The many faces of Conformal Bootstrap

Alessandro Vichi

ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

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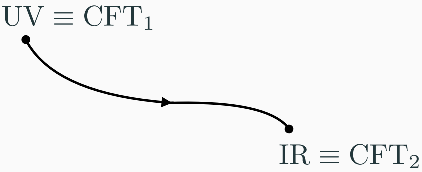
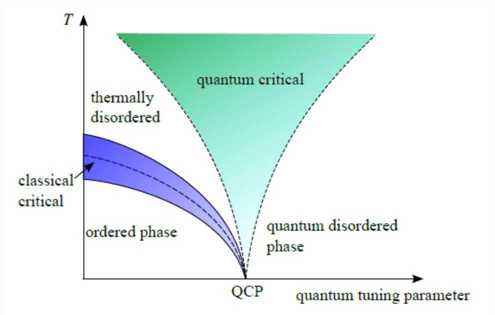
- CFT's are the building blocks of quantum field theories

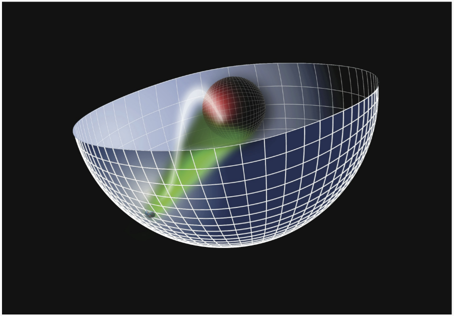


- Classical and quantum phase transitions in real physical systems



Ubiquitous CFT's

- Quantum gravity via AdS/CFT correspondence



Usually: Classical gravity $\Rightarrow$ Strongly coupled CFTs

Lately: 2D CFTs $\Rightarrow$ Black holes physics

How to study CFTs?

D=2 is a great deal:

buy 1 (scale invariance)	$\Rightarrow$	get ∞ (Virasoro algebra)

D>2, much harder. Usually rely on

- perturbative expansions (in couplings, dimensions, number of fields,...)
- supersymmetry
- strong/weak dualities (AdS/CFT, ...)
- Simulations (Lattice, MonteCarlo)

Today: Conformal Bootstrap

What are CFTs?

Theories invariant under the *conformal algebra* $SO(D|2)$ which includes:

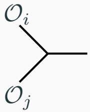
- translations
- Lorentz transformations
- dilatations
- "inversion"

They are described by three ingredients:

1) Spectrum: infinite set of operators $\mathcal{O}_{\Delta,\ell}$ (spin: ℓ; dimension in energy: Δ)

2) Interactions between operators:

$$\mathcal{O}_i \times \mathcal{O}_j \sim \sum_k$$



$$\mathcal{O}_k \sim \sum_k C_{ijk} \mathcal{O}_k$$

C_{ijk}: Operator Product Expansion (OPE) coefficients

3) Crossing symmetry constraints: see next slides...

The power of conformal invariance

In CFT we are interested in computing correlations functions $\langle \mathcal{O}_i(x_1)...\mathcal{O}_j(x_n)\rangle$:

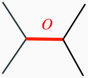
$$\text{fixed by symmetry} \quad \begin{cases} \langle \mathcal{O}_i(x_1)\mathcal{O}_j(x_2)\rangle & \checkmark \\ \langle \mathcal{O}_i(x_1)\mathcal{O}_j(x_2)O_k(x_3)\rangle & \checkmark \end{cases}$$

$$\text{encode dynamics} \quad \begin{cases} \langle \mathcal{O}_i(x_1)\mathcal{O}_j(x_2)O_k(x_3)O_l(x_4)\rangle \\ \dots \end{cases}$$

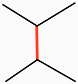
Four point functions

Use OPE to reduce higher point functions to smaller ones:

$$\langle \underbrace{\mathcal{O}(x_1)\mathcal{O}(x_2)}\underbrace{\mathcal{O}(x_3)\mathcal{O}(x_4)}\rangle \sim \sum_{\mathcal{O}}$$



$$\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\mathcal{O}(x_3)\mathcal{O}(x_4)\rangle \sim \sum_{\mathcal{O}}$$



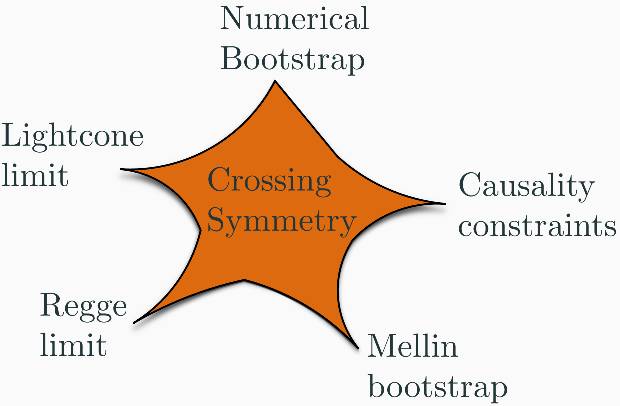
Crossing symmetry: two expansions must give the same result!
(Constraint on spectrum and interactions)

Definition of a CFT:

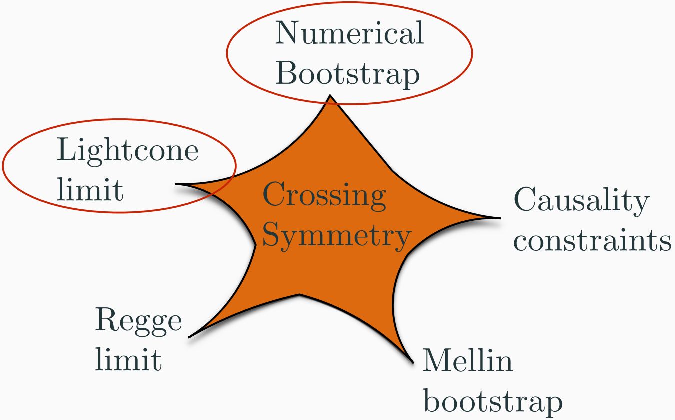
A CFT is an infinite set of primary operators $\mathcal{O}_{\Delta,\ell}$ and OPE coefficients C_{ijk} that satisfy crossing symmetry for all set of four-point functions.

Q: What choices of CFT data are consistent?

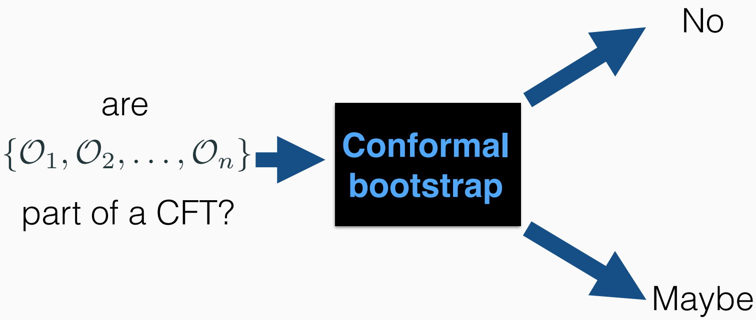
Conformal Bootstrap



Conformal Bootstrap



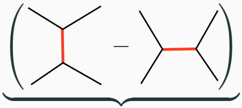
Numerical bootstrap



Numerical Bootstrap

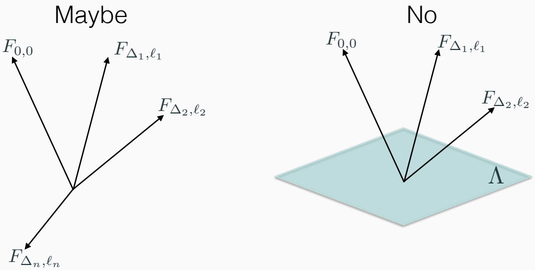
- Crossing equation for $\langle O(x_1)\mathcal{O}(x_2)\mathcal{O}(x_3)\mathcal{O}(x_4)\rangle$:

$$\sum_{\Delta,\ell} C^2_{\Delta,\ell} \left(\quad \right) = 0$$



Known functions $F_{\Delta,\ell}$

- Unitarity: $C^2_{\Delta,\ell} \geq 0$



Existence of Λ can be recast into a *linear (or semi-definite) programming problem* and checked numerically. [Rattazzi,Rychkov,Tonni, AV] 2008

Numerical Bootstrap

Rules of the game:

- Choose one or more operators $\mathcal{O}_1, \mathcal{O}_2, ...$
- Consider all four point functions containing those operators $< \mathcal{O}_1\mathcal{O}_1\mathcal{O}_1\mathcal{O}_1 >, < \mathcal{O}_1\mathcal{O}_1\mathcal{O}_2\mathcal{O}_2 >, ...$
- Make assumptions on the operators (and coefficients) appearing in the OPE's $\mathcal{O}_i \times \mathcal{O}_j$
- Check numerically if assumptions are consistent with crossing symmetry
- If not consistent: no CFT with that operator content

An application in 3D

Scalars σ ε, with OPE:

$$\begin{aligned} \sigma \times \sigma &\sim 1 + \epsilon + ... \\ \sigma \times \epsilon &\sim \sigma + ... \\ \epsilon \times \epsilon &\sim 1 + \epsilon + ... \end{aligned}$$

Q: given Δ_σ, how large can Δ_ε be?

An application in 3D

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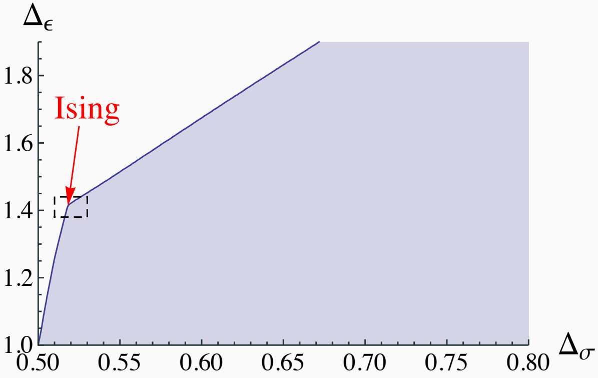
Q: given Δ_σ, how large can Δ_ε be?

A: study $< \sigma\sigma\sigma\sigma >, < \sigma\sigma\epsilon\epsilon >, < \epsilon\epsilon\epsilon\epsilon >$



[El-Showk,Paulos,Poland,Rychkov,Simmons-Duffin, AV] 2011

Can we narrow down the 3D Ising model?

So far we have assumed anything about the CFT besides unitarity.

The 3D Ising model has only two relevant perturbations

Q: if only σ and ϵ have dimension ≤ 3: allowed values for Δ_σ, Δ_ϵ?

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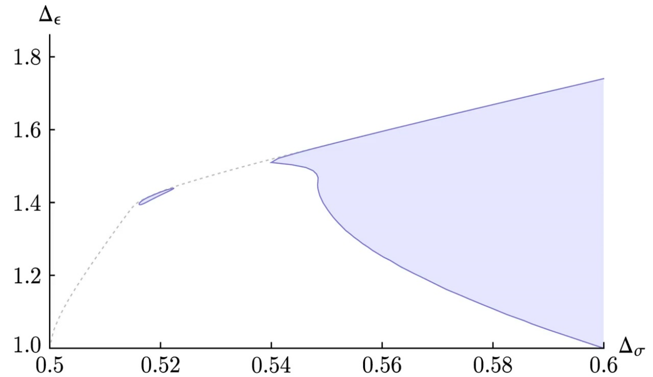

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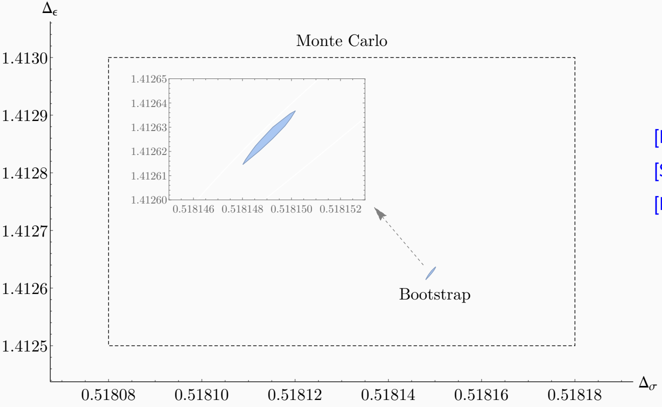


[Poland,Simmons-Duffin,Kos '14]

[Simmons-Duffin,'15]

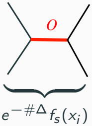
[Poland,Simmons-Duffin,Kos,AV,'16]

Why does it work?

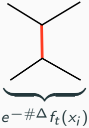
Only a finite number of operators effectively matter

Decoupling of high dimensional operators:

s-channel: $\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\mathcal{O}(x_3)\mathcal{O}(x_4)\rangle \sim \sum_{\mathcal{O}_{\Delta,\ell}}$



t-channel: $\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\mathcal{O}(x_3)\mathcal{O}(x_4)\rangle \sim \sum_{\mathcal{O}_{\Delta,\ell}}$



[Rychkov,Yvernay,'15]

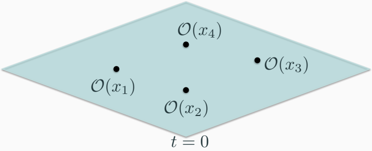
[Pappadopulo,Rychkov,Espin,Rattazzi,'12]

In the numerical bootstrap functions $f_{s,t} \sim O(1)$

Large dimension operators exponentially suppressed!

Euclidean VS lightcone limit

Euclidean configuration

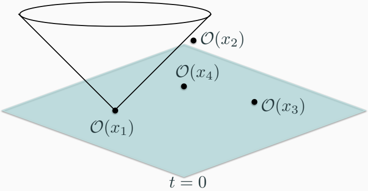


Maximal convergence

⇓

Good for numerical bootstrap

Lightcone limit



No exponential decoupling

⇓

Sensitive to high dimensional operators

Lightcone bootstrap

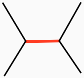
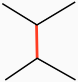
Goal: describe the sector of operators with large spin ℓ

Different use of crossing symmetry:

- numerical: carve out the CFTs parameter space
- lightcone: prove existence of certain towers of operators

Lightcone bootstrap

In lightcone limit:

$\underbrace{\sum_{\mathcal{O}_{\Delta,\ell}}}$  $=$ $\sum_{\mathcal{O}_{\Delta,\ell}}$ 

dominated by low twists — "all" important

Twist: $\tau \equiv \Delta - \ell$

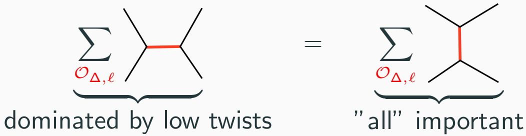
Unitarity:

$$\begin{aligned} \tau &\geq D-2 && \ell > 0 \\ \tau &\geq (D-2)/2 && \ell = 0 \\ \tau &= 0 && \text{identity operator} \end{aligned}$$

Main contribution from identity, conserved currents and scalars

Lightcone bootstrap

In lightcone limit:



Main results:

- RHS needs an infinite tower of operators to reproduce LHS leading singularities
- Can compute the anomalous dimensions and OPE coeff: $1/\ell^{\tau}$ expansion
- for $\langle\mathcal{O}\mathcal{O}\mathcal{O}\mathcal{O}\rangle$ the tower corresponds to double trace operators

$$\mathcal{O}\partial_{\mu_1}\partial_{\mu_\ell}\mathcal{O} \qquad \text{with} \qquad \Delta_\ell = 2\Delta_{\mathcal{O}} + \ell + O(\frac{1}{\ell^{\tau_{min}}})$$

- subleading sigularities can also be reproduced: more towers...

[Fitzpatrick, Kaplan, Poland, Simmons-Duffin,'12]

[Alday,Bissi,Lukowski,Zhiboedov,'14-'17]

[Li, Meltzer, Poland,'15,'16]

Many faces, one face?

Q:Are there evidences of lightcone results in the numerical bootstrap?

A: Yes! [Simmons-Duffin,'16]

The 3D ising model is super constrained: solution of crossing can be reconstructed up to O(100) operators

Many faces, one face?

Q:Are there evidences of lightcone results in the numerical bootstrap?

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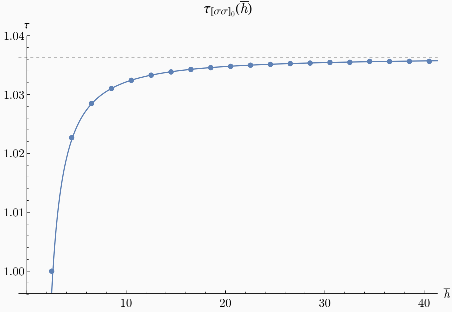
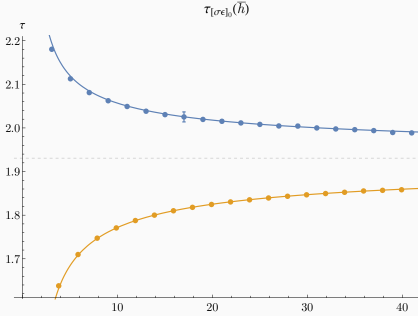
The 3D ising model is super constrained: solution of crossing can be reconstructed up to O(100) operators Compare numerical with lightcone results:





Conclusions

CFT's in $D \geq 3$ (neglected for many years) are now under siege

- Numerical techniques allow to precisely determine CFT data
- Lightcone limit provides insights on "large" spin operators
- Other complementary approaches available
- We have only scratched the surface:
  are we on the verge of a "CFT eight-fold way"?