Cosmological Imprints of the Superhorizon Universe (from Numerical Relativity)

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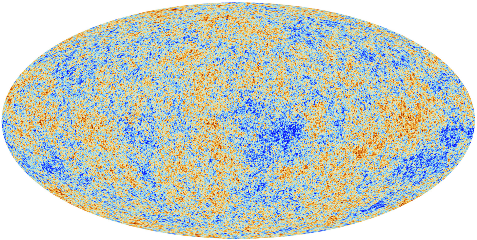
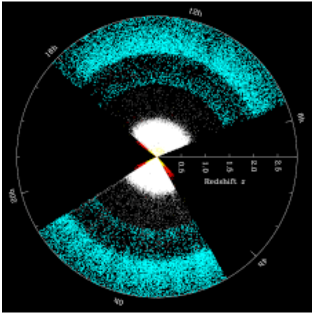
based on arXiv:1604.04001 and *in progress*

The CMB and LSS



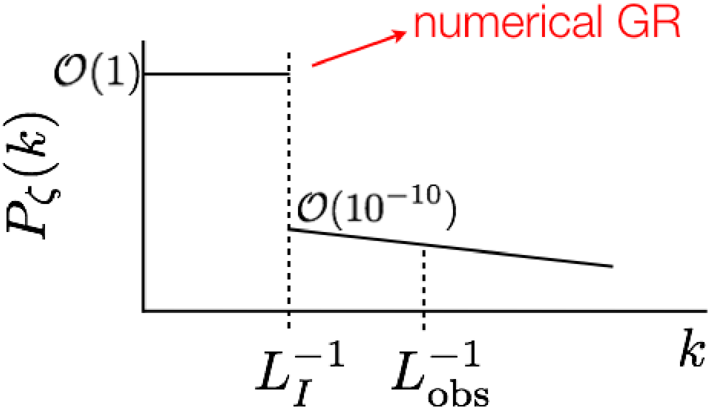


Inflation: Only a few parameters

$$P_\zeta(k) = Ak^{n_s - 1} \qquad r = 16\epsilon \qquad f_{NL}$$

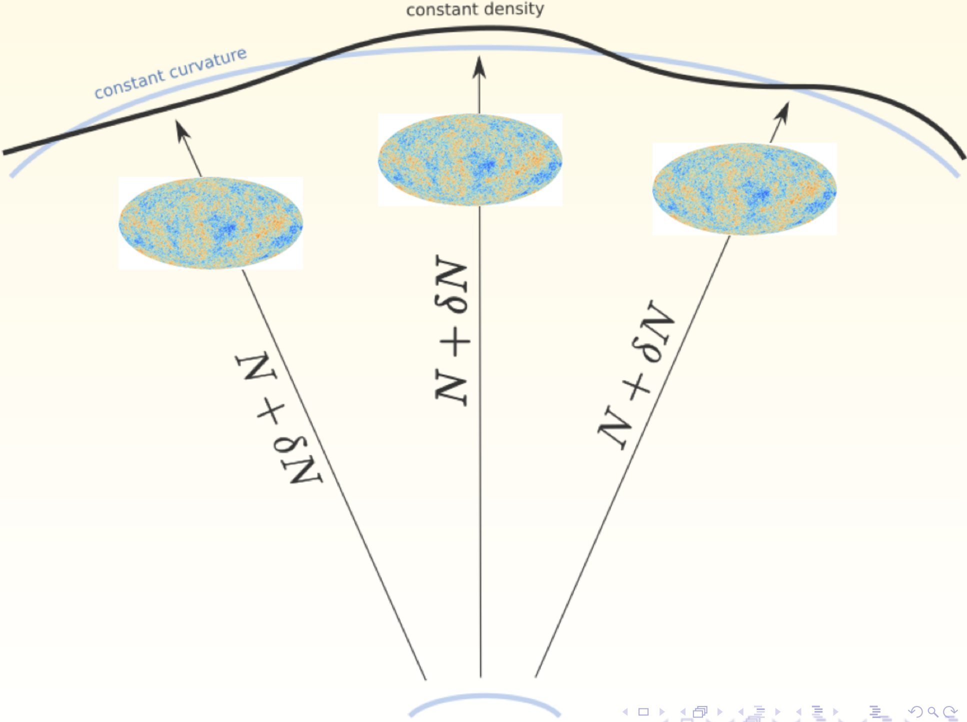
- Nature of Inflaton?
- Initial Conditions?

What About Ultra-Large Scales



Evolve long wavelength modes dynamically

CMB scales see locally homogeneous background


constant density
constant curvature
$N + \delta N$
$N + \delta N$
$N + \delta N$

Modelling Initial Conditions

Monte Carlo Sampling: Planar Symmetry

$$ds^2 = -d\tau^2 + a_{\|}^2(x,\tau)dx^2 + a_{\perp}^2(x,\tau)\left(dy^2 + dz^2\right)$$

Inflaton on $a_{\|}(\tau = 0) = 1 = a_{\perp}(\tau = 0)$

$$\phi(x) = \bar{\phi} + \delta\hat{\phi}$$

$$\bar{\phi} \text{ gives } \mathcal{N} \text{ e-folds} \qquad 3H_{\mathrm{I}}^2 \equiv V(\bar{\phi})$$

Field Fluctuations

$$\delta\hat{\phi}(x_i) = A_\phi \sum_{n=1} \hat{G} e^{ik_n x_i} \sqrt{P(k_n)} \qquad \hat{G} = \sqrt{-2\ln\hat{\beta}}\, e^{2\pi i\hat{\alpha}}$$

Modelling Initial Conditions

Monte Carlo Sampling: Planar Symmetry

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Field Fluctuations

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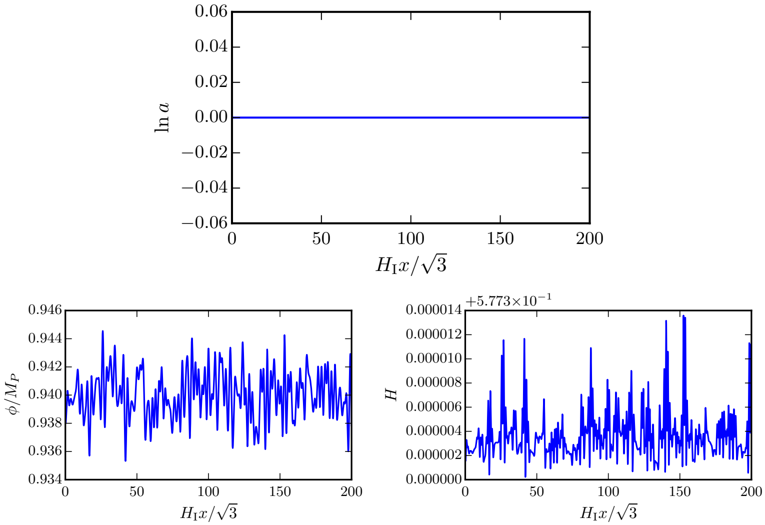
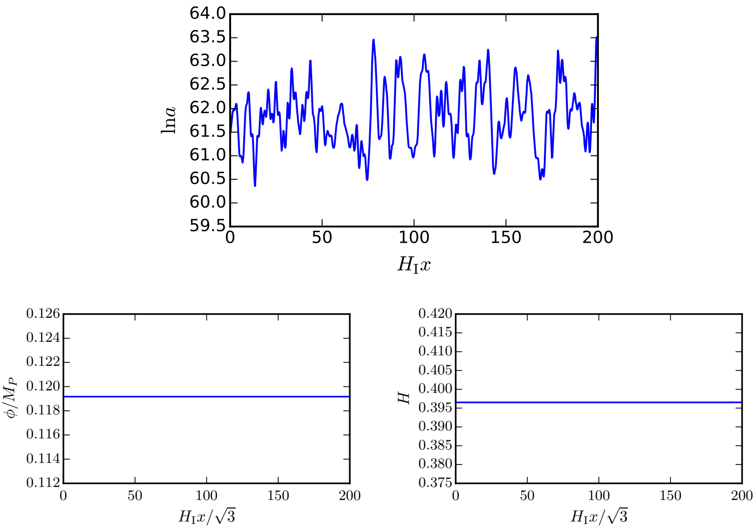
$$P(k) = \Theta(k_{\mathrm{max}} - k) \qquad H_{\mathrm{I}}^{-1} k_{\mathrm{max}} = 2\pi\sqrt{3}$$

Required Evolution



Initial Conditions ($\tau = 0$)

Required Evolution



End of Inflation ($\epsilon_H = -d \ln H / d \ln a = 1$)

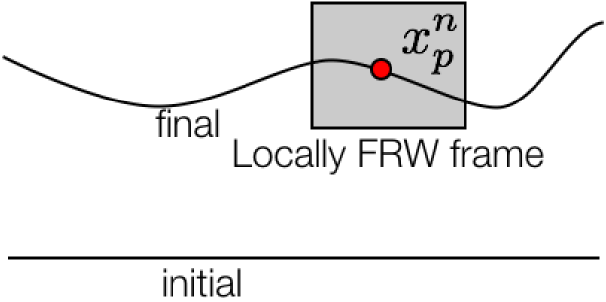
Ultra Large Scale Structure

$$\zeta(x_p) \simeq \underline{\zeta(x_p^n)} + \underline{\partial_{x_p}\zeta(x_p^n)\,(x_p - x_p^n)} + \underline{\partial^2_{x_p^n}\zeta(x_p^n)\,(x_p - x_p^n)^2/2} + \ldots$$

log(local scale factor)

gauge mode (not observable)

Maps to CMB quadrupole



Observational Constraints

$$\mathrm{Pr}(A_\phi, H_I L_{\mathrm{obs}} | C_2^{\mathrm{obs}}, \dots) \propto \mathcal{L}(A_\phi, H_I L_{\mathrm{obs}}) \mathrm{Pr}(A_\phi, H_I L_{\mathrm{obs}} | \dots)$$
$$\mathcal{L} = \mathrm{Pr}(C_2^{\mathrm{obs}} | A_\phi, H_I L_{\mathrm{obs}}, \dots)$$

- A_ϕ : Fluctuation Amplitude $P(k) \propto A_\phi^2$
- $H_I L_{\mathrm{obs}}$: Uncertain post-inflation expansion history
- $\dots$: $V(\phi)$, spectrum shape, IC hypersurface, $C_2^{\mathrm{high}-\ell}$, etc.

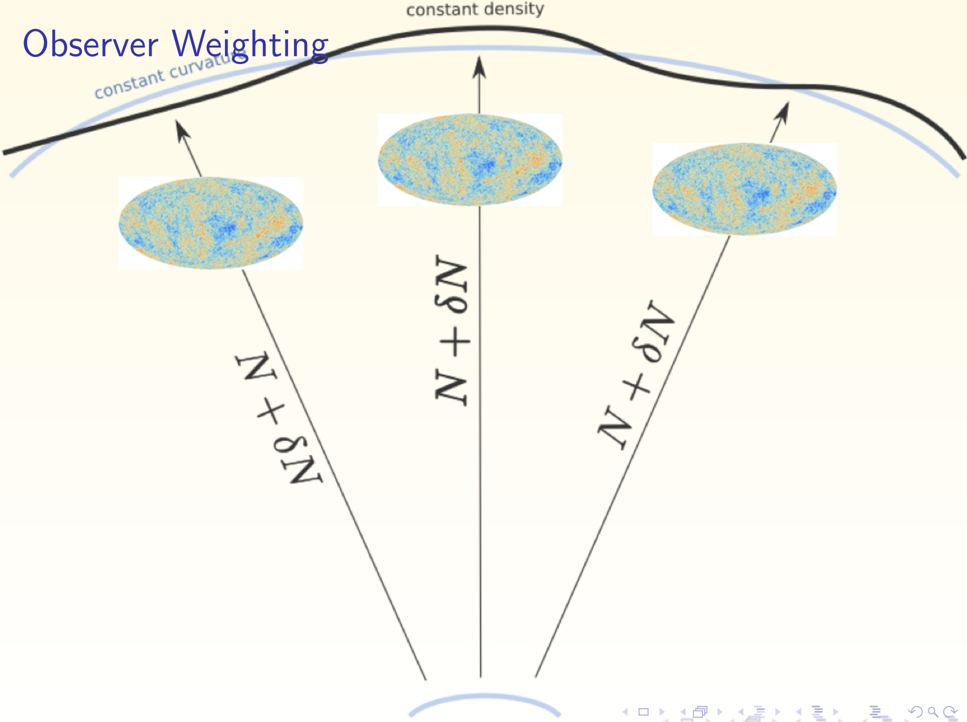
Planck measured C_ℓ

$$C_2^{\mathrm{obs}} = 253.6 \mu K^2 \qquad C_2^{\mathrm{high}-\ell} = 1124.1 \mu K^2$$

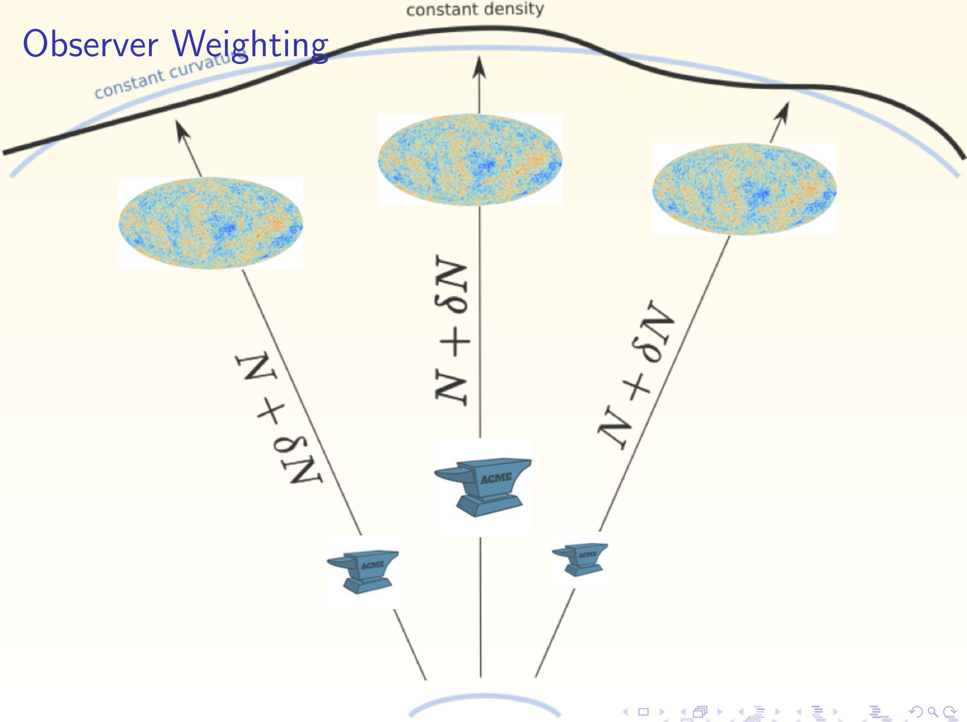
Numerical GR Input

$$\mathrm{Pr}\left(\hat{C}_2 | A_\phi, H_I L_{\mathrm{obs}}, \dots\right)$$

Observer Weighting



Observer Weighting



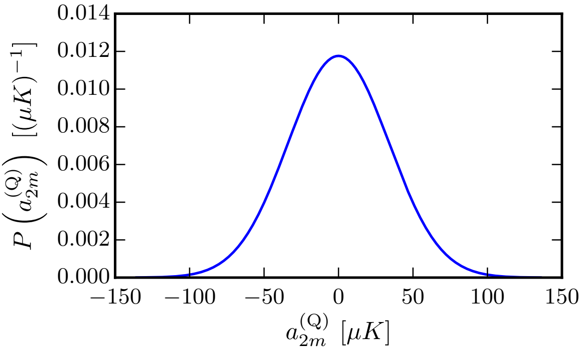
Evaluation of CMB Quadrupole

$$\hat{C}_2 = \frac{1}{5}\left[\left(a_{20}^{(\mathrm{UL})} + a_{20}^{(\mathrm{Q})}\right)^2 + \sum_{m=-2, m\neq 0}^{m=2}\left(a_{2m}^{(\mathrm{Q})}\right)^2\right]$$

Evaluation of CMB Quadrupole

$$\hat{C}_2 = \frac{1}{5}\left[\left(a_{20}^{(\mathrm{UL})} + a_{20}^{(\mathrm{Q})}\right)^2 + \sum_{m=-2,m\neq 0}^{m=2}\left(a_{2m}^{(\mathrm{Q})}\right)^2\right]$$

$$a_{2m}^{(\mathrm{Q})} \ : \ \text{Gaussian with } \langle (a_{2m}^{(\mathrm{Q})})^2 \rangle = 1124.1 \mu K^2$$



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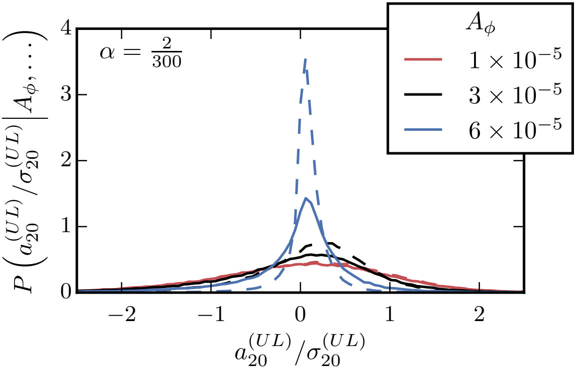
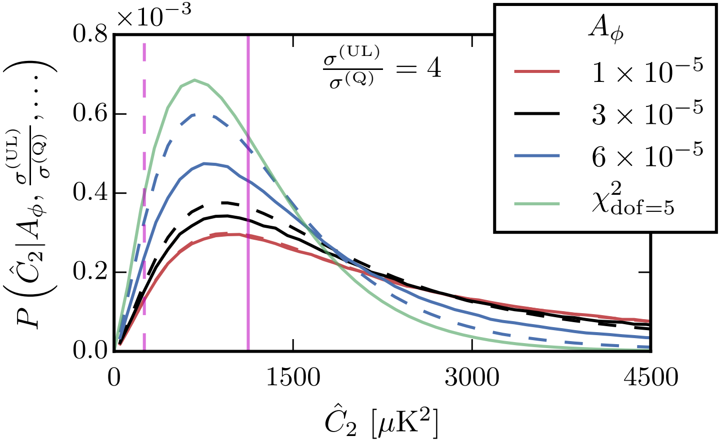
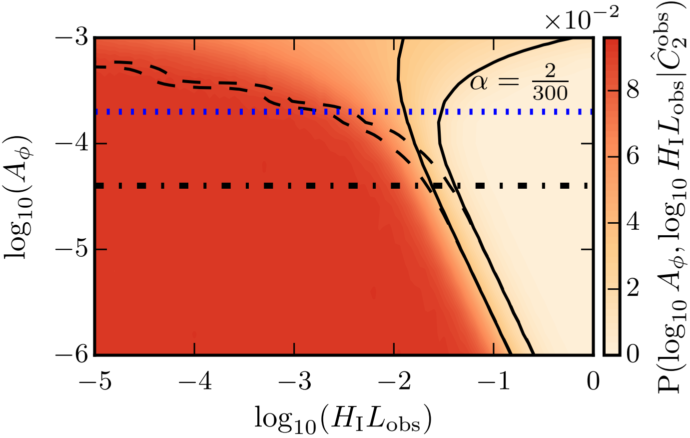
$a_{20}^{(\mathrm{UL})}$: NR simulations



Dependence of C_2 on Model Parameters



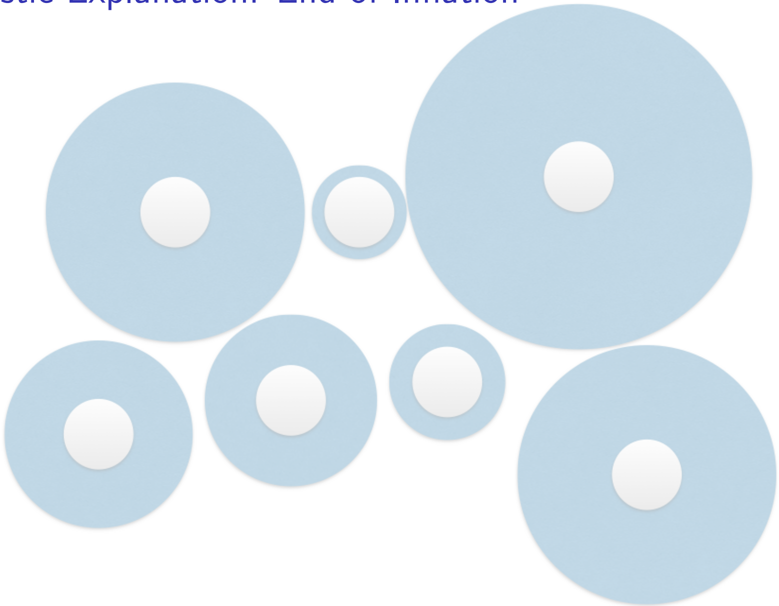
Final Posterior



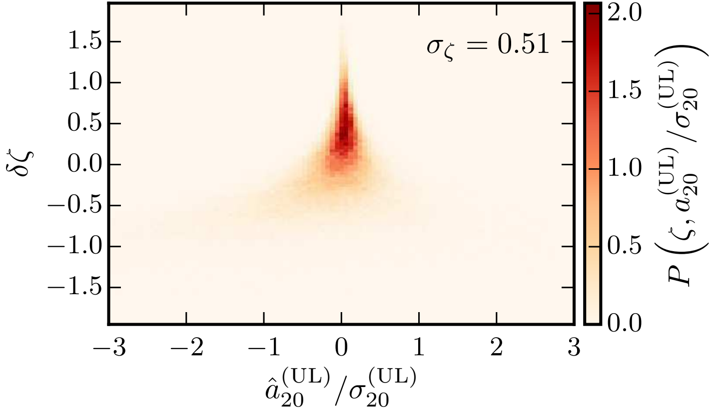
Significant deviations from Gaussian approximation

Heuristic Explanation: Initial Conditions

Heuristic Explanation: End-of-Inflation

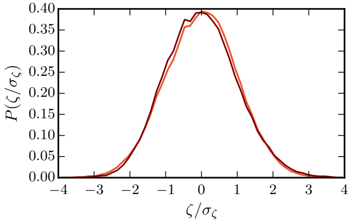


Distribution of a_{20} with $\delta\zeta$ dependence



Analytic Approximation

ζ and comoving derivatives nearly Gaussian



Treat as Gaussian random field

Analytic Approximation

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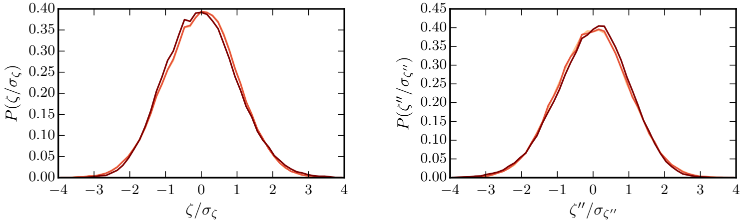
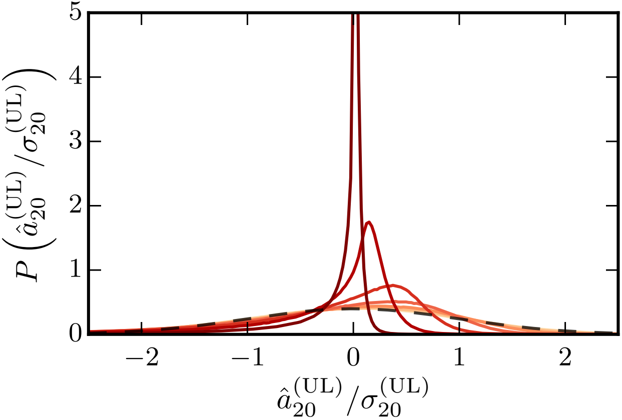


Treat as Gaussian random field

Large-Scale Approximation for a_{20}

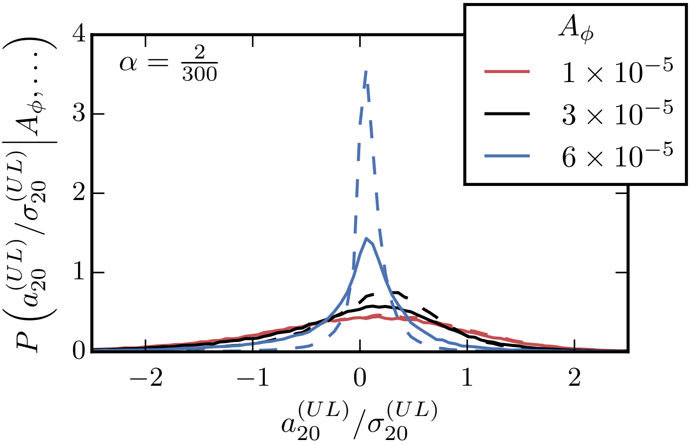
$$a_{20}(x_0) \sim -\mathcal{A}e^{-2\zeta(x_0)}\left(\zeta''(x_0) - \mathcal{O}(\zeta'(x_0)^2)\right)$$

Anaytic a_{20} Distributions



Vary σ_ζ at fixed $\sigma_{\zeta^{(p)}}/\sigma_\zeta$

Recall The Numerical Result



Conclusions

- Numerical relativity is a useful framework for making cosmological predictions
  - Sometimes it is a *necessary* tool (deviations from Gaussianity)
- Robust qualitative conclusions over a variety of inflationary models
- Constraining large amplitude superhorizon structure is hard even in the most optimistic case
- Inflation is effective at hiding large amplitude initial fluctuations
- Gaussianity of ζ in comoving coordinates suggests analytic approach in 3D