Diagnosing new physics in $b \to s$ transitions

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The facts

Talk by
F. Martínez Vidal

The $b \to s$ anomalies

2013 - Episode IV: A new hope

2014 - Episode V: LHCb strikes back

2015 - Episode VI: Return of the anomalies

2016 - Episode I: The Belle menace

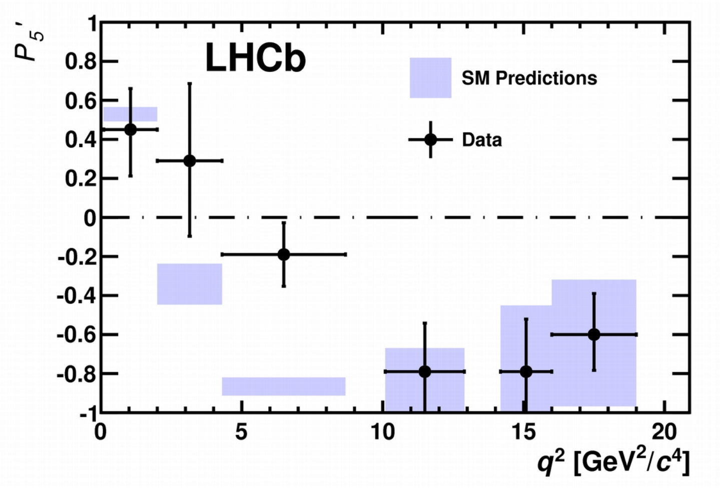
2017 - Episode II: Attack of $R_K{}^*$

2018 - Episode III: ???

The $b \to s$ anomalies

Episode IV: A new hope

2013 : First anomalies found by LHCb



Episode V: LHCb strikes back

2014 : Lepton universality violation

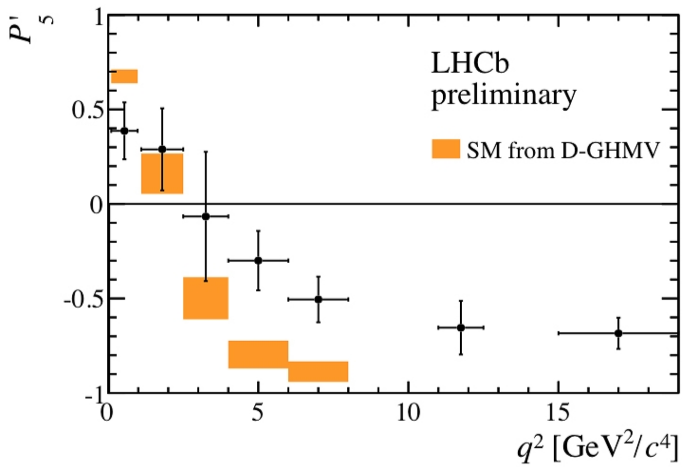
$$R_K = \frac{\mathrm{BR}(B \to K \mu^+ \mu^-)}{\mathrm{BR}(B \to K e^+ e^-)} = 0.745^{+0.090}_{-0.074} \pm 0.036$$

$$R_K^{\mathrm{SM}} \sim 1.00 \pm 0.01$$

2.6σ away from the SM

Episode VI: Return of the anomalies

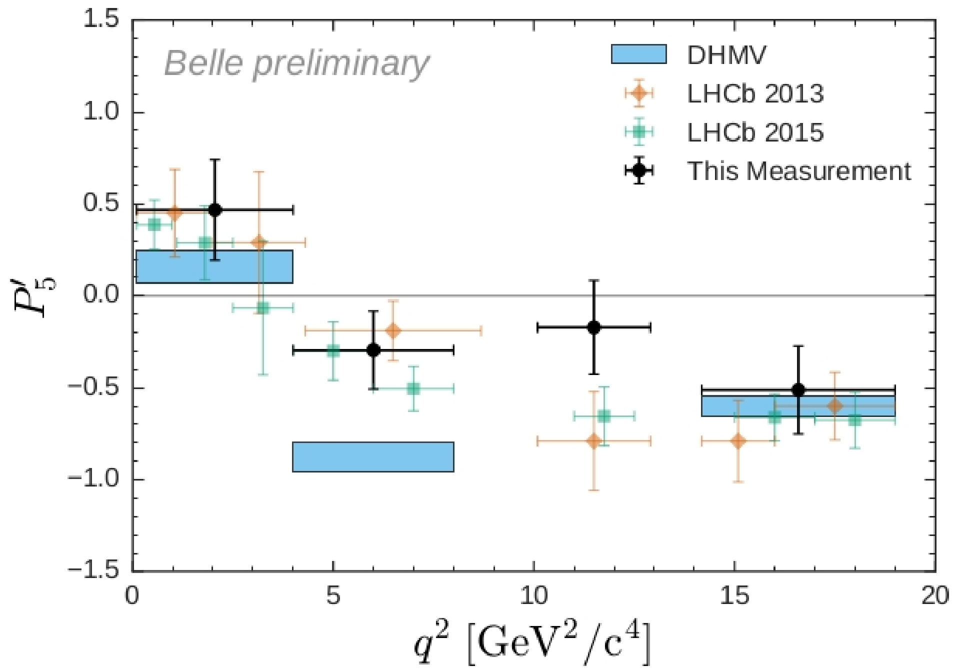
2015 : LHCb confirms first anomalies



The $b \to s$ anomalies

Episode I: The Belle menace

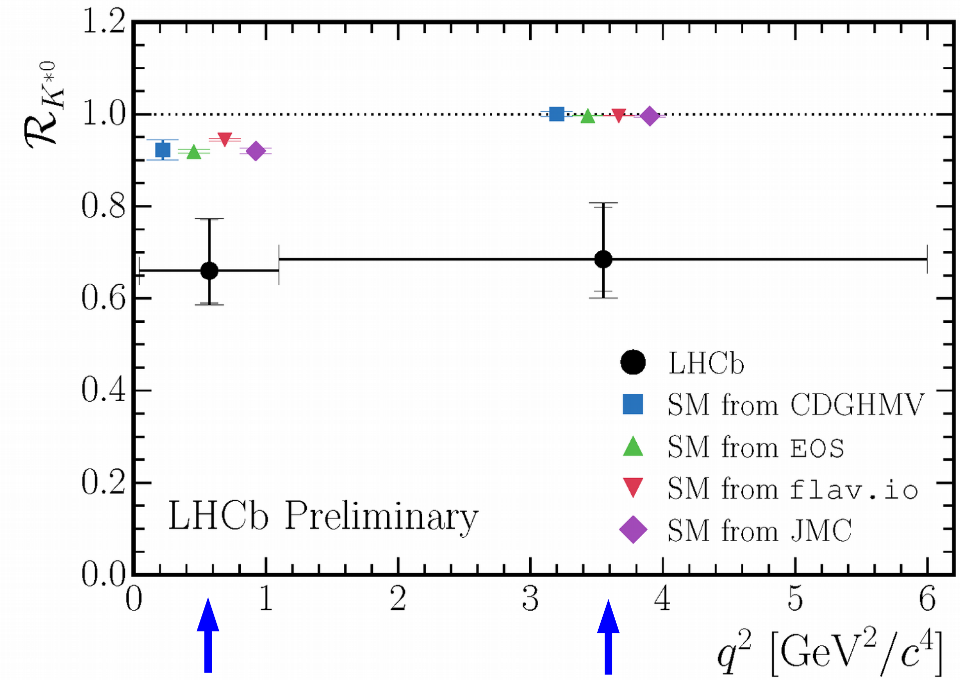
2016 : Belle finds additional hints



P_5' anomaly confirmed
+ little LFVU indication

Episode II: Attack of R_{K^*}

2017 : More universality violation in LHCb



low bin

central bin

[Announced on April 18th]

The $b \to s$ anomalies

LHCb measurement

$$[R_K]_{[1,6]} = 0.745^{+0.090}_{-0.074} \pm 0.036$$

$$[R_{K^*}]_{[0.045,1.1]} = 0.660^{+0.110}_{-0.070} \pm 0.024$$

$$[R_{K^*}]_{[1.1,6]} = 0.685^{+0.113}_{-0.069} \pm 0.047$$

SM prediction

$$[R_K]^{\rm SM}_{[1,6]} = 1.00 \pm 0.01$$ **2.6 σ**

$$[R_{K^*}]^{\rm SM}_{[0.045,1.1]} = 0.92 \pm 0.02$$ **2.2 σ**

$$[R_{K^*}]^{\rm SM}_{[1.1,6]} = 1.00 \pm 0.01$$ **2.4 σ**

Important: LFUV ratios are clean observables, free from hadronic uncertainties

If confirmed: dramatic implications for New Physics

Run-2 update eagerly awaited

The $b \to s$ anomalies

Episode III: Revenge of the Standard Model?

?

Hopefully not!

The interpretation

Talks by
P. Paradisi
T. Nomura

Interpreting the anomalies

$$\boldsymbol{b \to s}$$

Effective hamiltonian

$$\mathcal{H}_{\rm eff} = -\frac{4 G_F}{\sqrt{2}} V_{tb} V_{ts}^\ast \frac{e^2}{16\pi^2} \sum_i \left(C_i \mathcal{O}_i + C_i^\prime \mathcal{O}_i^\prime \right) + \text{h.c.}$$

C_i : Wilson coefficients

$\mathcal{O}_i$: Operators

$$\mathcal{O}_9 = (\bar{s} \gamma_\mu P_L b) \left(\bar{\ell} \gamma^\mu \ell \right)$$

$$\mathcal{O}_{10} = (\bar{s} \gamma_\mu P_L b) \left(\bar{\ell} \gamma^\mu \gamma_5 \ell \right)$$

$$\mathcal{O}_9^\prime = (\bar{s} \gamma_\mu P_R b) \left(\bar{\ell} \gamma^\mu \ell \right)$$

$$\mathcal{O}_{10}^\prime = (\bar{s} \gamma_\mu P_R b) \left(\bar{\ell} \gamma^\mu \gamma_5 \ell \right)$$

$$C_i = C_i^{\rm SM} + C_i^{\rm NP}$$

[analogous for primed operators]

Global fits

Table from Capdevila et al, 1704.05340

	All					LFUV				
1D Hyp.	Best fit	1 σ	2 σ	Pull_{SM}	p-value	Best fit	1 σ	2 σ	Pull_{SM}	p-value
$\mathcal{C}_{9\mu}^{\text{NP}}$	-1.10	$[-1.27, -0.92]$	$[-1.43, -0.74]$	5.7	72	-1.76	$[-2.36, -1.23]$	$[-3.04, -0.76]$	3.9	69
$\mathcal{C}_{9\mu}^{\text{NP}} = -\mathcal{C}_{10\mu}^{\text{NP}}$	-0.61	$[-0.73, -0.48]$	$[-0.87, -0.36]$	5.2	61	-0.66	$[-0.84, -0.48]$	$[-1.04, -0.32]$	4.1	78
$\mathcal{C}_{9\mu}^{\text{NP}} = -\mathcal{C}_{9\mu}'$	-1.01	$[-1.18, -0.84]$	$[-1.33, -0.65]$	5.4	66	-1.64	$[-2.12, -1.05]$	$[-2.52, -0.49]$	3.2	31
$\mathcal{C}_{9\mu}^{\text{NP}} = -3\mathcal{C}_{9e}^{\text{NP}}$	-1.06	[-1.23,-0.89]	[-1.39,-0.71]	5.8	74	-1.35	$[-1.82, -0.95]$	$[-2.38, -0.59]$	4.0	71

All observables
"clean" + "dirty"

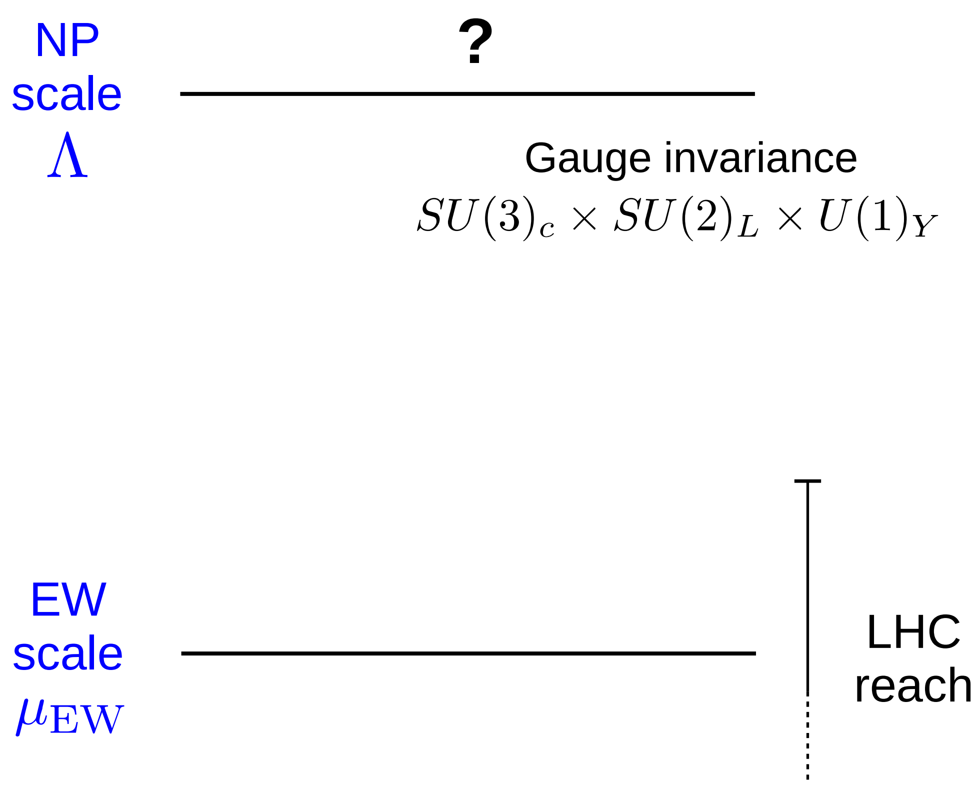
Only LFUV observables
"clean"

New Physics hypothesis preferred over SM by more than 5 σ (4 σ if only LFUV)

The $C_{9\mu}$ coefficient seems to be crucial

Qualitatively similar results in 1704.05435 and 1704.05438

Gauge invariant EFT



The hypothetical UV model must respect the SM gauge invariance

Analyses based on non-gauge-invariant EFTs miss relations among operators

Example: $\mathcal{O}_9$ and $\mathcal{O}_{10}$ from the same gauge invariant operator

$\Rightarrow$ Gauge invariant Effective Field Theory approach

SMEFT

The SMEFT

$$\mathcal{L} = \mathcal{L}_{\text{SM}}^{(4)} + \frac{1}{\Lambda} \sum_k C_k^{(5)} Q_k^{(5)} + \frac{1}{\Lambda^2} \sum_k C_k^{(6)} Q_k^{(6)} + \mathcal{O}\left(\frac{1}{\Lambda^3}\right)$$

Gauge invariant operators

Focus on dimension-6 operators

Warsaw basis

[Grzadkowski et al, 2010]

2499 real parameters (3045 with B-violation)

Full 1-loop RGEs computed

[Alonso, Chang, Jenkins, Manohar, Shotwell, Trott, 2013-2014]

The SMEFT

$$\mathcal{L} = \mathcal{L}_{\text{SM}}^{(4)} + \frac{1}{\Lambda} \sum_k C_k^{(5)} Q_k^{(5)} + \frac{1}{\Lambda^2} \sum_k C_k^{(6)} Q_k^{(6)} + \mathcal{O}\left(\frac{1}{\Lambda^3}\right)$$

Gauge invariant operators

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[Alonso, Chang, Jenkins, Manohar, Shotwell, Trott, 2013-2014]

Non-trivial coupled system

Computers are known to be good at complex games...

DsixTools

A. Celis, J. Fuentes-Martín, A. Vicente, J. Virto

Manual: arXiv:1704.04504
Website: https://dsixtools.github.io/

A Mathematica package for the matching and RGE evolution from the New Physics scale to the scale of low-energy observables

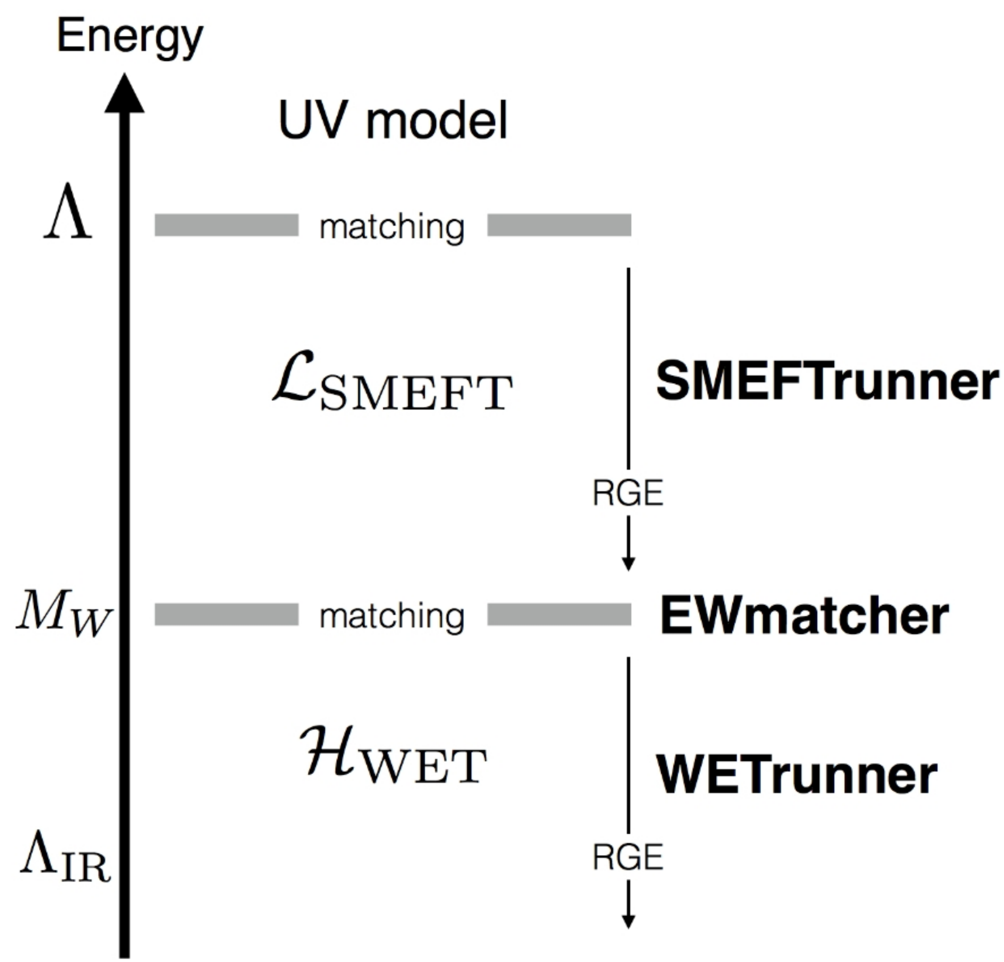
Public code with several extensions on the way

DsixTools

Mathematica package

Modular structure

Each module can be used independently



Interpretation in terms of the SMEFT

Gauge-invariant operators for the anomalies [Celis, Fuentes-Martin, AV, Virto, 2017]

SMEFT operator	Definition	Matching	Order
$[Q_{\ell q}^{(1)}]_{aa23}$	$(\bar{\ell}_a \gamma_\mu \ell_a)(\bar{q}_2 \gamma^\mu q_3)$	$\mathcal{O}_{9,10}$	Tree
$[Q_{\ell q}^{(3)}]_{aa23}$	$(\bar{\ell}_a \gamma_\mu \tau^I \ell_a)(\bar{q}_2 \gamma^\mu \tau^I q_3)$	$\mathcal{O}_{9,10}$	Tree
$[Q_{qe}]_{23aa}$	$(\bar{q}_2 \gamma_\mu q_3)(\bar{e}_a \gamma^\mu e_a)$	$\mathcal{O}_{9,10}$	Tree
$[Q_{\ell d}]_{aa23}$	$(\bar{\ell}_a \gamma_\mu \ell_a)(\bar{d}_2 \gamma^\mu d_3)$	$\mathcal{O}'_{9,10}$	Tree
$[Q_{ed}]_{aa23}$	$(\bar{e}_a \gamma_\mu e_a)(\bar{d}_2 \gamma^\mu d_3)$	$\mathcal{O}'_{9,10}$	Tree
$[Q_{\varphi \ell}^{(1)}]_{aa}$	$\left(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi\right)(\bar{\ell}_a \gamma^\mu \ell_a)$	$\mathcal{O}_{9,10}$	1-loop
$[Q_{\varphi \ell}^{(3)}]_{aa}$	$\left(\varphi^\dagger i \overleftrightarrow{D}^I_\mu \varphi\right)(\bar{\ell}_a \gamma^\mu \tau^I \ell_a)$	$\mathcal{O}_{9,10}$	1-loop
$[Q_{\ell u}]_{aa33}$	$(\bar{\ell}_a \gamma_\mu \ell_a)(\bar{u}_3 \gamma^\mu u_3)$	$\mathcal{O}_{9,10}$	1-loop
$[Q_{\varphi e}]_{aa}$	$\left(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi\right)(\bar{e}_a \gamma^\mu e_a)$	$\mathcal{O}_{9,10}$	1-loop
$[Q_{eu}]_{aa33}$	$(\bar{e}_a \gamma_\mu e_a)(\bar{u}_3 \gamma^\mu u_3)$	$\mathcal{O}_{9,10}$	1-loop

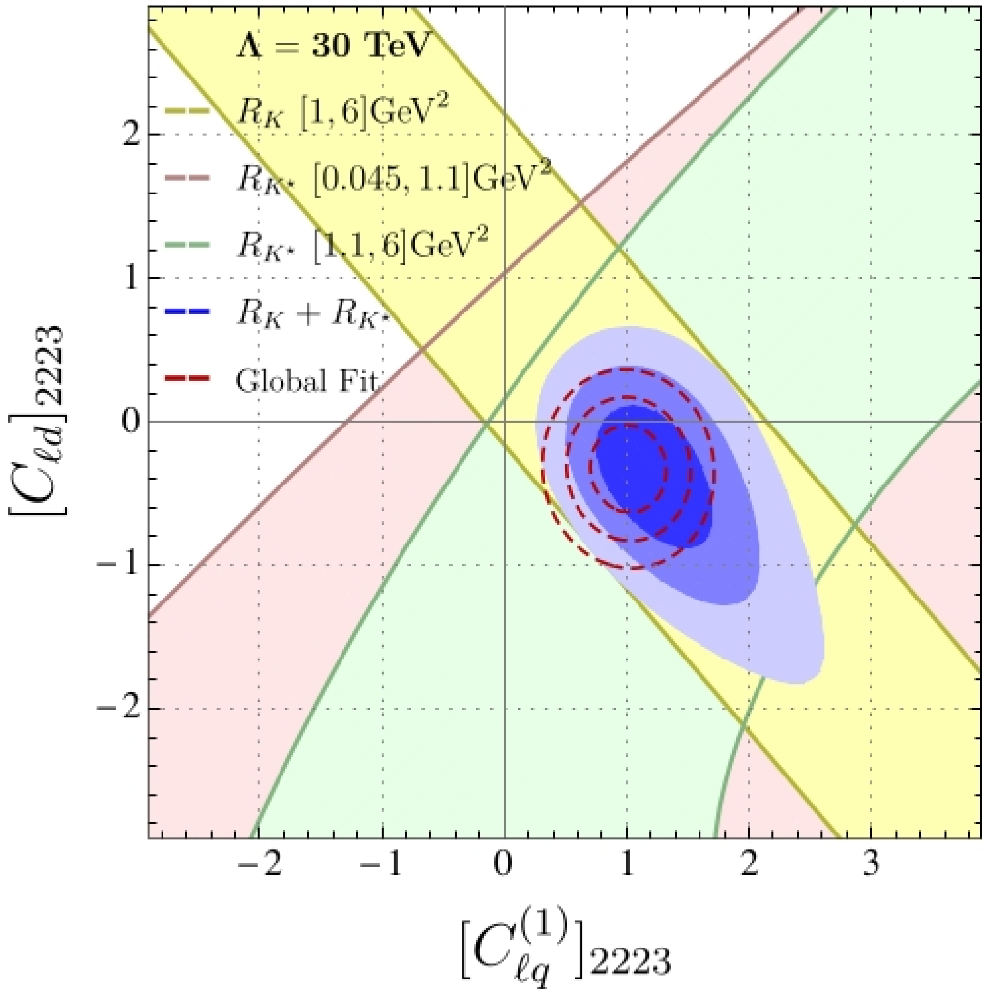
RGE effects (example):

$$[\mathcal{C}_{\ell q}^{(1)}(\mu_{\rm EW})]_{aa23} = [\mathcal{C}_{\ell q}^{(1)}(\Lambda)]_{aa23} - \frac{y_t^2 \lambda_t^{sb}}{16\pi^2} \log\left(\frac{\Lambda}{\mu_{\rm EW}}\right)\left([\mathcal{C}_{\varphi \ell}^{(1)}(\Lambda)]_{aa} - [\mathcal{C}_{\ell u}(\Lambda)]_{aa33}\right)$$

Interpretation in terms of the SMEFT

[Celis, Fuentes-Martin, AV, Virto, 2017]

Global fit in terms of the SMEFT



At $\mu = \mu_{\mathrm{EW}}$

Anomalies explained by $\left[Q_{\ell q}^{(1,3)}\right]_{2223}$

Other operators fail since they predict opposed deviations in R_K and R_{K^*}

At $\mu = \Lambda$

Tree-level: $\left[Q_{\ell q}^{(1,3)}\right]_{2223}$

$$\Rightarrow \quad \Lambda \sim 1 - 50\,\mathrm{TeV}$$

New viable operator: $[Q_{\ell u}]_{2233}$

$$\Rightarrow \quad \Lambda \sim 1\,\mathrm{TeV}$$

Summary

Summary

The anomalies in b-s transitions constitute an interesting set of hints for NP: an intriguing global pattern

Model-independent interpretation in terms of EFTs: guideline for model builders

SMEFT operators that can do the job:

At $\mu = \mu_{\rm EW}$

$$\left[Q_{\ell q}^{(1)}\right]_{2223} = \left(\bar{\ell}_2 \gamma_\mu \ell_2\right) \left(\bar{q}_2 \gamma^\mu q_3\right)$$

$$\left[Q_{\ell q}^{(3)}\right]_{2223} = \left(\bar{\ell}_2 \gamma_\mu \tau^I \ell_2\right) \left(\bar{q}_2 \gamma^\mu \tau^I q_3\right)$$

At $\mu = \Lambda$

$$\left[Q_{\ell q}^{(1)}\right]_{2223} = \left(\bar{\ell}_2 \gamma_\mu \ell_2\right) \left(\bar{q}_2 \gamma^\mu q_3\right)$$

$$\left[Q_{\ell q}^{(3)}\right]_{2223} = \left(\bar{\ell}_2 \gamma_\mu \tau^I \ell_2\right) \left(\bar{q}_2 \gamma^\mu \tau^I q_3\right)$$

$$\left[Q_{\ell u}\right]_{2233} = \left(\bar{\ell}_2 \gamma_\mu \ell_2\right) \left(\bar{u}_3 \gamma^\mu u_3\right)$$

Backup slides

The $b \to s$ anomalies

[LHCb, 2013]
1305.2168, 1308.1707, 1403.8044

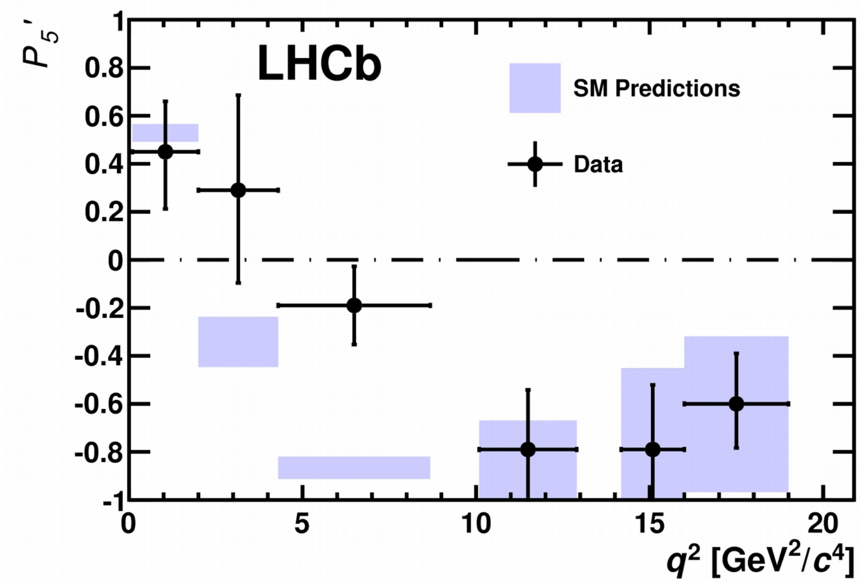
Episode IV: A new hope

2013 : First anomalies found by LHCb

- Data collected: 1 fb^{-1} $(3 \text{ fb}^{-1}$ in some observables$)$
- Decrease (w.r.t. the SM) in several branching ratios
- Several anomalies in angular observables

Popular example: P_5' in $B \to K^* \mu^+ \mu^-$

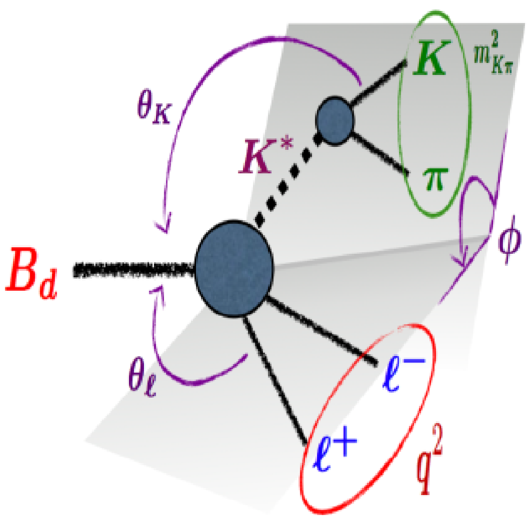
arXiv:1308.1707



The $b \to s$ anomalies

$B \to K^* (\to K\pi)\, \mu^+\mu^-$ differential angular distribution

$$\frac{d^4\Gamma}{dq^2\, d\cos\theta_K\, d\cos\theta_l\, d\phi} = \frac{9}{32\pi}\Big[J_{1s}\sin^2\theta_K + J_{1c}\cos^2\theta_K + (J_{2s}\sin^2\theta_K + J_{2c}\cos^2\theta_K)\cos 2\theta_l$$
$$+J_3\sin^2\theta_K\sin^2\theta_l\cos 2\phi + J_4\sin 2\theta_K\sin 2\theta_l\cos\phi + J_5\sin 2\theta_K\sin\theta_l\cos\phi$$
$$+(J_{6s}\sin^2\theta_K + J_{6c}\cos^2\theta_K)\cos\theta_l + J_7\sin 2\theta_K\sin\theta_l\sin\phi$$
$$+J_8\sin 2\theta_K\sin 2\theta_l\sin\phi + J_9\sin^2\theta_K\sin^2\theta_l\sin 2\phi\Big]$$



[Figure borrowed from Javier Virto]

J_i : functions of q^2, C_i, FF

Optimized observables
[Descotes-Genon et al, 2012, 2013]

$$P_5' = \frac{J_5}{2\sqrt{-J_{2s}J_{2c}}}$$

The $b \to s$ anomalies

[LHCb, 2014]
arXiv:1406.6482

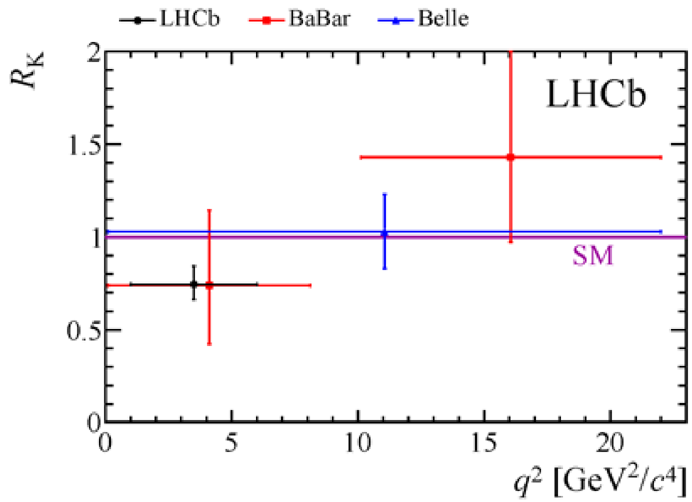
Episode V: LHCb strikes back

2014 : Lepton universality violation

Obtained with 3 fb^{-1}

$$R_K = [R_K]_{[1,6]} = \left.\frac{\text{BR}(B \to K \mu^+ \mu^-)}{\text{BR}(B \to K e^+ e^-)}\right|_{q^2 \in [1,6] \text{GeV}^2} = 0.745^{+0.090}_{-0.074} \pm 0.036$$

$$R_K^{\text{SM}} \sim 1.00 \pm 0.01$$



2.6σ away from the SM

The $b \to s$ anomalies

[LHCb, 2015]
1512.04442

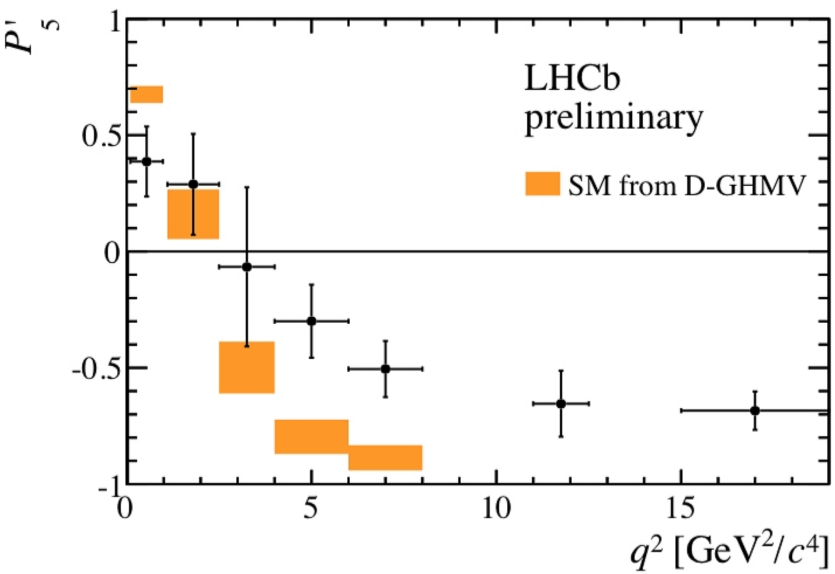
Episode VI: Return of the anomalies

2015 : LHCb confirms first anomalies

$$\text{All observables updated to } 3 \text{ fb}^{-1}$$

[Complete LHC Run I dataset]

Errors shrunk...
... anomalies persist

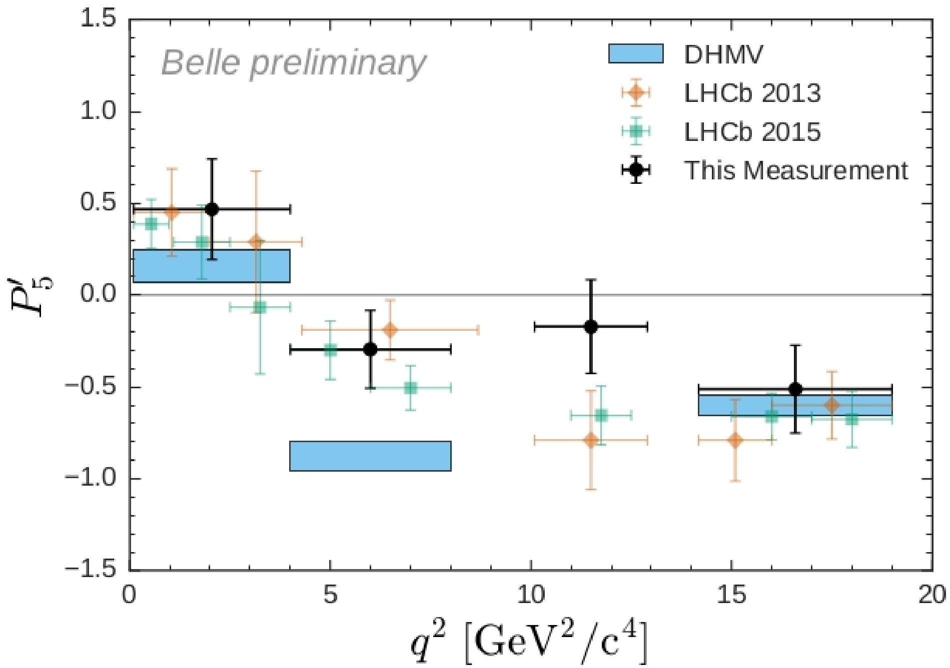


The $b \to s$ anomalies

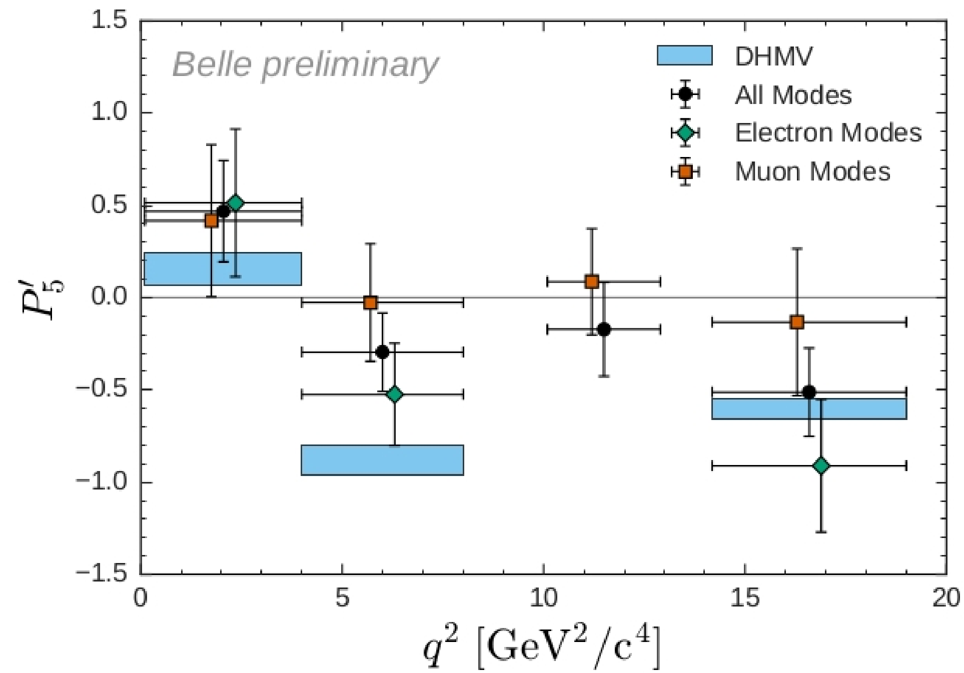
[Belle, 2016]
1612.05014

Episode I: The Belle menace

2016 : Belle finds additional hints



P_5' anomaly confirmed



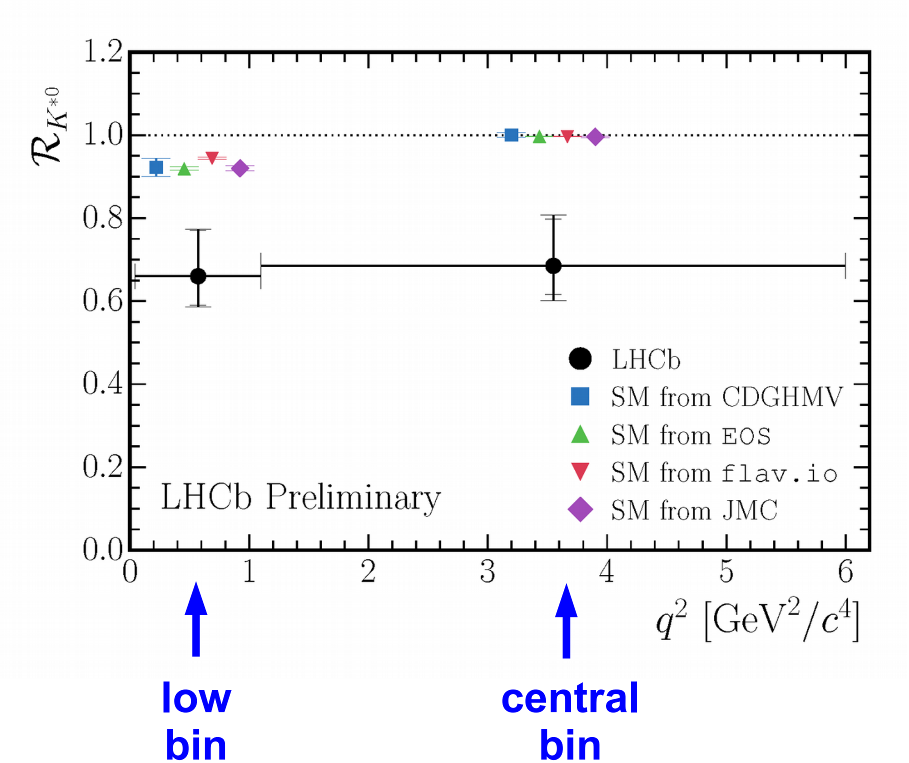
Little LFVU indication

[LHCb, 2017]
Talk by S. Bifani
April 18th
1705.05802

Episode II: Attack of R_{K^*}

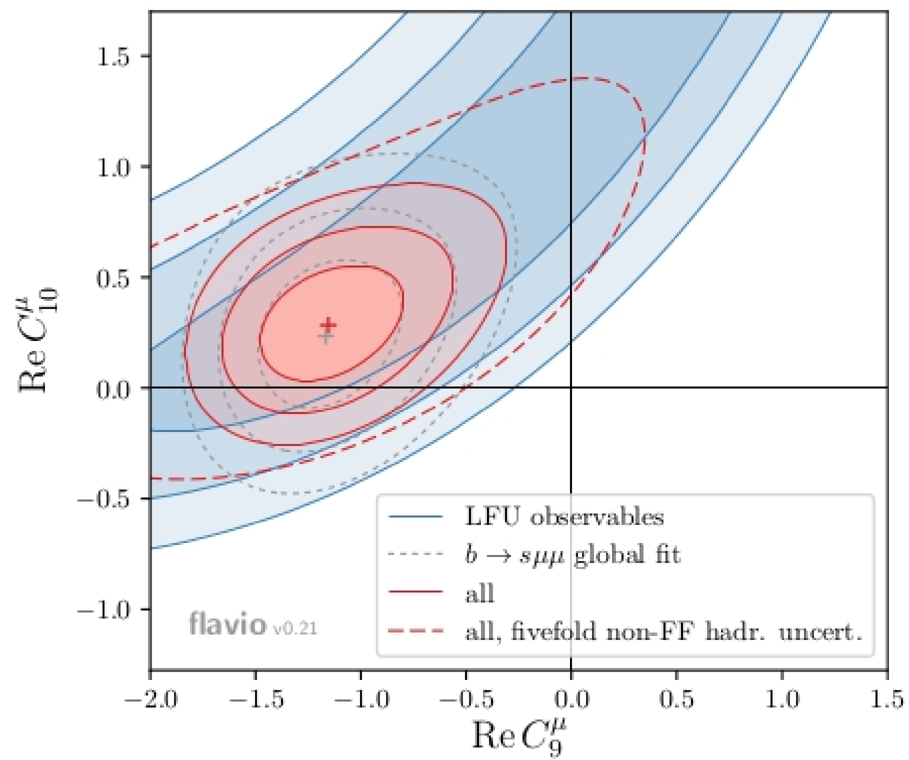
2017 : More universality violation in LHCb

Obtained with 3 fb^{-1}



Global fits

Plot from Altmannshofer et al, 1704.05435



The “clean” and “dirty” anomalies are compatible

A DsixTools Program

This notebook loads DsixTools and shows how to use the SMEFTrunner module.

```
SetDirectory[NotebookDirectory[]];
```

Start DsixTools

```
Needs["DsixTools`"]
```

Read input files

```
ReadInputFiles["Options.dat", "WCsInput.dat", "SMInput.dat"];
```

Load SMEFTrunner module

```
LoadModule["SMEFTrunner"]
```

Use SMEFTrunner module

```
LoadBetaFunctions;
```

```
RunRGEsSMEFT;
```

SMEFT WCs input file

```
Block WC4
6 1.0          # phiBtilde
Block IMWCDPHI
1 1 0.1        # dphi(1,1)
1 2 0.2        # dphi(1,2)
1 3 0.3        # dphi(1,3)
2 1 0.1        # dphi(2,1)
2 2 0.2        # dphi(2,2)
2 3 0.3        # dphi(2,3)
3 1 0.4        # dphi(3,1)
3 2 0.5        # dphi(3,2)
3 3 0.6        # dphi(3,3)
Block WCDD
2 3 2 3 1.0    # dd(2,3,2,3)
Block WCPHIQ3
1 3 1.0        # phiq3(1,3)
```

WCsInput.dat

Simple text file

Inspired by the SLHA

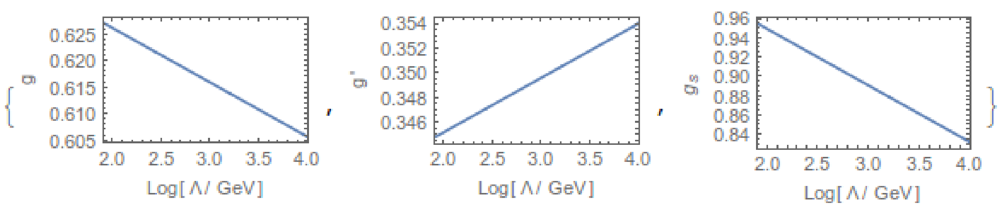
Similar format for the output file

Also possible to give input directly on the notebook

Results after SMEFTrunner

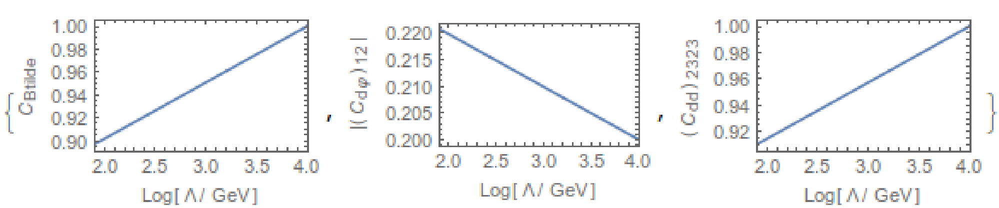
```
In[7]:= (* The results can also be plotted as a function of the energy scale *)

In[8]:= (* Gauge couplings *)
plotGauge1 = Plot[outSMEFTrunner[[1]], {t, tLOW, tHIGH}, Frame → True, Axes → False, PlotRange → {{tLOW, tHIGH}, Automatic},
   FrameLabel → {"Log[Λ/GeV]", "g", None, None}];
plotGauge2 = Plot[outSMEFTrunner[[2]], {t, tLOW, tHIGH}, Frame → True, Axes → False, PlotRange → {{tLOW, tHIGH}, Automatic},
   FrameLabel → {"Log[Λ/GeV]", "g'", None, None}];
plotGauge3 = Plot[outSMEFTrunner[[3]], {t, tLOW, tHIGH}, Frame → True, Axes → False, PlotRange → {{tLOW, tHIGH}, Automatic},
   FrameLabel → {"Log[Λ/GeV]", "gs", None, None}];
plotGauge = {plotGauge1, plotGauge2, plotGauge3}
```

Out[11]=




```
In[12]:= (* Wilson coefficients *)
plotWC1 = Plot[outSMEFTrunner[[48]], {t, tLOW, tHIGH}, Frame → True, Axes → False, PlotRange → {{tLOW, tHIGH}, Automatic},
   FrameLabel → {"Log[Λ/GeV]", "CBtilde", None, None}];
plotWC2 = Plot[Abs[outSMEFTrunner[[61]]], {t, tLOW, tHIGH}, Frame → True, Axes → False, PlotRange → {{tLOW, tHIGH}, Automatic},
   FrameLabel → {"Log[Λ/GeV]", "|(Cdφ)12|", None, None}];
plotWC3 = Plot[outSMEFTrunner[[443]], {t, tLOW, tHIGH}, Frame → True, Axes → False, PlotRange → {{tLOW, tHIGH}, Automatic},
   FrameLabel → {"Log[Λ/GeV]", "(Cdd)2323", None, None}];
plotWC = {plotWC1, plotWC2, plotWC3}
```

Out[15]=



Another simple program: analytics

A DsixTools Program

This notebook shows how to use the SMEFTrunner module to study SMEFT β functions analytically.

```
SetDirectory[NotebookDirectory[]];
```

Start DsixTools

```
Needs["DsixTools`"]
```

Set CP conservation

```
CPV = 0;
```

Load SMEFTrunner module

```
LoadModule["SMEFTrunner"]
```

Compute β functions

```
GetBeta;
```

Another simple program: analytics

Results

```
In[6]:= (* Let us compute β1q(1) and β1q(3) assuming top dominance and no NP effects in the 1st fermion family *)

In[7]:= (* Top dominance approximation *)
        top = {GD[i_, j_] :> 0, GE[i_, j_] :> 0, GU[i_, j_] :> If[i == j == 3, Vtb yt, If[i == 2 && j == 3, Vts yt, 0]]};

In[8]:= (* No NP in 1st family *)
        WCs2F = {φL1, φL3, φQ1, φQ3};
        WCs4F = {LQ1, LQ3, LU, QE, QU1, QU8, QD1, QD8, QQ1, QQ3};
        nofirst2F = Table[Part[WCs2F, i][a_, b_] → If[AnyTrue[{a, b}, # == 1 &], 0, 1] Part[WCs2F, i][a, b], {i, 1, Length[WCs2F]}];
        nofirst4F = Table[Part[WCs4F, i][a_, b_, c_, d_] → If[AnyTrue[{a, b, c, d}, # == 1 &], 0, 1] Part[WCs4F, i][a, b, c, d],
           {i, 1, Length[WCs4F]}];
        nofirst = Join[nofirst2F, nofirst4F];

In[13]:= βlq1 = β[lq1][[2, 2, 2, 3]] /. top /. nofirst // Expand
```

Out[13]=
$$\frac{1}{2}\,\text{Vtb Vts yt}^2\,\text{LQ1}[2,2,2,2] - \frac{1}{3}\,\text{gp}^2\,\text{LQ1}[2,2,2,3] + \frac{1}{2}\,\text{Vtb}^2\,\text{yt}^2\,\text{LQ1}[2,2,2,3] + \frac{1}{2}\,\text{Vts}^2\,\text{yt}^2\,\text{LQ1}[2,2,2,3] + \frac{1}{2}\,\text{Vtb Vts yt}^2\,\text{LQ1}[2,2,3,3] + \frac{2}{3}\,\text{gp}^2\,\text{LQ1}[3,3,2,3] + 9\,g^2\,\text{LQ3}[2,2,2,3] - \text{Vtb Vts yt}^2\,\text{LU}[2,2,3,3] + \frac{2}{3}\,\text{gp}^2\,\text{QD1}[2,3,2,2] + \frac{2}{3}\,\text{gp}^2\,\text{QD1}[2,3,3,3] + \frac{2}{3}\,\text{gp}^2\,\text{QE}[2,3,2,2] + \frac{2}{3}\,\text{gp}^2\,\text{QE}[2,3,3,3] - \frac{2}{9}\,\text{gp}^2\,\text{QQ1}[2,2,2,3] - \frac{4}{3}\,\text{gp}^2\,\text{QQ1}[2,3,2,2] - \frac{14}{9}\,\text{gp}^2\,\text{QQ1}[2,3,3,3] - \frac{2}{3}\,\text{gp}^2\,\text{QQ3}[2,2,2,3] - \frac{2}{3}\,\text{gp}^2\,\text{QQ3}[2,3,3,3] - \frac{4}{3}\,\text{gp}^2\,\text{QU1}[2,3,2,2] - \frac{4}{3}\,\text{gp}^2\,\text{QU1}[2,3,3,3] + \text{Vtb Vts yt}^2\,\varphi\text{L1}[2,2] - \frac{1}{3}\,\text{gp}^2\,\varphi\text{Q1}[2,3]$$

```
In[14]:= βlq3 = β[lq3][[2, 2, 2, 3]] /. top /. nofirst // Expand
```

Out[14]=
$$3\,g^2\,\text{LQ1}[2,2,2,3] + \frac{1}{2}\,\text{Vtb Vts yt}^2\,\text{LQ3}[2,2,2,2] - \frac{16}{3}\,g^2\,\text{LQ3}[2,2,2,3] - \text{gp}^2\,\text{LQ3}[2,2,2,3] + \frac{1}{2}\,\text{Vtb}^2\,\text{yt}^2\,\text{LQ3}[2,2,2,3] + \frac{1}{2}\,\text{Vts}^2\,\text{yt}^2\,\text{LQ3}[2,2,2,3] + \frac{1}{2}\,\text{Vtb Vts yt}^2\,\text{LQ3}[2,2,3,3] + \frac{2}{3}\,g^2\,\text{LQ3}[3,3,2,3] + \frac{2}{3}\,g^2\,\text{QQ1}[2,2,2,3] + \frac{2}{3}\,g^2\,\text{QQ1}[2,3,3,3] - \frac{2}{3}\,g^2\,\text{QQ3}[2,2,2,3] + 4\,g^2\,\text{QQ3}[2,3,2,2] + \frac{10}{3}\,g^2\,\text{QQ3}[2,3,3,3] - \text{Vtb Vts yt}^2\,\varphi\text{L3}[2,2] + \frac{1}{3}\,g^2\,\varphi\text{Q3}[2,3]$$