Constant-roll inflation

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Martin, HM, Suyama, PRD 87, 023514 (2013) , arXiv:1211.0083
HM, Starobinsky, Yokoyama, JCAP 1509, 018 (2015), arXiv:1411.5021
HM, Starobinsky, EPL 117, 39001 (2017), arXiv:1702.05847
HM, Starobinsky, arXiv:1704.08188
HM, Hu, arXiv:1706.06784

2017.06.22 PASCOS at IFT, Madrid

Canonical single field Inflation

$$3H^2 = \frac{\dot{\phi}^2}{2} + V$$

$$-2\dot{H} = \dot{\phi}^2$$

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V}{\partial \phi} = 0$$

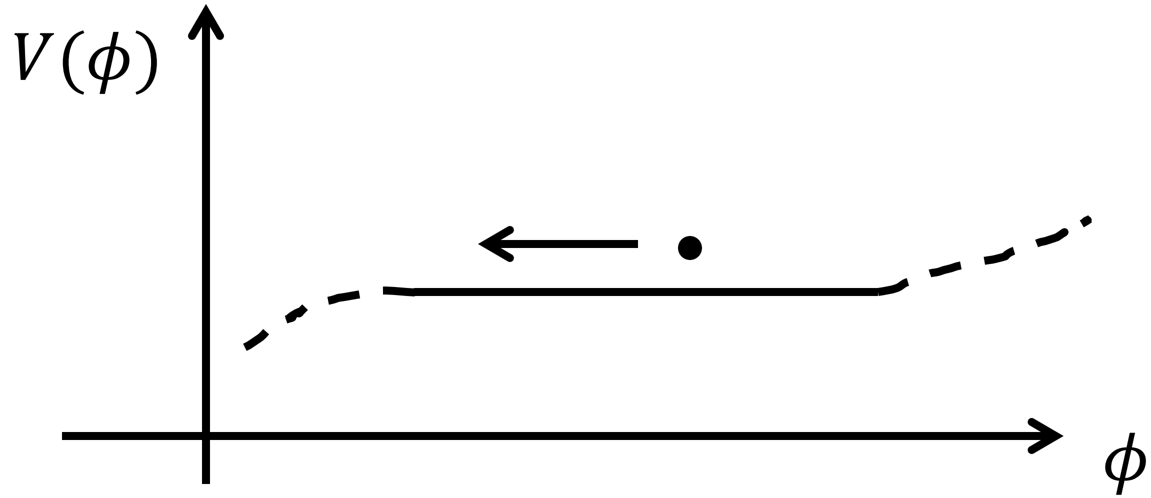
Slow-roll approximation $\ddot{\phi} \simeq 0$

Ultra slow-roll : constant potential $\ddot{\phi} = -3H\dot{\phi}$

Constant-roll generalization $\ddot{\phi} = \beta H \dot{\phi}$ (β: constant)

Ultra slow-roll inflation

Kinney, gr-qc/0503017



Constant potential $V \simeq V_0$

$$\ddot{\phi} = -3H\dot{\phi} \Rightarrow \dot{\phi} \propto a^{-3}$$

$$3H^2 = \frac{\dot{\phi}^2}{2} + V \simeq V_0$$

Slow-roll parameter

$$\epsilon_H \equiv -\frac{\dot{H}}{H^2} = \frac{\dot{\phi}^2}{2H^2} \propto a^{-6}$$

Ultra slow-roll inflation

Ultra slow roll $\ddot{\phi} = -3H\dot{\phi} \Rightarrow \epsilon_H \propto a^{-6}$

Curvature perturbation on superhorizon scales

$$\zeta_k = A_k + B_k \int \frac{dt}{a^3 \epsilon_H}$$

Constant mode

Slow-roll $\epsilon_H \simeq$ const $\ll 1$: decaying mode ✓

Ultra slow-roll $\epsilon_H \propto a^{-6}$: growing mode ✗

While ϵ_H itself is small, $\frac{d\ln\epsilon_H}{dN} = -6$ is not small.

$\Rightarrow$ Violation of slow roll.

Generalization to constant roll

Martin, HM, Suyama, 1211.0083

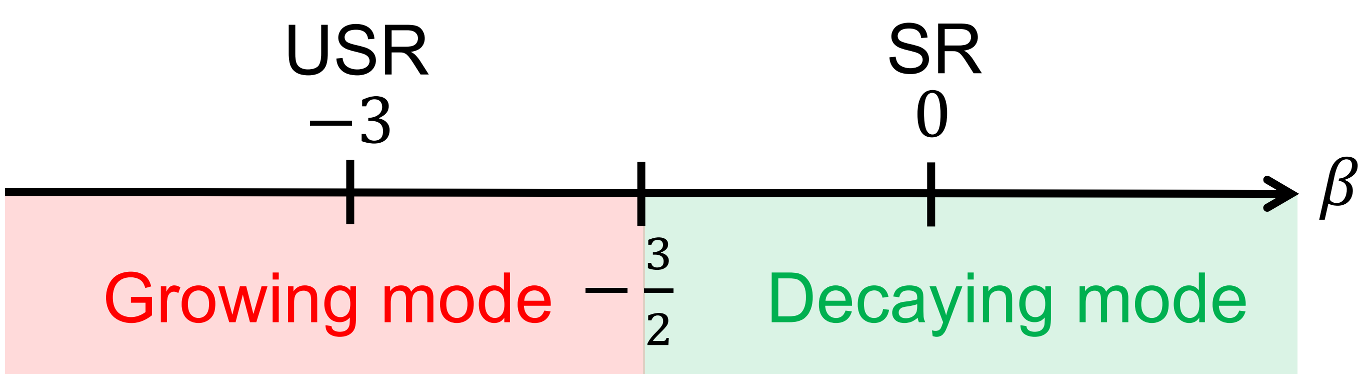
Constant roll condition $\ddot{\phi} = \beta H \dot{\phi} \Rightarrow \epsilon_H \propto a^{2\beta}$

Curvature perturbation on superhorizon scales

$$\zeta_k = A_k + B_k \int \frac{dt}{a^3 \epsilon_H}$$

Constant mode (A_k)

$$\frac{d\ln\epsilon_H}{dN} = 2\beta \gtrless -3$$



PBH & slow-roll violation

HM, Hu, 1706.06784

Closely related notion is enhancement of $\Delta_\zeta^2(k)$ for PBH (subhorizon evolution).

Leading order slow roll

$$\Delta_\zeta^2(k) \approx \frac{H^2}{8\pi^2\epsilon_H}$$

Blue tilt

$$\Leftrightarrow \frac{d\ln\epsilon_H}{dN} < 0$$

No go for slow roll: $\frac{d\ln\epsilon_H}{dN} < -0.38$ is necessary for PBH as DM from single field inflation. Since

$$\frac{\epsilon_V}{\epsilon_H} = \left(1 + \frac{1}{6 - 2\epsilon_H}\frac{d\ln\epsilon_H}{dN}\right)^2$$

$$\epsilon_H \equiv -\dot{H}/H^2$$

$$\epsilon_V \equiv (V'/V)^2/2$$

it is not appropriate to use

$$\epsilon_H \approx \epsilon_V, \qquad \Delta_\zeta^2(k) \approx \frac{V}{24\pi^2\epsilon_V}$$

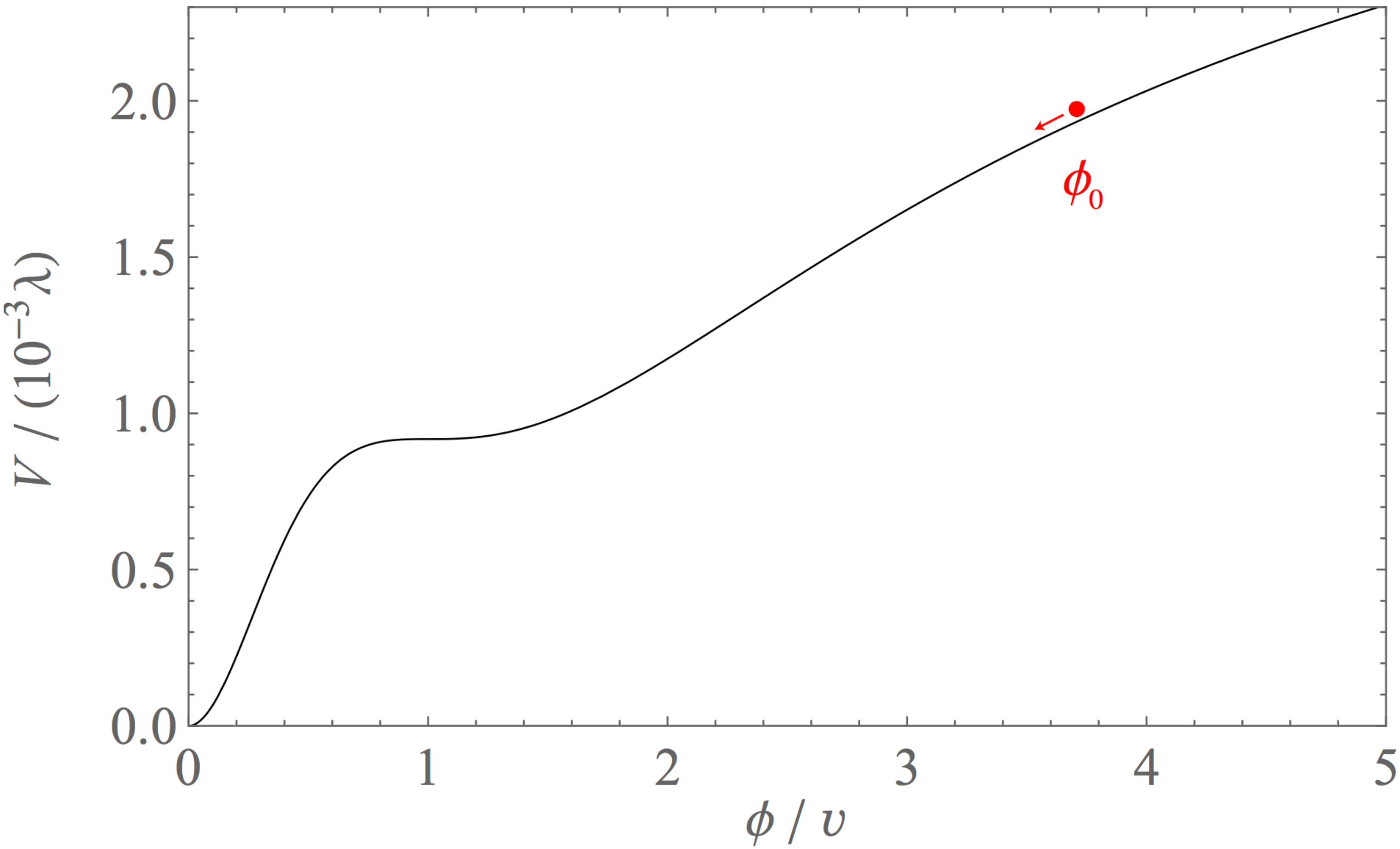
Case study: Inflection model

HM, Hu, 1706.06784

Garcia-Bellido, Morales, 1702.03901

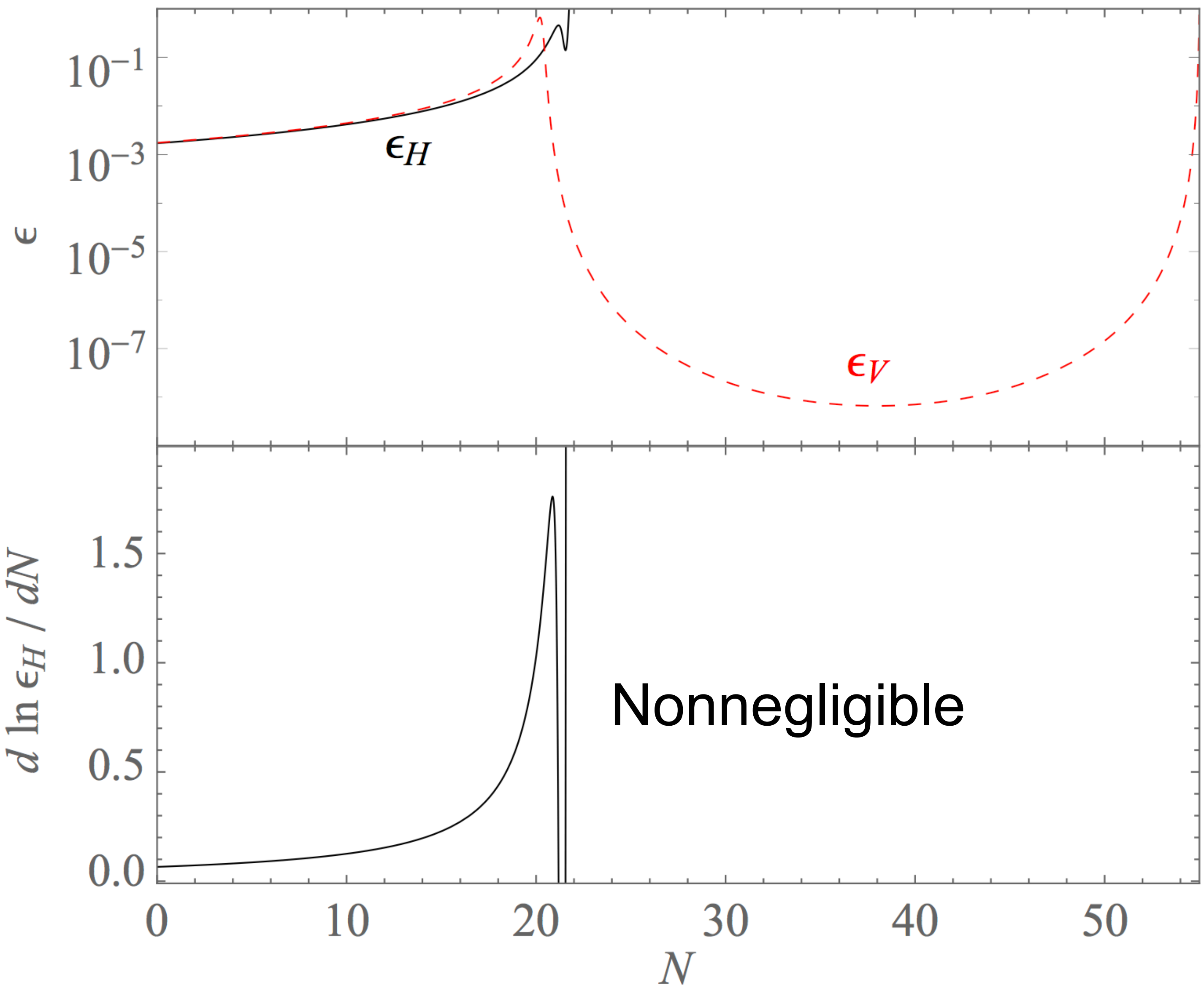
Ezquiaga, Garcia-Bellido, Morales, 1705.04861

Bezrukov, Pauly, Rubio, 1706.05007



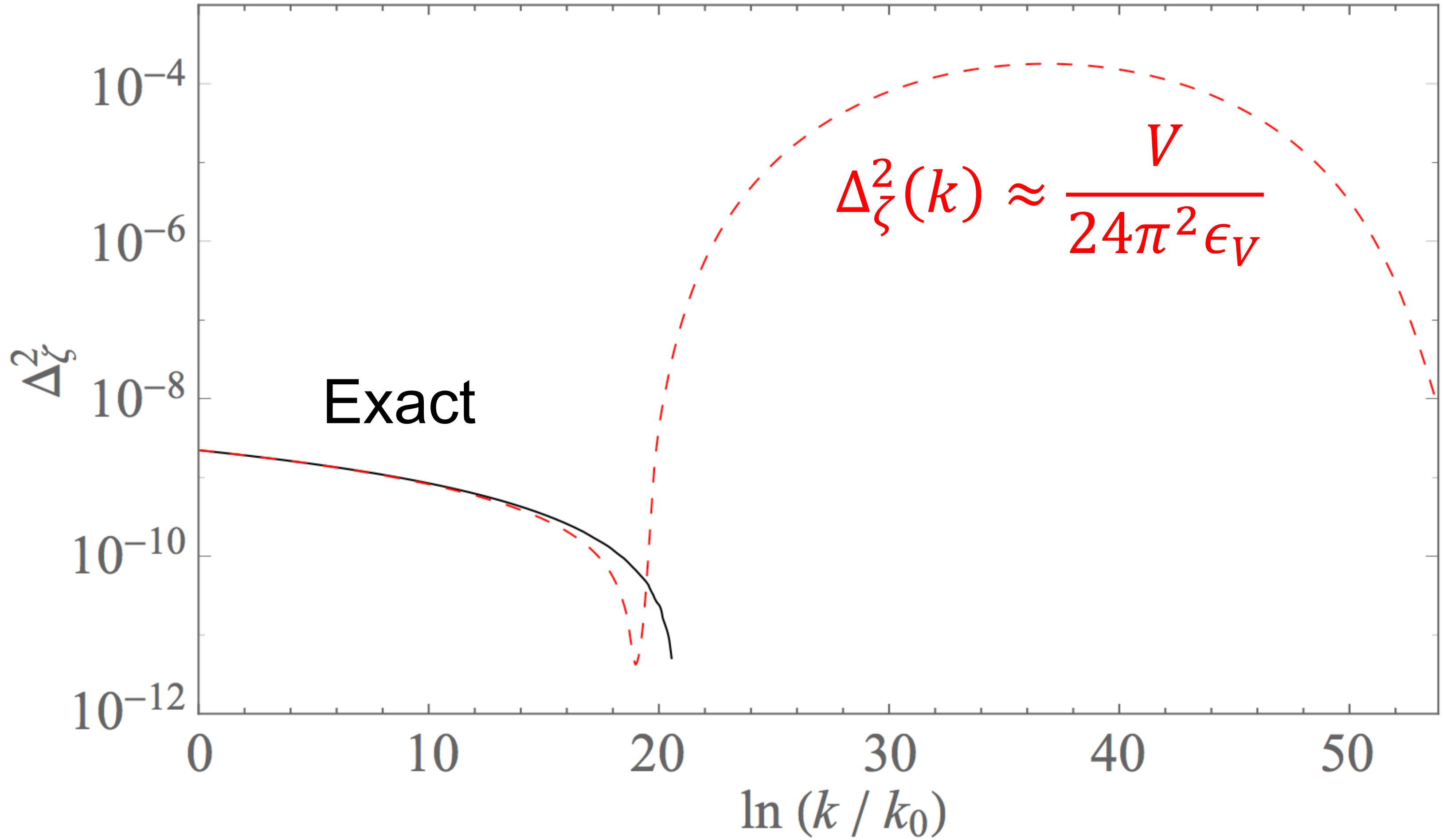
Case study: Inflection model

HM, Hu, 1706.06784



Case study: Inflection model

HM, Hu, 1706.06784



Constant-roll potential

HM, Starobinsky, Yokoyama, 1411.5021

With constant-roll condition

$$\ddot{\phi} = \beta H \dot{\phi}$$

the evolution equation $-2\dot{H} = \dot{\phi}^2$ or $\dot{\phi} = -2\frac{dH}{d\phi}$ implies

$$\frac{d^2H}{d\phi^2} = -\frac{\beta}{2}H$$

Thus, H is given by linear combination of $e^{\pm\sqrt{-\beta/2}\phi}$.

The potential is then given exactly by

$$V = 3H^2 - 2\left(\frac{dH}{d\phi}\right)^2$$

which is linear combination of $e^{\pm\sqrt{-2\beta}\phi}$.

For each $V(\phi)$, we can get $\phi(t), H(t), a(t)$ analytically.

Constant-roll potential

The potential includes

Abbott, Wise, 1984
Lucchin, Matarese, 1985

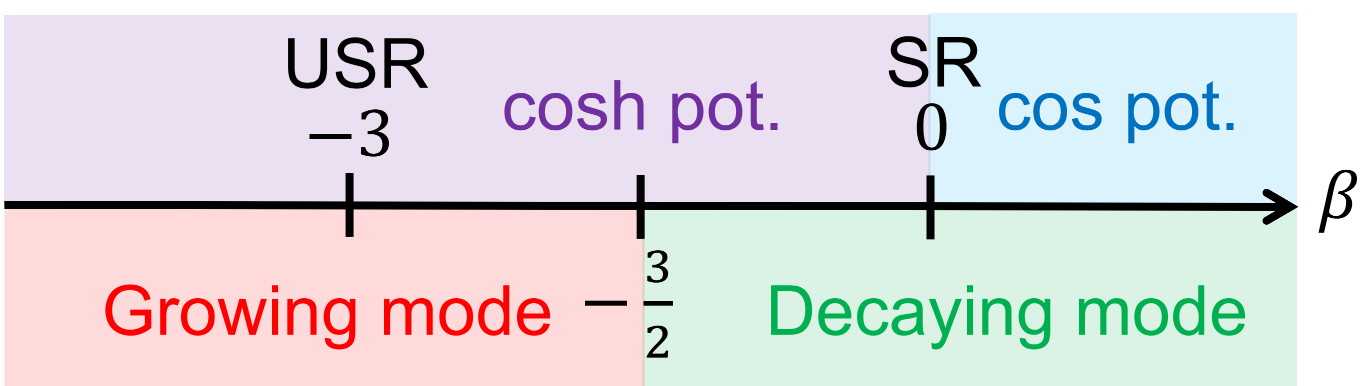
a) $V \propto e^{\sqrt{-2\beta}\phi}$ with $\beta < 0$: Power-law inflation

❌ but $r = 8(1 - n_s) \approx 0.28$ is too large.

Barrow, 1994

b) $V \propto \cosh(\sqrt{-2\beta}\phi)$ + const with $\beta < 0$: Known

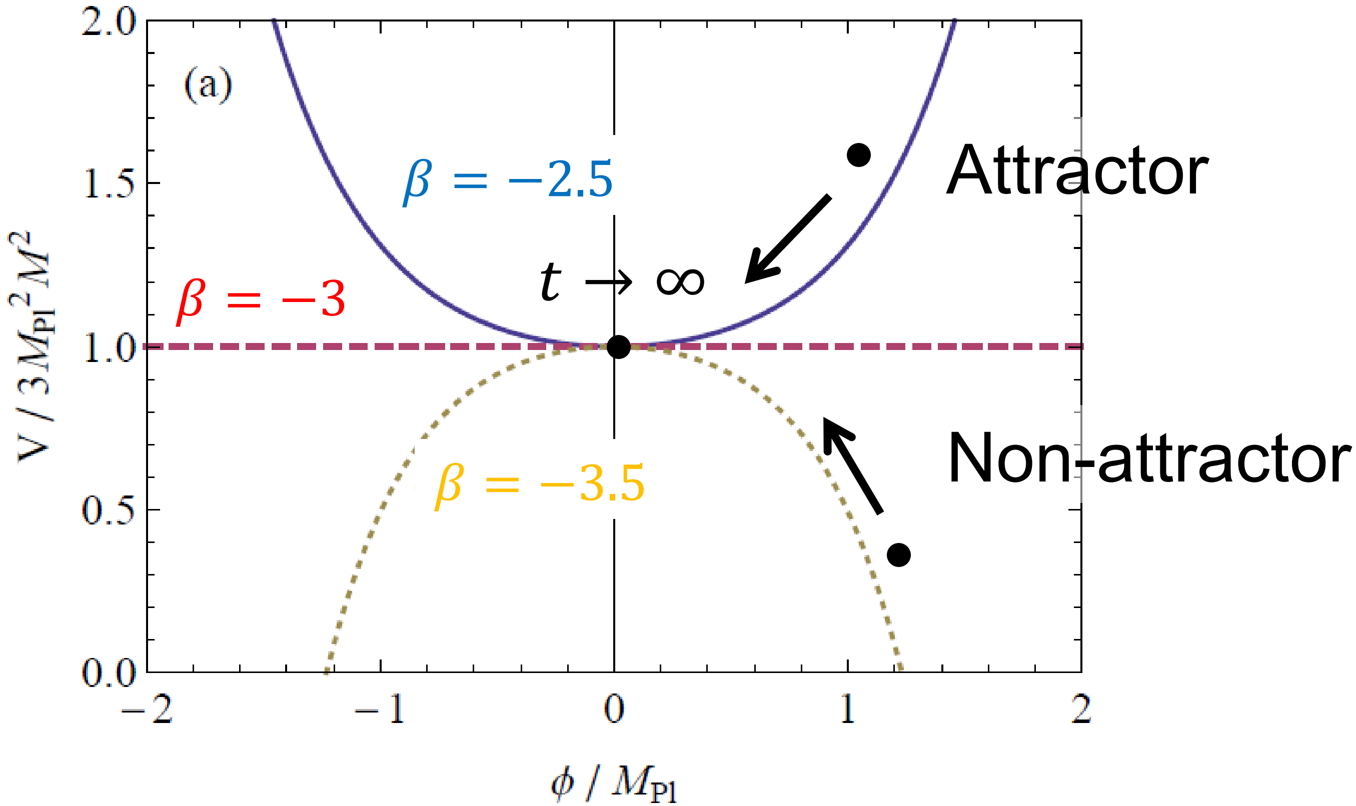
c) $V \propto \cos(\sqrt{2\beta}\phi)$ + const with $\beta > 0$: New potential



cosh potential ($\beta < 0$)

$$\frac{V(\phi)}{M^2 M_{Pl}^2} = 3\left[1 - \frac{3+\beta}{6}\left\{1 - \cosh\left(\sqrt{-2\beta}\,\frac{\phi}{M_{Pl}}\right)\right\}\right]$$

Assume inflation ends before $\phi = 0$.



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Analytic solution

$$\frac{\phi(t)}{M_{Pl}} = \sqrt{\frac{2}{-\beta}}\ln\left[\coth\left(\frac{-\beta}{2}Mt\right)\right] \to 0 \quad (t \to \infty)$$

$$\frac{H(t)}{M} = \coth(-\beta M t) \to 1$$

$$a(t) \propto \sinh^{-1/\beta}(-\beta M t) \to e^{Mt}$$

Slow-roll parameters ($\epsilon_1 \equiv -\dot{H}/H^2, \epsilon_{n+1} \equiv \dot{\epsilon}_n / H\epsilon_n$)

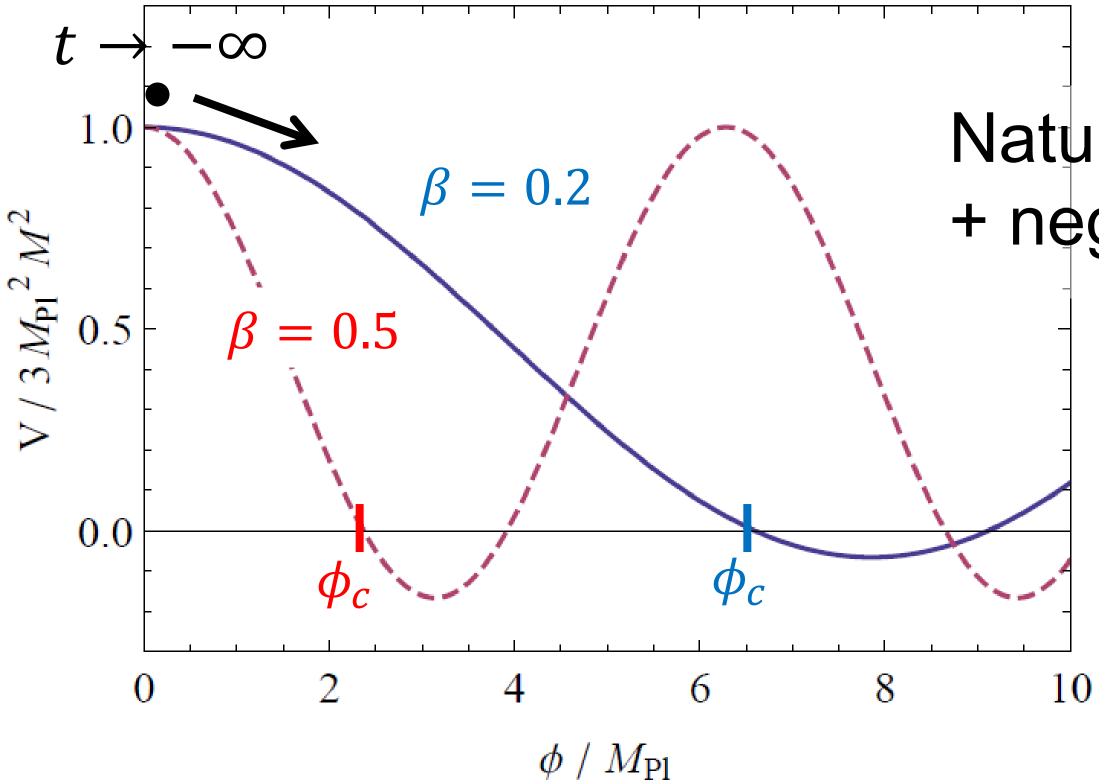
$$2\epsilon_1 = \epsilon_{2n+1} = -\beta/\cosh^2(-\beta M t) \to 0$$

$$\epsilon_{2n} = 2\beta\tanh^2(-\beta M t) \to 2\beta$$

New cos potential ($\beta > 0$)

$$\frac{V(\phi)}{M^2 M_{Pl}^2} = 3\left[1 - \frac{3+\beta}{6}\left\{1 - \cos\left(\sqrt{2\beta}\,\frac{\phi}{M_{Pl}}\right)\right\}\right]$$

Assume inflation ends before $\phi = \phi_c$.



Natural inflation
+ negative CC

New cos potential ($\beta > 0$)

$$\frac{V(\phi)}{M^2 M_{Pl}^2} = 3\left[1 - \frac{3+\beta}{6}\left\{1 - \cos\left(\sqrt{2\beta}\frac{\phi}{M_{Pl}}\right)\right\}\right]$$

Assume inflation ends before $\phi = \phi_c$.

Analytic solution

$$\frac{\phi(t)}{M_{Pl}} = 2\sqrt{\frac{2}{\beta}}\arctan(e^{\beta M t}) \to 0 \qquad (t \to -\infty)$$

$$\frac{H(t)}{M} = -\tanh(\beta M t) \to 1$$

$$a(t) \propto \cosh^{-1/\beta}(\beta M t) \to e^{Mt}$$

Slow-roll parameters

$$2\epsilon_1 = \epsilon_{2n+1} = 2\beta/\sinh^2(\beta M t) \to 0$$

$$\epsilon_{2n} = 2\beta\tanh^2(\beta M t) \to 2\beta$$

Same asymptotic values

Curvature perturbation

Mukhanov-Sasaki equation

$$v_k'' + \left(k^2 - \frac{z''}{z}\right) v_k = 0$$

where $v_k = \sqrt{2} z \zeta_k$ with $z = a\sqrt{\epsilon_1}$.

$$\epsilon_1 \equiv -\dot{H}/H^2$$

$$\epsilon_{n+1} \equiv \dot{\epsilon}_n / H\epsilon_n$$

Without approximation,

$$\frac{z''}{z} = a^2 H^2 \left(2 - \epsilon_1 + \frac{3}{2}\epsilon_2 + \frac{1}{4}\epsilon_2^2 - \frac{1}{2}\epsilon_1\epsilon_2 + \frac{1}{2}\epsilon_2\epsilon_3\right)$$

For both cosh potential and cos potential,

$$\frac{z''}{z} \to \frac{(\beta+2)(\beta+1)}{\tau^2} = \frac{\nu^2 - 1/4}{\tau^2}$$

where

$$\nu \equiv \sqrt{(\beta+2)(\beta+1) + 1/4} = |\beta + 3/2|$$

Curvature perturbation

Since spectral index is given by

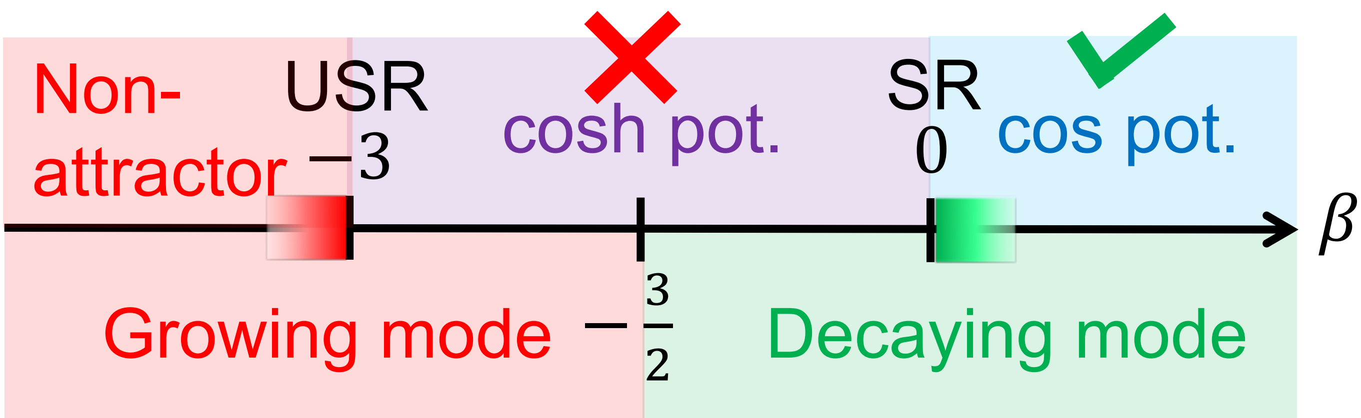
$$n_s - 1 = 3 - 2\nu = 3 - |2\beta + 3|$$

we obtain

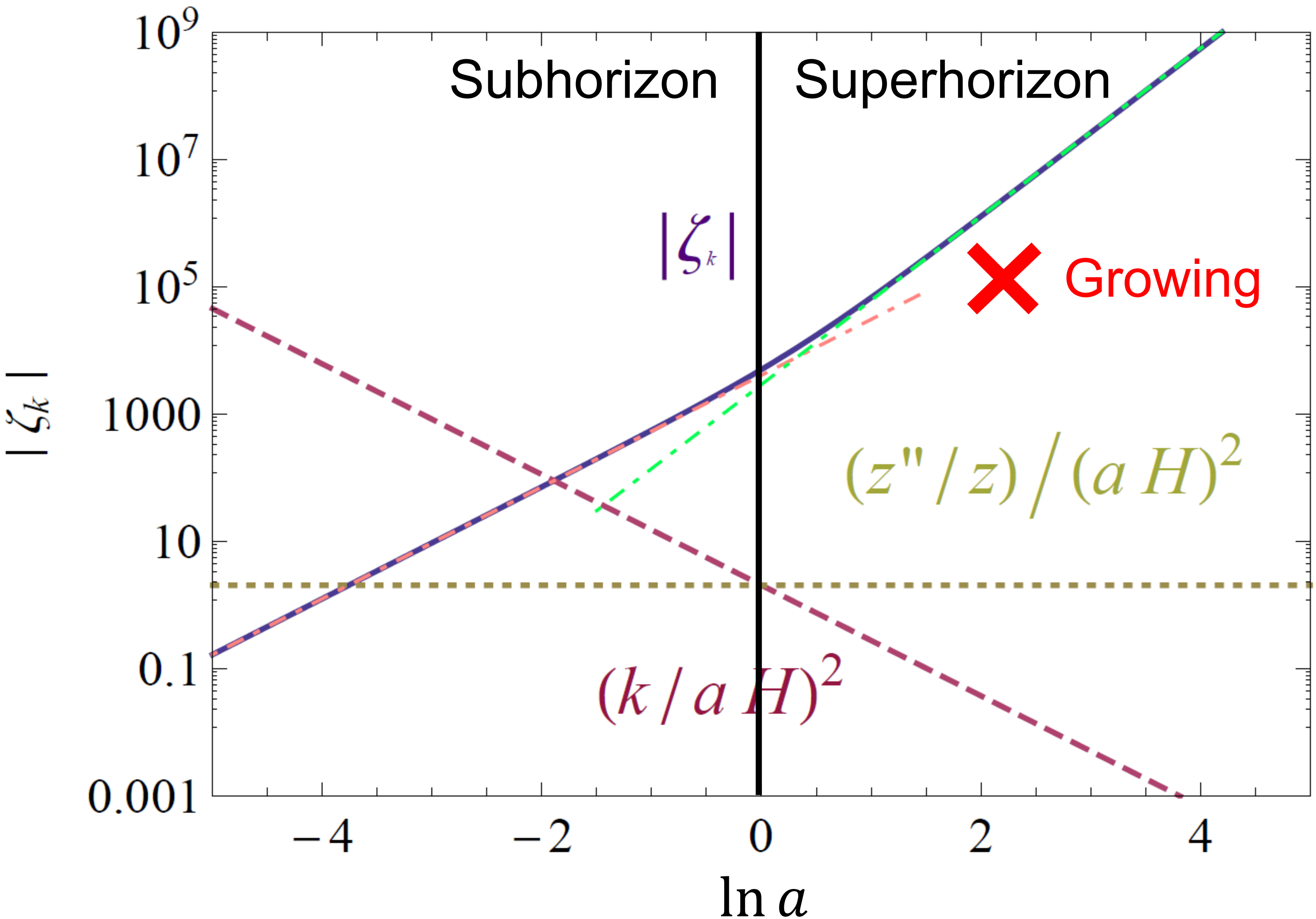
$$\beta = \frac{n_s - 7}{2} \quad \text{or} \quad \frac{1 - n_s}{2}$$

For $n_s = 0.96$,

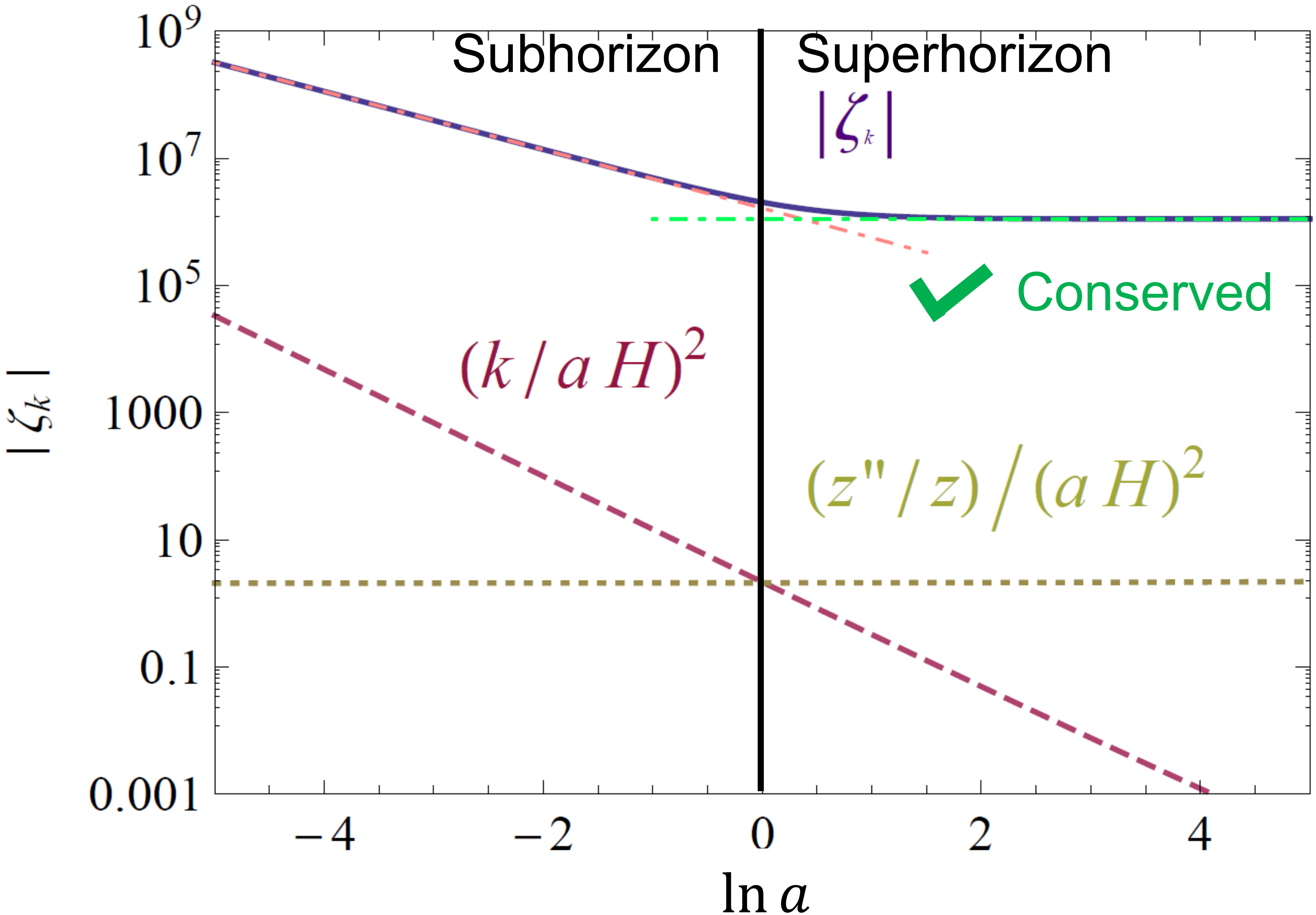
$$\beta = -3.02 \quad \text{or} \quad 0.02$$



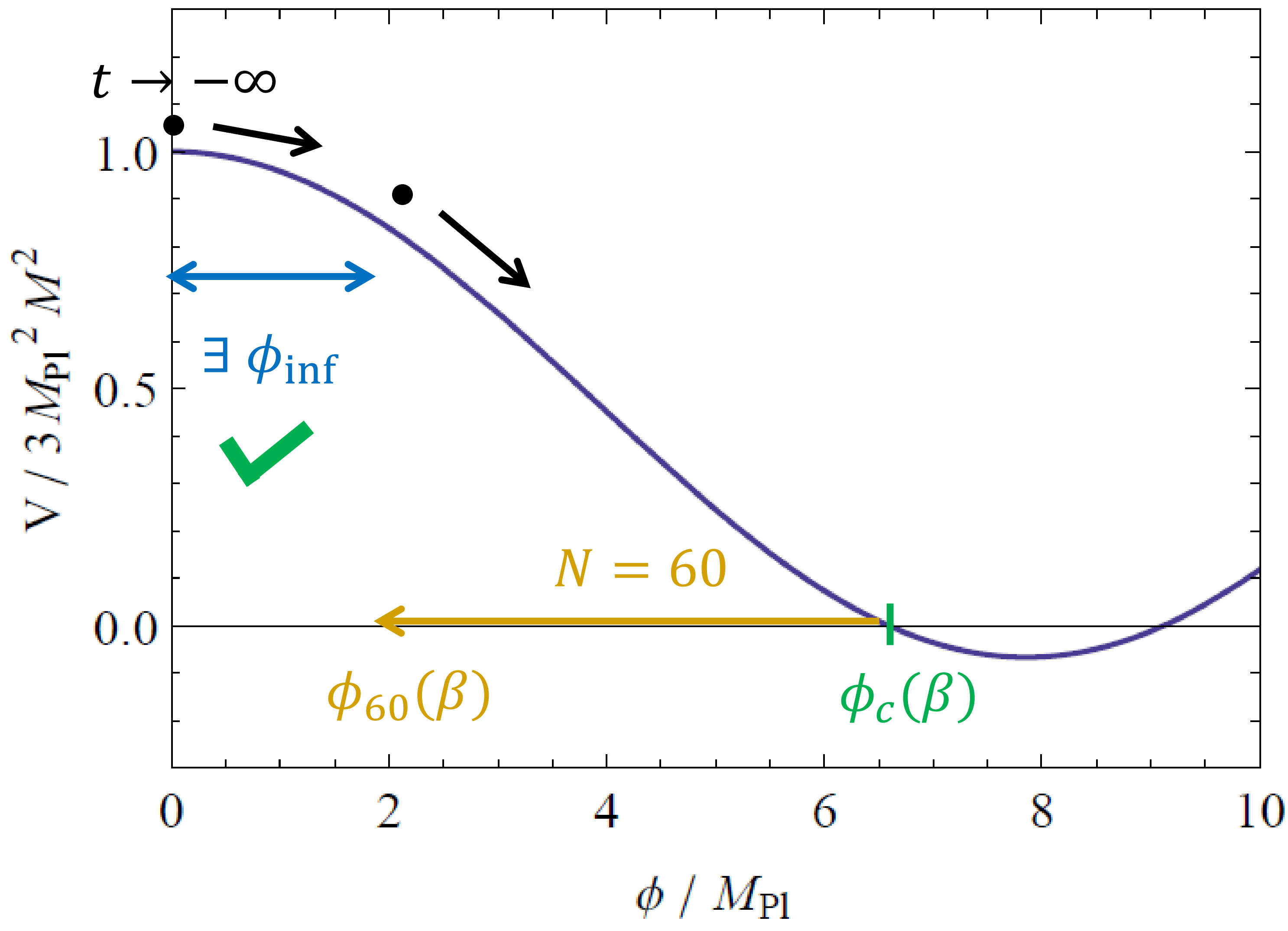
Curvature perturbation $\beta = -3.02$



Curvature perturbation $\beta = 0.02$

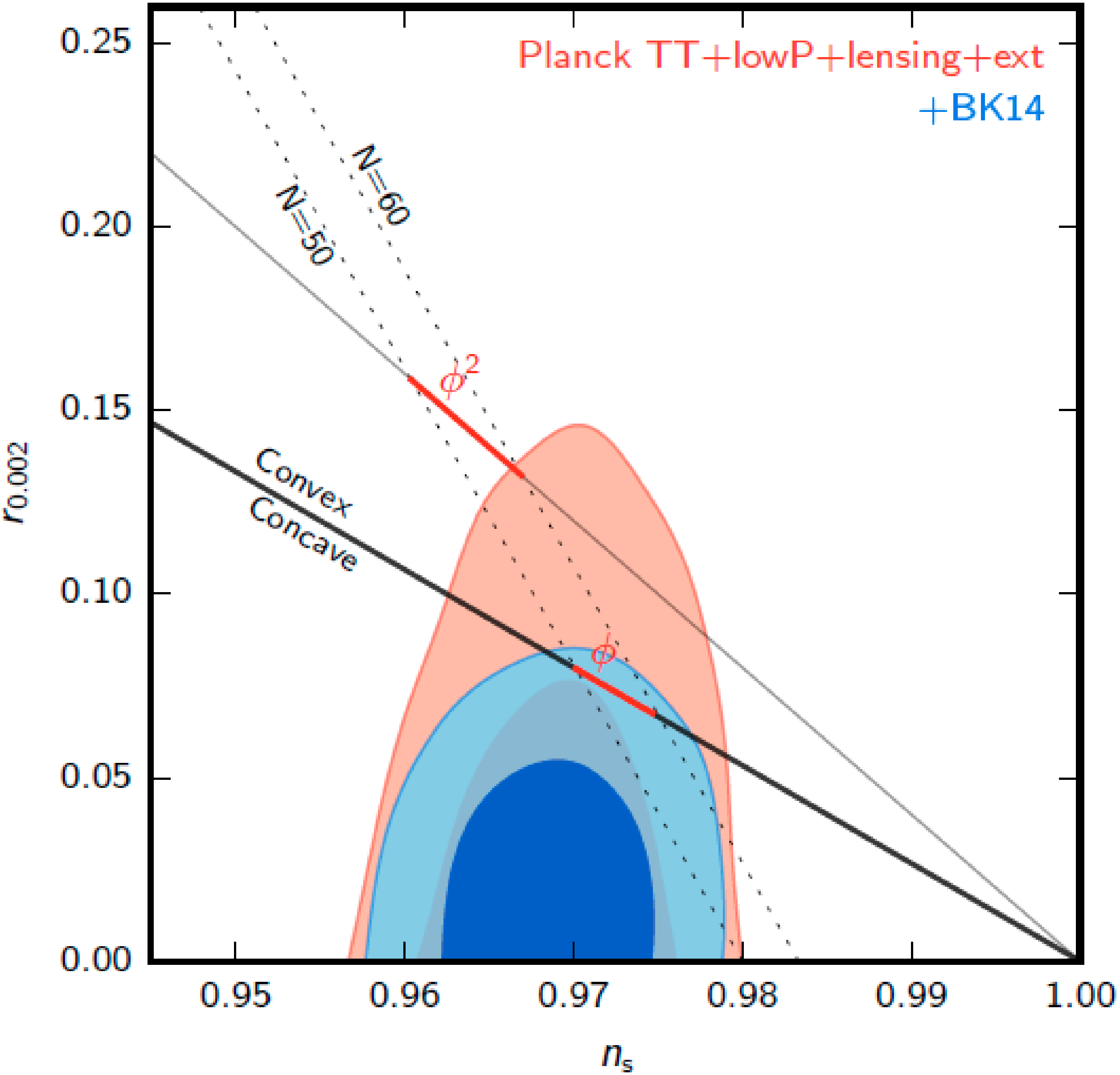


Sufficient number of efolds



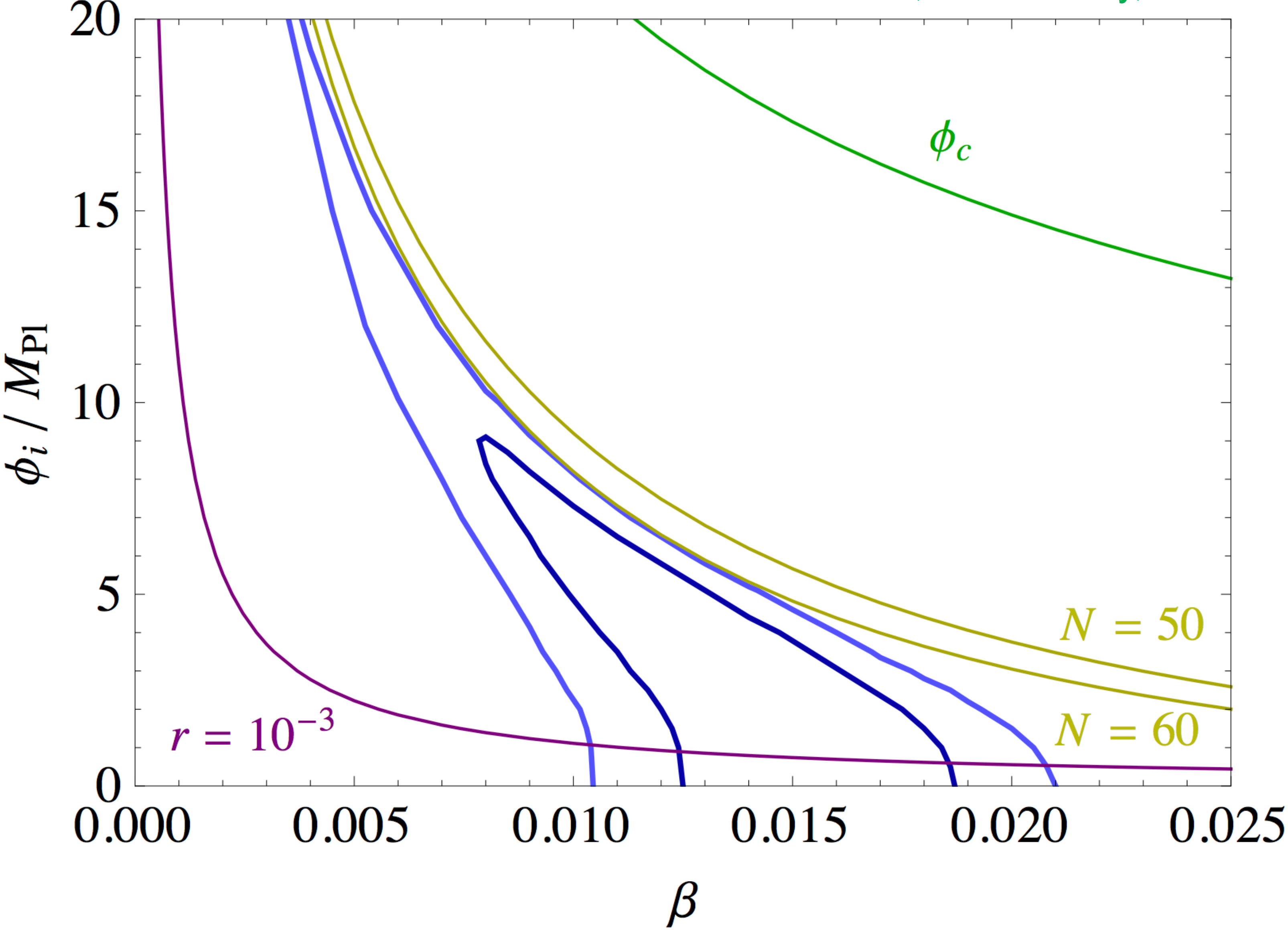
Observational constraint

Ade et al., 1510.09217



Observational constraint

HM, Starobinsky, 1702.05847



$f(R)$ constant-roll inflation

HM, Starobinsky, 1704.08188

$$S = \int d^4x \sqrt{-g} \frac{f(R)}{2}$$

EOM

$$F \equiv df/dR$$

$$3FH^2 = \frac{1}{2}(RF - f) - 3H\dot{F}$$

$$2F\dot{H} = -\ddot{F} + H\dot{F}$$

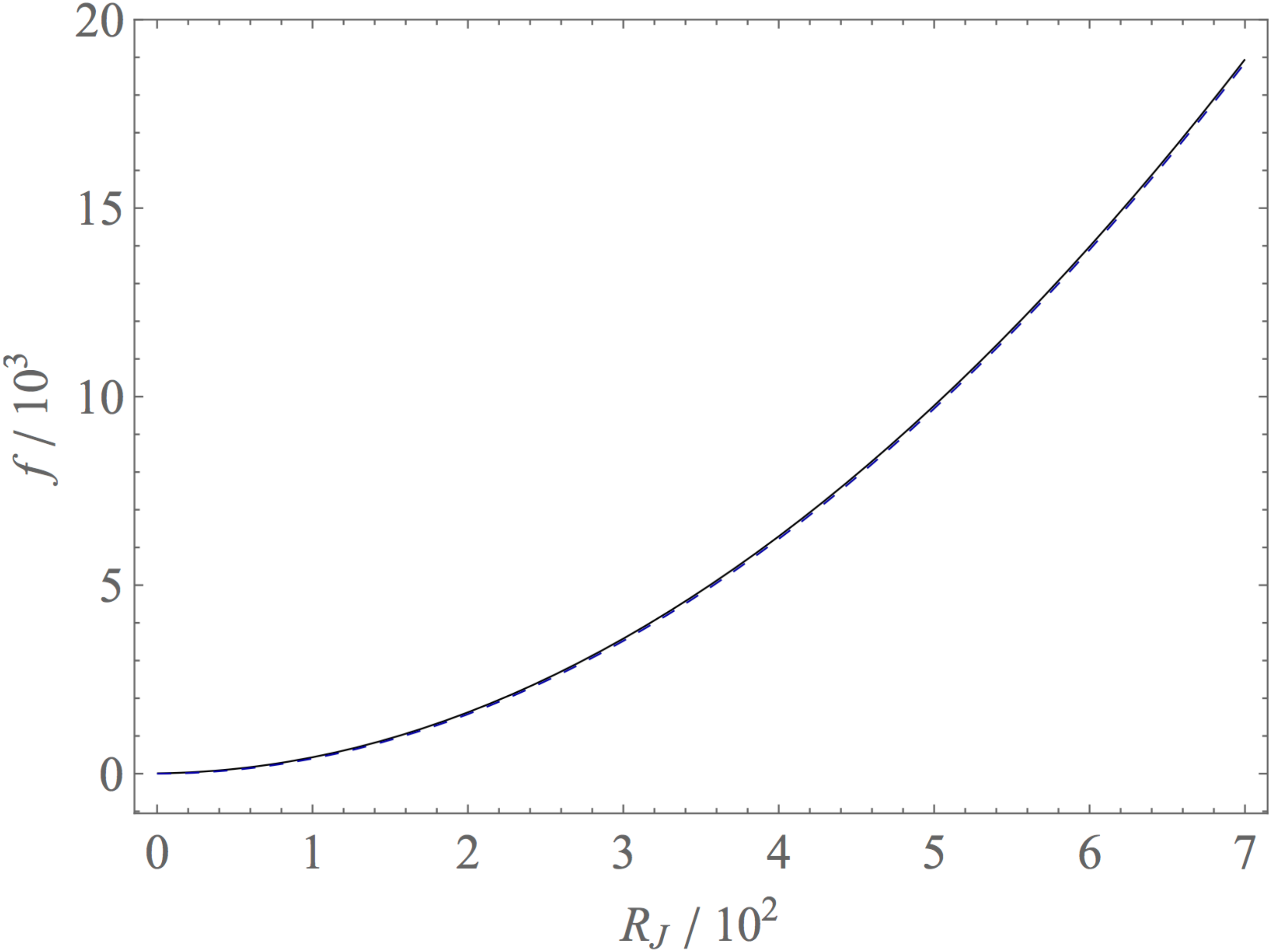
Constant-roll condition

$$\ddot{F} = \beta H \dot{F}$$

allows one to find analytic solution for $f(R)$ (parametric form) and evolution or equivalently Einstein frame potential $V(\phi)$ and $\phi(t_E)$.

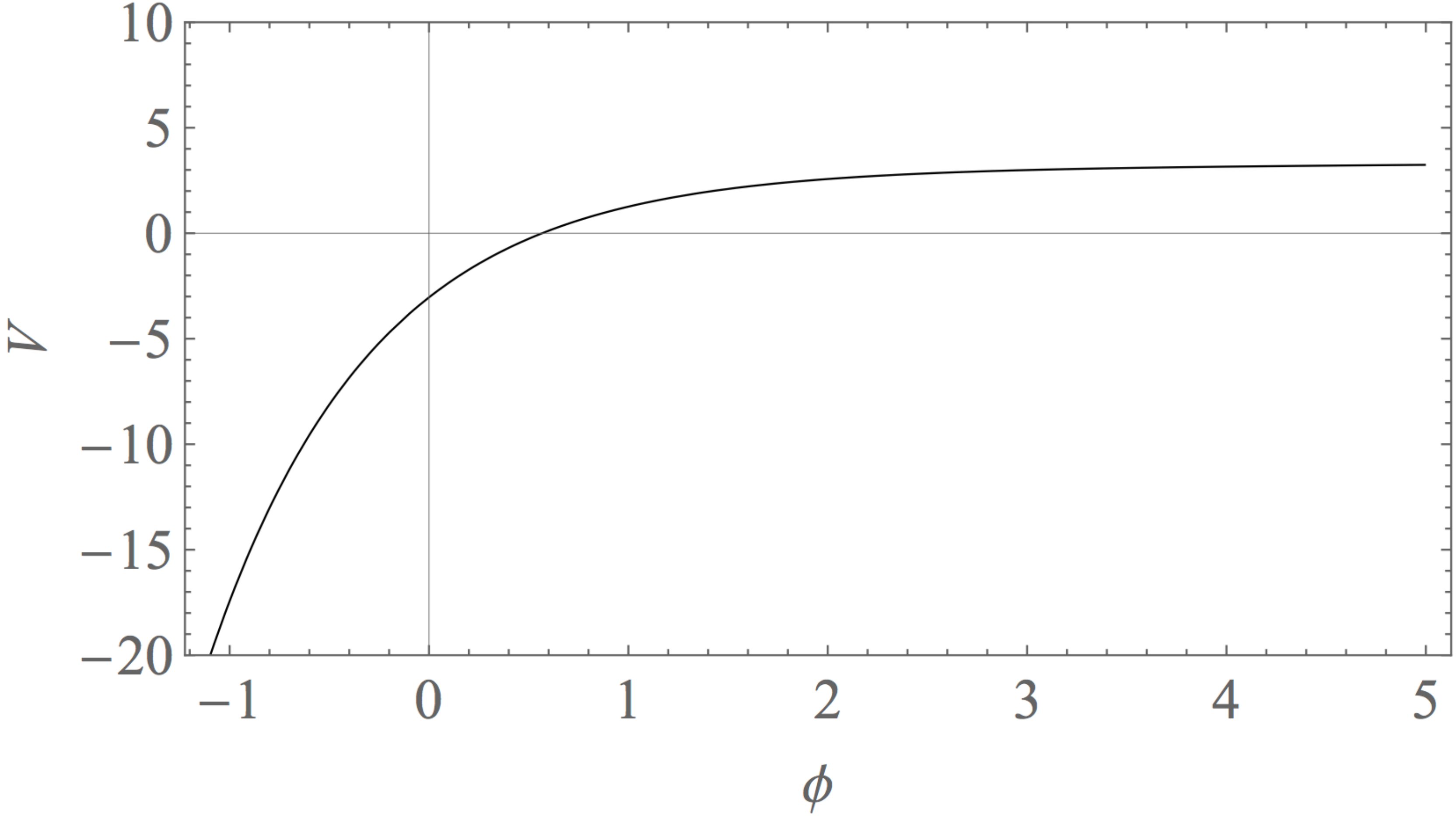
$f(R)$

HM, Starobinsky, 1704.08188



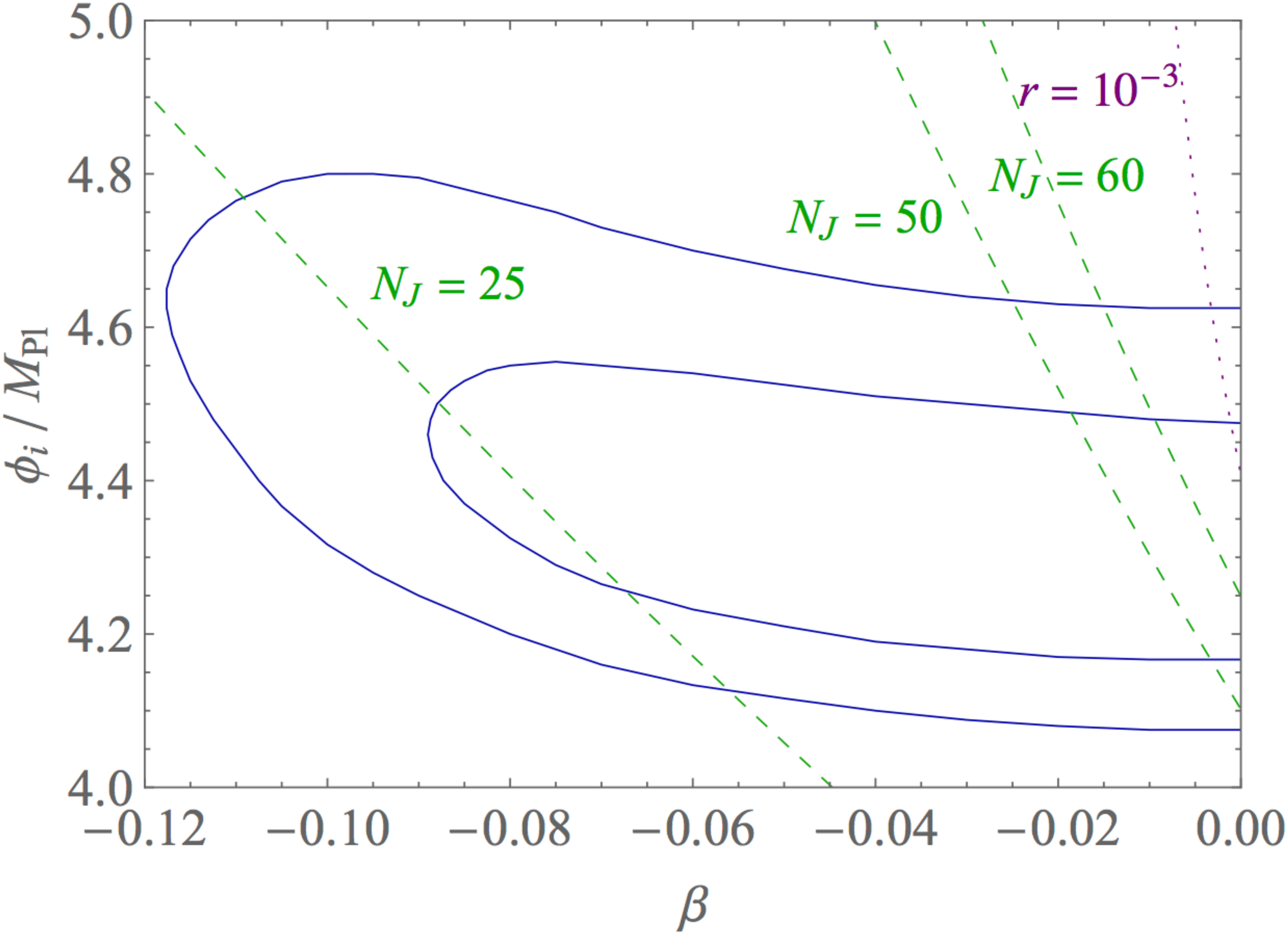
Einstein frame potential

HM, Starobinsky, 1704.08188



Observational constraint

HM, Starobinsky, 1704.08188



Summary

- Even if ϵ_H itself is small, slow roll approximation breaks down by $\frac{d\ln\epsilon_H}{dN}$. If $\frac{d\ln\epsilon_H}{dN} < -3$, ζ_k grows on superhorizon scales.

- Constant-roll condition $\ddot{\phi} = \beta H \dot{\phi}$ or $\ddot{F} = \beta H \dot{F}$ allows one to reconstruct potential and solve inflationary dynamics fully analytically.

- Two-parametric phenomenological constant-roll potentials have parameter regions that satisfy observational constraint.