Bubbles of Nothing and Supersymmetric Compactifications

with J.J. Blanco-Pillado, B. Shlaer and J. Urrestilla, (arXiv:1606.03095)

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Bubbles of Nothing (BON)

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Supersymmetric compactifications

- They are protected against perturbative and non-perturbative instabilities,
- while BON production seems to be generic in non-supersymmetric compactifications *(Fabinger '00, Dine '04, Yang '09, Blanco-Pillado '10, Brown '11)*.
- what happens after spontaneous breaking of supersymmetry?.
  - If SUSY provides a **topological** protection the compactification remains stable,
  - but this is not always the case...

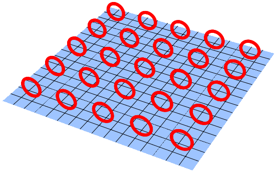
(*Witten '81*)

- The Kaluza-Klein vacuum is a solution to the **5D** Einstein's equations

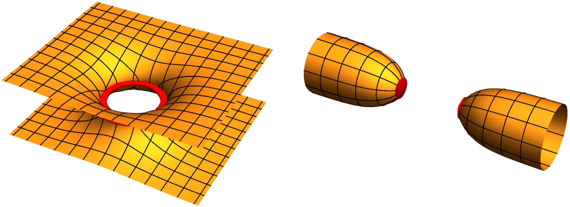
$$ds^2 = -dt^2 + dr^2 + r^2 d\Omega_2^2 + d\theta^2, \qquad \theta \in [0, 2\pi R_{kk}).$$

- One spatial dimension is compactified to a circle of radius R_{kk}

$$M_5 \to M_4 \times S^1.$$



- The Bubble of Nothing (BON) is a non-perturbative instability of the KK vacuum –a gravitational instanton–.
- The nucleation rate is determined by the bubble size: $\Gamma \sim e^{-R_B^2}$



- The bubble materialises with radius $R_B = R_{kk}/2$.
- Initially static, accelerates rapidly almost reaching the speed of light.
- Far from the bubble the geometry approaches the KK vacuum.
- At the bubble the KK direction is smoothly sealed off.

Bubbles of Nothing

Stabilisation with twisted fermions (generalized KK reduction)

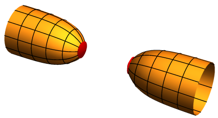
The KK reduction is not unique when

- the theory contains more fields besides the metric and
- it is invariant under global symmetries. *(Sherk and Schwartz '78, '79)*

- The fields only have to be periodic along the compact direction up to a global symmetry transformation

Example: Any Lorentz invariant theory is left unchanged by flipping the sign of all fermions

$$\chi(\theta + 2\pi) = \pm\chi(\theta)$$



Only KK compactifications with anti-periodic fermions can decay into BON.

Condition for unbroken supersymmetry:

$$D_\mu \epsilon = (\partial_\mu + \tfrac{1}{2}\gamma_{ab}\omega_\mu^{ab})\epsilon = 0$$

- The KK background has no curvature $\omega_\mu^{ab} = 0 \implies \partial_\mu \epsilon = 0$.
- ϵ is a constant spinor field: it satisfies periodic boundary conditions.

- Only KK compactifications with periodic fermions can be supersymmetric.
- Supersymmetric compactifications cannot decay into BON.

- The **topological protection** ensures stability even after low energy SUSY breaking.

Dressed Bubbles of Nothing

KK compactifications with Wilson lines.

It is possible to construct supersymmetric compactifications with anti-periodic fermions: no topological protection.

The idea: adding a Wilson line on the compactification

- Fermions transported around a loop in the compact direction must to pick a phase $e^{\mathrm{i}\pi}$.
- We turn on a $U(1)$ gauge field coupled to all fermions.

$$D_\theta \epsilon = (\partial_\theta - \mathrm{i}qA_\theta)\epsilon = 0 \quad \Longrightarrow \quad \epsilon(\theta) = \mathrm{e}^{\mathrm{i}qA_\theta \theta}\epsilon_0$$

- This supersymmetric solution would be consistent with anti-periodic boundary conditions when $qA_\theta = 1/2$.

Dressed Bubbles of Nothing

Explicit example: 4D locally supersymmetric Abelian-Higgs model

Bosonic sector of the action

$$S = \int d^4x\sqrt{-g}\left(\frac{1}{2\kappa^2}R - |D_\mu\phi|^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{\beta e^2}{2}(\eta^2 - \phi\bar{\phi})^2\right),$$

$$\delta_g\phi = \mathrm{i}e\phi\,\alpha \qquad D_\mu\phi = (\partial_\mu - \mathrm{i}eA_\mu)\phi, \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

- The couplings are determined by two dimensionless parameters $\gamma \equiv \kappa^2\eta^2$ and β.

- The supersymmetry parameter has a $U(1)$ charge:

$$\delta_g\psi_{\mu L} = -\mathrm{i}e\frac{\eta^2\kappa^2}{2}\psi_{\mu L}\,\alpha\,.$$

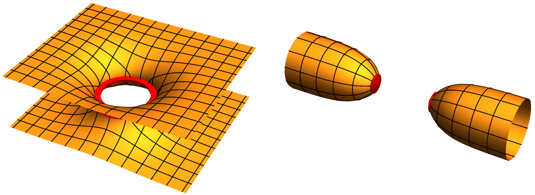
$$\delta_\epsilon\psi_\mu = D_\mu\epsilon = (\partial_\mu + \tfrac{1}{2}\gamma_{ab}\omega_\mu^{ab} + \tfrac{\mathrm{i}}{2}\kappa^2\eta^2A_\mu)\epsilon$$

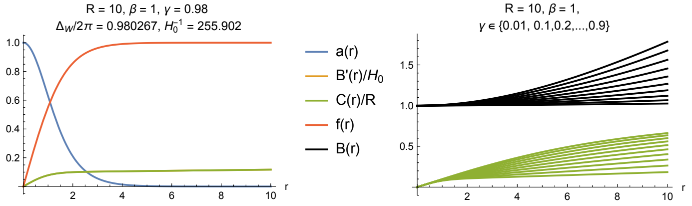
Dressed Bubbles of Nothing

Winding compactifications.

The gauge potential deforms the BON:

- The stokes theorem requires that the BON to be dressed with magnetic flux,
- the energy of the magnetic field produces a backreaction on the BON space-time.





- In the supersymmetric limit $\gamma \to 1$ the bubble radius diverges $R_B \to \infty$, and the bubble becomes static.
- In the supersymmetric limit the decay is suppressed by a *Coleman-De Luccia mechanism*:

$$\Gamma \sim e^{-S_E} \to 0$$

Conclusions

- We have revisited non-perturbative stability of supersymmetric compactifications with respect to the decay into bubbles of nothing.
- Previous analyses had suggested a topological obstruction for the decay due to the fermionic structure of the theory.
- This protection would persist when supersymmetry is broken below the KK scale.

- Contrary to these expectations we find that it is possible to construct superymmetric compactifications with no topological obstruction.
- The decay of supersymmetric compactifications is still suppressed by a Coleman-de Lucia type of mechanism.
- If supersymmetry is broken at low energies this instability can have a non-vanishing decay rate.