Backreaction in Axion Monodromy, 4-forms and the Swampland

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I.V., [arXiv:1611.00394 [hep-th]]

Bielleman,Ibanez,Pedro,I.V.,Wieck [arXiv:1611.07084 [hep-th]]

Blumenhagen,I.V.,Wolf [arXiv:1703.05776 [hep-th]]

PASCOS 2017, Madrid

Transplanckian field ranges

Relevant for:

- Large field inflation (detectable tensor modes)
- Cosmological relaxation [Graham,Kaplan,Rajendran'15]

$\rightarrow$ Axion $\Delta\phi > M_p$

Challenge for string theory:

- Technical difficulties $\longrightarrow$ effective theory out of control [Banks et al.'03] (strong coupling or small volume)
- In conflict with quantum gravity

  Weak Gravity Conjecture (WGC) [Arkani-Hamed et al.'06]

Are they possible in a consistent theory of quantum gravity?

Transplanckian field ranges

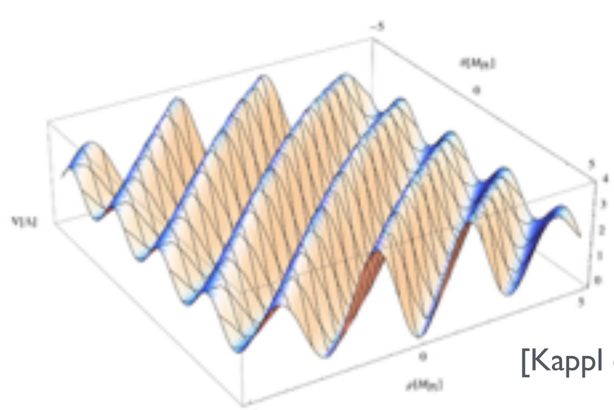
Current proposals in string theory:

(starting point: subplanckian axions $f < M_p$)

Natural inflation with multiple axions

[Dimopoulos et al,Kim-Nills-Peloso,McAllister et al…]

Periodic potential
(compact moduli space)



[Kappl et al]

Constraints from WGC

[Rudelius,Heidenreich,Reece,Montero,Ibanez,Uranga,IV,Brown,
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Axion monodromy

[Silverstein et al., Flauger et al…]

Multi-branched potential
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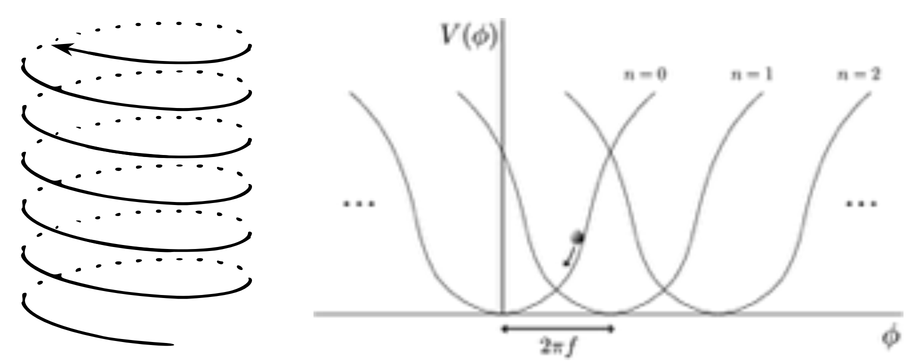

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[Oouri-Vafa et al.'06] [Klaewer-Palti,'16]

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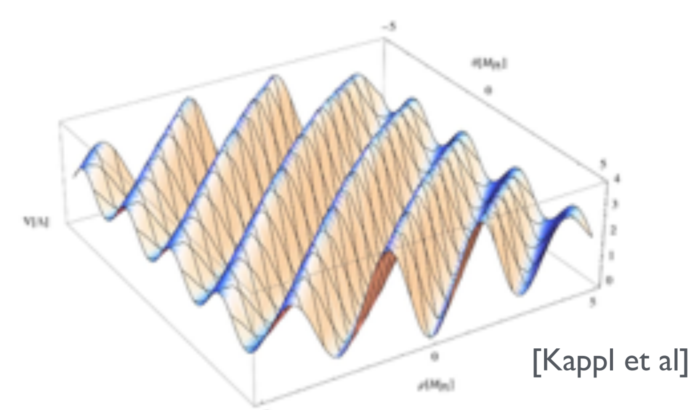

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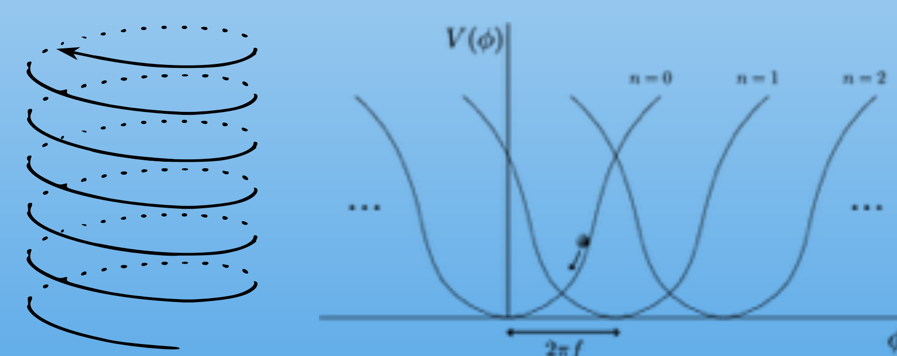

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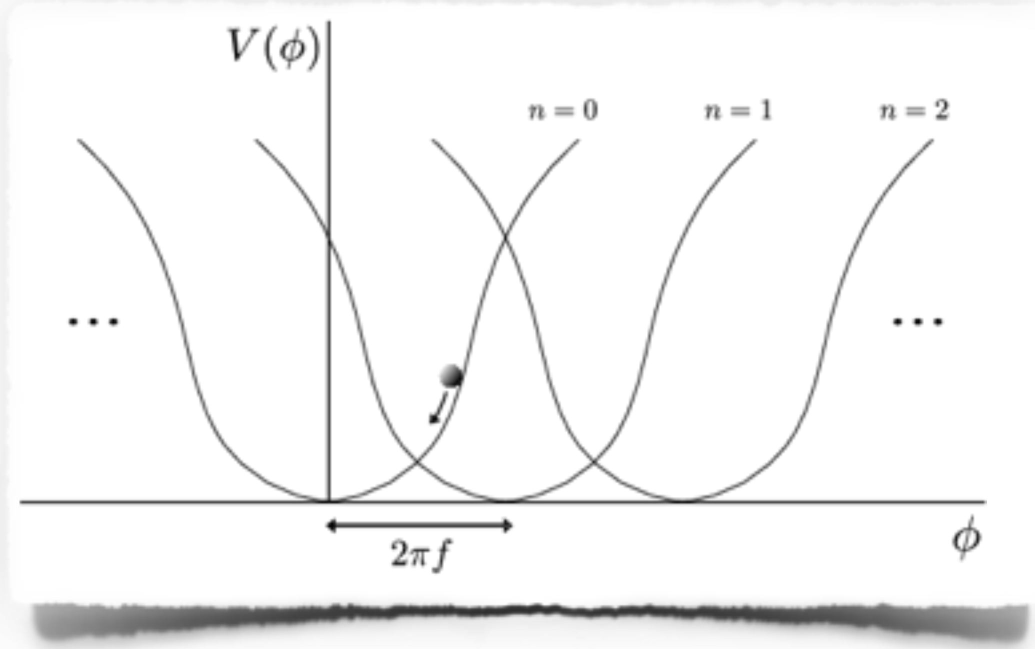

Axion Monodromy

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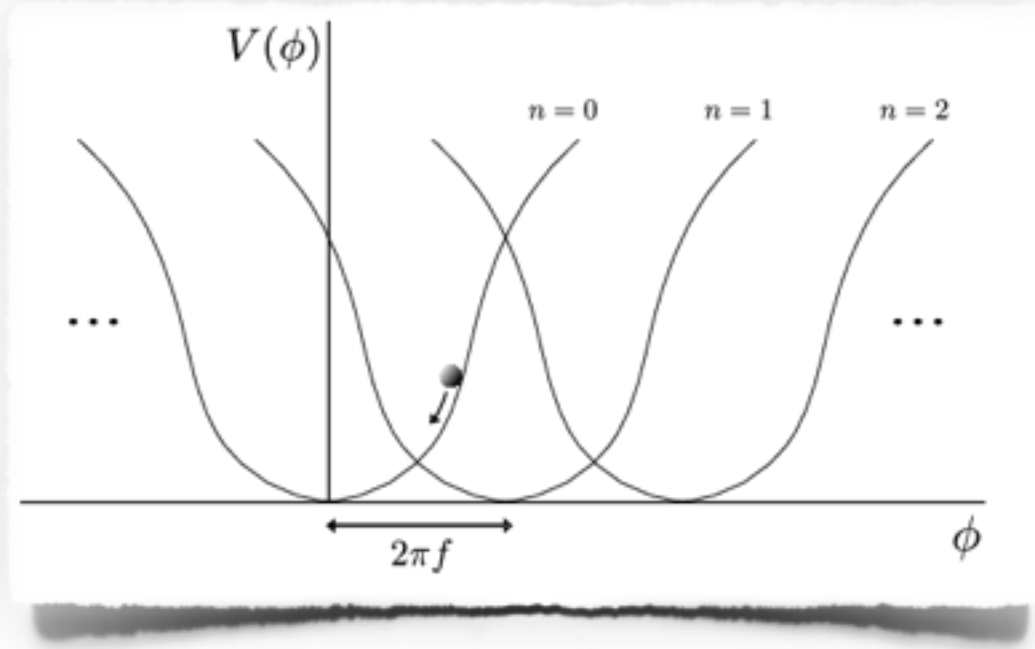

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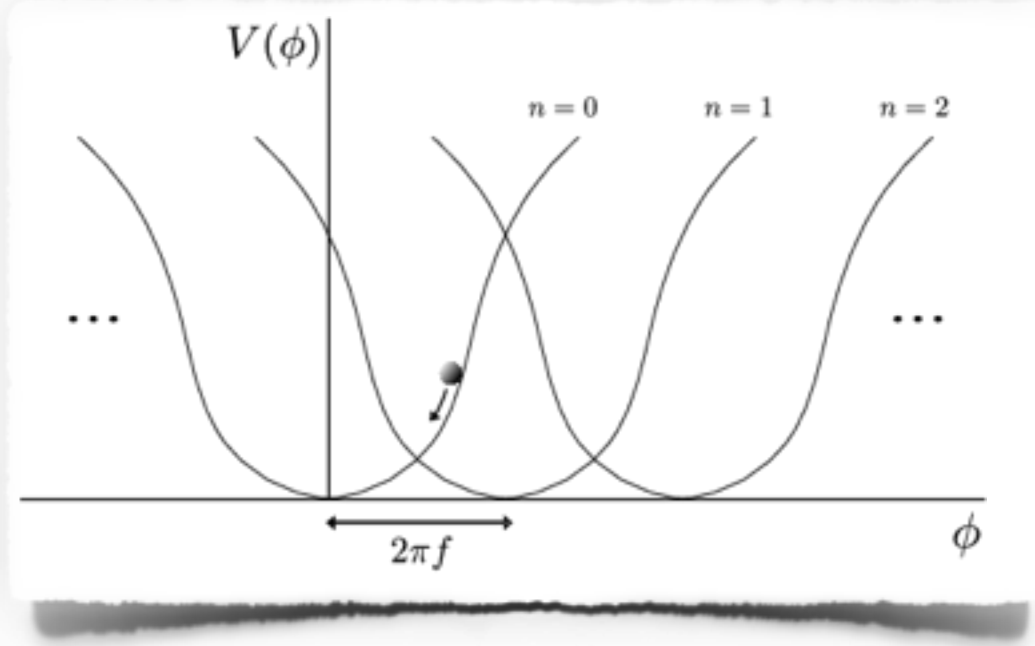

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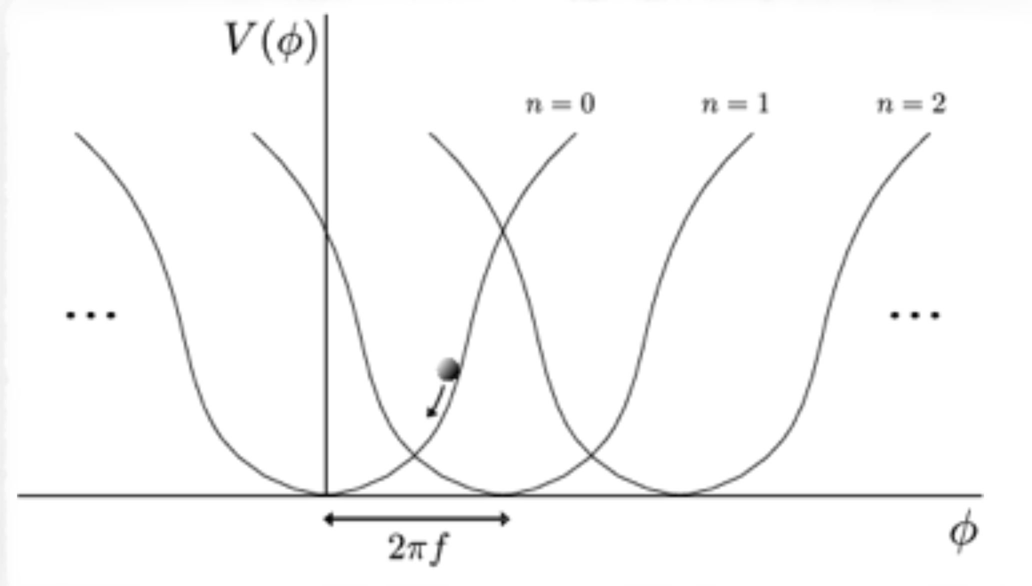

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We induce a multi-branched potential $V = \frac{1}{2}(n + m\phi)^2$ invariant under

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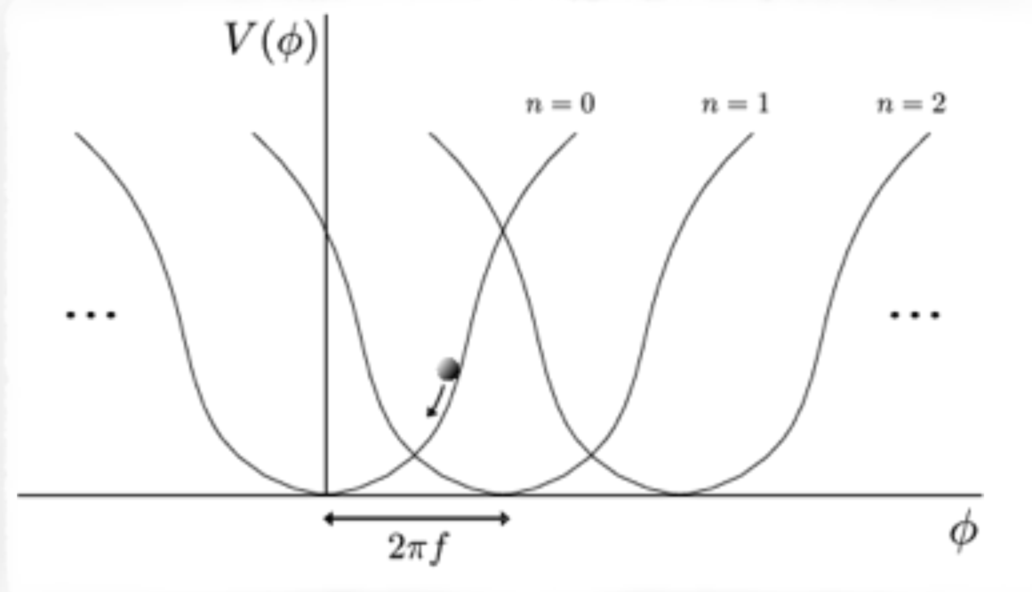

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4-form description: $\mathcal{L} = -f^2(d\phi)^2 - |F_4|^2 + mF_4\phi$ where $F_4 = dC_3$

[Dvali'05] [Kaloper,Sorbo,Lawrence'09-'11]

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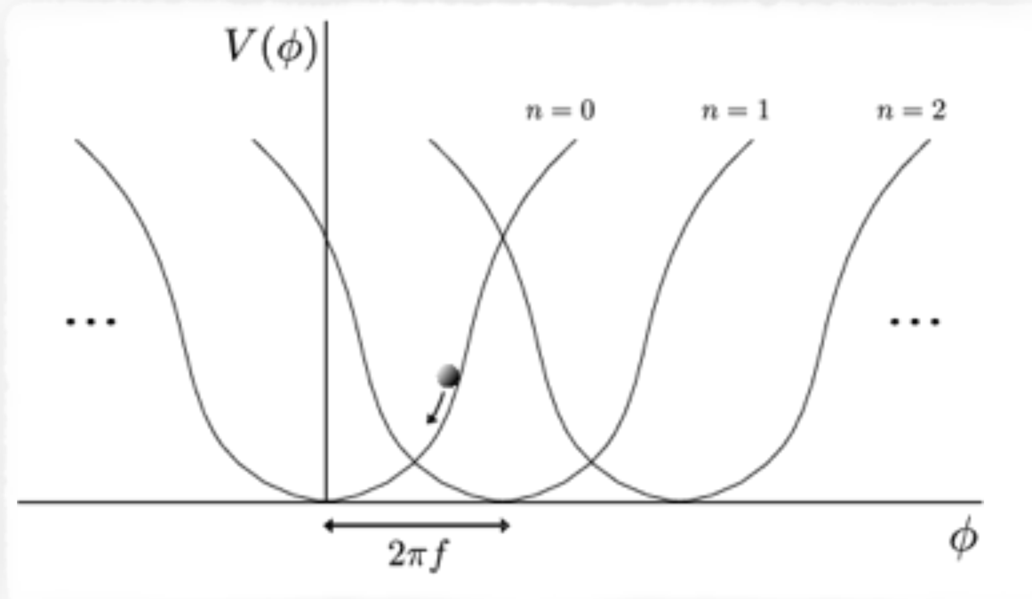

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String Theory embedding: Technical difficulties related to backreaction

[McAllister et al] [Blumenhagen et al] [Hebecker et al.] [Dudas,Wieck] [Buchmuller et al] [Marchesano et al.]

Backreaction in terms of 4-forms

$$V = |F_4|^2 - mF_4\phi$$

$$*F_4 = n + m\phi \longrightarrow V = \frac{1}{2}(n + m\phi)^2$$

Backreaction in terms of 4-forms

$$\boxed{V = \sum_i Z_{ij}(s^a) F_4^i F_4^j + \sum_i F_4^i \rho_i(\phi^a) + V_{loc}(s^a)}$$

saxions

axions

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Field-dependent metrics $\Rightarrow$ Backreaction effects

Truncation: Freeze all saxions and study only axion/inflaton dynamics

$\longrightarrow$ Not valid away from minimum

Compute $\partial_s V = 0 \rightarrow s = s(\phi)$ and plug it into effective theory

$\longrightarrow$ Modify scalar potential and kinetic term

Back-reaction on the inflaton metric

Kinetic term of the inflaton: $K_{\phi\phi}(s)(\partial\phi)^2$

Minima of the saxions: $\langle s \rangle \propto \rho(\phi)$

$\rightarrow$ Backreacted $K_{\phi\phi}(s(\phi))$

s : saxion

ϕ : inflaton

If $K = -log(s)$; $S = s + i\phi$

$$\Delta\phi = \int K_{S\bar{S}}^{1/2} d\phi \simeq \int \frac{1}{s(\phi)} d\phi \sim \int \frac{1}{\rho(\phi)} d\phi$$

$\rightarrow$ Reduce physical field range

At best, $\rho(\phi) \propto \phi \rightarrow \Delta\phi \propto log(\phi)$ for large field

[Bumenhagen et al.'15]
[Baume,Palti'16]
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Exponential drop-off of mass scale $M_{KK} \sim s(\phi)^{-n} \sim e^{-n\lambda\Delta\varphi}$

→ Invalidity of the effective theory

Behaviour predicted by the **Swampland Conjecture** [Ooguri-Vafa et al.'06]

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An effective theory is valid only in a finite domain Δr on the moduli space, because an infinite tower of states becomes exponentially light

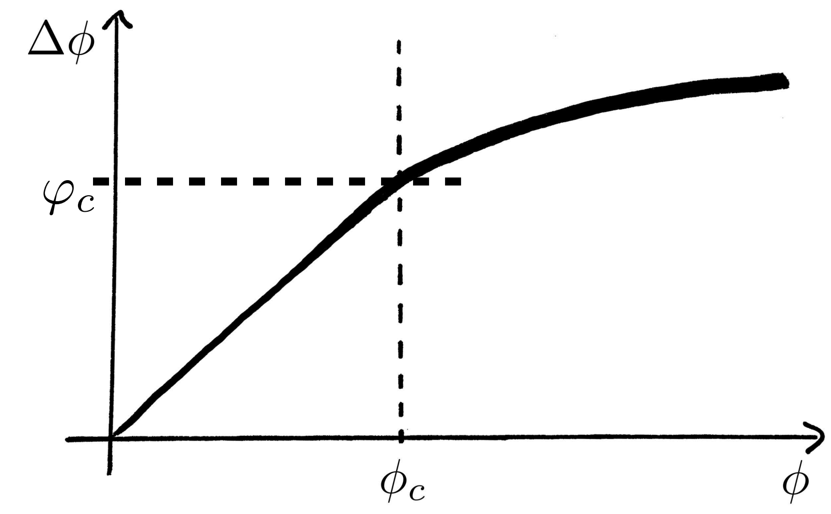
$M \sim M_0 e^{-\lambda\Delta r}$ when $\Delta r \to \infty$

When does this effect appear?

How far can we delay backreaction? Logarithmic behaviour tied to M_p ?

$$\partial_s V = 0 \to s = s_0 + \delta s(\phi) \;\; ; \;\; \delta s(\phi) \simeq \lambda\phi$$

Critical value ϕ_c : $\delta s(\phi_c) \approx s_0$



Effective field range before backreaction effects dominate:

$$\varphi_c \simeq \int_0^{\phi_c} K_{S\bar{S}}^{1/2}(s_0) d\phi \simeq \frac{\phi_c}{s_0} \sim \frac{1}{\lambda} \qquad \text{(in } M_p \text{ units)}$$

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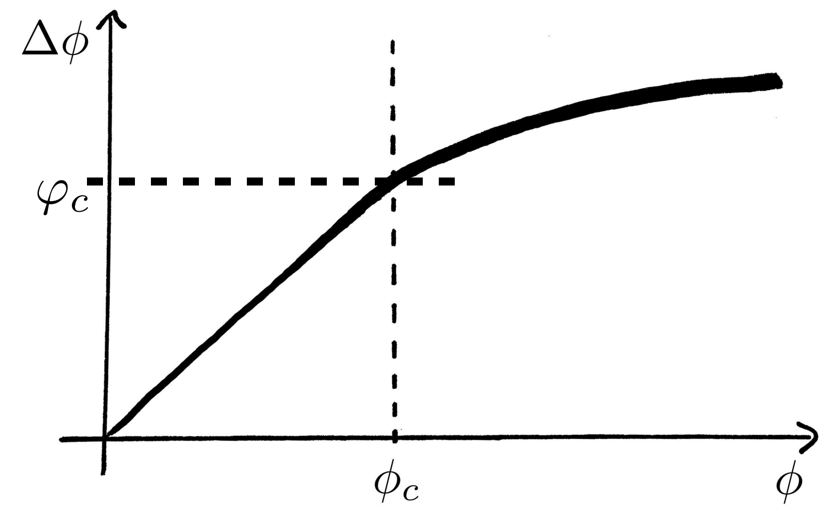

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Possible counterexamples $\longrightarrow$ Open string fields [I.V.,'16]

Inflaton $\phi =$ D7 position modulus on a Type IIB orientifold flux background [Hebecker et al/Arends et al'14] [Ibanez,Marchesano,I.V.'14] ...

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  $$\varphi_c \sim \lambda^{-1} \gg 1 \quad \text{if} \quad \mu \ll \text{fluxes}$$

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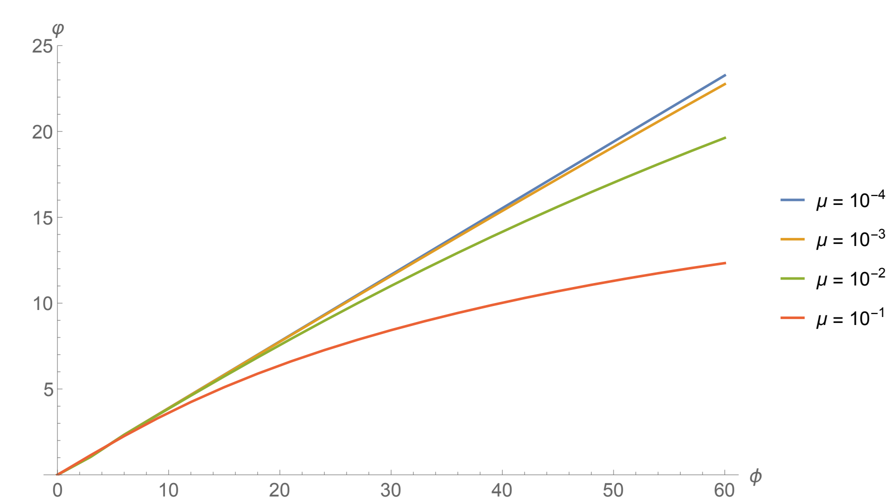

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In general:

$$s = s_0 + \delta s(\phi) \;\; ; \quad \delta s(\phi) \simeq \lambda \phi$$

- $\lambda \sim \mathcal{O}(1) \rightarrow$ IIA closed string sector, non-geometric IIB backgrounds…
  (General for N=2 special Kahler manifolds [Palti'17])
- $\lambda \sim \frac{m_\phi}{m_s} \rightarrow$ D7-branes on IIB (+ KKLT, LVS or non-geometric fluxes)…

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Is such a mass hierarchy possible in a global compactification?

Open string models

- This is an example where the logarithmic behaviour can (in principle) be pushed far away in field distance by tuning the fluxes

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Caution! Stabilisation of the Kahler moduli: [Blumenhagen,I.V.,Wolf'17]

➡ KKLT/Large Volume Scenario: $\delta t(\phi) < t_0 \to m_\phi < m_t$ implies

KKLT $\longrightarrow \mu \ll W_0$

LVS $\longrightarrow \mu \ll 1/\mathcal{V}^{-1/6}$

Not possible!

➡ Non-geometric IIB fluxes:

If $\lambda < 1$ then $M_{KK} >> m_{\text{moduli}}$

KK states become light when trying to get a transplanckian field range!

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It forces $\lambda \sim 1 \longrightarrow$ Consistent with the Refined Swampland Conjecture

It is not possible to get $\varphi_c > M_p$ without loosing parametric control of the effective theory

Open string models

Loophole ➡ Can we generate $\mu \ll 1$?

➡ μ can be an effective parameter depending also on moduli vevs.

Example: $W \sim (\mu_1 - \mu_2 U^2)\Phi^2 + \ldots \qquad \to \mu_{\rm eff}^2 \geq \frac{63}{64}\mu_1^2$

Bound is flux independent $\longrightarrow$ No parametric suppression.

What about more elaborated Calabi-Yau compactifications?

Saved by tuning in the landscape? [Hebecker et al.'14]

➡ Other option: Alignment of fluxes? [Landete et al.'17]

Summary

✦ Axion monodromy models in string theory can be described in terms of couplings to 3-form gauge fields with non-canonical metrics parametrised by the saxions.

✦ These field dependent metrics give rise to back-reaction problems. They can backreact on the Kahler metric of the inflaton leading to a logarithmic scaling of the field distance at large values.

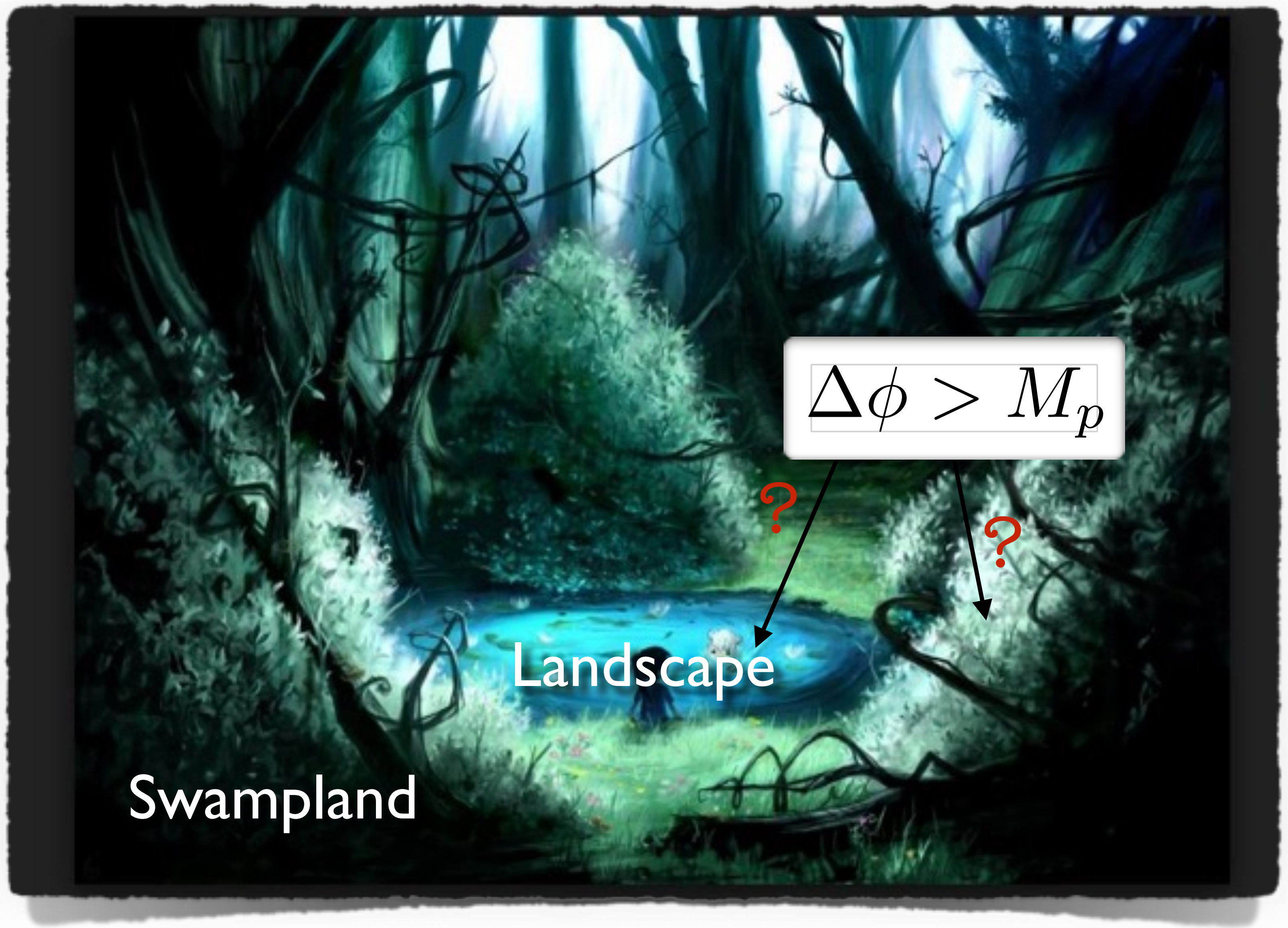
Inflaton → Closed string sector IIA/B: $\Delta\varphi \sim \lambda^{-1} \sim 1$

Inflaton → Open string sector: $\Delta\varphi \sim \lambda^{-1} \sim \dfrac{m_s}{m_\phi}$ flux-dependent

✦ In a toroidal compactification (or simple CY's), the required flux tuning to get a mass hierarchy cannot be achieved without loosing parametric control of the effective theory.

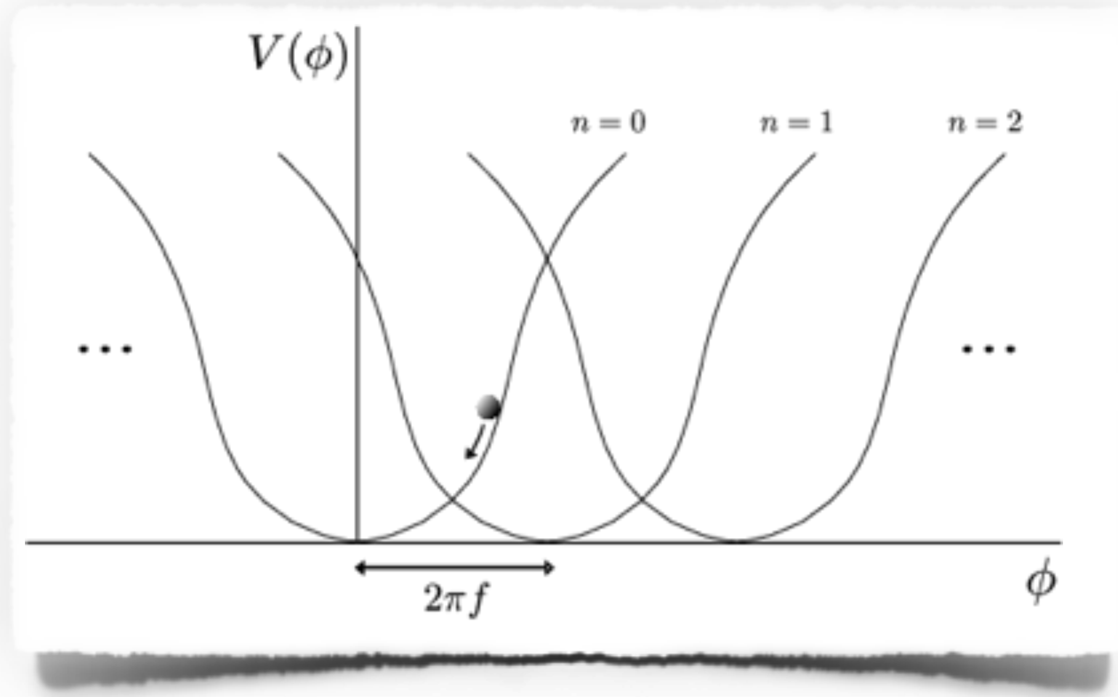
This supports the Refined Swampland Conjecture.

Thank you!



back-up slides

Axion Monodromy



Higher dimensional operators

✓ Potential protected by gauge invariance of the 3-form field

$$\delta V \sim \sum_n \left(\frac{F_4^2}{M_p^4}\right)^n \sim \sum_n \left(\frac{V_0}{M_p^4}\right)^n$$

[Kaloper,Sorbo,Lawrence]

Tunneling between branches

Mediated by membranes charged under C_3 that shift n

WGC $\longrightarrow$ membrane tension

✓ Tunneling negligible for inflation

[Franco et al/Brown et al/Hebecker et al.]
[Ibanez,Montero,Uranga,IV]

Sugra generalisation of Kaloper-Sorbo

$$V = \sum_i Z_{ij}(s^a) F_4^i F_4^j + \sum_i F_4^i \rho_i(\phi^a) + V_{loc}(s^a)$$

saxions

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$$*F_4^i = Z^{ij}(s^a)\rho_j(\phi^a) \; \rightarrow \; V = Z^{ij}(s^a)\rho_i(\phi^a)\rho_j(\phi^a)$$

2 **Multiple 4-forms** → Higher order corrections

$$\delta V = \sum_n (\prod_i (F_4^2)^i)^n \longrightarrow$$ not as powers of V, but of the different $\rho_i(\phi^a)$

Discrete shift symmetry cannot be broken (it is a large gauge transf.)

Periodic corrections

$$\cancel{\left(\frac{\phi}{M_p}\right)^n} \quad \left(\frac{\rho(\phi)}{M_p}\right)^n \checkmark$$

Open string models

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Inflaton: Position modulus of a D7-brane on a Type IIB orientifold flux background

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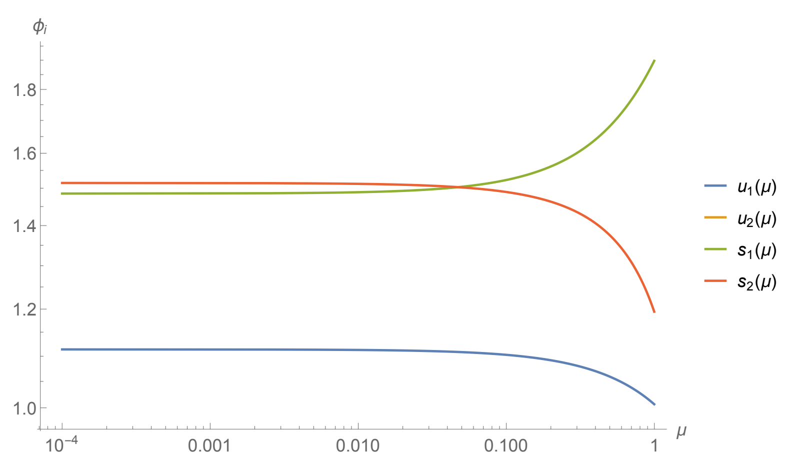
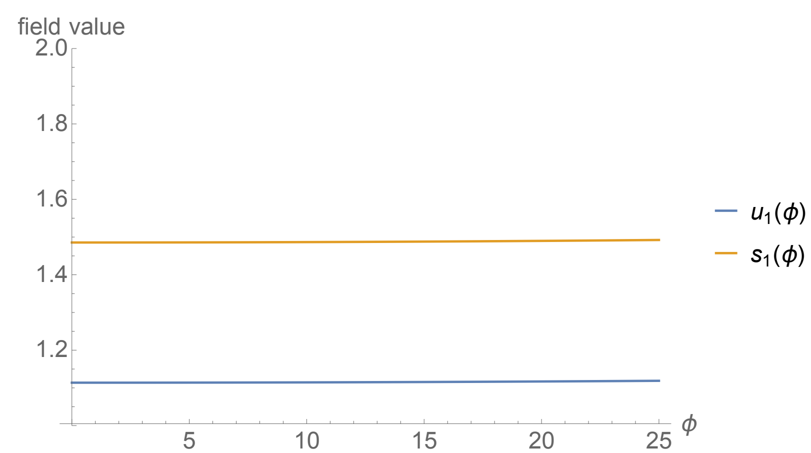
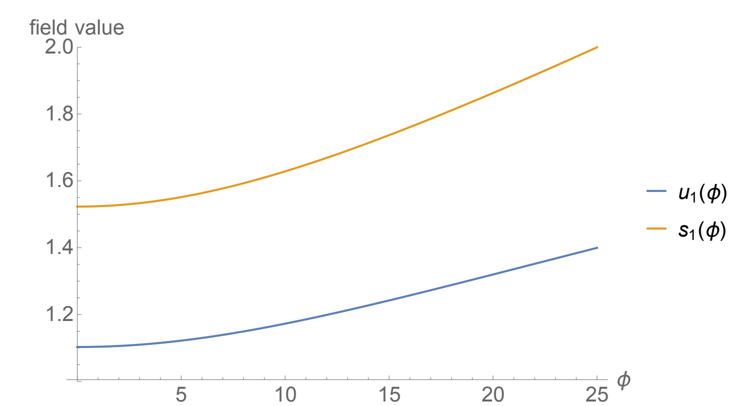
$$W = \mu\Phi^2 + W_0 + Ae^{-aT} \quad \text{with} \quad W_0 = e_0 + ie_1U + imU^3 + ih_0S + \mu SU + \bar{h}_0SU^3$$







Masses and vevs of u, s almost independent of μ

Critical value ϕ_c highly depends on μ

4-forms in IIA flux compactifications

4d scalar potential of IIA on orientifold flux CY compactifications:

$$V_{RR} = \frac{e^{K_{cs}}}{2s}\left[\overbrace{-kF_4^0 \wedge *F_4^0 + 2F_4^0\rho_0}^{F_4} - \overbrace{4kg_{ij} * F_4^i \wedge F_4^i + 2F_4^i\rho_i}^{F_6} - \overbrace{\frac{1}{4k}g_{ij}\tilde{F}_4^i \wedge *\tilde{F}_4^j + 2\tilde{F}_4^i\tilde{\rho}_j}^{F_8} + \overbrace{kF_4^m\rho_m}^{F_{10}}\right]$$

$$V_{NS} = e^{K_{cs}}\frac{s^2}{k}c_{IJ}H_4^I \wedge *H_4^J$$

$$\rho_0 = e_0 + b^i e_i - \frac{m}{6}k_{ijk}b^i b^j b^k + k_{ijk}\frac{1}{2}q_i b^j b^k - h_0 c_3^0 - h_i c_3^i$$

$$\rho_i = e_i + k_{ijk}b^j q^k - \frac{m}{2}k_{ijk}b^j b^k$$

$$\tilde{\rho}_i = q_i - mb_i$$

$$\rho_m = m$$

Axions: b^i, c_3^0, c_3^i

Internal fluxes: e_0, e_i, q_i, m, h_I

$$V_{RR} + V_{NS} = \frac{e^{K_{cs}}}{s}\left[\frac{1}{2k}|\rho_0|^2 + \frac{g_{ij}}{8k}\rho^i\rho^j + 2kg_{ij}\tilde{\rho}^i\tilde{\rho}^j + k|\rho_m|^2 + \frac{1}{k}c_{IJ}\rho_h^I\rho_h^J\right]$$

Minimisation of the potential:

Minima of axions $\longrightarrow$ $\rho_0 = 0$ and

$$(I) \quad \tilde{\rho}^i = 0$$

$$(II) \quad k_{ijl}\frac{g^{jk}}{8k}\rho_k + 2kg_{il}\rho^m = 0$$

Minima of saxions $\longrightarrow$ $s_0 \sim \dfrac{\rho_i^{3/2}}{\rho_{h_0}\sqrt{\rho_m}}\,, \quad u_0 \sim \dfrac{\rho_i^{3/2}}{\rho_{h_1}\sqrt{\rho_m}}\,, \quad t_0 \sim \dfrac{\rho_i^{1/2}}{\sqrt{\rho_m}}$

IIA flux compactifications

Inflaton = RR axion: $\phi = c_3^0 \longrightarrow \rho_0 \neq 0$

$$K_{\phi\phi} = \frac{1}{s_0}$$

At large field: $s = \dfrac{\rho_0(\phi)}{h_0} \longrightarrow$ $\Delta\phi \to log(\phi)$

Critical value: $\rho_0 > t\rho_i \longrightarrow \phi_c = s_0 \longrightarrow$ $\delta\phi \sim 1$ in M_p units

$(\lambda \sim 1)$

Back-reaction effects tied to M_p

in agreement with [Baume,Palti]

(Same for Inflaton = NS axion)

Open string models

We add non-geometric (geometric) fluxes on IIB (IIA) to stabilise all fields at tree level.

[Blumenhagen,I.V.,Wolf'17]

Toroidal compactification:

$$K = -2\log\left[(U+\bar{U})\right] - \log\left[(U+\bar{U})(S+\bar{S}) - \frac{1}{2}(\Phi+\bar{\Phi})^2\right] - 3\log(T+\bar{T})$$

$$W_{\rm tot} = W_{\rm RR} + W_{\rm NS} + W_{\rm non-geom}$$

where $W_{\rm RR} = f_0 + if_1 U + f_2 U^2 + if_3 U^3 \;\; ; \;\; W_{\rm non-geom} = iq_0 T - q_1 3UT$

$$W_{\rm NS} = ih_0 S + \mu_1(3US - \Phi^2) - i\mu_2(3U^2 S - 2U\Phi^2) - \mu_3(U^3 S - U^2\Phi^2)$$

Toy example I

$$W = \mathfrak{f}_0 + 3\mathfrak{f}_2\, U^2 - h\, S\, U - q\, T\, U - \mu\, \Phi^2$$

$$s_0(\theta) \sim \frac{(\mathfrak{f}_0 + \mu\theta^2)^{\frac{1}{2}}\, \mathfrak{f}_2^{\frac{1}{2}}}{h} \;\; \rightarrow \lambda \sim \sqrt{\frac{\mu}{h}} \sim \frac{M_\Theta}{M_{\rm mod}} \ll 1$$

$$\frac{M^2_{\rm KK}}{M^2_{\rm mod}} \sim \frac{1}{h\, q} \gtrsim 1$$

Incompatible for $\mu > 1$

Toy example II

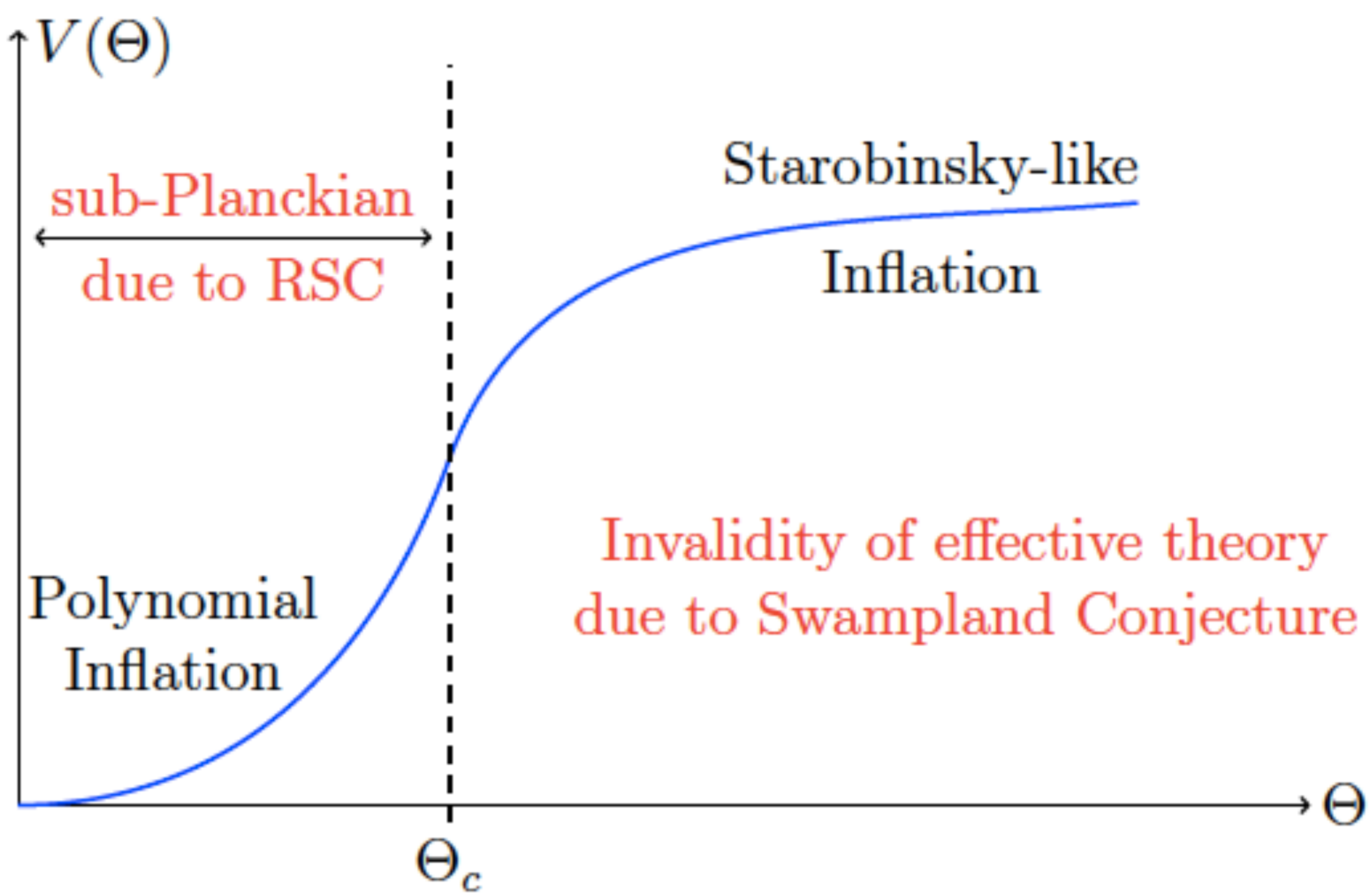
$$W = \left(i\mathfrak{f}_1 U + i\tilde{\mathfrak{f}}_0\, U^3 + ih\, S + iq\, T\right) - \mu_1\,(3US - \Phi^2) - q_1\, 3UT$$

$$\lambda \sim \frac{M_\Theta}{M_{\rm mod}} \sim \frac{\mu_1\, \mathfrak{f}_1^{\frac{1}{2}}}{h\, \tilde{\mathfrak{f}}_0^{\frac{1}{2}}} \ll 1$$

$$\frac{M^2_{\rm KK}}{M^2_{\rm mod}} \sim \frac{1}{q^2}\left(\frac{q\mathfrak{f}_1}{h\tilde{\mathfrak{f}}_0}\right) \gtrsim 1$$

Incompatible for $\mu > 1$

Scalar potential for the canonically normalised field:



$$\Theta = \Theta_c \log\left(\frac{\theta}{\theta_c}\right) \to V(\Theta) \simeq |V_0| \left[1 - \exp\left(-\gamma\frac{\Theta}{\Theta_c}\right)\right]$$