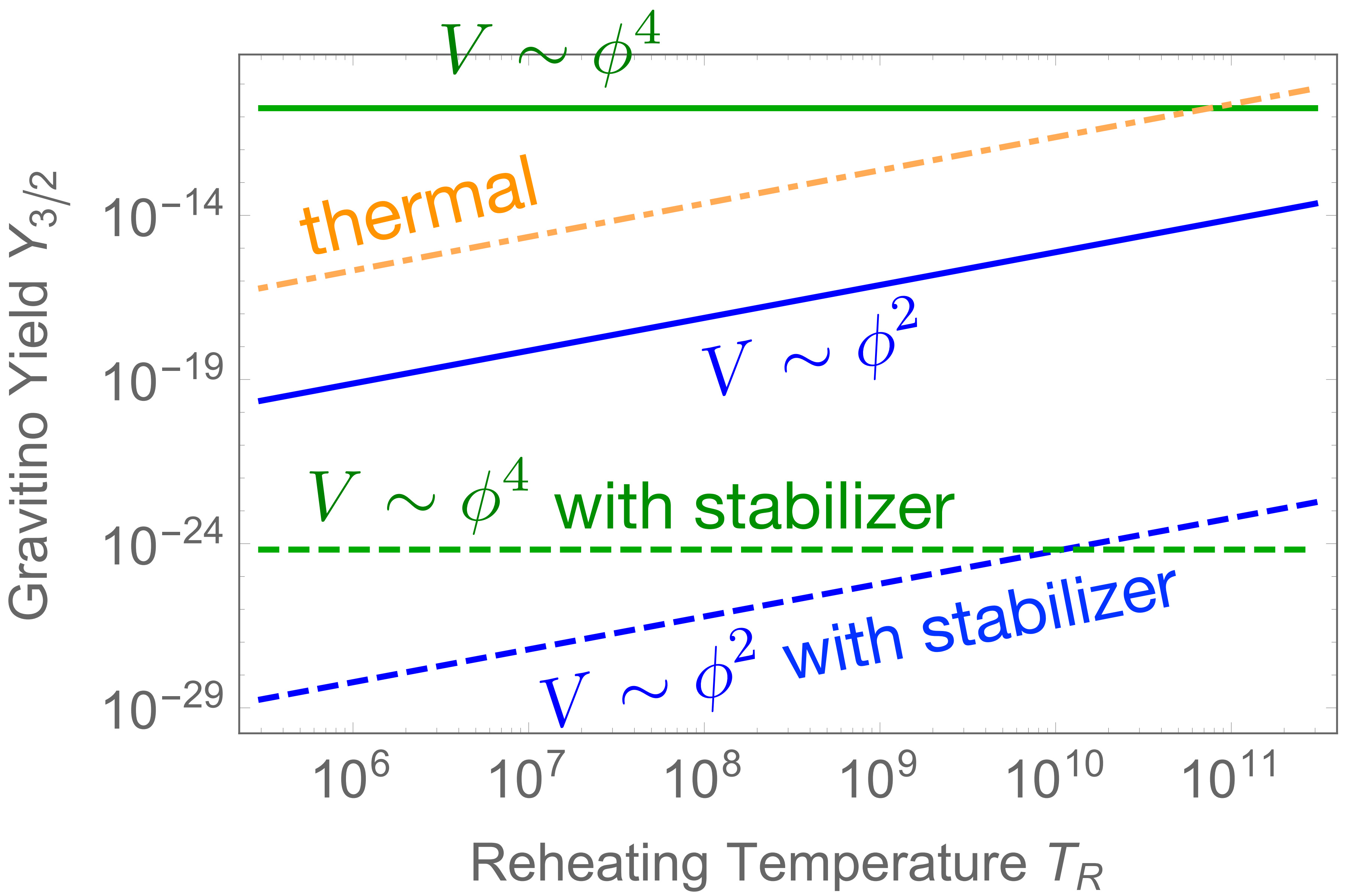
Small induced oscillation

$$X_{\rm amp} \sim \frac{m^0_{3/2}}{H}\,\phi_{\rm amp}$$



$$Y^{(\rm t)}_{3/2} \simeq Y^{(\ell)}_{3/2} \simeq 3\times 10^{-22}\,\mathcal{C}\left(\frac{m^0_{3/2}}{10^6\ {\rm GeV}}\right)\left(\frac{T_{\rm R}}{10^{10}\ {\rm GeV}}\right).$$

Results
(Gravitino Abundance)



$$Y_{3/2} \equiv \frac{n_{3/2}}{s}$$

$$H_{\rm inf} = 10^{13}\,\mathrm{GeV}$$

$$m_{3/2} = 10^{3}\,\mathrm{GeV}$$

Summary & Conclusions

Summary & Conclusions

- We studied non-thermal gravitino production
  in modern inflation models in supergravity.

Summary & Conclusions

- We studied non-thermal gravitino production in modern inflation models in supergravity.
- Typically, (non-thermal) << (thermal).

Summary & Conclusions

- We studied non-thermal gravitino production in modern inflation models in supergravity.
- Typically, (non-thermal) << (thermal).
- For models **without** the stabilizer field, the inflaton potential around the origin should be quadratic.

Appendix A

Difficulties of inflation in supergravity and their solutions

Very steep potential

$$V = e^{K}\left(K^{\bar{\Phi}\Phi}\,|W_{\Phi} + K_{\Phi}W|^{2} - 3|W|^{2}\right)$$

Very steep potential

Exponentially steep!

$$V = e^{K}\left(K^{\bar{\Phi}\Phi}\left|W_{\Phi} + K_{\Phi}W\right|^{2} - 3|W|^{2}\right)$$

Very steep potential

Exponentially steep!

Negative semi-definite!

$$V = e^{K}\left(K^{\bar{\Phi}\Phi}\left|W_{\Phi} + K_{\Phi}W\right|^{2} - 3|W|^{2}\right)$$

Very steep potential

Exponentially steep!

Negative semi-definite!

$$V = e^{K}\left(K^{\bar{\Phi}\Phi}\left|W_{\Phi} + K_{\Phi}W\right|^{2} - 3|W|^{2}\right)$$

Positive semi-definite.
F-term SUSY breaking term.

Very steep potential

Exponentially steep!

Negative semi-definite!

$$V = e^K \left(K^{\bar{\Phi}\Phi} \left| W_\Phi + K_\Phi W \right|^2 - 3|W|^2 \right)$$

Positive semi-definite.
F-term SUSY breaking term.

slow-roll conditions

$$\epsilon = \frac{1}{2}\left(\frac{V'}{V}\right)^2 \ll 1$$

$$|\eta| = \frac{|V''|}{V} \ll 1$$

Very steep potential

Exponentially steep!

Negative semi-definite!

$$V = e^{K}\left(K^{\bar{\Phi}\Phi}\left|W_{\Phi} + K_{\Phi}W\right|^{2} - 3|W|^{2}\right)$$

Positive semi-definite.
F-term SUSY breaking term.

slow-roll conditions

$$\epsilon = \frac{1}{2}\left(\frac{V'}{V}\right)^{2} \ll 1$$

$$|\eta| = \frac{|V''|}{V} \ll 1$$

SUGRA corrections

$$V = e^{K}V_{\text{global}} + \cdots = V_{\text{global}} + V_{\text{global}}\frac{|\phi|^{2}}{M_{\text{G}}^{2}} + \ldots,$$

$$\eta = \eta_{\text{global}} + 1 + \ldots$$

Flat but Negative

Shift symmetry [Kawasaki, Yamaguchi, Yanagida, hep-ph/0004243]

$$K(\Phi, \bar{\Phi}) = K(i(\Phi - \bar{\Phi})) \qquad \Phi \to \Phi + c$$

$V_{\mathrm{Im}\,\Phi} = 0$ implies $K_\Phi \simeq 0$, so,

$$\begin{aligned} V =& e^{K(i(\Phi - \bar{\Phi}))} \left(K^{\bar{\Phi}\Phi} \left| W_\Phi + K_\Phi W \right|^2 - 3|W|^2 \right) \\ \simeq & -3 e^{K(i(\Phi - \bar{\Phi}))} |W|^2 \end{aligned}$$

For large $\mathrm{Re}\,\Phi$, the potential tends to be negative.

Solutions

More fields

[Kawasaki, Yamaguchi, Yanagida, hep-ph/0004243]
[Kallosh, Linde, 1008.3375]
[Kallosh, Linde, Rube, 1011.5945]

[Farakos, Kehagias, Riotto, 1307.1137]
[Ferrara, Kallosh, Linde, Porrati, 1307.7696]

More terms

[Goncharov, Linde, PLB 139 (1984) 27]
[Roest, Scalisi, 1503.07909]
[Linde, 1504.00663]

[Izawa, Shinbara, 0710.1141]
[Ketov, Terada, 1406.0252]
[Ketov, Terada, 1408.6524]

Stabilizer field

Introduce another superfield X s.t. $X = 0$ during inflation.

[Kawasaki, Yamaguchi, Yanagida, hep-ph/0004243]

[Kallosh, Linde, 1008.3375]

[Kallosh, Linde, Rube, 1011.5945]

Stabilizer field

Introduce another superfield X s.t. $X = 0$ during inflation.

Take $W = X f(\Phi)$ cf. R-symmetry

[Kawasaki, Yamaguchi, Yanagida, hep-ph/0004243]

[Kallosh, Linde, 1008.3375]

[Kallosh, Linde, Rube, 1011.5945]

Stabilizer field

Introduce another superfield X s.t. $X = 0$ during inflation.

Take $W = X f(\Phi)$ cf. R-symmetry

Then $W = W_\Phi = 0$ while $W_X = f(\Phi) \neq 0$

[Kawasaki, Yamaguchi, Yanagida, hep-ph/0004243]

[Kallosh, Linde, 1008.3375]

[Kallosh, Linde, Rube, 1011.5945]

Stabilizer field

Introduce another superfield X s.t. $X = 0$ during inflation.

Take $W = X f(\Phi)$ cf. R-symmetry

Then $W = W_\Phi = 0$ while $W_X = f(\Phi) \neq 0$

$$V = e^K \left(K^{\bar{\Phi}\Phi} |W_\Phi + K_\Phi W|^2 + K^{\bar{X}X} |W_X + K_X W|^2 - 3|W|^2 \right)$$

[Kawasaki, Yamaguchi, Yanagida, hep-ph/0004243]

[Kallosh, Linde, 1008.3375]

[Kallosh, Linde, Rube, 1011.5945]

Stabilizer field

Introduce another superfield X s.t. $X = 0$ during inflation.

Take $W = X f(\Phi)$ cf. R-symmetry

Then $W = W_\Phi = 0$ while $W_X = f(\Phi) \neq 0$

$$V = e^K \left(K^{\bar{\Phi}\Phi} |W_\Phi + K_\Phi W|^2 + K^{\bar{X}X} |W_X + K_X W|^2 - 3|W|^2 \right)$$

[Kawasaki, Yamaguchi, Yanagida, hep-ph/0004243]

[Kallosh, Linde, 1008.3375]

[Kallosh, Linde, Rube, 1011.5945]

Stabilizer field

Introduce another superfield X s.t. $X = 0$ during inflation.

Take $W = Xf(\Phi)$ cf. R-symmetry

Then $W = W_\Phi = 0$ while $W_X = f(\Phi) \neq 0$

$$V = e^K \left(K^{\bar{\Phi}\Phi} |W_\Phi + K_\Phi W|^2 + K^{\bar{X}X} |W_X + K_X W|^2 - 3|W|^2 \right)$$

$$= e^K K^{\bar{X}X} |f(\Phi)|^2$$

[Kawasaki, Yamaguchi, Yanagida, hep-ph/0004243]

[Kallosh, Linde, 1008.3375]

[Kallosh, Linde, Rube, 1011.5945]

Stabilizer field

Introduce another superfield X s.t. $X = 0$ during inflation.

Take $W = X f(\Phi)$ cf. R-symmetry

Then $W = W_\Phi = 0$ while $W_X = f(\Phi) \neq 0$

$$V = e^K \left(K^{\bar{\Phi}\Phi} |W_\Phi + K_\Phi W|^2 + K^{\bar{X}X} |W_X + K_X W|^2 - 3|W|^2 \right)$$

$$= e^K K^{\bar{X}X} |f(\Phi)|^2 = |f(\mathrm{Re}\,\Phi)|^2$$

Additional Z_2 symmetries.

[Kawasaki, Yamaguchi, Yanagida, hep-ph/0004243]

[Kallosh, Linde, 1008.3375]

[Kallosh, Linde, Rube, 1011.5945]

Appendix B

Constrained superfields and violent gravitino production

Based on F. Hasegawa, K. Mukaida, K. Nakayama, **TT** and Y. Yamada,
PLB 767 (2017) 392 [arXiv:1701.03106 [hep-ph]]

Remove unnecessary fields!

Scalars: stability in *all the directions* in the scalar manifold

Spinors: diagonalization with *time-dependent* gravitino

Make them heavy and let them decouple!

→ Non-linearly realized SUSY

Constrained superfields are useful ingredients.

ex.) Nilpotent superfield X $\quad X^2 = 0$

$$X = \frac{\widetilde{X}\widetilde{X}}{2F^X} - \sqrt{2}\theta\widetilde{X} + \theta\theta F^X$$

Minimal Supergravity Inflation

Minimal Supergravity Inflation

Orthogonal nilpotent superfields

$$X^2 = 0 \qquad X(\Phi - \bar{\Phi}) = 0$$

[Komargodski and Seiberg, 2009]

Minimal Supergravity Inflation

Orthogonal nilpotent superfields

$$X^2 = 0 \qquad X(\Phi - \bar{\Phi}) = 0$$

[Komargodski and Seiberg, 2009]

Particle spectrum

sgoldstino

$$\left(\cancel{X}, \widetilde{X}, F^X\right)$$

goldstino

inflatino

$$\left(\mathrm{Re}\Phi + \cancel{i\mathrm{Im}\Phi}, \cancel{\widetilde{\Phi}}, \cancel{F^\Phi}\right)$$

inflaton

sinflaton

$$F^\Phi \equiv 0$$

in addition to graviton and gravitino.

[Kahn, Roberts, and Thaler, 2015] [Ferrara, Kallosh, and Thaler, 2015]
[Delacretaz, Gorbenko, and Senatore, 2016] [Carrasco, Kallosh, and Linde, 2015]

Minimal Supergravity Inflation

Orthogonal nilpotent superfields

$$X^2 = 0 \qquad X(\Phi - \bar{\Phi}) = 0$$

[Komargodski and Seiberg, 2009]

Particle spectrum

$$\left(\overset{\text{sgoldstino}}{\cancel{X}}, \underset{\text{goldstino}}{\widetilde{X}}, F^X\right)$$

$$\left(\underset{\text{inflaton}}{\mathrm{Re}\Phi} + \underset{\text{sinflaton}}{\cancel{i\,\mathrm{Im}\Phi}}, \overset{\text{inflatino}}{\cancel{\widetilde{\Phi}}}, \cancel{F^\Phi}\right) \qquad F^\Phi = 0$$

in addition to graviton and gravitino.

[Kahn, Roberts, and Thaler, 2015] [Ferrara, Kallosh, and Thaler, 2015]
[Delacretaz, Gorbenko, and Senatore, 2016] [Carrasco, Kallosh, and Linde, 2015]

General Kähler and superpotentials (after field redefinition)

$$K = \bar{X}X + k(\Phi, \bar{\Phi}) \qquad W = X f(\Phi) + g(\Phi)$$

[Ferrara, Kallosh, Thaler, 2015] [Carrasco, Kallosh, Linde, 2015]

Minimal Supergravity Inflation

Goldstino

$$v = -F^X \widetilde{X} + \frac{1}{\sqrt{2}} \partial_\mu \phi \gamma^\mu \widetilde{\Phi}$$

Unitary gauge

$$v = 0$$

Gravitino Lagrangian density

$$ds^2 = a(\eta)(-d\eta^2 + d\vec{x}^2)$$

$$\mathcal{L}_t = -\frac{1}{2}\overline{\vec{\psi}^T} \cdot \left[\gamma^0 \partial_0 + \left(\vec{\gamma} \cdot \vec{\nabla}\right) + a m_{3/2}\right] \vec{\psi}^T$$

$$\mathcal{L}_\ell = -\frac{1}{2}\overline{\psi^L} \left[\gamma^0 \partial_0 - \widehat{c}_{3/2} \left(\vec{\gamma} \cdot \vec{\nabla}\right) + a \widehat{m}_{3/2}\right] \psi^L$$

$$\widehat{c}_{3/2} \equiv \frac{p_{\mathrm{SB}} - \gamma^0 p_W}{\rho_{\mathrm{SB}}} \qquad \widehat{m}_{3/2} \equiv \frac{3H p_W + m_{3/2}(\rho_{\mathrm{SB}} + 3p_{\mathrm{SB}})}{2\rho_{\mathrm{SB}}},$$

$$\rho_{\mathrm{SB}} \equiv \rho + 3m_{3/2}^2 M_{\mathrm{P}}^2 \qquad p_{\mathrm{SB}} \equiv p - 3m_{3/2}^2 M_{\mathrm{P}}^2 \qquad p_W \equiv 2\dot{m}_{3/2} M_{\mathrm{P}}^2$$

Varying "sound speed"

$$\mathcal{L}_t = -\frac{1}{2}\overline{\vec{\psi}^T} \cdot \left[\gamma^0 \partial_0 + \left(\vec{\gamma} \cdot \vec{\nabla}\right) + a m_{3/2}\right] \vec{\psi}^T$$

$$\mathcal{L}_\ell = -\frac{1}{2}\overline{\psi^L} \left[\gamma^0 \partial_0 - \widehat{c}_{3/2} \left(\vec{\gamma} \cdot \vec{\nabla}\right) + a \widehat{m}_{3/2}\right] \psi^L$$

The "speed of sound" parameter for the gravitino field
= The maximum speed of the longitudinal gravitino

$$\widehat{c}_{3/2} \equiv \frac{p_{\rm SB} - \gamma^0 p_W}{\rho_{\rm SB}} \simeq \frac{p_{\rm SB}}{\rho_{\rm SB}} \equiv w_{\rm SB}$$

Varying “sound speed”

$$\mathcal{L}_t = -\frac{1}{2}\overline{\vec{\psi}^T} \cdot \left[\gamma^0 \partial_0 + \left(\vec{\gamma} \cdot \vec{\nabla}\right) + a m_{3/2}\right] \vec{\psi}^T$$

$$\mathcal{L}_\ell = -\frac{1}{2}\overline{\psi^L} \left[\gamma^0 \partial_0 - \underset{\uparrow}{\widehat{c}_{3/2}} \left(\vec{\gamma} \cdot \vec{\nabla}\right) + a \widehat{m}_{3/2}\right] \psi^L$$

The “speed of sound” parameter for the gravitino field
= The maximum speed of the longitudinal gravitino

“sound speed” parameter = “Eq. of state” parameter

$$\widehat{c}_{3/2} \equiv \frac{p_{\mathrm{SB}} - \gamma^0 p_W}{\rho_{\mathrm{SB}}} \simeq \frac{p_{\mathrm{SB}}}{\rho_{\mathrm{SB}}} \equiv w_{\mathrm{SB}}$$

Gravitino eq. and Solution

Gravitino eq. and Solution

Consider the simplest case $g(\Phi) = m_{3/2}$ and take the limit $m_{3/2} \to 0$

Gravitino eq. and Solution

Consider the simplest case $g(\Phi) = m_{3/2}$ and take the limit $m_{3/2} \to 0$

Expansion of the longitudinal gravitino (Dirac rep.)

annihilation op.

$$\psi^L = \sum_h \int \frac{d^3k}{(2\pi)^{3/2}} e^{i\vec{k}\cdot\vec{x}} \begin{pmatrix} u^+_{\vec{k},h}(t) \\ u^-_{\vec{k},h}(t) \end{pmatrix} \otimes \xi_{\vec{k},h} \hat{b}_{\vec{k},h} + \mathrm{H.c.},$$

helicity sum Fourier transform eigenvector of helicity

$(\vec{\sigma}\cdot\vec{k})\xi_{\vec{k},h} = h|\vec{k}|\xi_{\vec{k},h}$

Gravitino eq. and Solution

Consider the simplest case $g(\Phi) = m_{3/2}$ and take the limit $m_{3/2} \to 0$

Expansion of the longitudinal gravitino (Dirac rep.)

annihilation op.

$$\psi^L = \sum_h \int \frac{d^3k}{(2\pi)^{3/2}} e^{i\vec{k}\cdot\vec{x}} \begin{pmatrix} u^+_{\vec{k},h}(t) \\ u^-_{\vec{k},h}(t) \end{pmatrix} \otimes \xi_{\vec{k},h}\, \hat{b}_{\vec{k},h} + \mathrm{H.c.},$$

helicity sum

Fourier transform

eigenvector of helicity

$(\vec{\sigma}\cdot\vec{k})\xi_{\vec{k},h} = h|\vec{k}|\xi_{\vec{k},h}$

Equation of motion for this u

$$c_{3/2}u'' - c'_{3/2}u' + c_{3/2}\widetilde{\omega}_k^2 u = 0$$

Gravitino eq. and Solution

Consider the simplest case $g(\Phi) = m_{3/2}$ and take the limit $m_{3/2} \to 0$

Expansion of the longitudinal gravitino (Dirac rep.)

annihilation op.

$$\psi^L = \sum_h \int \frac{d^3k}{(2\pi)^{3/2}} e^{i\vec{k}\cdot\vec{x}} \begin{pmatrix} u^+_{\vec{k},h}(t) \\ u^-_{\vec{k},h}(t) \end{pmatrix} \otimes \xi_{\vec{k},h} \hat{b}_{\vec{k},h} + \mathrm{H.c.},$$

helicity sum

Fourier transform

eigenvector of helicity

$$(\vec{\sigma}\cdot\vec{k})\xi_{\vec{k},h} = h|\vec{k}|\xi_{\vec{k},h}$$

Equation of motion for this u

$$c_{3/2}u'' - c'_{3/2}u' + c_{3/2}\widetilde{\omega}_k^2 u = 0$$

Solution

$$u(\eta) = \frac{1}{\sqrt{2}} \exp\left[ik\int_0^\eta \mathrm{d}\eta' c_{3/2}(\eta')\right]$$

Gravitino eq. and Solution

Consider the simplest case $g(\Phi) = m_{3/2}$ and take the limit $m_{3/2} \to 0$

Expansion of the longitudinal gravitino (Dirac rep.)

annihilation op.

$$\psi^L = \sum_h \int \frac{d^3k}{(2\pi)^{3/2}} e^{i\vec{k}\cdot\vec{x}} \begin{pmatrix} u^+_{\vec{k},h}(t) \\ u^-_{\vec{k},h}(t) \end{pmatrix} \otimes \xi_{\vec{k},h}\, \hat{b}_{\vec{k},h} + \text{H.c.},$$

helicity sum Fourier transform eigenvector of helicity

$$(\vec{\sigma}\cdot\vec{k})\xi_{\vec{k},h} = h|\vec{k}|\xi_{\vec{k},h}$$

Equation of motion for this u

$$c_{3/2}u'' - c'_{3/2}u' + c_{3/2}\widetilde{\omega}_k^2 u = 0$$

Solution

$$u(\eta) = \frac{1}{\sqrt{2}} \exp\left[ik\int_0^\eta \mathrm{d}\eta' c_{3/2}(\eta')\right]$$

Phase space density

$$f_{3/2}(\vec{k};t) \equiv \frac{1}{2\omega_k(t)}\left(2\mathrm{Im}\left(u^{+*}_{\vec{k}}(t)\dot{u}^+_{\vec{k}}(t)\right) - \widehat{m}_{3/2}(t)\right) + \frac{1}{2}$$

$$= \frac{1}{2}\left(1 + \mathrm{sgn}\left(c_{3/2}(t)\right)\right)$$

Momentum-**in**dependent!

Eq. of state & gravitino production

$$c_{3/2} = w_{\rm SB} = \frac{p - 3m_{3/2}^2}{\rho + 3m_{3/2}^2}$$

inflation
-1 (<0)

→ *

gravitino production

radiation dominant
1/3 (>0)

→ *

gravitino annihilation

late time
-1 (<0)

* This order depends on the parameters.

Eq. of state & gravitino production

$$c_{3/2} = w_{\rm SB} = \frac{p - 3m_{3/2}^2}{\rho + 3m_{3/2}^2}$$

inflation
-1 (<0)

*

gravitino production

radiation dominant
1/3 (>0)

*

gravitino annihilation

late time
-1 (<0)

* This order depends on the parameters.

Cosmic expansion dilutes the gravitino,

and makes the annihilation **in**complete!

Eq. of state & gravitino production

$$c_{3/2} = w_{\rm SB} = \frac{p - 3m_{3/2}^2}{\rho + 3m_{3/2}^2}$$

inflation		radiation dominant		late time
-1 (<0)	⇒ *	1/3 (>0)	⇒ *	-1 (<0)
	gravitino production		gravitino annihilation	

* This order depends on the parameters.

Cosmic expansion dilutes the gravitino,

and makes the annihilation **in**complete!

Gravitino yield $Y_{3/2} \equiv \dfrac{n_{3/2}}{s} \sim \mathcal{O}(1)$ ⋙ cf. BBN bound $Y_{3/2} \lesssim \mathcal{O}(10^{-13} \sim 10^{-16})$

[e.g. Kawasaki et al., 2008]

Summary

1609.04716

1701.03106

Summary

1609.04716

1701.03106

- We studied non-thermal gravitino production in modern inflation models in supergravity.

Summary

1609.04716

1701.03106

- We studied non-thermal gravitino production
  in modern inflation models in supergravity.
- Typically, (non-thermal) << (thermal).

Summary

1609.04716

1701.03106

- We studied non-thermal gravitino production in modern inflation models in supergravity.
- Typically, (non-thermal) << (thermal).
- For models **without** the stabilizer field, the inflaton potential around the origin should be quadratic.

Summary

1609.04716

1701.03106

- We studied non-thermal gravitino production in modern inflation models in supergravity.
- Typically, (non-thermal) << (thermal).
- For models **without** the stabilizer field, the inflaton potential around the origin should be quadratic.
- In "Minimal Supergravity Inflation", extremely many longitudinal gravitinos can be produced.