Solitons, bounces, and tunneling with non-canonical kinetic terms

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Solitons: arXiv:1607.05260 (PRD)
Tunneling: arXiv:1703.00909 (PRD, in review)

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Outline

Motivation: Scalar Field Theories

Tunneling

Solitons

Why non-canonical kinetic terms?

Scalars with non-canonical kinetic terms are ubiquitous in particle physics and cosmology, e.g.,

- Dark energy
- Modified gravity
- Supergravity/string theory

Interesting non-perturbative physics

Long known that canonical scalars have zoo of interesting nonlinear phenomena

Will focus in this talk on two:

- Quantum tunneling
- Solitons

Both arise when potentials have non-degenerate minima

These have been well-understood for decades. What changes when we introduce newly-discovered kinetic structures?

Galileons

Discovered in response to DE problem, starting with DGP

Properties of the galileons

- Second-order equations of motion
- Galilean invariance $\phi \to \phi + c + b_\mu x^\mu$
- Vainshtein mechanism: nonlinearities kill "fifth force"
- Non-renormalization theorem

Galileon Lagrangians

The five Lagrangians in $D = 4$ consistent with $\phi \to \phi + c + b_\mu x^\mu$ and with second-order eoms:

$$
\begin{aligned}
\mathcal{L}_2 &\sim (\partial\phi)^2 && \sim \varepsilon\varepsilon\partial\phi\partial\phi \\
\mathcal{L}_3 &\sim (\partial\phi)^2\Box\phi && \sim \varepsilon\varepsilon\partial\phi\partial\phi\partial^2\phi \\
\mathcal{L}_4 &\sim (\partial\phi)^2\left[(\Box\phi)^2 - \phi_{\mu\nu}^2\right] && \sim \varepsilon\varepsilon\partial\phi\partial\phi\partial^2\phi\partial^2\phi \\
\mathcal{L}_5 &\sim (\partial\phi)^2\left[(\Box\phi)^3 - 3\phi_{\mu\nu}^2\Box\phi + 2\phi_{\mu\nu}^3\right] && \sim \varepsilon\varepsilon\partial\phi\partial\phi\partial^2\phi\partial^2\phi\partial^2\phi
\end{aligned}
$$

where $\phi_\mu \equiv \partial_\mu\phi$, $\phi_{\mu\nu} \equiv \partial_\mu\partial_\nu\phi$

Generalizing galileons

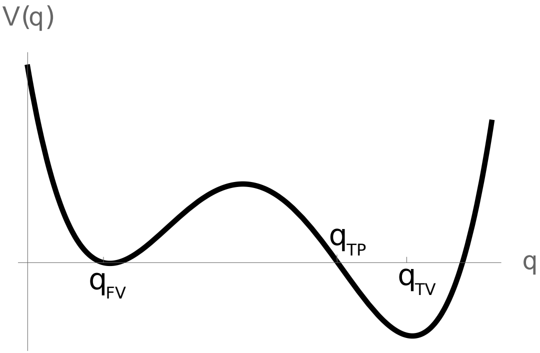
Most general flat-space scalars with second-order eoms: $f_n(\phi, (\partial\phi)^2) \times \mathcal{L}_n$.

Lose galilean invariance (though may have other interesting symmetries)

Some special cases—DBI, conformal, and (A)dS galileons—have interesting origins in higher-dimensional physics

Vainshtein mechanism! How does this affect nonperturbative solutions?

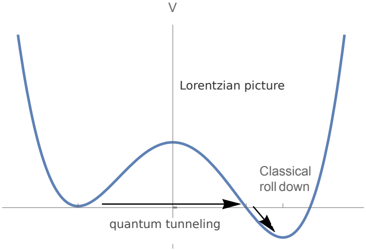
Consider a potential with two minima at different heights:

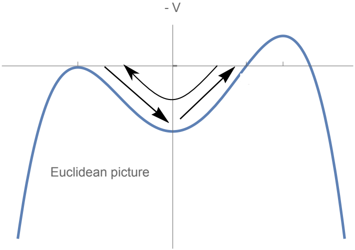


Classically both minima are stable, but quantum mechanics induces decay of false vacuum via tunneling

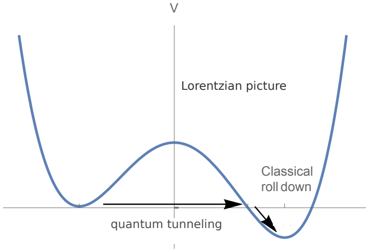
Prescription for determining decay rate (Coleman 1977):

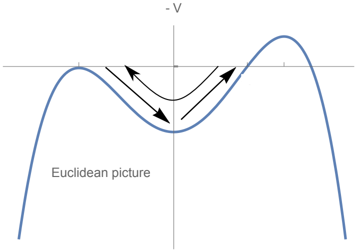
Transform to Euclidean time $t \to i\tau$, *i.e.*, invert potential





Tunneling: Computing the decay rate





The action of the Euclidean "bounce" solution determines the decay rate:

$$\frac{\Gamma}{V} \sim e^{-B}$$

with

$$B = S_{\mathrm{E}}(\mathrm{bounce}) - S_{\mathrm{E}}(\mathrm{FV})$$

The result $\Gamma/V \sim e^{-B}$ was proven for canonical scalar field using WKB approximation

By solving for wavefunctional $\psi[\phi]$ in semi-classical limit, we show that for general

$$L = L(\phi, \dot{\phi}, \partial_i\phi, \partial_i\partial_j\phi)$$

the dominant contribution to the decay rate comes from the solution to the Euclidean equation of motion

Explicitly see: non-canonical kinetic terms do not change Coleman prescription for decay rate*

*Assumption: second-order eoms

Decay rates

Problem of finding decay rate Γ amounts to solving Euclidean eoms with $O(4)$ symmetry

Warm up: $\mathcal{L} = P(X) + V(\phi)$ with $X = -(\partial\phi)^2$. Euclidean action:

$$S_{\mathrm{E}} = 2\pi^2 \int \rho^3 (P + V) \mathrm{d}\rho$$

Define non-standard Lagrangian with volume factor removed:

$$S_{\mathrm{E}} \equiv 2\pi^2 \int \rho^3 L \mathrm{d}\rho$$

and similarly non-standard canonical momentum:

$$\pi_\phi \equiv \frac{\partial L}{\partial \dot{\phi}}$$

$P(X)$ decay rates in the thin-wall limit

Consider thin-wall limit (small potential difference between the two minima):

$$\epsilon \equiv V_{\mathrm{FV}} - V_{\mathrm{TV}} \ll V$$

The bounce factor in this limit is

$$B = \frac{27\pi^2 S_1^4}{2\epsilon^3}$$

with S_1 the tension of the bubble wall,

$$S_1 \equiv \int_{\mathrm{wall}} \pi_\phi \mathrm{d}\phi$$

In the canonical case $P(X) = X/2$ this reduces to Coleman's famous result ✓

However, $\Gamma/V \sim e^{-B}$ depends extremely sensitively on even small changes in S_1

Galileon decay rates

Finally consider galileons (incl. generalizations). For concreteness consider cubic galileon,

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 + \frac{1}{\Lambda^3}(\partial\phi)^2\Box\phi - V(\phi)$$

Two regimes:

- Standard: Canonical kinetic term dominates, standard decay rate:

$$B_{\rm can} = \frac{27\pi^2 (S_1^{\rm can})^4}{2\epsilon^3}, \qquad S_1^{\rm can} = \int_{\rm wall} \dot{\phi}\,{\rm d}\phi$$

- Vainshtein: Galileons dominate, qualitatively different decay rate

$$B_{\rm gal} = \frac{2\pi^2 (S_1^{\rm gal})^2}{\epsilon} \qquad S_1^{\rm gal} = \int_{\rm wall} \frac{6}{\Lambda^3}\dot{\phi}^2{\rm d}\phi$$

Galileon decay rates: Vainshtein mechanism

Which regime we're in depends on free parameters:

- ϵ: difference between potential heights
- $\Delta\phi$: difference between location of the two minima
- Λ: energy scale associated to the galileon

Canonical(/galileon) term dominates if $\frac{\epsilon}{\Delta\phi} \gg (\ll) \Lambda^3$

Equivalently: depends on whether Euclidean bubble size ρ is larger or smaller than a threshold value, akin to a Vainshtein radius

If the galileon dominates, the decay rate can be many orders of magnitude larger than for a canonical scalar with the same potential

Solitons are defined to be

- Non-trivial field configurations
- Finite energy
- Localized in space
- Do not dissipate

Exist due solely to nonlinearities in the field, no external source

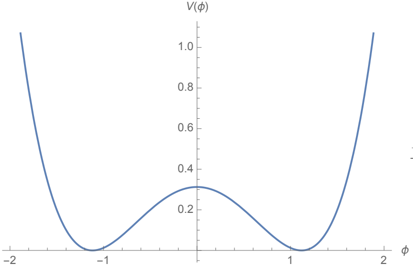
E.g., if two non-degenerate minima, can have domain wall

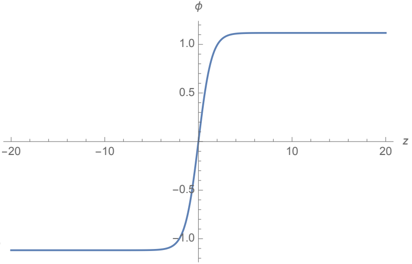
For example,

$$V(\phi) = -\frac{1}{2}m^2\phi^2 + \frac{1}{4}\lambda\phi^4$$

Solved by

$$\phi(z) = \frac{m}{\sqrt{\lambda}}\tanh\left(\frac{mz}{\sqrt{2}}\right)$$





Derrick's theorem: a no-go

Zero-mode/scaling arguments (Derrick's theorem)* tell us we need a potential or time dependence.

Let's focus on each of these in turn.

*Derrick's theorem extended to galileons: Endlich et al. arXiv:1002.4873;
Extended to generalized galileons: us

Stable static walls difficult to obtain:

- Standard galileons with a potential: galileon $\mathcal{L}_n$ are total derivatives in 1D, so no difference from canonical case
- Conformal galileons with a potential: domain walls do not exist
- (A)dS galileons: naturally possess a potential due to their symmetries. Leads to bubbles, but they shrink and suffer from ghosts

Time dependence—light-speed domain walls

Time dependence requires $v = c$ so that there isn't a frame where the soliton is at rest

Masoumi & Xiao, arXiv:1201.3132—standard galileons have stable light-speed solitons

Generalized galileons:

- DBI galileons: stable solitons ✓
- Conformal galileons: unstable X
- (A)dS galileons: naturally include a potential, so should not include solutions with $v = c$

Summary

- Tunneling rates very sensitive to kinetic structure, especially for galileons outside a Vainshtein region
- Non-canonical kinetic terms can severely boost vacuum decay rate
- Difficult to construct static galileon domain walls
- Easier for walls moving at speed of light