Towards a non-perturbative approach to the (EW) Hierarchy problem

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PASCOS 2017
June 22, Madrid

Introduction

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The quantum face of the problem:

- $\delta m^2_{H,\,X} \sim M_X^2$ is expected, whenever new physics comes about with M_X a new particle's threshold.

  $m_H \ll M_X \longrightarrow$ Fine-tuning or some mechanism to suppress the quadratic contributions

- $\delta m^2_{H,\,grav.} \sim M_P^2$ is naively expected, with M_P the gravity interaction scale.

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List of proposals:

SUSY, Composite Higgs, Extra dimensions, …
Clockwork, Relaxion, …
…

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Approaches to the hierarchy problem:

EFT approach

Questioned by the LHC data.

G. F. Giudice, PoS EPS -HEP2013 (2013) 163
[arXiv:1307.7879 [hep-ph]].

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Approaches to the hierarchy problem:

EFT approach $\xrightarrow{\text{Ways out}}$ Something else $\rightarrow$ Non-perturbative physics

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G. F. Giudice, PoS EPS -HEP2013 (2013) 163 [arXiv:1307.7879 [hep-ph]].

Some deep principle that shape the behavior of a theory both at HE and LE…

Examples: MPCP, Higgs criticality

C. D. Froggatt and H. B. Nielsen, Phys. Lett. B 368, 96 (1996) [hep-ph/9511371]
M. Shaposhnikov and C. Wetterich, Phys. Lett. B 683 (2010) 196 [arXiv:0912.0208 [hep-th]]

Or much more casual non-perturbative effects

Example: EW vacuum decay

V. Branchina, E. Messina and M. Sher, Phys. Rev. D 91, 013003 (2015) [arXiv:1408.5302 [hep-ph]]
F. Bezrukov and M. Shaposhnikov, J. Exp. Theor. Phys. 120, 335 (2015) [arXiv:1411.1923 [hep-ph]]

Let's look for the effect of a similar kind: generation of the Higgs vev via instanton.

To make this scenario possible, we must have a framework, in which m_H is not generated perturbatively.

Setup

Conjectures:

- Scale Invariance (SI)

Motivation: No scales – no problems
Spontaneous breaking of SI – technically natural m_H

G. 't Hooft, NATO Sci. Ser. B 59 (1980) 135

- No heavy BSM DoFs

Motivation: We want to solve the problem of the m_H/M_P ratio;
No quadratic corrections associated with the heavy particle's mass scales

S. W. Hawking and W. Israel, General Relativity : An Einstein Centenary Survey, Chapter 16

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A good candidate for the theory that obeys the conjectures above and is acceptable phenomenologically is a Higgs-Dilaton model.

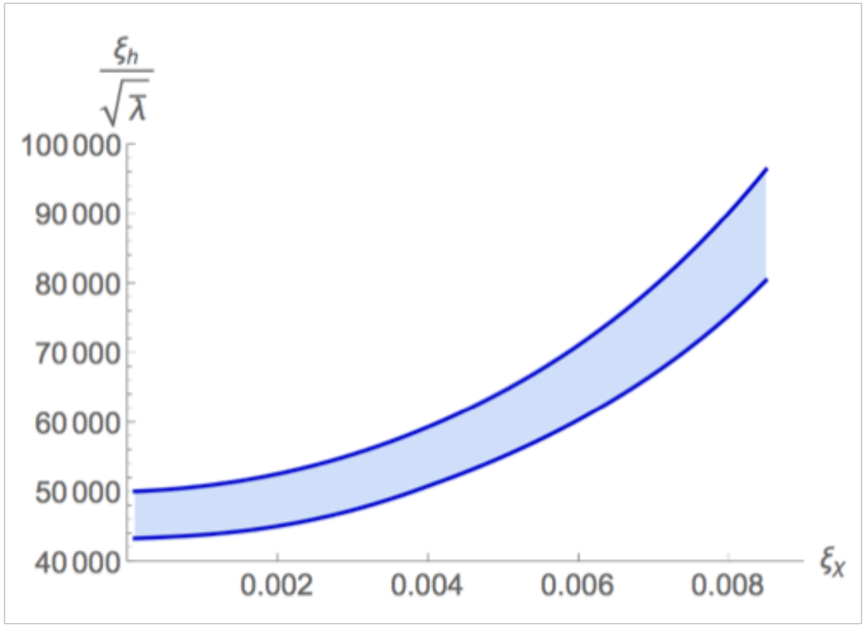
J. Garcia-Bellido, J. Rubio, M. Shaposhnikov and D. Zenhausern, Phys. Rev. D 84 (2011) 123504 [arXiv:1107.2163 [hep-ph]]

F. Bezrukov, G. K. Karananas, J. Rubio and M. Shaposhnikov, Phys. Rev. D 87 (2013) no.9, 096001 [arXiv:1212.4148 [hep-ph]]

Higgs-Dilaton model: classical theory

Lagrangian of the model:
$$\mathcal{L} = \frac{1}{2}(\xi_\chi \chi^2 + 2\xi_h \phi^\dagger \phi) R - \frac{1}{2}(\partial_\mu \chi)^2 - V(\chi, \phi^\dagger \phi) + \mathcal{L}_{SM,\lambda \to 0}$$
$$V(\chi, \phi^\dagger \phi) = \lambda \left(\phi^\dagger \phi - \frac{\alpha}{2\lambda}\chi^2 \right)^2 + \beta \chi^4$$

Constraints on the parameters of the model:



Bounds on non-minimal couplings come from inflationary data.
$\bar{\lambda}$ denotes the Higgs self-coupling taken at the inflationary scale $\sim M_P/\xi_h$.

Classical ground state of the model:

$$h_0^2 = \frac{\alpha}{\lambda}\chi_0^2 + \frac{\xi_h}{\lambda} R, \quad R = \frac{4\beta\lambda\chi_0^2}{\lambda\xi_\chi + \alpha\xi_h}$$
$$M_P^2 = \xi_\chi \chi_0^2 + \xi_h h_0^2$$
$$\Downarrow$$
$$m_H^2 \sim \frac{\alpha M_P^2}{\xi_\chi} \quad \Rightarrow \quad \boxed{\alpha \sim 10^{-34}\xi_\chi}$$

EW hierarchy problem

$$\Lambda \sim \frac{\beta M_P^4}{\xi_\chi^2} \quad \Rightarrow \quad \boxed{\beta \sim 10^{-56}\alpha^2}$$

CC problem

Higgs-Dilaton model: quantum corrections

HD Lagrangian is to be supplemented with the set of subtraction rules.

Non-renormalizability: how to choose the correct rules?

SI → Take dim.reg. with the field-dependent normalization point $\mu^2 \sim F(\chi, h)$.

The choice of *F* has to be made by hand. For example:

$$F_I(\chi, h) = \xi_\chi \chi^2 + \xi_h h^2\,, \quad F_{II}(\chi, h) = \xi_\chi \chi^2$$

Notice also the approximate invariance of the theory under the dilaton shifts $\chi \longrightarrow \chi + \text{Const}$.

Then, it was shown that the Higgs mass receives no large contributions.

M. Shaposhnikov and D. Zenhausern, Phys. Lett. B 671 (2009) 162 [arXiv:0809.3406 [hep-th]].

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Let us put $\alpha = \beta = 0$ classically. Then, the Higgs mass is not generated perturbatively.

Hence, the HD model provides a suitable framework for looking for a non-perturbative mechanism of generation of the Higgs mass.

Scalar field vev in the Dilaton model

The Dilaton sector of the HD model as a toy theory:

- The theory in the Jordan frame: $$S = \int d^4x \sqrt{g}\mathcal{L} + I_{GH,J} \qquad I_{GH,J} = -\int d^3x \sqrt{\gamma} K \xi \chi^2$$
$$\mathcal{L} = -\frac{1}{2}\xi\chi^2 R + \frac{1}{2}(\partial_\mu\chi)^2 + \frac{1}{4}\lambda_0\chi^4$$
- Change of variables: $$\chi = \bar{\chi}\Omega\,, \quad \tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}\,, \quad \Omega = e^{\sigma/M_P}\,, \quad \bar{\chi}^2 = \frac{M_P^2}{\xi}$$
- The theory in the Einstein frame: $$S = \int d^4x \sqrt{\tilde{g}}\tilde{\mathcal{L}} + I_{GH,E}\,, \quad I_{GH,E} = -M_P^2 \int d^3x \sqrt{\tilde{\gamma}}\tilde{K}$$
$$\tilde{\mathcal{L}} = -\frac{1}{2}M_P^2 \tilde{R} + \frac{a}{2}\tilde{g}^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma + \frac{1}{4}\lambda_0\frac{M_P^4}{\xi^2}\,, \quad a = \frac{1}{\xi} + 6$$
- vev of the dilaton: $$\langle\chi\rangle \sim \int D\tilde{g}_{\mu\nu} D\sigma e^{\sigma(0)/M_P - S} \Big/ \int D\tilde{g}_{\mu\nu} D\sigma e^{-S}$$
- Saddle-point approximation: $$\langle\chi\rangle \sim e^{-(W - S_{vac.})}\,, \quad W = \sigma(0)/M_P - S$$

Instanton in the Dilaton model

- The vacuum solution:

$$ds^2 = f^2 d\rho^2 + \rho^2 d\Omega_3^2\,, \quad f^2 = \frac{1}{1 \pm b^2\rho^2}\,, \quad \sigma = 0\,, \quad b^2 = \frac{|\Lambda|}{3M_P^2}\,, \quad \Lambda = \frac{M_P^4\lambda_0}{4\xi^2}$$

- The solution with the source:

$$ds^2 = f^2 d\rho^2 + \rho^2 d\Omega_3^2\,, \quad f^2 = \frac{1}{1 + (M_P^*\rho)^{-4} \pm b^2\rho^2}\,, \quad \sigma' = -\frac{1}{2\pi^2}\frac{1}{aM_P}\frac{f}{\rho^3}\,, \quad M_P^* = \pi(24a)^{1/4}M_P$$

Strong effect of gravity at $M_P^*\rho \lesssim 1$

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Strong effect of gravity at $M_P^* \rho \lesssim 1$

The short distance behavior of the solution with the source: $\sigma \sim -\log\rho\,, \quad \rho \to 0$

$$W - S_{vac} = \boxed{B} + \boxed{\mathcal{B}}$$

Contributions from the bulk and boundary
$B \approx 0$

Contribution from the singularity

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Contribution from the singularity

- Interpretation of the singularity:

The singular term comes from the HE physics. We do not know details of it, but we accept the conjectures about it. They allow us to make predictions about possible values of $\mathcal{B}$.

SI $\longrightarrow$ "dimensional regularization" of the instanton $\longrightarrow$ $\mathcal{B} = 0$ Hence, $\langle\chi\rangle = \bar{\chi}$

Instanton in the model with two fields

- Lagrangian of the model: $$\mathcal{L} = -\frac{1}{2}(\xi_\chi \chi^2 + \xi_h h^2)R + \frac{1}{2}(\partial_\mu \chi)^2 + \frac{1}{2}(\partial_\mu h)^2 + \frac{1}{4}\lambda h^4$$

  $\lambda = \lambda(\sigma, \bar{h})$ – aka Higgs self-coupling

- Change of variables: $$(\chi,\, h) \to (\sigma,\, \bar{h})$$

  $$\chi = \bar{\chi} e^{\sigma/M_P}\,, \quad h = \bar{h} e^{\sigma/M_P}\,, \quad \bar{\chi}^2 = \frac{M_P^2}{\xi_\chi}$$

- The vacuum solution: $\bar{h} = 0\,, \quad \sigma = 0$

- The solution with the source: The full analysis is left for numerics.

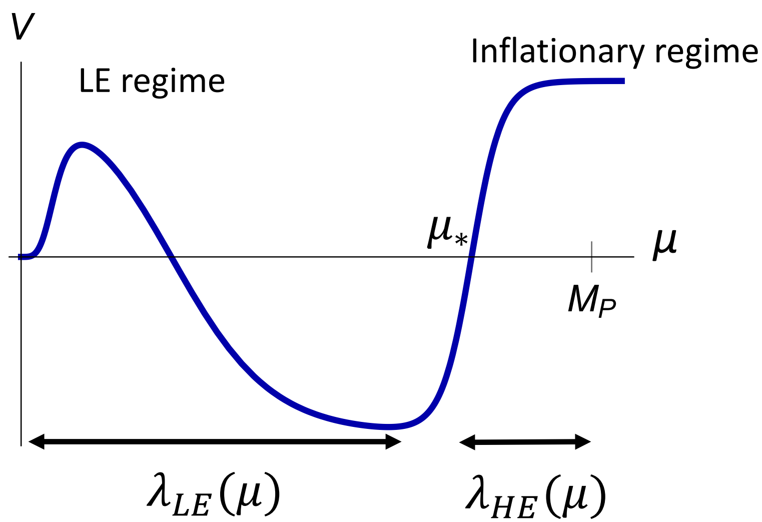
  The short distance asymptotics are $\bar{h} \sim \frac{a}{\rho} + A + \mathcal{O}(\rho^2)\,, \quad \sigma' \sim -\frac{1}{\rho} + \mathcal{O}(1)$

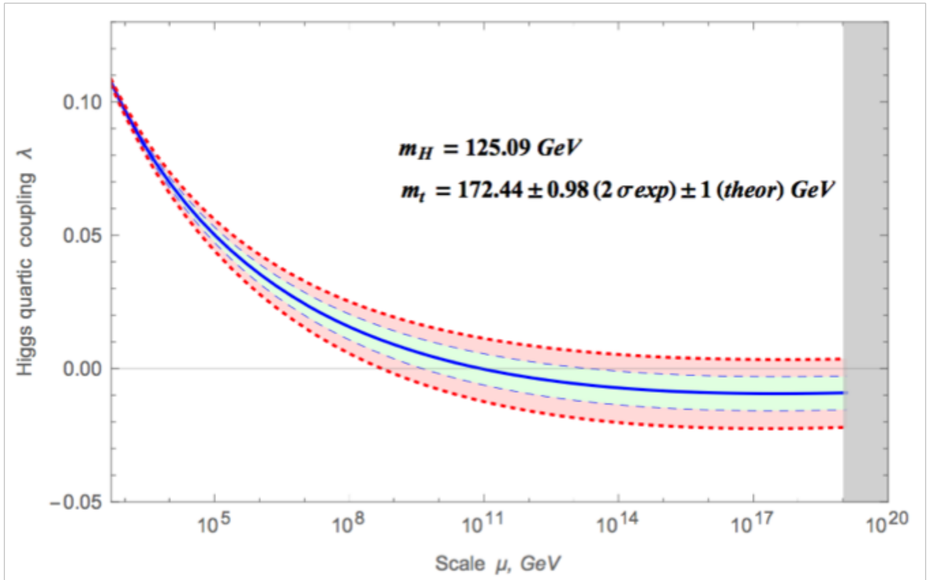
  $$A \sim M_P$$

- The vev of the *h*-field: $\langle h \rangle = A e^{-B}\,, \quad B = S - S_{vac}$

HD model: phenomenology

- The effective Higgs potential in the HD model





Possible form of the effective Higgs potential in the HD model, schematically.

Running of the Higgs self-coupling in the SM, at NNLO.

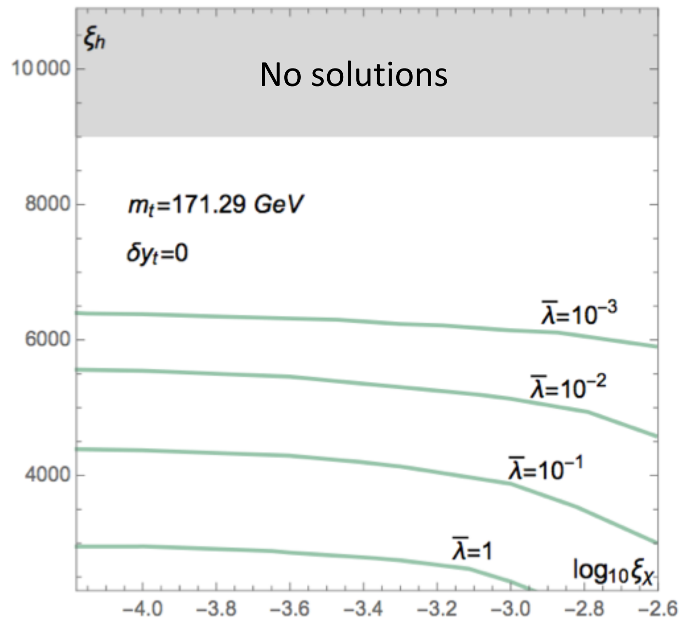
The effect of radiative corrections to the potential is encoded by the ``jumps'' $\delta\lambda, \delta y_t$ at the inflationary scale.

F. Bezrukov, J. Rubio and M. Shaposhnikov, Phys. Rev. D 92 (2015) no.8, 083512 [arXiv:1412.3811 [hep-ph]]

The transition between the LE and inflationary regimes can be modeled as

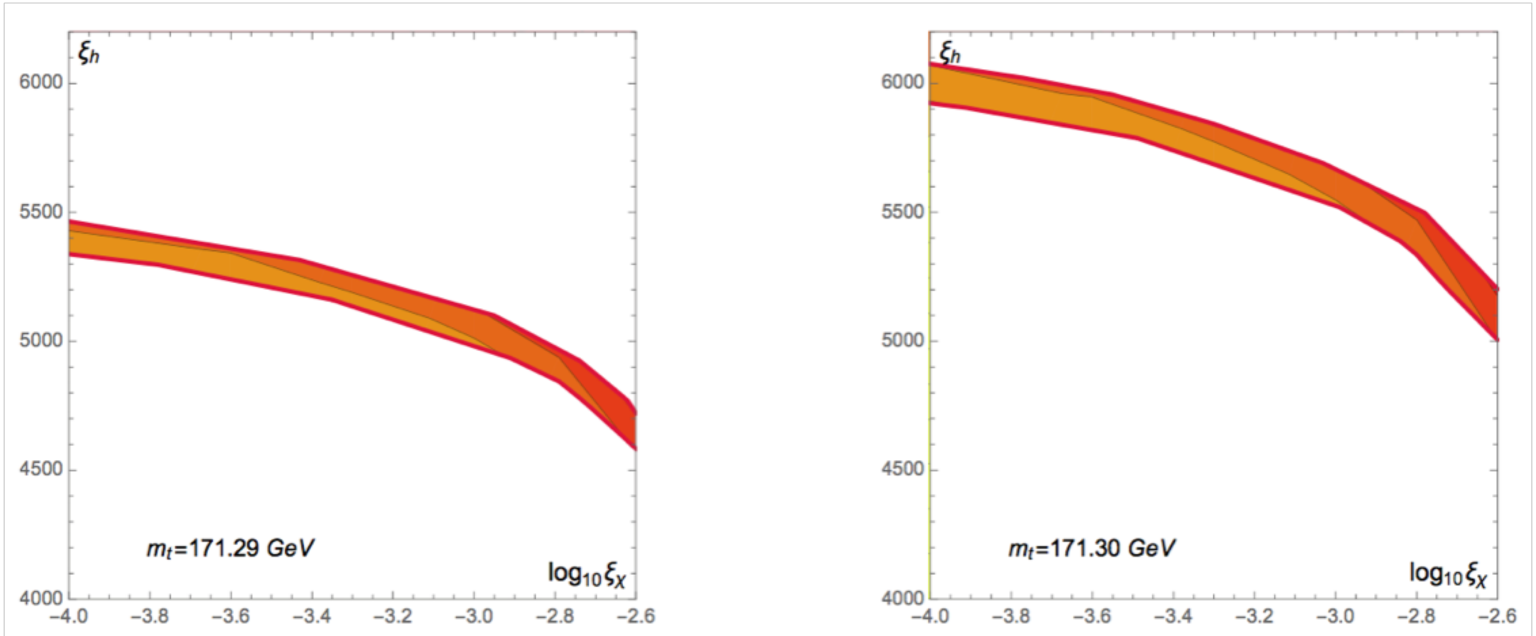
$$\lambda(\mu) = \frac{1}{2}\lambda_{LE}(\mu)\left(1 - \tanh(\gamma(\mu - \mu_*)\right) + \frac{1}{2}\lambda_{HE}(\mu)\left(1 + \tanh(\gamma(\mu - \mu_*)\right)$$

HD model: calculation of the Higgs vev



The plots displaying the parameter regions at which the contribution from the gravitational instanton gives the observed value of the Higgs mass, $m_H = 125.09\ GeV$. We take $\delta y_t = 0, \gamma = 10^5$. $\bar{\lambda}$ denotes the HE asymptotics of the Higgs self-coupling.

HD model: calculation of the Higgs vev



The regions of the non-minimal couplings, where the generation of the Higgs mass matches with the constraints from inflationary observations. Here we take, as an example, $\delta y_t = 0, \gamma = 10^5$. The borders between different colors correspond to the asymptotic Higgs self-coupling crossing the values $\log\bar{\lambda} = -1.8, -1.9$ (the left panel), $\log\bar{\lambda} = -1.9, -2.0$ (the right panel).

Discussion

- Is Λ generated in the same way? – **No** (at least in the HD model)
- Is the value of m_H special? – **No** (one can generate it smaller or larger than the observed value)
- Are the results sensitive to the unknown UV completion of the model? – **Yes** (although, the mere existence of the effect can well be universal)
- Note also that the instanton prefers the region of parameters, where the EW vacuum is not absolutely stable but very close to be so.
- Can one generate something else via gravitatioinal instanton, in the HD model? – **Probably, yes** (e.g., the small non-minimal dilaton coupling)

Thank you!