# Quantum scale-invariant effective potentials

*broccoli romanesque*

Paweł Olszewski

Based on:

D.M. Ghilencea, Z. Lalak, PO 1612.09120 [hep-ph]
*Standard Model with spontaneously broken quantum scale invariance*

D.M. Ghilencea, Z. Lalak, PO 1608.05336 [hep-th]
*Two-loop scale-invariant potential and quantum effective operators*

D.M. Ghilencea 1508.00595 [hep-ph]
*Manifest scale-invariant regularisation and quantum effective operators*

*Supported by National Science Centre, Poland,*
*under research grant DEC-2016/21/N/ST2/03312*

# Plan

1) Scale symmetry vs quantum corrections

2) Scale invariant (SI) regularisation

3) We did an exercise, a toy-model-higgs-sector with *quasi*-dim-reg:

   a) Spontaneous scale symmetry breaking

   b) Effective potential

   c) RGEs and Callan-Symanzik

4) Scale invariant Standard Model

**Scale symmetry**

$$\phi \rightarrow s^{d_\phi}\phi$$
$$x^\mu \rightarrow \frac{1}{s}x^\mu$$

Flat version of the global Weyl symmetry

$$\phi \rightarrow s^{d_\phi}\phi$$
$$g_{\mu\nu} \rightarrow \frac{1}{s^2}g_{\mu\nu}$$

Dilatation component of the conformal transformation of coordinates

$$\phi \rightarrow \Omega(x)^{-1}\phi$$
$$g_{\mu\nu}(x) \rightarrow \Omega^2(x)g_{\mu\nu}(x)$$

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A symmetric action $S = \int \mathrm{d}x^4 \mathcal{L}$ , e.g. $\mathcal{L} = \frac{1}{2}(\partial\varphi)^2 - \frac{\lambda}{4!}\varphi^4$

Noether current $D^\mu = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_j)}(x^\nu \partial_\nu \phi_j + d_\phi \phi_j) - x^\mu \mathcal{L}$

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trace of the E-M tensor

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trace of the E-M tensor

Homogeneity of the potential ?

**Scale symmetry**

Homogeneity of the potential $V(\phi_i)$

Dimensionfull quantities are forbidden

$$C.C. + m^2\phi^2 + \frac{\phi^6}{\Lambda^2}$$

## Scale symmetry

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But quantum corrections!

A dimensionful regulator,

$$V = \frac{\lambda\phi^4}{4!} + \frac{1}{4(4\pi)^2}\left(\frac{\lambda\varphi^2}{2}\right)^2\left(\log\frac{\lambda\varphi^2}{2\,M^2} - \frac{3}{2}\right)$$

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The scale anomaly

# Proposition: cure the scale anomaly with nonrenormalizable interactions

$$m^2\phi^2$$

$$\downarrow$$

$$\lambda_m \left(\langle\sigma\rangle + \sigma\right)^2 \phi^2$$

$$\left(\lambda\phi^2\right)^2 \log\frac{\lambda\phi^2}{2\,M^2}$$

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$$\left(\lambda\phi^2\right)^2 \log\frac{\lambda\phi^2}{2\,M^2} + \left(\lambda^2\frac{\phi^4\,\sigma}{\Lambda} + \dots\right)$$

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Added at the tree level

$$\left(\lambda\phi^2\right)^2 \log\frac{\lambda\phi^2}{2\,M^2} + \left(\lambda^2\frac{\phi^4\,\sigma}{\Lambda} + \ldots\right)$$

nonrenorm. series such that ...

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the **1PI effective action** is to all orders **invariant wrt.**

$$\left\{\begin{array}{rrcl} & \sigma & \rightarrow & \sigma + \Delta \\ & \langle\sigma\rangle & \rightarrow & \langle\sigma\rangle - \Delta \\ \text{including} & \Lambda & \rightarrow & \Lambda - \Delta \\ \text{and} & M & \rightarrow & M - z\,\Delta \end{array}\right.$$

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$M = z\,\Lambda = z\,\langle\sigma\rangle\,, \quad z \in \mathbb{R}$ is charged under scaling: $\langle\sigma\rangle + \sigma(x) \;\to\; s\left(\langle\sigma\rangle + \sigma(sx)\right)$

Way to organize the calculation: make the **regulator dynamical**

→

Use a propagating and interacting field

$$M \sim \sigma(x)$$

←

In the end: assume the field has a VEV

$$\sigma(x) \rightarrow \langle\sigma\rangle + \sigma(x)$$

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Arbitrariness of the prescription.

Consistency/universality of physical predictions
***???***

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$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \mu^{2\epsilon}V(\phi)\ , \qquad T^\mu_\mu = -2\epsilon V(\phi) \neq 0$$


M.Shaposhnikov, D. Zenhäusern, arXiv:0809.3406 [hep-th]
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$$\mu = z\,\sigma^{\frac{1}{1-\epsilon}}$$

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# Simple setup

Scale invariant SM + $\sigma$,

$$\mathcal{L}_{SM}\Big|_{m^2=0} + \frac{1}{2}\left(\partial\sigma\right)^2 - \lambda_m |H|^2\sigma^2 - \frac{\lambda_\sigma}{4}\sigma^4 \qquad H = \begin{pmatrix} 0 \\ \frac{\phi}{\sqrt{2}} \end{pmatrix}$$

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focus on the scalar sector

Toy-model

$$\frac{1}{2}(\partial\phi)^2 + \frac{1}{2}(\partial\sigma)^2 - \underbrace{\left(\frac{\lambda_\phi}{4!}\phi^4 + \frac{\lambda_m}{4}\phi^2\sigma^2 + \frac{\lambda_\sigma}{4!}\sigma^4\right)}_{V(\phi,\sigma)}$$

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consider regularisation

$$d = 4 - 2\epsilon\,, \quad \mu(\sigma) = z\,\sigma^{\left(\frac{1}{1-\epsilon}\right)}$$

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$\sigma$ is a field with its own **anomalous dim.**

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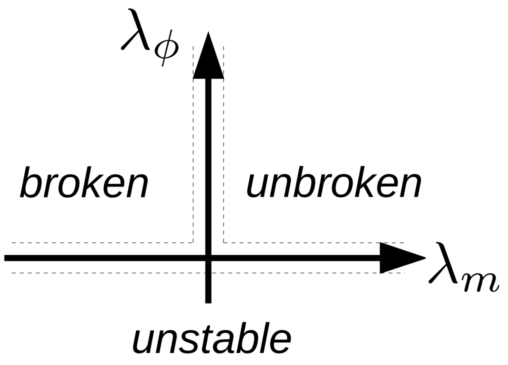
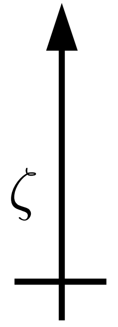
Infinite number of new running couplings

# Accomodating S(Scale)SB & tuning *C.C.* in the effective potential

$$\lambda_\sigma - \frac{9\lambda_m^2}{\lambda_\phi}$$

$$V(\phi,\sigma) = \\ = \frac{\lambda_\phi}{4!}\left(\phi^2 + \frac{3\lambda_m}{\lambda_\phi}\sigma^2\right)^2 + \frac{1}{4!}\,\zeta\,\sigma^4$$



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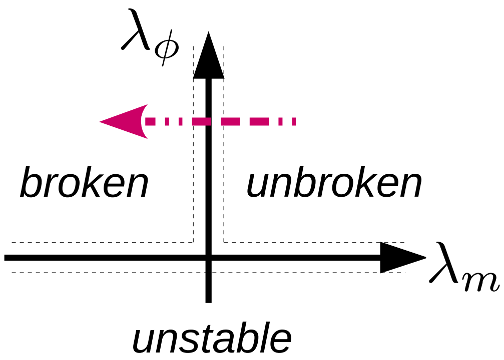


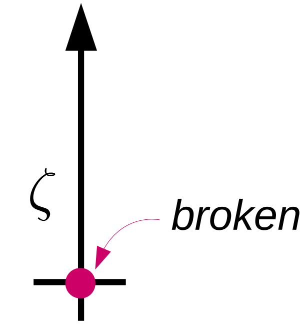


$$\langle\phi\rangle\,,\langle\sigma\rangle \neq 0 \;\Leftrightarrow\; \begin{cases} \zeta = 0 \\ \lambda_m < 0 \end{cases}, \qquad \lambda_\sigma = \frac{9\lambda_m^2}{\lambda_\phi}\,[1 + loops]$$

## Accomodating S(Scale)SB & tuning *C.C.* in the effective potential

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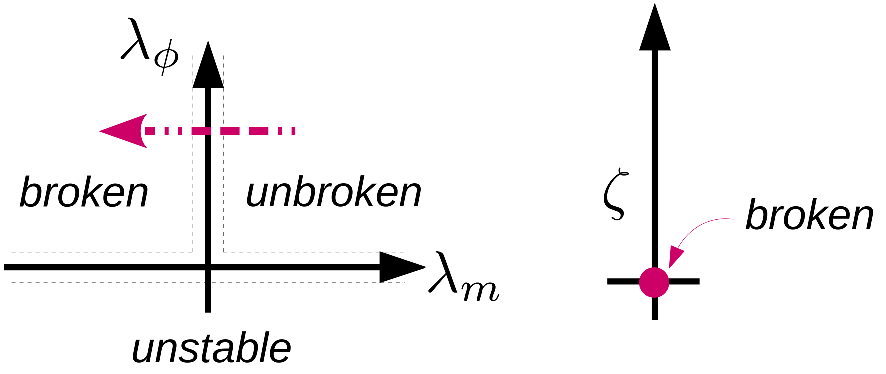
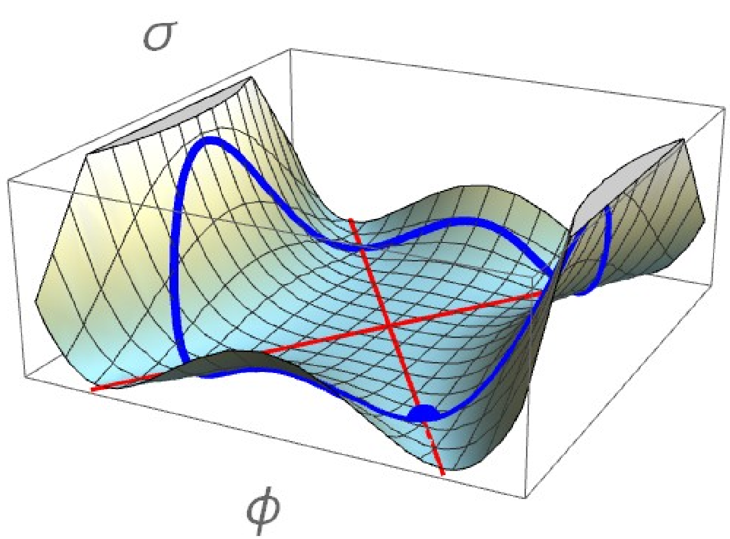
$\lambda_\sigma - \frac{9\lambda_m^2}{\lambda_\phi}$




$$\langle\phi\rangle\,,\langle\sigma\rangle \neq 0 \Leftrightarrow \begin{cases}\zeta = 0\\ \lambda_m < 0\end{cases}, \qquad \lambda_\sigma = \frac{9\lambda_m^2}{\lambda_\phi}\left[1 + loops\right]$$

$$\begin{pmatrix}\phi\\ \sigma\end{pmatrix} = \Lambda\begin{pmatrix}\cos\theta\\ \sin\theta\end{pmatrix}\,,\; V_{eff} = \Lambda^4\, W(\theta)\,,$$

$$\langle\tan^2\theta\rangle = \frac{\langle\phi\rangle^2}{\langle\sigma\rangle^2} = -\frac{3\lambda_m}{\lambda_\phi}\left[1 + loops\right]$$



# Calculating corrections with evanescent interactions

*Broken phase* lagrangian: $\mathcal{L} = \frac{1}{2}(\partial\phi) + \frac{1}{2}(\partial\sigma)^2 - \widetilde{V}(\phi,\sigma)$ ,

where

$$\widetilde{V}(\phi,\sigma) = \mu_0^{2\epsilon}\left[1 + 2\epsilon\frac{\sigma}{M_2} - \epsilon\frac{\sigma^2}{M_2^2} + \ldots\right]\cdot$$

$$\cdot\left[\frac{\lambda_\phi}{4!}(M_1+\phi)^4 + \frac{\lambda_m}{4}(M_1+\phi)^2(M_2+\sigma)^2 + \frac{\lambda_\sigma}{4!}(M_2+\sigma)^4 + \lambda_6\frac{(M_1+\phi)^6}{(M_2+\sigma)^2} + \ldots\right]$$

# Calculating corrections with evanescent interactions

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$$\cdot\left[\frac{\lambda_\phi}{4!}(M_1+\phi)^4 + \frac{\lambda_m}{4}(M_1+\phi)^2(M_2+\sigma)^2 + \frac{\lambda_\sigma}{4!}(M_2+\sigma)^4 + \lambda_6\frac{(M_1+\phi)^6}{(M_2+\sigma)^2} + \ldots\right]$$

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$\lambda_6, \lambda_8, \ldots = 0$

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evanesc. part & minimal subtraction
= not a very typical regular. scheme

$$\cdot\left[\frac{\lambda_\phi}{4!}(M_1+\phi)^4 + \frac{\lambda_m}{4}(M_1+\phi)^2(M_2+\sigma)^2 + \frac{\lambda_\sigma}{4!}(M_2+\sigma)^4 + \underbrace{\lambda_6\frac{(M_1+\phi)^6}{(M_2+\sigma)^2} + \ldots}_{\lambda_6,\ \lambda_8,\ \ldots\ =\ 0}\right]$$

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evanesc. part & minimal subtraction
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Compute:

- $V_{eff}^{(\text{n-loop})}(\phi,\sigma)$ (in the 1PI sense)
- $\text{RGEs}^{(\text{n-loop})}$ : $\beta_{\lambda\text{'s}}$ , $\gamma_\phi$ , $\gamma_\sigma$

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*new*

$$\widetilde{V}(\phi,\sigma) = \mu_0^{2\epsilon}\left[1 + 2\epsilon\frac{\sigma}{M_2} - \epsilon\frac{\sigma^2}{M_2^2} + \ldots\right]\cdot$$

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*$\lambda_6, \lambda_8, \ldots = 0$*

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with

$$M_1 = \phi_0 \equiv \langle\phi\rangle$$
$$M_2 = \mu_0/z = \sigma_0 \equiv \langle\sigma\rangle$$

# Calculating corrections with evanescent interactions

$$\lambda_\phi\, \sigma^{\frac{2\epsilon}{1-\epsilon}} \phi^4 \;\to\; \lambda_\phi\, \sigma_0^{\frac{2\epsilon}{1-\epsilon}} \left(1 + \frac{\delta\sigma}{\sigma_0}\right)^{\frac{2\epsilon}{1-\epsilon}} (\phi_0 + \delta\phi)^4 \;\to\; \ldots$$

# Calculating corrections with evanescent interactions

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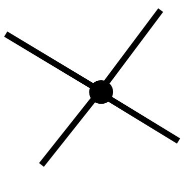
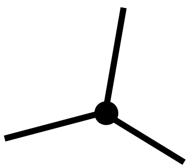
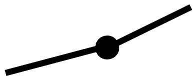


$\sim \lambda_\phi$



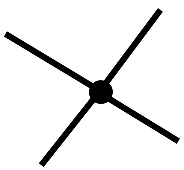
$\sim \lambda_\phi \phi_0$



$\sim \lambda_\phi \phi_0^2$

# Calculating corrections with evanescent interactions

$$\lambda_\phi \, \sigma^{\frac{2\epsilon}{1-\epsilon}} \phi^4 \;\rightarrow\; \lambda_\phi \, \sigma_0^{\frac{2\epsilon}{1-\epsilon}} \left(1 + \frac{\delta\sigma}{\sigma_0}\right)^{\frac{2\epsilon}{1-\epsilon}} (\phi_0 + \delta\phi)^4 \;\rightarrow\; \dots$$



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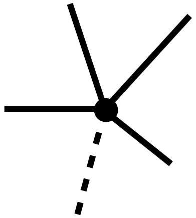
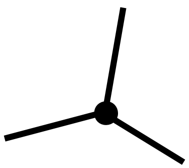
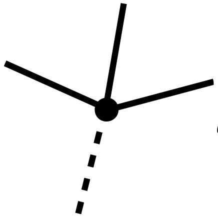


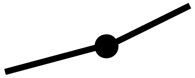
$\sim \frac{\lambda_\phi}{\sigma_0}\left(\epsilon + \epsilon^2 + \dots\right)$



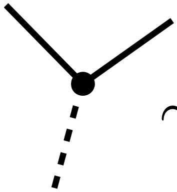
$\sim \lambda_\phi \phi_0$



$\sim \lambda_\phi \frac{\phi_0}{\sigma_0}\left(\epsilon + \epsilon^2 + \dots\right)$



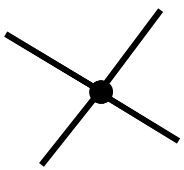
$\sim \lambda_\phi \phi_0^2$



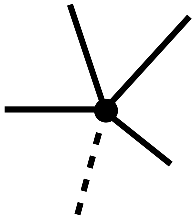
$\sim \lambda_\phi \frac{\phi_0^2}{\sigma_0}\left(\epsilon + \epsilon^2 + \dots\right)$

# Calculating corrections with evanescent interactions

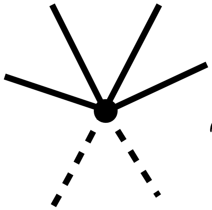
$$\lambda_\phi \, \sigma^{\frac{2\epsilon}{1-\epsilon}} \phi^4 \;\to\; \lambda_\phi \, \sigma_0^{\frac{2\epsilon}{1-\epsilon}} \left(1 + \frac{\delta\sigma}{\sigma_0}\right)^{\frac{2\epsilon}{1-\epsilon}} (\phi_0 + \delta\phi)^4 \;\to\; \dots$$



$\sim \lambda_\phi$

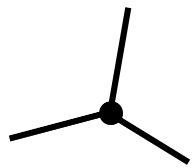


$\sim \frac{\lambda_\phi}{\sigma_0}\left(\epsilon + \epsilon^2 + \dots\right)$

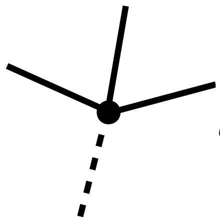


$\sim \frac{\lambda_\phi}{\sigma_0^2}\left(\epsilon + \epsilon^2 + \dots\right)$

$\vdots$



$\sim \lambda_\phi \phi_0$



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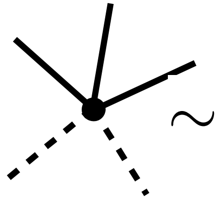
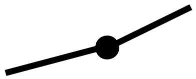
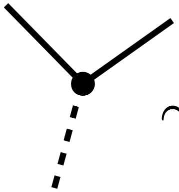


$\sim \lambda_\phi \frac{\phi_0}{\sigma_0^2}\left(\epsilon + \epsilon^2 + \dots\right)$

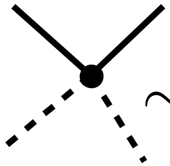
$\vdots$



$\sim \lambda_\phi \phi_0^2$



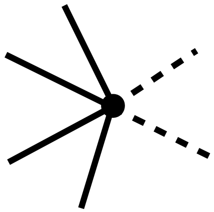
$\sim \lambda_\phi \frac{\phi_0^2}{\sigma_0}\left(\epsilon + \epsilon^2 + \dots\right)$



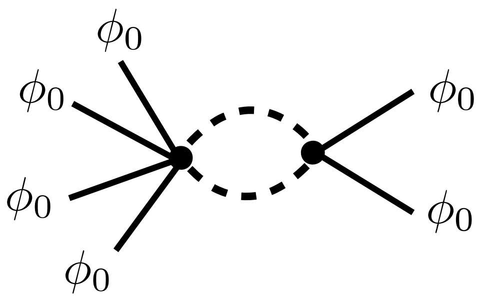
$\sim \lambda_\phi \frac{\phi_0^2}{\sigma_0^2}\left(\epsilon + \epsilon^2 + \dots\right)$

$\vdots$

# Calculating corrections with evanescent interactions



$$\sim \; \epsilon \, \lambda_\phi \frac{1}{\sigma_0^2}$$



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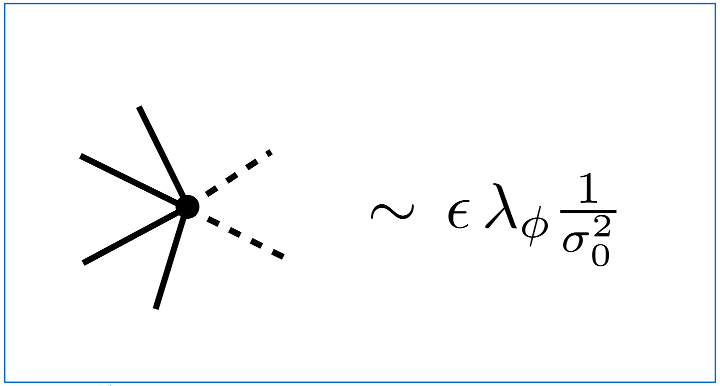


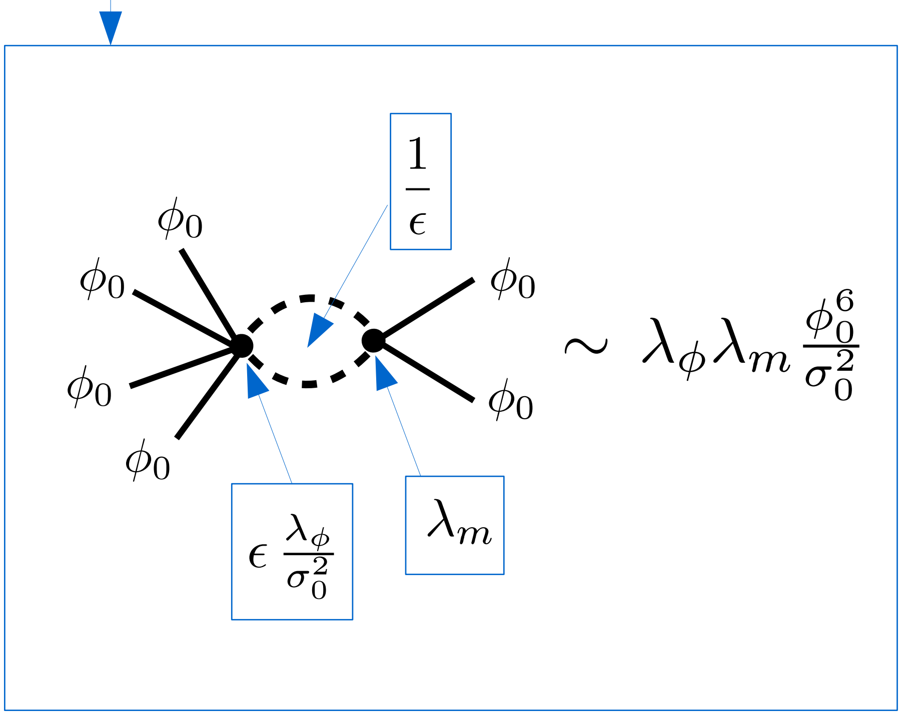

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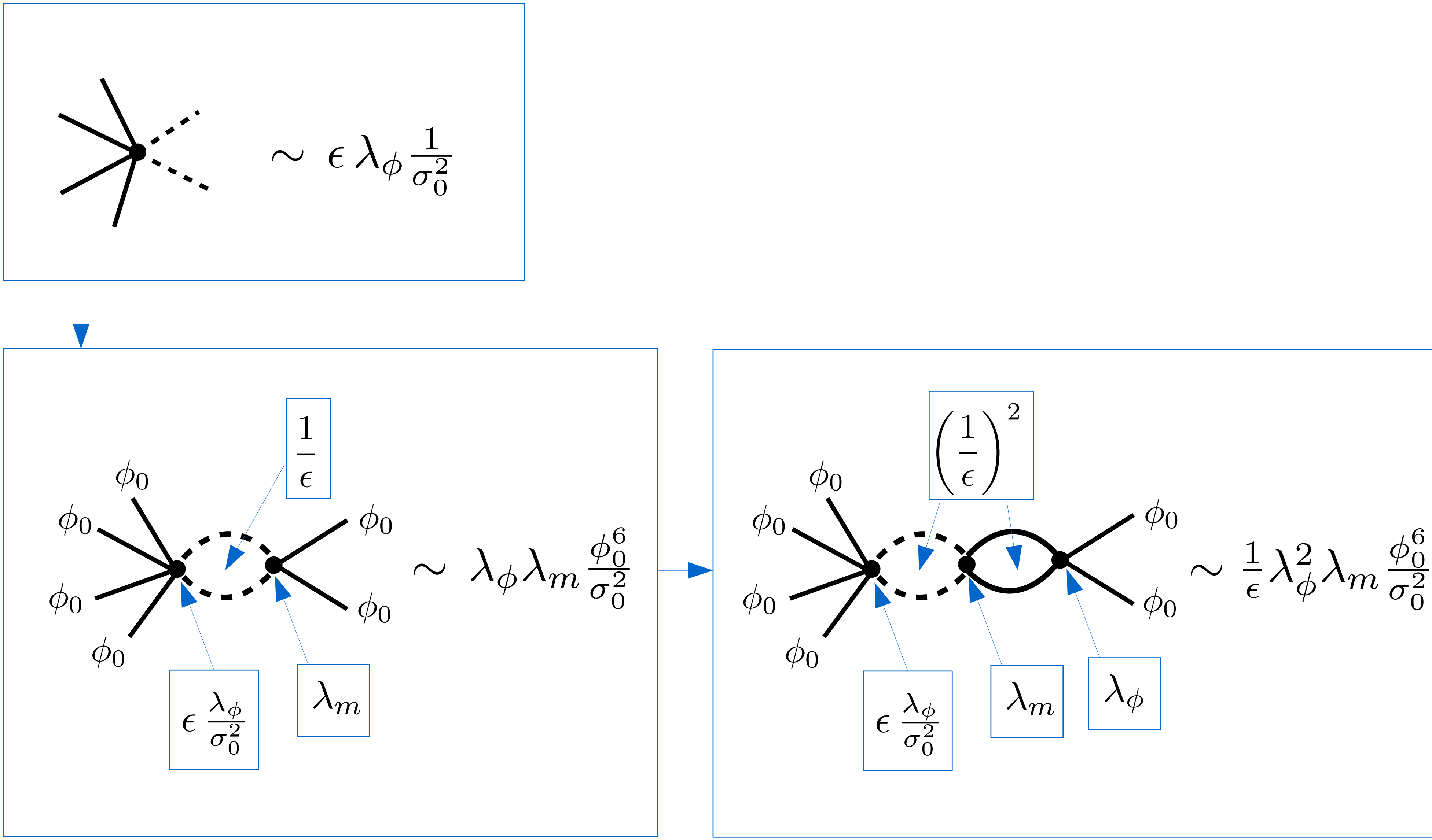
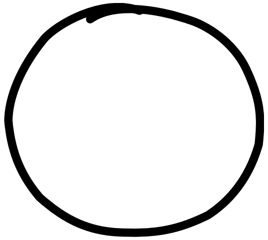


## **Effective potential** (by the background field method)

- 1-loop



$$\frac{1}{64\pi^2}\sum_{s=(+,-)} m_s^4\left(\frac{1}{\epsilon}-\log\frac{m_s^2}{z^2\,\sigma^2}+\frac{3}{2}\right)+V^{new}_{\text{1-loop}}$$

$$V^{new}_{\text{1-loop}}=\frac{1}{(4\pi)^2}\left[\ldots+\lambda_\phi\lambda_m\frac{\phi^6}{\sigma^2}\right]$$

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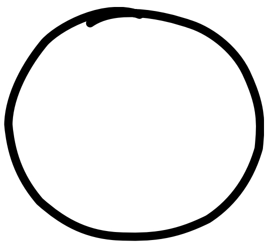
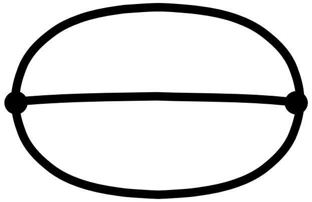
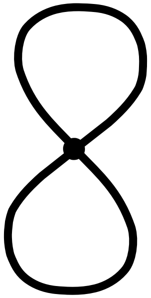
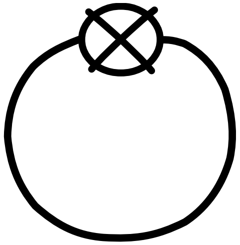


$$V^{new}_{1\text{-loop}} = \frac{1}{(4\pi)^2}\left[\ldots + \lambda_\phi\lambda_m\frac{\phi^6}{\sigma^2}\right]$$

- 2-loop

$$V^{usual}_{2\text{-loop}} + V^{new}_{2\text{-loop}}\,, \qquad V^{new}_{2\text{-loop}} \supset \frac{1}{\epsilon}\left[\frac{\sim\lambda^3}{(4\pi)^3}\,\frac{\phi^6}{\sigma^2} + \frac{\sim\lambda^3}{(4\pi)^3}\,\frac{\phi^8}{\sigma^4}\right]$$

Davydychev, Tausk, *„Two loop selfenergy diagrams with different masses and the momentum expansion”, Nucl. Phys. B 397 (1993) 123*

## **Effective potential** (by the background field method)

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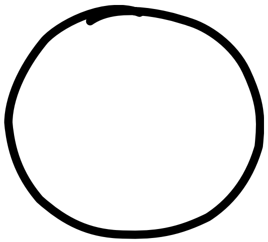
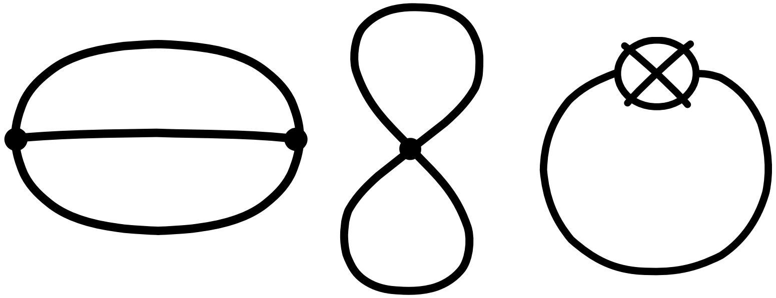


$$V_{\text{1-loop}}^{new} = \frac{1}{(4\pi)^2}\left[\ldots + \lambda_\phi\lambda_m\frac{\phi^6}{\sigma^2}\right]$$

- 2-loop

$$V_{\text{2-loop}}^{usual} + V_{\text{2-loop}}^{new}\,,$$

$$V_{\text{2-loop}}^{new} \supset \frac{1}{\epsilon}\left[\frac{\sim\lambda^3}{(4\pi)^3}\,\frac{\phi^6}{\sigma^2} + \frac{\sim\lambda^3}{(4\pi)^3}\,\frac{\phi^8}{\sigma^4}\right]$$



Infinitely many new terms suppressed by $\sigma$

$$\lambda_{(4+2n)}\frac{\phi^{4+2n}}{\sigma^{2n}}\,,\quad n=1,2,\ldots$$

$$\beta_{\lambda_6} \neq 0$$

$$\beta_{\lambda_8} \neq 0\,,\quad \text{even at } \lambda_{(4+2n)} = 0$$

Davydychev, Tausk, „*Two loop selfenergy diagrams with different masses and the momentum expansion*”, *Nucl. Phys. B 397 (1993) 123*

# RGEs and Callan-Symanzik equation (at 2-loop)

Recall that $\mu(\sigma) = z\,\sigma^{\frac{1}{1-\epsilon}}$

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Recall that $\mu(\sigma) = z\,\sigma^{\frac{1}{1-\epsilon}}$

arbitrary number playing the role of $\mu/\mu_0$

$$0 = \frac{\mathrm{d}\; z^{2\epsilon}\lambda_\phi(z) Z_\phi^{-2} Z_\sigma^{-\frac{2\epsilon}{1-\epsilon}} Z_{\lambda_\phi}}{\mathrm{d}\;\log z}$$

etc.

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etc.

$$0 = \frac{\mathrm{d}\, V_{eff}(\lambda,\, z)}{\mathrm{d}\log z}$$

$$0 = \left( z\frac{\partial}{\partial z} + \beta_{\lambda_j}\frac{\partial}{\partial \lambda_j} - \phi\gamma_\phi\frac{\partial}{\partial\phi} - \sigma\gamma_\sigma\frac{\partial}{\partial\sigma}\right) V_{eff}(\lambda, z)$$

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**New RGE's**

$$0 = \frac{\mathrm{d}\, V_{eff}(\lambda,\; z)}{\mathrm{d}\log z}$$

$$0 = \left( z\frac{\partial}{\partial z} + \beta_{\lambda_j}\frac{\partial}{\partial \lambda_j} - \phi\gamma_\phi\frac{\partial}{\partial \phi} - \sigma\gamma_\sigma\frac{\partial}{\partial \sigma}\right) V_{eff}(\lambda, z)$$

Summation over lambda's, including $\lambda_{(4+2n)}$

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**New RGE's**

**New corrections to *V***

$$0 = \left( z\frac{\partial}{\partial z} + \beta_{\lambda_j}\frac{\partial}{\partial \lambda_j} - \phi\gamma_\phi\frac{\partial}{\partial\phi} - \sigma\gamma_\sigma\frac{\partial}{\partial\sigma}\right) V_{eff}(\lambda, z)$$

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assumed to contain $\lambda_{(4+2n)}$

# RGEs and Callan-Symanzik equation (at 2-loop)

Recall that $\mu(\sigma) = z\,\sigma^{\frac{1}{1-\epsilon}}$

arbitrary number playing the role of $\mu/\mu_0$

$$0 = \frac{\mathrm{d}\; z^{2\epsilon}\lambda_\phi(z) Z_\phi^{-2} Z_\sigma^{-\frac{2\epsilon}{1-\epsilon}} Z_{\lambda_\phi}}{\mathrm{d}\;\log z}$$ etc.

$$0 = \frac{\mathrm{d}\, V_{eff}(\lambda,\, z)}{\mathrm{d}\log z}$$

**New RGE's**

**New corrections to *V***

***C-S eq. holds***

$$0 = \left( z\frac{\partial}{\partial z} + \beta_{\lambda_j}\frac{\partial}{\partial \lambda_j} - \phi\gamma_\phi\frac{\partial}{\partial \phi} - \sigma\gamma_\sigma\frac{\partial}{\partial \sigma}\right) V_{eff}(\lambda, z)$$

Summation over lambda's, including $\lambda_{(4+2n)}$

assumed to contain $\lambda_{(4+2n)}$

# Few remarks

$\lambda_m = 0$
is a fixed point of the RGEs

$$\beta_{\lambda_{(4+2n)}}\Big|_{\lambda_m=0} \neq 0$$

One could've equally well use a more complicated $\mu(\{\Phi\})$

Nonrenormalizable interactions suppressed by powers of $\mu$

One needs to include all nonrenormalisable terms

Including the ***p*²-dependent** ones, like

$$\overline{H}\left(\partial_\mu H\right)\left(\partial_\mu \sigma\right)\frac{1}{\sigma}$$

But, thanks to „evanescent presence" of *σ* in every interaction,
**anomalous dim. of *σ*** contributes to the ***β* function of each coupling**

$$\lambda_B \phi_B^{2n} \sigma_B^{2m} \cdot \left(z\sigma_B^{\frac{1}{1-\epsilon}}\right)^{2\epsilon}, \quad \lambda_B = z^{2\epsilon} Z_\lambda \lambda(z)\, Z_\phi^{-n} Z_\sigma^{-(m+\frac{\epsilon}{1-\epsilon})}$$

$$0 = \frac{\mathrm{d}\log\lambda_B}{\mathrm{d}\log z} = 2\epsilon + \ldots - \left(m + \frac{\epsilon}{1-\epsilon}\right)\frac{\partial \log Z_\sigma}{\partial \lambda_i}\frac{\mathrm{d}\,\lambda_i}{\mathrm{d}\log z}$$

# Few remarks

$\lambda_m = 0$

is a fixed point of the RGEs

$$\beta_{\lambda_{(4+2n)}}\Big|_{\lambda_m=0} \neq 0$$

One could've equally well use a more complicated $\mu(\{\Phi\})$

Nonrenormalizable interactions suppressed by powers of $\mu$

One needs to include all nonrenormalisable terms

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contributes even for *m*=0

# Scale invariant Standard Model and vacuum stability

$$\mathcal{L}_{SM}\Big|_{m^2=0} + \frac{1}{2}(\partial\sigma)^2 - \lambda_m |H|^2\sigma^2 - \frac{\lambda_\sigma}{4}\sigma^4 - \lambda_6\frac{|H|^6}{\sigma^2} + \dots$$

$$H = \begin{pmatrix} 0 \\ \frac{\phi}{\sqrt{2}} \end{pmatrix}$$

Infinite series: $\lambda_6\,,\ \lambda_8\,,\ \lambda_{10}\,,\ \dots$

Hence, generically I cannot make trips into $\{\phi > \sigma\}$.

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Problem of explaining spont. broken scale symmetry comes before the issue of instability.

b) $\langle\sigma\rangle \gtrsim \Lambda_{instability}$

There may exist a tunneling field configuration (Coleman's bounce) such that $\phi < \sigma$ along this configuration.

# Scale invariant Standard Model and vacuum stability

E.g.

$$\langle\sigma\rangle \sim 10^{10}\, GeV$$

$$\lambda_m \sim \left(\frac{100\, GeV}{\langle\sigma\rangle}\right)^2 \sim 10^{-16}$$

$$\lambda_\sigma \sim \lambda_m^2$$

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Renormalizale interaction with $\sigma$ switched off at the tree level.

$$V_{eff} = \frac{\lambda_\phi}{4}\phi^4 - 12\frac{1}{4(4\pi)^2}\left(\frac{y_t\,\phi}{\sqrt{2}}\right)^2\left[\log\frac{y_t^2\phi^2}{2\,(4\pi \mathrm{e}^{-\gamma_E}\,z\sigma)^2} - \frac{3}{2}\right] + \dots$$

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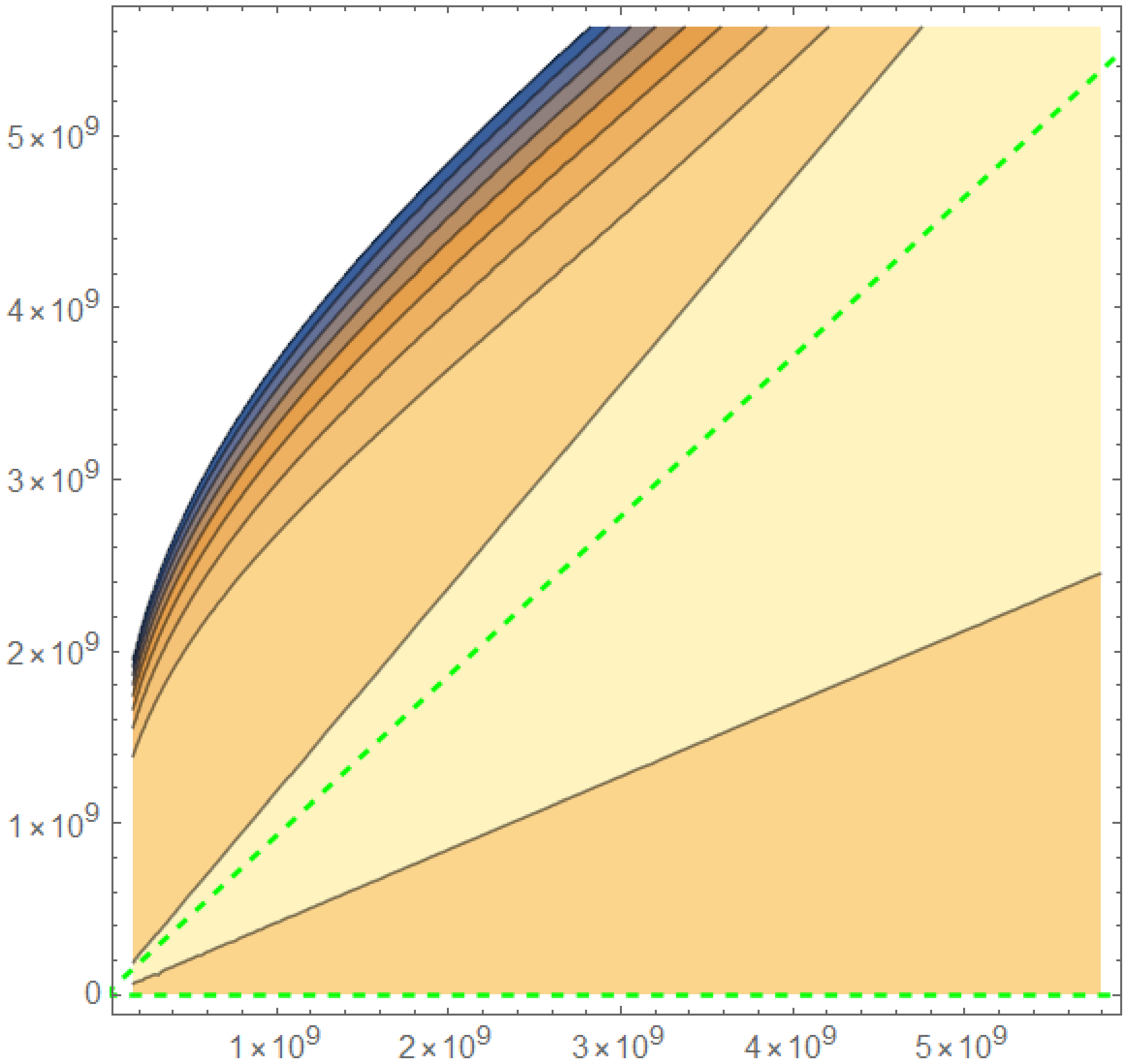
Running with the ***z*** parameter.

**RG-improv.:** $z \sim \phi/\sigma$

$$V_{eff}(\phi,\sigma) = \\ = \frac{\lambda(\phi/\sigma)}{4}\phi^4 + \lambda_6(\phi/\sigma)\frac{\phi^6}{\sigma^2} + \ldots$$

Does the tunneling rate feel the new degree of freedom?

# Scale invariant Standard Model and vacuum stability



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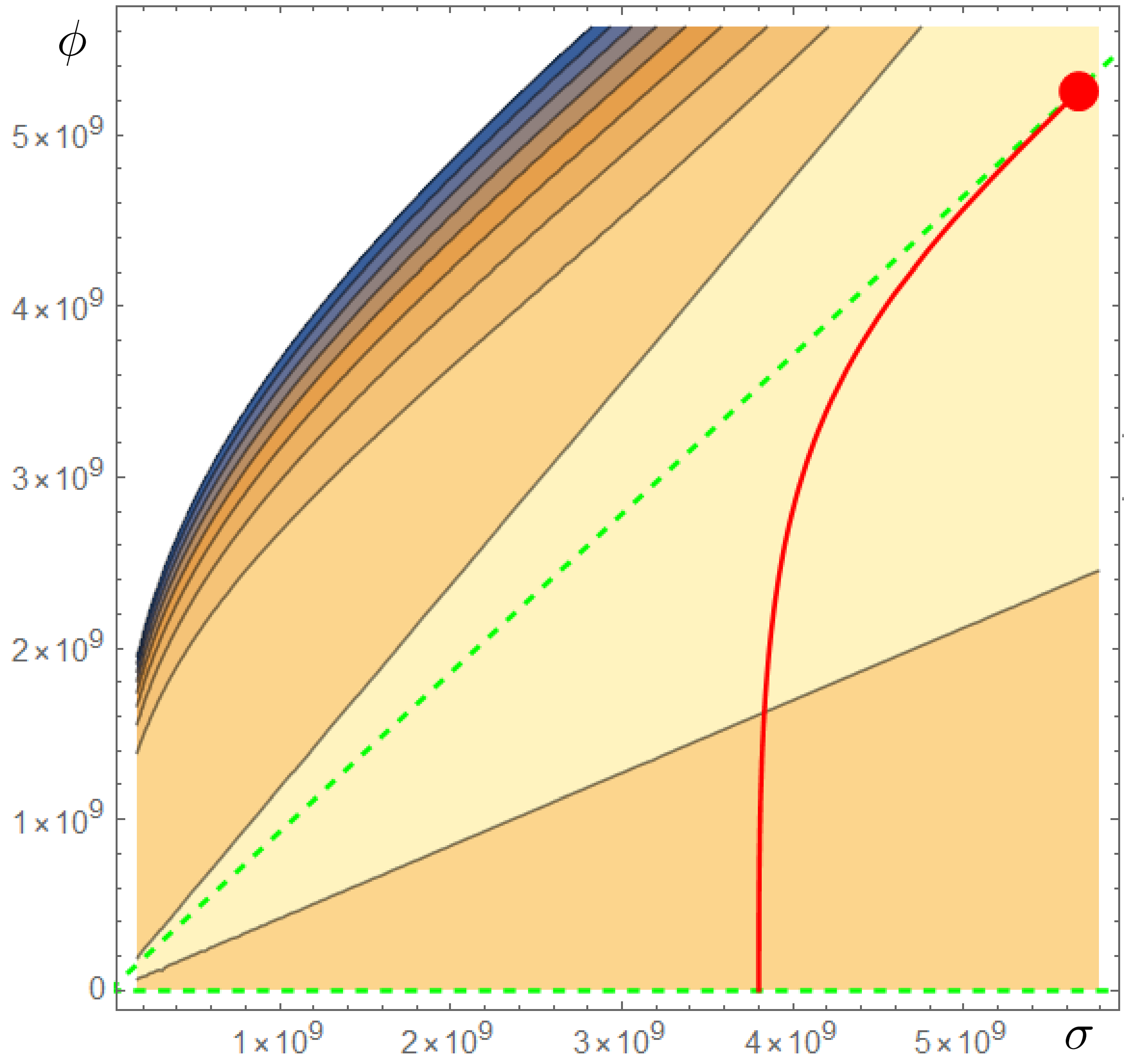


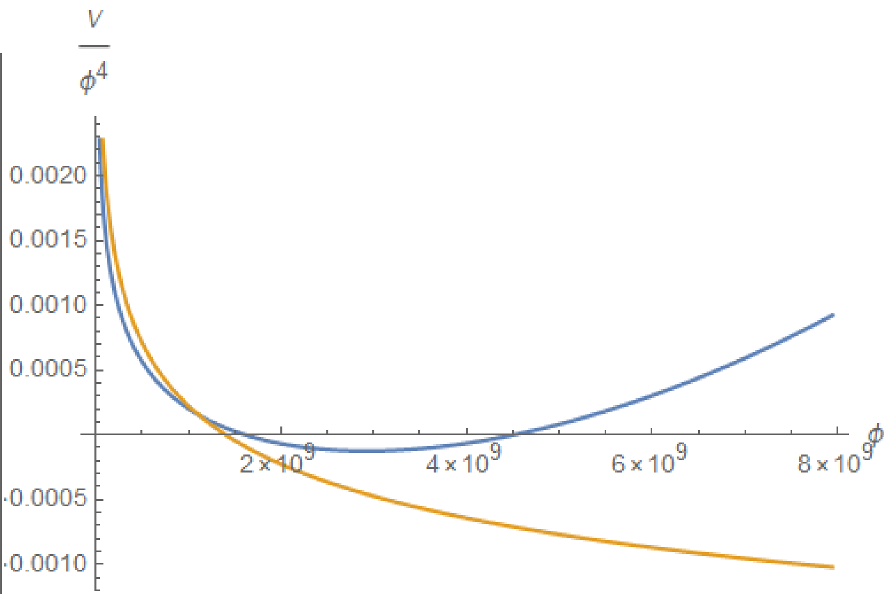


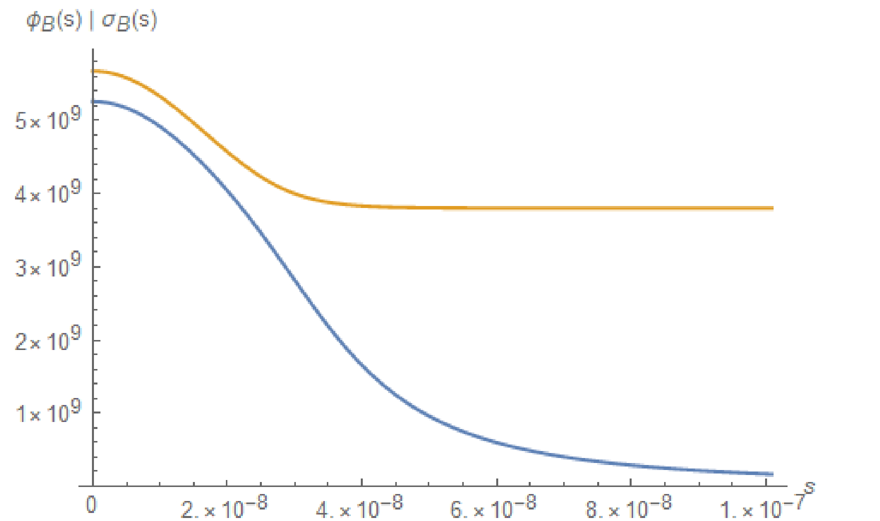

# I was staying in a flat spacetime.
# See: other pleople's existing work in cosmology

Dilatations

$$\phi \rightarrow \Omega(x)^{-1}\phi$$
$$g_{\mu\nu}(x) \rightarrow \Omega^2(x) g_{\mu\nu}(x)$$

$$\int d^4x \sqrt{-g} \left( \frac{1}{2} g^{\mu\nu} (\partial_\mu \phi_i)(\partial_\nu \phi_i) - V(\phi_i) - \frac{\alpha_i}{12} \phi_i^2 R \right)$$

„Higgs-Dilaton theory":

J. Garcia-Belido, J. Rubio, M. Shaposhnikov, D. Zenhaeusern, arXiv:1107.2163 [hep-ph]
*Higgs-Dilaton Cosmology: From the Early to the Late Universe*

F. Bezrukov, G. K. Karananas, J. Rubio, M. Shaposhnikov, arXiv:1212.4148 [hep-ph]
*Higgs-Dilaton Cosmology: an effective field theory approach*

A. Shkerin, arXiv:1701.02224 [hep-ph]
*Electroweak vacuum stability in the Higgs-Dilaton theory*

„Scale-invariant inflation":

K. Allison, C.T. Hill, G.G. Ross, arXiv:1404.6268 [hep-ph]
*Ultra-weak sector, Higgs boson mass, and the dilaton*

P.G. Ferreira, C.T. Hill, G.G. Ross, arXiv:1603.05983 [hep-th]
*Scale-Independent Inflation and Hierarchy Generation*

P.G. Ferreira, C.T. Hill, G.G. Ross, arXiv:1610.09243 [hep-th]
*Weyl Current, Scale-Invariant Inflation and Planck Scale Generation*

P.G. Ferreira, C.T. Hill, G.G. Ross, arXiv:1612.03157 [gr-qc]
*No fifth force in a scale invariant universe*

# Conclusions

1) You may use a field as a $\mu$ in dimreg to preserve scale symmetry at the quantum level.

2) The price to pay: infinitely many nonpolynomial operators suppressed by $\mu$ and corresponding running couplings.

3) Not a MS scheme: all Your RGE's are different (Callan-Symanzik still holds).

4) Instability equals unboundness from below.

5) If You assume perturbativity of SM, the story of vacuum decay is generically not modified.

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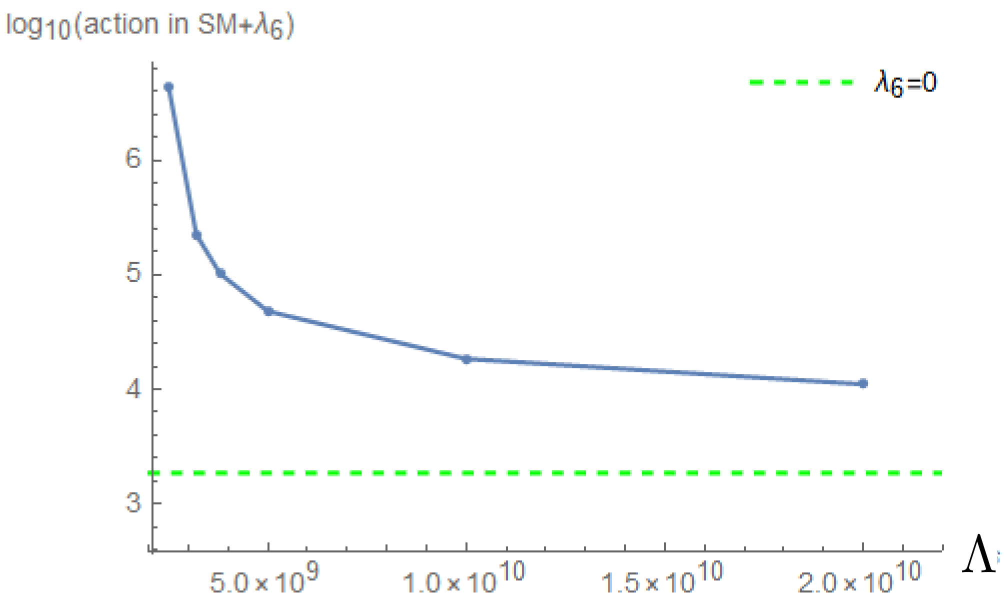
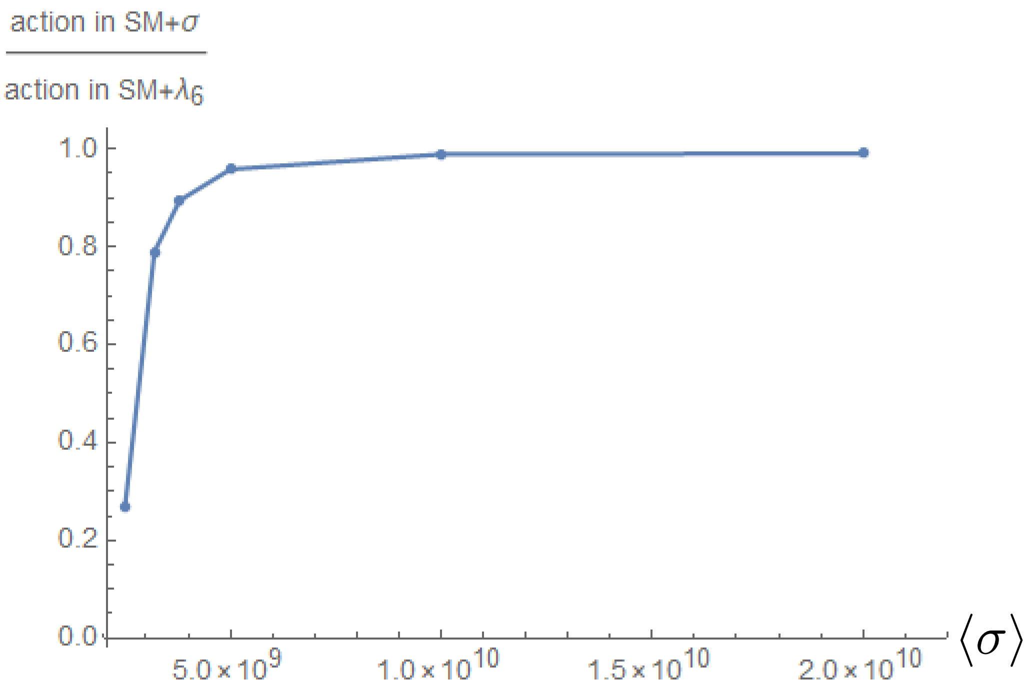
***Thank You!***

# Backup



log10(action in SM+λ6)
λ6=0
6
5
4
3
5.0×10^9
1.0×10^10
1.5×10^10
2.0×10^10
Λ

action in SM+σ
action in SM+λ6
1.0
0.8
0.6
0.4
0.2
0.0
5.0×10^9
1.0×10^10
1.5×10^10
2.0×10^10
⟨σ⟩