Einstein-Yang-Mills-Dirac Systems from the discretized Kaluza-Klein Theory

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Abstract

A unified theory of the non-Abelian gauge interactions with gravity in the framework of a discretized Kaluza-Klein theory is constructed with modified Dirac Operator and wedge product. All the couplings of the chiral spinors to non-Abelian gauge fields emerge naturally as components of the couplings of the chiral spinors to the generalized gravity together with some new interactions. In particular, the currently prevalent gravity-QCD-quark and gravity –electroweak –quark –lepton models are shown to follow as special cases of the general framework.

OUTLINE

- Introductory Remarks
- Fundamental Basics of NCG; Spectral Triple
- Algebraic Formulation of NCG Geometry
- Equivalence Principle and Vielbeins
- A set of constraints; Cartan Structure Equations
- Determination of connection coefficients
- Ricci curvature leading to Lagrangians
- Physical consequences and Conclusions

Introductory Remarks

- Our current picture of space-time is inadequate for the description of phenomena at short distances. The twin pillars of modern physics, GR and QFT are both structured on a continuum picture of space-time.
- Algebraically geometrized pseudo-Riemannian manifold with a metric based on a continuum picture underlies General Relativity.
- Likewise, Quantum Fields and their interactions are local operators based on continues functions of space-time coordinates.

Noncommuatative Geometry (NCG) a la Connes has provided a new suitable framework for a Geometrical description of all elementary particle interactions.

General Framework of NCG

Consists of Three Basic Points: Spectral Triple

$$(A, H, D)$$

A- An involutive algebra, commutative or noncommutative.
H- A Hilbert space as the carrier space for A
D- A self-adjoint operator acting on A
The Algebra A generalizes the smooth functions C(M). The Operator D allows one to build a Differential structure associated with any associative algebra.

Extended space-time $\mathcal{M}^4 \times Z_2$ and Dirac operator

- Z_2 algebra has only two elements represented as follows

$$\mathbf{e} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \mathbf{r} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \tag{1}$$

- Generalized spinors and function operators

$$\Psi = \begin{bmatrix} \psi_L \\ \psi_R \end{bmatrix}, \quad F = \begin{bmatrix} f_L & 0 \\ 0 & f_R \end{bmatrix} \tag{2}$$

- Generalized Dirac operator and Dirac matrices $M, N = \mu, 5$

$$\begin{aligned} D &= \Gamma^M \partial_M = \Gamma^\mu \partial_\mu + \Gamma^5 \partial_5 = d + Q = \begin{bmatrix} \gamma^\mu \partial_\mu & i m_2 \gamma^5 \\ -i m_2 \gamma^5 & \gamma^\mu \partial_\mu \end{bmatrix}, \quad D_5 = \begin{bmatrix} 0 & m \\ -m & 0 \end{bmatrix}, \\ \partial_5 F &= \sigma^\dagger [D_5, F] = m(f_L - f_R)\mathbf{r}, \quad \Gamma^\mu = \begin{bmatrix} \gamma^\mu & 0 \\ 0 & \gamma^\mu \end{bmatrix}, \quad \Gamma^\mu = \begin{bmatrix} 0 & i\gamma^5 \\ -i\gamma^5 & 0 \end{bmatrix} \end{aligned} \tag{3}$$

Differential forms and wedge product

- Exterior derivative of 0-form

$$DF = [D,F] = \begin{bmatrix} df_L^u & im_2\gamma^5(f_L^u - f_R^u) \\ im_2\gamma^5(f_L^u - f_R^u) & df_R^u \end{bmatrix}, D^2 = 0. \quad (4)$$

- Generalized 1-form $U = \Gamma^M U_M = \Gamma^\mu U_\mu + \Gamma^5 U_5 = \begin{bmatrix} \gamma^\mu u_{L\mu} & im_2\gamma^5 u_{5L} \\ -im_2 u_{5R} & u_{R\mu} \end{bmatrix}$. For gravity it enough to impose the hermitian 1-form $u_{5L} = u_{5R}$. For nonabelian gauge fields $u_{L,R\mu}$ can have values in some nonabelian Lie-algebra, while $u_{L5} = u_{R5}^\dagger$.
- Wedge product

$$\Gamma^\mu \wedge \Gamma^M = -\Gamma^M \wedge \Gamma^\mu, \ \ \Gamma^5 \wedge \Gamma^5 \neq 0, \ \ U \wedge V = \Gamma^M \wedge \Gamma^N (U \wedge V)_{MN} \quad (5)$$

- It is straight forward to write down all the components of $DU = [D,U] = \Gamma^M \wedge \Gamma^N (DU)_{MN}$ and $U \wedge V = \Gamma^M \wedge \Gamma^N (U \wedge V)_{MN}$.
- Generalized 2-form $S = \Gamma^M \wedge \Gamma^N S_{MN}$
- NCG is determined by a spectral quartet not triplet!

Generalized equivalence principle and vielbeins

- In curved space-time the curvi-linear basis is $\Gamma^M(x)$
- The new equivalence principle: there exist a locally flat basis $\Gamma^A, A = a, \dot{5}$ which is related to Γ^M by the local transformation

$$\Gamma^A = \Gamma^M E^A_M(x), \quad \Gamma^M = \Gamma^A E^M_A(x) \tag{6}$$

- $E^M_A(x)$ vielbeins defined the metric as follows

$$G^{MN} = Tr(\Gamma^M \Gamma^N) \quad = \quad E^M_A(x) Tr(\Gamma^A \Gamma^B) E^N_B(x) = E^M_A(x) \eta^{AB} E^N_B(x) \tag{7}$$

- We will use the hermitian vielbein

$$E^a_\mu \quad = \quad \begin{bmatrix} e^a_{L,\mu} & 0 \\ 0 & e^a_{R,\mu} \end{bmatrix}, \; E^a_{\mu} = 0 \;, \; E^{\dot{5}}_\mu = \begin{bmatrix} a_{L\mu} & 0 \\ 0 & a_{R\mu} \end{bmatrix}, \; E^{\dot{5}}_5 = \begin{bmatrix} \phi & 0 \\ 0 & \phi \end{bmatrix}, \tag{8}$$

Levi-Civita connection and Cartan structure equation

- Metric compatible and hermitian Levi-Civita connection $\Omega_{AB} = -\Omega_{BA} = \Omega^{\dagger}_{AB}$
- Generalized structure equation

$$T^A \;=\; DE^A + E^B \wedge \Omega^A_B \;,\; R^{AB} = D\Omega^{AB} + \Omega^{AC} \wedge \Omega^B_C \tag{9}$$

- Viet-Wali's constraint (1995): The torsion free $T^E = 0$ leads to restricted metric. Therefore, we have

$$T^a = 0 \;,\; T^{\dot{5}}_{AB} = t^{\dot{5}}_{AB}\mathbf{r} \tag{10}$$

  allows us to compute torsion and connection and then Ricci tensor from vielbeins.
- Generalized Hilbert-Einstein action

$$\mathcal{S}_{HE} \;=\; M^2_{5,Pl} \int \sqrt{-detG}\, Tr(R_5), \quad R_5 = \eta^{AC} R_{ABCD} \eta^{BD} \tag{11}$$

- Finite spectrum of bigravity, one Brans-Dicke and the gauge sector from $a_{L,R\mu}$ possibly non-abelian

Bigravity coupled to Brans-Dicke

- In this case we choose

$$a_{L,R} = 0, \quad \phi = \phi_0 \exp\left(\frac{\sigma(x)}{\sqrt{2}M_{Pl}}\right), \tag{12}$$

$$e^{\mu}_{a}(x) = \frac{1}{2}(e^{\mu}_{La}(x) + e^{\mu}_{Ra}(x)), \quad h_{\mu\nu} = \frac{1}{2m_h}(e^{a}_{L\mu} - e^{a}_{R\mu})e_{a\nu}, \tag{13}$$

- The Hilbert-Einstein action reduces to

$$\begin{aligned} S_{HE}(5) &= \int dx^4 (\sqrt{-detg}\, (\frac{M^2_{Pl}}{2}(r_{L4} + r_{R4}) + \frac{1}{2}g^{\mu\nu}\partial_\mu\sigma(x)\partial_\nu\sigma_(x) \\ &+ \frac{2m^2 M^2_{Pl}}{\phi^2_0 m^2_h}(h^\mu_\nu h^\nu_\mu - (h_{\mu\nu}g^{\mu\nu})^2) + \mathcal{L}_{int}(\sigma(x), h_{a\mu}(x), e^\mu_a(x)), \end{aligned} \tag{14}$$

- The Pauli-Fierz mass term for the massive gravity $h_{\mu\nu}$ with mass $\frac{\sqrt{2}mM_{Pl}}{\phi_0 m_h}$ and $M_{Pl} = M_{5,Pl}\sqrt{\phi_0}$

Viet-Du's theorem

- The case of $e_L = e_R$ Einstein's gravity, Brans-Dicke coupled to the gauge fields

$$R_5 = r_4 - \frac{1}{16}\phi^2 g^{\mu\rho} g^{\nu\tau} \mathcal{F}_{\mu\nu}\mathcal{F}_{\rho\tau} + g^{\mu\nu}\frac{\partial_\mu\phi}{\phi}\frac{\partial_\nu\phi}{\phi}, \quad (15)$$

$$\mathcal{F}_{\mu\nu} = f_{L\mu\nu} + f_{R\mu\nu} - m([a_{R\nu}, a_{L\mu}] + [a_{L\nu}, a_{R\mu}]), \quad (16)$$

$$f_{L,R\mu\nu} = \partial_\mu a_{L,R\nu} - \partial_\nu a_{L,R\mu} - m[a_{L,R\mu}, a_{L,R\nu}] \quad (17)$$

- R_5 and S_{HE} is gauge invariant with nonabelian gauge fields only in two cases:
  1. Gauge field on one sheet is abelian $a_{R\mu}$. Electroweak case.
  2. $a_{R\mu} \sim a_{L\mu}$. Two gauge fields are the same on both sheets. Strong interaction case.
- Viet-Du's results are recovered with the new Dirac operator and wedge product.

Coupling to the chiral spinors

- The curved Dirac operator with spinor connection

$$\mathcal{D} = D + \Omega = D - \tfrac{1}{8}\Gamma^C \Omega_{ABC}[\Gamma^A, \Gamma^B], \tag{18}$$

$$D = \Gamma^\mu \partial_\mu + \Gamma^5 \sigma^\dagger D_5 = \Gamma^a E_a^\mu(x)\partial_\mu - \Gamma^a E_a^\mu(x) A_\mu(x)\sigma^\dagger D_5 + \Gamma^{\dot{5}} \phi^{-1}(x)\sigma^\dagger D_5, \tag{19}$$

- The Einstein-Dirac parts can be derived from the following generalized Lagrangian

$$\mathcal{L}_f = \bar{\Psi}\mathcal{D}\Psi = \mathcal{L}_{d+m} + \mathcal{L}_{f-g} + \mathcal{L}_\omega + \mathcal{L}_\Omega(2) + \mathcal{L}_\Omega(3) \tag{20}$$

where the Brans-Dicke modifies the quark-lepton mass by a small amount since ϕ_0 is large

$$\mathcal{L}_{d+m} = i\bar{\psi}(\gamma^a e_a^\mu(x)\partial_\mu - m\phi^{-1}(x))\psi, \quad \mathcal{L}_\omega = -\frac{1}{8}\bar{\psi}\gamma^c \omega_{abc}[\gamma^a, \gamma^b]\psi \tag{21}$$

$$\mathcal{L}_\Omega(2) = \frac{i}{16}\bar{\psi} e_a^\mu e_b^\nu (\hat{f}_{+\mu\nu} + 2m[a_{-\nu}, a_{-\mu}])[\gamma^a, \gamma^b]\psi, \tag{22}$$

$$\mathcal{L}_\Omega(3) = \frac{1}{\sqrt{2}M_{Pl}}\bar{\psi}\gamma^a e_a^\mu \partial_\mu \sigma(x)\psi \tag{23}$$

Coupling to the gauge fields

- All the gauge-chiral spinor coupling can be derived from generalized Einstein-Dirac Lagrangian.
- Matching with the SM terms, based on the currently known quark-leptons we can derived the relations

$$g = \frac{2\sqrt{2}m}{\phi_0 M_{Pl}} \ , \quad g' = \sqrt{1.2}\, g, \quad g_s = \sqrt{2}g \tag{24}$$

- Parity violation by QCD can be transfered into P-violation by gravity

$$i\bar{\psi}_L \gamma^a e^{\mu}_{La}(\partial_\mu + ia_{L\mu})\psi_L + i\bar{\psi}_R \gamma^a e^{\mu}_{Ra}(\partial_\mu + ia_{R\mu})\psi_R \tag{25}$$

Summary

- With new Dirac operator and wedge product, one can derive nonabelian gauge fields from the generalized gravity under two different conditions, which allows to build the Einstein-Yang-Mills-Dirac systems for QCD and Electroweak interactions.
- The coupling of generalized gravity to chiral spinors can be reduced to couplings of gravity and gauge interactions to chiral spinors, together with the spinor connection terms and new terms.
- There are some relations between the coupling constants.
- Bigravity can be derived fron the framework.
- The theory is based on a new equivalence principle for spacetime extended by discrete dimension.
- The theory has a finite spectrum without truncation
- The hierachy problem can be solved by Brans-Dicke scalar.

Questions for further studies

- The energy scale, where the theory becomes valid? (Frome the relations between the coupling constant)
- Since two conditions of Viet-Du theorem cannot be satisfied at the same time, one can must go with the extended space-time $\mathcal{M}^4 \times Z_2 \times Z_2$ (Viet 2015)
- w-type and v-type chiral matter (SM and dark matter?)